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Incompatibility of the Observer-Based Vacuum with Canonical Quantum Field Theory

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Three new stationary coordinate systems in flat spacetime, the last of six possible types, are described. The vacuum state found by canonical quantization of the scalar field in two of these systems is identical to that of Rindler coordinates, yet detectors at rest in these systems are excited in different amounts. A definition of the vacuum based on these detectors is therefore incompatible with canonical quantum field theory. The vacuum of the third system is identical to the Minkowski coordinate vacuum, thus there are only two different vacua in flat spacetime.

Fulling¹ has shown that the vacuum state as defined by canonical quantum field theory is not coordinate independent. In particular, he showed that when particles are defined as normal mode excitations of the field, the no-particle state in a coordinate system adapted to an accelerating observer (Rindler coordinates²) is filled with particles as defined in Minkowski coordinates. We shall call this state the Fulling vacuum.

Fulling's results seem to necessitate a coordinate or observer dependent definition of the vacuum. The detailed analysis of an accelerating detector by Unruh³ showed that it is excited in the Minkowski vacuum to a temperature $T = \hbar \kappa / 2\pi k c$ (κ = acceleration), yet in the Fulling vacuum in Rindler coordinates (where the detector is at rest) no excitation takes place. One is therefore led to the purely observer dependent definition of the vacuum as that state of the field in which a detector moving with the observer is unexcited.

In this paper we discuss scalar field quantization in two new coordinate systems which are essentially different from Rindler coordinates. We find that the vacuum state in these systems is the Fulling vacuum, but detectors at rest in these coordinates are excited in the Fulling vacuum and the excitation spectra in the Minkowski vacuum are not Planckian. We conclude that a definition of the vacuum based on these

detectors is incompatible with canonical quantum field theory at this level (stationary coordinates in flat space). The vacuum of the third system is the Minkowski vacuum showing by exhaustion that there are only two distinct vacua in flat space.

Rindler coordinates (τ, ξ, y, z) are related to Minkowski coordinates (t, x, y, z) by the transformation $\tau = \tanh^{-1}(t/x)$ and $\xi = (x^2 - t^2)^{1/2}$. The metric is static and the line element is

$$ds^2 = \xi^2 d\tau^2 - d\xi^2 - dy^2 - dz^2 \quad (1)$$

Surfaces $\xi=0$ are event horizons. Lines of constant ξ, y , and z are world lines with proper acceleration ξ^{-1} . Normal mode solutions to the Klein-Gordon equation¹ are

$$\phi = (2\pi^2)^{-1} (\sinh \pi E)^{1/2} e^{-iE\tau} e^{ipy} e^{ikz} \kappa_{iE}(Q\xi) \quad (2)$$

E is the energy and p, k , and Q are momenta satisfying $Q^2 = p^2 + k^2 + M^2$ where M is the mass. The field operator may be decomposed into a sum over momenta of the form

$$\phi = \int b \phi_+ + b^\dagger \phi_- \quad (3)$$

where ϕ_+ and ϕ_- are modes of positive and negative norm with the usual scalar product

$$\langle \phi | \phi' \rangle = i \int \phi^* (g^{1/2} g^{\mu\nu} \partial_\nu - \partial_\nu g^{\mu\nu} g^{1/2}) \phi' d\Sigma_\mu. \quad (4)$$

b and $b^\dagger$ are therefore necessarily annihilation and creation operators. It also happens that this is a decomposition into positive and negative frequency modes relative to the Killing vector $\partial/\partial\tau$, a coincidence which we find only in static coordinate systems. An expansion of the normal modes ϕ_+ in

terms of Minkowski normal modes has positive and negative frequency parts, hence Bogoliubov transformations mix annihilation and creation operators and the vacua are inequivalent.

We define coordinate system I $(\tau, \xi, \bar{y}, z)$ by the transformation $\bar{y} = y - \bar{\Omega}\tau$ where $\bar{\Omega}$ is a constant. It should be noted that this is not a Lorentz transformation. The metric is stationary with line element

$$ds^2 = \xi^2 d\tau^2 - d\xi^2 - (d\bar{y} + \bar{\Omega}d\tau)^2 - dz^2 \quad (5)$$

The surfaces $\xi=0$ are event horizons and the surfaces $\xi^2 = \bar{\Omega}^2$ are stationary limits. There is an analogy between this system and the rotating black hole because these surfaces are distinct. Normal modes satisfying the Klein-Gordon equation in this system are

$$\bar{\phi} = (2\pi^2)^{-1} (\sinh\pi(\bar{E} + \bar{p}\bar{\Omega}))^{\frac{1}{2}} e^{-i\bar{E}\tau} e^{i\bar{p}\bar{y}} e^{ikz} K_{\frac{1}{2}(\bar{E} + \bar{p}\bar{\Omega})}(Q\xi) \quad (6)$$

where $Q^2 = \bar{p}^2 + k^2 + M^2$. As in (3) we expand the field operator in positive and negative norm parts

$$\bar{\phi} = \sum \bar{b}\bar{\phi}_+ + \bar{b}^\dagger\bar{\phi}_- \quad (7)$$

and $\bar{b}$ and $\bar{b}^\dagger$ are necessarily annihilation and creation operators respectively. The sign of the energy of the modes, relative to the Killing vector $\partial/\partial\tau$ (which is not the same field as used in Rindler coordinates), is not always the sign of the normalization. Thus if $\bar{E} + \bar{p}\bar{\Omega} > 0$ the normalization is positive, but $\bar{E}$ is not necessarily positive in this expression.

The vacuum state in system I is defined in the usual manner: $\bar{b}|0\rangle = 0$ for all $\bar{b}$. Bogoliubov transformations between Rindler and system I annihilation and creation operators show no mixing, thus the vacua are equivalent. The (normal ordered) Hamiltonian

$$\mathcal{H} = \int \bar{E} \bar{b}^\dagger \bar{b} \quad (8)$$

derived from the integral over constant time hypersurfaces of the stress energy tensor dotted into the Killing vector is not positive definite; therefore, the vacuum in coordinate system I is not the lowest energy state. The vacuum is stable, however, because there are no particle states with both $\bar{E}$ and $\bar{p}$ equal to zero.⁴ Stability, not least energy, is the essential requirement for a vacuum. Canonical quantum field theory therefore defines a satisfactory vacuum state in both Rindler coordinates and coordinate system I. This is the Fulling vacuum in both cases.

Coordinate system II (τ, ξ, r, θ) is related to Rindler coordinates by the transformation $r^2 = y^2 + z^2$ and $\theta = \tan^{-1}(z/y) - \tilde{\Omega}\tau$. The metric is stationary with line element

$$ds^2 = \xi^2 d\tau^2 - d\xi^2 - dr^2 - r^2 (d\theta + \tilde{\Omega} d\tau)^2 \quad (9)$$

This system has event horizons at $\xi=0$ and the surfaces $\xi^2 = \tilde{\Omega}^2 r^2$ are stationary limits. The normal modes are

$$\tilde{\phi} = (\sinh \pi (\tilde{E} + m\tilde{\Omega}) / 2\pi^3)^{1/2} e^{-i\tilde{E}\tau} e^{im\theta} J_m(qr) K_{i(\tilde{E} + m\tilde{\Omega})}(Q\xi) \quad (10)$$

with $Q^2 = q^2 + M^2$. Following the quantization procedure applied to coordinate system I, one finds that the vacuum state (no-particle state) in these coordinates is identical to the vacuum of Rindler coordinates and coordinate system I.

Each of the three coordinate systems discussed here is based on a fundamentally different Killing vector field. The excitation spectra of detectors moving along the trajectories of these fields in the Minkowski vacuum have been calculated⁵ and are different. For example, a detector passing through the point $\tau=0$, $\xi=\kappa^{-1}$, $y=0$, and $z=0$ along a Killing orbit associated with Rindler coordinates (Rindler coordinate line) has a Planckian excitation spectrum with temperature as given above, but a detector on a system I coordinate line at the same point has a non-Planckian excitation which exceeds the excitation of the Rindler detector at all frequencies. Likewise, in the Fulling vacuum the detectors are excited differently as can be seen from the autocorrelation (Wightman) function⁶ taken along world lines with constant ξ and z

$$\langle 0 | \phi(y, \tau) \phi(0, 0) | 0 \rangle = -\alpha \tanh^{-1} \alpha / \pi^2 y^2 \{ \tau^2 - 4 (\tanh^{-1} \alpha)^2 \} \quad (11)$$

where $\alpha = y / (y^2 + 4\xi^2)^{1/2}$. On a Rindler coordinate line ($y=0$) this function goes as τ^{-2} just as an inertial detector in the Minkowski vacuum, that is, there is no excitation. On a system I coordinate line ($y=\Omega\tau$) the function has a more complicated dependence on τ and the detector is excited.

The third new stationary coordinate system is defined by the transformation

$$\begin{aligned} t &= \frac{1}{6}\kappa^2 \tilde{t}^3 + (\kappa \tilde{x} + \frac{1}{2}) \tilde{t} + \tilde{y} & x &= \frac{1}{2}\kappa \tilde{t}^2 + \tilde{x} - \frac{1}{2\kappa} \\ y &= \frac{1}{6}\kappa^2 \tilde{t}^3 + (\kappa \tilde{x} + \frac{1}{2}) \tilde{t} + \tilde{y} & z &= \tilde{z} \end{aligned}$$

Normal modes in this system are Airy functions. The canonical quantization is complicated by the constant time hypersurfaces being null. Details will be given at a later time.⁷ The vacuum in system III is the Minkowski vacuum.

The three new coordinate systems presented here - along with Minkowski, Rindler, and rotating coordinates - exhaust the possible types of stationary coordinate systems in flat spacetime. This follows from the classification of timelike Killing vector fields in reference 5. Since nonstationary coordinate systems cannot have a stable vacuum⁸ there are only two different vacua in flat spacetime: the Minkowski vacuum and the Fulling vacuum. The Minkowski vacuum is found in Minkowski and rotating coordinates, and system III, none of which have event horizons. The Fulling vacuum is found in Rindler coordinates and system I and system II, all of which have event horizons identical to the wedge in Rindler coordinates. Our results therefore suggest that the event horizon plays a greater part in the definition of the vacuum than the world lines of individual observers.

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contain fields of this type.