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"Ballistic Damping" - A Proposed Method of Stabilizing
Resonant Ion Cyclotron Modes

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INTRODUCTION

It has been observed in 2XIIB and in other mirror experiments that the r.f. activity detected is frequently of a highly coherent nature, with a frequency close to the central ion cyclotron frequency.¹ It is proposed to take advantage of the coherent nature of the instability mode as observed (presumably the DCLC mode), stabilizing it by a method which we will call "ballistic damping" (BD), a variant on the idea of warm plasma stabilization. The essence of the idea is as follows:

One or more beams of energetic ions (or neutral atoms that will become ionized) are injected, parallel to the field lines, at radii where the instability electric field has maximum amplitude (say roughly halfway between the axis and the outer radius of the plasma). While in transit through the plasma these ions acquire transverse energy by resonant acceleration, thereby extracting energy from the wave. This imparted energy is then carried out of the confined plasma by the beam particles as they exit through the far mirror. In this way the BD process introduces a damping mechanism that can be used to inhibit the growth of unstable waves and/or to damp them out before they reach unacceptably high amplitude. The scaling laws for the BD process, derived below, indicate that the beam current densities required for damping to be effective should be low enough to avoid stimulating beam-plasma instabilities, particularly considering that the beams need not be monoenergetic for the BD process to operate. It is also shown that

the beam power required is substantially lower than the plasma powers involved, scaling in a favorable way with increase in the size of the mirror cell.

The essence of the physics involved is that single-transiting beam particles will be acted on coherently by the instability wave, whereas the wave interacts in the main incoherently with the bulk of the trapped ions of the plasma. Thus a relatively small number of transiting ions can exert a disproportionately large effect on the wave and its growth. This being the case it seems not unlikely that the BD effect may be operative in 2XIIB and that it might even provide a more significant effect on the plasma than the usual warm plasma stabilization mechanism under the present circumstances.

In the derivation given below we will use, insofar as possible, the 2XIIB data as a guide to the parameters involved, i.e. the wave numbers, instability growth rates, etc.

1. DERIVATION OF EQUATIONS

We have, from 2XIIB,¹ that the instability wave has an azimuthal phase velocity given by the ion diamagnetic drift frequency

$$\omega^* = k_{\perp} \frac{e\bar{W}_i}{eB} \frac{1}{R_p}, \quad (1)$$

where R_p is the plasma radius and $\bar{W}_i$ is the mean ion energy. Perpendicular wave numbers, $k_{\perp}$, are of order

$$k_{\perp} a_i = \alpha, \quad 3. < \alpha < 6. \quad (2)$$

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corresponding (in 2XIIB) to azimuthal mode numbers, m , of order 7 for $R_p = 7$ cm. Further, the frequency observed is close to the (depressed) central cyclotron frequency.

Resonant Energy Gain

We first evaluate the gain in perpendicular energy for an ion in cyclotron resonance with the wave, injected through the mirror parallel to the field lines ($W_{\perp} = 0$ upon entering the plasma). The requirement that $W_{\perp} \approx 0$ at entry arises from the desirability of insuring that the rotational energy gained by the beam particles is insensitive to the phase of the wave electric field at the point of entry of the particle.

Under the above stated condition the gain in $W_{\perp}$ after N cyclotron periods in the wave electric field is given by the easily derived expression:

$$W_N = \frac{1}{2} \pi^2 (Mc^2) (E/B)^2 N^2. \quad (3)$$

We will estimate the wave electric field, E , from the magnitude of the potential fluctuations and the perpendicular wavelength:

$$E \approx \Delta\phi/(\lambda/2) \approx (k_{\perp}/\pi)\Delta\phi. \quad (4)$$

The number of cyclotron rotation periods, N , during which the transiting beam particles are exposed to the wave electric field depends on the time history of their transit through the plasma. Since these particles must ascend the ambipolar potential, $\phi(z)$, en route to the center of the plasma their longitudinal energy will be reduced as they approach the plasma center. In fact if the initial beam particle

energy $W_b < e\phi(0)$ the particle will be stopped and reflected back, gaining perpendicular energy the while from the wave. Under some circumstances the particle might even gain enough perpendicular energy to become trapped, thereby joining the confined particle population. For the purposes of the present discussion, however, we will assume that the injected particles do not become trapped, i.e. they either have a sufficiently high injection energy, or the wave electric field is sufficiently low to limit the ratio W_N/W_b to a value less than that required to trap the particles. Under these assumptions we may then represent the number of cyclotron rotation periods of the beam particles by the relation

$$2\pi N = (\lambda L \omega_{ci} / v_b), \quad (5)$$

where L is the length of the central plasma region (where the wave exists) and v_b is the (parallel) velocity of the beam particles at entry. The parameter λ quantifies the effect of the ambipolar potential and/or the mirrors on the parallel component of beam velocity, thus on N . In the limit where $W_b \gg e\phi(0)$ and $W_N \ll W_b$, $\lambda \rightarrow 1$; for cases where $W_b \approx e\phi(0)$, λ could typically take on values of order 3 or more.* As will be seen in what is to follow, "tuning", accomplished through adjustment in the beam entrance energy to optimize λ , may represent an important empirical means of maximizing the BD effect.

From (2), (4) and (5) we find

$$W_N \approx \frac{1}{16\pi^2} (e\Delta\phi/W_i)^2 (\alpha \lambda L \omega_{ci} / v_b)^2 W_i \quad (6)$$

*See Appendix A for an example calculation of λ .

The power extracted by the beam ions from the wave will then be given by the product $I_b W_N$, where I_b is the total particle current of the injected BD beams.

Energy Stored in Wave Field

To estimate the damping effect of the beams on the wave we will first evaluate the total energy stored in the wave at a given amplitude. We will then compare this amount of energy with that which can be removed at that amplitude by the damping beams during a time equal to the characteristic growth time of the wave. When these two energies are comparable the BD process would be expected to be sufficient to limit the further growth of the wave, and/or to reduce its amplitude to a tolerable level.

The energy stored in the wave is, in general, given by²

$$U = K \left(\frac{E^2}{8\pi} \right) \text{ ergs cm}^{-3}, \quad (7)$$

where $K = \frac{\partial}{\partial \omega} (\omega E).$

For the ion mode here involved, at $\omega \approx \omega_{ci}$, we have²

$$K \approx \left(1 + \frac{\omega_{pi}^2}{\omega_{ci}^2} \right) \approx \frac{4\pi n M c^2}{B^2} \quad (8)$$

Using the previously derived estimate for E , and integrating over the plasma volume (assuming both K and $E \approx$ constant throughout the plasma, which thereby overestimates the wave energy), we find as an estimate of the total wave energy:

$$\int U dv \approx \pi R_p^2 L \left(1 + \frac{\omega_{pi}^2}{\omega_{ci}^2} \right) \frac{(\Delta\phi)^2}{8\pi^3} \cdot \left(\frac{\alpha^2}{a_i^2} \right). \quad (9)$$

substituting for a_i we then have

$$\int U dv \sim \frac{1}{2\pi} R_p^2 L a^2 \left(1 + \frac{\omega_{pi}^2}{\omega_{ci}^2}\right) \left(\frac{e\Delta\phi}{W_i}\right)^2 \cdot \left(\frac{W_i}{Mc^2}\right) \cdot \left(\frac{B^2}{8\pi}\right) \quad (10)$$

Per the earlier discussion we then have, from (10), (8), and (6):

$$\tau_{\text{damping}} = \left[\int U dv \right] / (I_b W_N) \quad (11)$$

$$= \frac{1}{2} \left(\frac{R_p^2}{L \lambda^2} \right) \cdot K \cdot (W_b / e^2 I_b) \quad (12)$$

i.e.

$$\tau_{\text{damping}} \omega_{ci} = 2 \left(R_p^2 / L \lambda^2 \right) \cdot (K W_b / Mc^2) (B / I_b) (c/e) \quad (13)$$

$$= 2.2 \times 10^{-5} \left(R_p^2 / L \lambda^2 \right) \cdot K \cdot \left[W_b (\text{keV}) / A \right] \left[B / I_b (\text{amps}) \right] \quad (14)$$

$$\text{i.e. } I_b = 2.2 \times 10^{-5} \left(R_p^2 / L \lambda^2 \right) \cdot K \cdot \left[W_b (\text{kW}) / A \right] \left[B / \omega_{ci} \tau_d \right] \text{ amps} \quad (14a)$$

Note that the magnitude of $\Delta\phi$ does not appear in these expressions since both the wave energy and the coherent acceleration process depend in the same way on the ratio $(e\Delta\phi/W_i)$. This means that the wave amplitude need only be allowed to become large enough to satisfy the coherency requirement of the acceleration process (Eq. (3)) in order for the BD process to become effective, assuming the BD beam current level is high enough.

Beam Power Required Per Unit Volume of Plasma

For future applications the scaling of BD beam power per unit volume of plasma is of interest. From (14), dividing by plasma volume, we see that

$$P_b/Vol \sim KW_b^2 B/L^2 \sim (BW_b^2) / (W_i L^2) \quad (15)$$

This scaling law is favorable for large, high ion temperature systems (scales as $W_i^{-1} L^{-2}$), but this favorable scaling will be partially offset by the increased W_b that may be required as ambipolar potentials scale up with increasing W_i . Note that the scaling gains indicated in (15) are in addition to any benefit arising from reductions in the ion matrix elements associated with scaleup in the radial dimension of the plasma (weakens DCLC mode).

III. Example from 2XIIB

To see how large a beam current would be required to test the BD idea in 2XIIB we assume the following typical parameters:

$$\begin{aligned} R_p &= 7 \text{ cm}, L = 20 \text{ cm}, B = 5 \times 10^3 \text{ gauss}, \\ A &= 2 \text{ (deuterons)}, K \approx 10^5 \text{ (} n \approx 10^{14} \text{ cm}^{-3} \text{)}. \end{aligned}$$

Take $W_b = 1 \text{ keV}$ (i.e. chosen to be greater than the ambipolar potential). From (14) we find

$$\omega_{ci} \tau_{\text{damping}} \approx (1.37 \times 10^4) / \lambda^2 I_b (\text{amps}) \quad (16)$$

$$\text{From 2XIIB data } \tau_{\text{growth}} \approx 10 - 20 \tau_{ci} \approx 60 - 120 \omega_{ci}^{-1}.$$

Thus the requirement that BD should exceed the wave energy growth is roughly that

$$I_b \approx 1.37 \times 10^4 / [(60 - 120)\lambda^2] \approx (115 - 230)/\lambda^2 \text{ amperes.}$$

The maximum injected beam power required, occurring when $\lambda = 1$, is therefore of order 100 to 200 kilowatts. Note that in 2XIIB, with a plasma length $L = 20$ cm, i.e. at $z = \pm L/2$, we find $B(z)/B(0) \approx 1.05$, i.e. coherent acceleration can occur for of order $N = 20$ revolutions.* For the parameters assumed $N \approx 3\lambda$, so that this coherency requirement can be well satisfied up to λ values $\approx 6 - 7$.

It is of some interest to estimate the maximum value that W_N might reach, assuming that the wave attained saturation amplitude, here estimated as $(e\perp\phi)_{\max} \approx kT_e$. From (6) we have, with $kT_e = 140$ eV and $W_i = 13$ keV, $\omega_{ci} = 2\pi \times 4.9 \times 10^6$ radians/sec and $\alpha \approx 4.5$; $W_{N(\max)} \approx 80\lambda^2$ eV.

At a BD beam current level of, say, 100 amperes, at $\lambda = 1$ this would represent a transport of wave energy out of the plasma at a rate of about 8 kilowatts. This incremental energy gain of the beam particles should be easily detectable on an external detector (calorimeter?), thereby providing an independent measure of the average wave energy at its saturation level.

IV. SUGGESTED TEST OF BD PRINCIPLE

It may be possible to test the efficacy of the BD idea in 2XIIB. If chopped, energy-spread, particle beams (square-wave chopped at about a 100 KHz rate) are injected through the mirrors in 2XIIB at a current level of several tens of amperes the following should be observed:

*See Appendix B for example calculations of the effects of field non-uniformity.

During the "off" period of the BD beams oscillations at the ion cyclotron frequency should begin to build up, only to be damped out during the "on" periods. Note that neutral beams or molecular ion (D_2^+) beams could be used (They will be ionized upon entering the plasma.), provided the beam particles are moving essentially parallel to the field lines at the point of ionization. Double ionization of He^0 beams, or single ionization of He^+ beams (He^{++} has same e/M as deuterons.) could also be employed, with some quantitative advantage (slower transit velocity, therefore larger N values for a given particle energy).

From Eq. (16) it can be seen that relatively modest currents should be sufficient to cause a substantial effect, provided care is taken to satisfy the coherency requirements while maximizing N. This end can be accomplished through care in choosing the beam energy (adjusts γ) and in aiming the beams (lowers $W_{\perp}$ at entrance). There may also be simpler ways to test the idea that will give preliminary indications as to its effectiveness. These ideas will no doubt occur upon further thought.

If it becomes desirable to enhance the coherence of the wave-beam interaction a possible alternative technique would be to use two types of beams, as follows: Beams A (ion or electron) would be intensity modulated at the local ion cyclotron frequency. The phase-determined instability wave thus stimulated would then convect azimuthally, meanwhile growing in amplitude. This wave would then encounter one or more Beams B spaced azimuthally around the axis of the plasma. Beams B would interact with the azimuthally growing wave, gaining transverse energy at the expense of the waves and carrying this energy out of the system, thereby preventing further growth of the wave. This method,

though more complicated, has the advantage that it would not only guarantee the coherency of the wave-beam interaction but that it would also provide "knobs" that could be used to optimize the BD action through adjustments in Beams A and/or B, including the possibility of employing closed-loop feedback to maximize the BD effect.

Appendix A

Example Calculations of Transit Time Factor

As an example of the increase in the transit time factor, λ , that can arise because of the presence of the ambipolar potential, we will consider the following:

Assume that the shape of the ambipolar potential can be represented by a parabola:

$$\phi = \phi_0 \left[1 - (2z/L)^2 \right] - \frac{L}{2} \left(z - \frac{L}{2} \right) \quad (A1)$$

The equation of motion is then

$$\ddot{z} = \gamma^2 z, \quad \gamma^2 = \left(\frac{8\phi_0}{L^2} \right) \left(\frac{e}{M} \right), \quad (A2)$$

with the boundary conditions (at $t = 0$) $z = -L/2$, $\dot{z} = +v_b$. Take

$$W = \frac{1}{2} M v_b^2 = G \cdot (e\phi_0);$$

$G > 1$ corresponds to transmission over the potential barrier; $G < 1$ corresponds to reflection before attaining the top of the barrier.

From the general solution we can determine the transit time factor $\lambda = t_1/t_0$, actual time within the plasma compared to the transit time when $\phi_0 = 0$, in the form of a transcendental equation:

$$\frac{\sinh(2\lambda/G^{1/2})}{\cosh(2\lambda/G^{1/2}) \mp 1} = G^{1/2} \quad (A3)$$

The minus sign corresponds to transiting particles; the plus sign to reflected particles. In the limit $G \gg 1$, $\lambda \rightarrow 1$; in the limit $G \rightarrow 1.0$, $\lambda \rightarrow \infty$, logarithmically. Example cases are given in Table A1.

Table A1

Transiting Particles ($G > 1$)			Reflected Particles ($G < 1$)		
G	λ	λ^2	G	λ	λ^2
1.01	3.02	9.08	0.99	2.98	8.87
1.05	2.26	5.10	0.90	1.73	3.00
1.10	1.96	3.84	0.80	1.30	1.68
1.20	1.69	2.86	0.70	1.01	1.02
1.50	1.41	1.97	0.60	0.80	0.64
2.00	1.25	1.56	0.50	0.62	0.38

Appendix B

Resonant Acceleration in a Non-Uniform B Field

We here evaluate the gain in perpendicular energy of a quasi-resonant particle passing through a region within which B_z varies with respect to the value corresponding to cyclotron resonance with the exciting electric field. The results can be used to estimate the sensitivity of the results given earlier to spatial variations in the confining field.

In component form, the equation of motion of the particle is:

$$M\ddot{x} = e \left[E_x + \dot{y}B_z/c \right] \quad (B1)$$

$$M\ddot{y} = e \left[E_y - \dot{x}B_z/c \right] \quad (B2)$$

We here take $E_x = E_0 \sin \omega t$, $E_y = 0$. With the substitutions $s = c(E_0/B_z)\dot{y}$, $\Omega(z) = (eB_z/Mc)$, $\gamma = \Omega/\omega$ and $\tau = \omega t$ we obtain the differential equation

$$\ddot{s} + \gamma^2(s + \sin \tau) = 0 \quad (B3)$$

We have here assumed that $E_0/B_z = \text{constant}$, which does not imply a significant error relative to actual cases. For the special case $\gamma^2 = \text{constant} = 1$ (resonance in a constant B field) the solution to Eq. (B3) is

$$s = \frac{1}{2}(\tau \cos \tau - \sin \tau) \quad (B4)$$

Reinserting the definitions of the variables, squaring and eliminating secular terms we find the reference case result:

$$w_N = \frac{1}{2} \gamma^2 (E_0/B_z)^2 N^2 \quad (B5)$$

To solve Eq. (B3) in the general case where γ is a function of position along the trajectory, thus of time, we convert (B3) to an integral equation and solve by iteration. The integral equation is

$$s = - \int_0^t dt \int_0^t \gamma^2(s + s \sin t') dt' , \quad (B6)$$

Subject to the initial conditions $s = \dot{s} = 0$. Solutions to this equation have been obtained using the UCSB On-line Computer Terminal for a parabolic functional dependence of γ , to wit

$$\gamma = \gamma_0 \left\{ 1 + \epsilon \left[t - t_{\max}/2 \right]^2 \cdot (4/t_{\max}^2) \right\}. \quad (B7)$$

This form of magnetic field variation would model the case of a particle moving at uniform longitudinal speed through a field that varies parabolically with distance, having its minimum point halfway along the particle's trajectory.

Solutions of Eq. (B6) for various values of ϵ and γ_0 are shown in Fig. B1. It can be seen from the example cases that provided B_z does not vary with respect to its resonant value by more than 10 to 20 per cent, s and thus $s^2 \sim w_N$ do not deviate greatly from the reference values, Eq. (B5), for N values of interest. The lines shown on the plots are the envelopes of the reference case values of s , Eq. (B4),

corresponding to the maxima and minima of that function. They therefore provide a convenient reference against which to gauge the deviations arising from the non-constancy of γ . As noted in the text, roughly speaking the energy gain is not seriously affected for transits in which N is less than or of order $(\gamma B/B)^{-1}$.

Figure B1

Examples of Resonant Energy Gain in Non-Uniform B Fields

$$\gamma = \gamma_0 \left\{ 1 + \epsilon \left[\tau - \tau_{\max}/2 \right]^2 (4/\tau_{\max}^2) \right\}$$

$\gamma_0 = 1.0, \epsilon = 0, \tau_{\max} = 50.$
(Reference Case)

$\gamma_0 = 0.9, \epsilon = 0.2, \tau_{\max} = 50.$

$\gamma_0 = 1.0, \epsilon = 0.1, \tau_{\max} = 50.$

$\gamma_0 = 0.9, \epsilon = 0, \tau_{\max} = 50.$

$\gamma_0 = 1.0, \epsilon = 0.2, \tau_{\max} = 50.$

$\gamma_0 = 1.1, \epsilon = 0, \tau_{\max} = 50.$

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