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CORRECTIONS TO PRIMORDIAL NUCLEOSYNTHESIS

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ABSTRACT

The changes in primordial nucleosynthesis resulting from small corrections to rates for weak processes that connect neutrons and protons are discussed. The weak rates are corrected by improved treatment of Coulomb and radiative corrections, and by inclusion of plasma effects. The calculations lead to a systematic decrease in the predicted ⁴He abundance of about $\Delta Y = 0.0025$. The relative changes in other primordial abundances are also 1-2%.

In this talk I will discuss improvements in the calculation of the primordial ${}^4\text{He}$ abundance. The corrections are related to improved treatment of the weak $n \leftrightarrow p$ rates. I will compare the new values of the ${}^4\text{He}$ abundance to the standard calculation of Wagoner.¹ For details of our calculations, the reader is referred to our original paper.²

The standard hot big-bang model of the universe seems to provide a reliable framework for understanding the origin and evolution of our universe.³ One of the features of our present universe which is naturally explained in this model is the large abundance of ${}^4\text{He}$. The success of primordial nucleosynthesis in predicting the large primordial abundance of ${}^4\text{He}$, and the relatively large abundance of D, is usually considered to be the strongest evidence that the universe can be described by a Friedmann-Robertson-Walker cosmology at very early times. Because of this concordance, it is attractive to assume that the Friedmann-Robertson-Walker cosmology was applicable at the time of nucleosynthesis, and then demand that the resulting primordial abundances of the light elements be within bounds extrapolated from present observations.

This latter approach has been recently employed to limit the number of neutrinos at nucleosynthesis, and to put constraints on a number of exotic particles, such as photinos, gravitinos, goldstinos, monopoles, quarks, stable leptons, etc. The ${}^4\text{He}$ abundance has also been recently used to argue that the standard cosmological model does not fit the data.⁴ The same phenomenon has been called by others the "crowning achievement of the standard model."

Because primordial nucleosynthesis provides such a powerful probe of the conditions in the universe at early times, it is important to have precise predictions of the primordial light element abundances, particularly that of ${}^4\text{He}$. In this talk, I will consider modifications to the calculation of the ${}^4\text{He}$ abundance due to: 1) use of an explicit numerical integration of the rates for n - p transitions rather than fits to the numerical rates; 2) correct treatment of Coulomb corrections; 3) inclusion of radiative corrections--both the usual radiative corrections, and the finite temperature and finite density radiative corrections that depend on the presence of a plasma; 4) inclusion of the effect of the plasma on the mass of the electron; and 5)

heating of electron neutrinos in e^+e^- annihilation. We find that the above five effects result in a systematic decrease in the ${}^4\text{He}$ abundance of about 0.003, or about a one percent relative decrease. (Addition of a light neutrino species leads to a ΔY of $\Delta Y \approx 0.01$.) There are similar 1-2% changes in the abundances of the other light elements which are produced (D, ${}^3\text{He}$, ${}^7\text{Li}$, etc.).

Although we find that the corrections to the weak rates are in general temperature dependent, we can get a rough estimate of the sensitivity of the primordial ${}^4\text{He}$ abundance, Y , on the weak rates by considering the change in Y resulting from a change in the neutron half-life.⁵ For instance, a decrease in the neutron half-life from 10.6 minutes to 10.4 minutes ($\Delta\lambda/\lambda \approx +0.02$ where $\lambda \equiv \tau_{1/2}^{-1}$) results in a decrease in the ${}^4\text{He}$ abundance of about 0.004. This decrease in the neutron half-life is equivalent to an increase in G_F , the Fermi constant, hence a temperature independent increase in all the weak rates. This suggests that the dependence of Y on $\Delta\lambda$ may be approximated as $\Delta Y \approx 0.2 \Delta\lambda/\lambda$.

It should be emphasized that the corrections 1-5 above are universal in the sense that they must be applied regardless of the values of the neutron lifetime, the number of neutrinos, or the ratio of baryons to photons.

The six weak reactions which interconvert neutrons and protons are $n \leftrightarrow p e \bar{\nu}$, $n \bar{\nu} \leftrightarrow p e$, and $n \nu \leftrightarrow p e$. The rates for these processes were first calculated in the context of the early universe by Alpher, Follin, and Herman,⁵ and we will adopt their notation (see also ref. 1). The rates depend on $Q = m_n - m_p$, the photon temperature T , the neutrino temperature T_ν , and the electron energy E_e . The rates depend on these quantities through the dimensionless variables $z = m_e/T$, $z_\nu = m_e/T_\nu$, $q = Q/m_e$, and $\varepsilon = E_e/m_e$. The weak rates may be written in terms of these variables. For instance, the rate for neutron decay is

$$\lambda_{n \rightarrow p e \bar{\nu}} = (\tau \lambda_0)^{-1} \int_1^q d\varepsilon \frac{\varepsilon(\varepsilon - q)^2(\varepsilon^2 - 1)^{1/2}}{[1 + \exp(-\varepsilon z)]\{1 + \exp[(\varepsilon - q) z_\nu]\}} \quad , \quad (1)$$

where τ is the neutron lifetime and λ_0 is defined as $\lambda_0 \equiv \int_1^q d\varepsilon \varepsilon(\varepsilon - q)^2(\varepsilon^2 - 1)^{1/2} = 1.63615$. The factor $(\tau\lambda_0)^{-1}$ fixes the nuclear matrix element to be used.

In the computer code developed by Wagoner,¹ the rates (2.7) are fit by an analytic function of the photon temperature. The fit to λ_n is good to an accuracy of about 1.4% over the temperature range $T_9 = 100$ to $T_9 = 0.3$ (T_9 is the temperature in units of 10^9 K). The fit to λ_p is good to about 1.4% over a range $T_9 = 100$ to $T_9 = 3$. This fit is rather remarkable considering that λ_n and λ_p change by more than seven orders of magnitude over the above temperature range. However, based on our approximate formula for ΔY given in Eq. (1.1), a 1.4% error in the weak rates lead to a potential ΔY of $|\Delta Y| = (0.2)(1.4\%) \cong .003$.

We have modified Wagoner's code to evaluate the rates numerically every time step. We evaluate the integrals to an accuracy of better than 0.005%, which should introduce an error in ΔY of $|\Delta Y| \lesssim 0.0001$. The change in Y is given in Table I.

Our next change is to correctly treat the Coulomb corrections. Coulomb corrections are taken into account by multiplying the phase space density for $n \leftrightarrow pe\nu$ and $n\bar{\nu} \leftrightarrow pe$ by F_+ , where F_+ is the Fermi function given in terms of the electron velocity, β , as

$$F_+(\beta) \cong \frac{2\pi\alpha/\beta}{1 - e^{-2\pi\alpha/\beta}} \quad (2)$$

It is also necessary to include F_+ in the phase space for λ_0 , which changes λ_0 from 1.63615 to about 1.69. Wagoner accounted for Coulomb corrections by decreasing λ_0 by 2%. The effect of correctly treating the Coulomb corrections is shown in Table I.

Our next correction is to include the radiative corrections. The Radiative corrections have been considered by many authors,⁶ the effect is to multiply the phase space for all processes by $[1 + \frac{\alpha}{2\pi} C(\beta, y)]$ where β is the electron's velocity, and y is the neutrino energy divided by m_e (y is different for the different weak processes). The function C is given by⁶

$$\begin{aligned}
C(\beta, y) \cong & 40 + 4(R - 1)(y/3\epsilon - 3/2 + \ln 2y) \\
& + R[2(1 + \beta^2) + y^2/6\epsilon^2 - 4\beta R] \\
& - 4[2 + 11\beta + 25\beta^2 + 30\beta^3 + 20\beta^4 + 8\beta^5]/(1 + \beta)^6. \quad (3)
\end{aligned}$$

In Eq. (3) the last term in the brackets is from expansions of Spence functions, and R is defined to be

$$R \equiv \beta^{-1} \tanh^{-1} \beta. \quad (4)$$

Including radiative corrections changes λ_0 to 1.75321. Wagoner ignored radiative corrections. The effect of including the radiative corrections is given in Table I.

We next correct the rates by doing the radiative corrections at finite temperature. These corrections were also done independently by Cambier, Primack, and Sher.⁷ The real-time formalism⁸ is particularly useful for this purpose, since there is a clean separation between temperature dependent and the temperature independent parts of the radiative corrections.

The finite temperature radiative corrections involve double integrals which must be done numerically. The integrals are too tedious to write here; they are given elsewhere.^{2,7} The finite temperature radiative corrections cause the rates to be multiplied by an additional factor of the form

$$1 + \frac{\alpha}{\pi} C'_i(T), \quad (5)$$

where the C'_i differ for different processes.

It turns out that the finite temperature corrections depend strongly on the temperature and on the individual reaction considered. It also turns out that the rate for neutron decay receives the largest correction. This large correction is mostly due to the effect of a photon in the initial state: $\gamma n \rightarrow p e \nu$. The effect of the photon can be thought of as an effective increase in the Q value for $n \rightarrow p e \nu$, $Q \rightarrow Q + \omega$, where ω is the energy of the photon, $\langle \omega \rangle \cong 2.7T$. Since the neutron decay rate is a more sensitive

function of Q than the $2 \rightarrow 2$ weak processes, this particular consequence of finite temperature is larger for $\lambda_{n \rightarrow p \nu}$. However, at the time of the freeze out of the neutron-proton ratio, $\lambda_{n \rightarrow p \nu}$ makes only a small contribution to the total rate. The effect of the finite temperature radiative corrections is shown in Table I.

The next effect we consider is the mass shift of the electron due to finite temperature effects. The electron mass at finite temperature is found by calculating the electron self-energy diagram with the temperature dependent real time propagators. If one defines mass renormalization at zero temperature by the addition of a counter term $\delta m = -\text{Re } \Sigma(p^2 = m_e^2)$ with $\Sigma(p)$ the electron self-energy, then finite temperature effects add to the electron mass a term δm_T defined by⁹

$$\delta m_T = \delta m + \text{Re } \Sigma_T(p^2 = m_e^2) \quad , \quad (6)$$

where Σ_T is the electron self-energy evaluated with the finite temperature photon and electron propagators. This results in a temperature dependent effective electron mass, m_T , of the form⁹

$$\begin{aligned} m_T &= m_e + \delta m_T \\ &= m_e + B\alpha T^2/m_e \quad , \quad (\alpha T^2/m_e^2 \leq 1) \quad , \end{aligned} \quad (7)$$

where m_e is the zero temperature electron mass (0.511 MeV), and B is a slowly varying function of the temperature with a value between 1 and 2.

It is easy to include this correction in the rates by the substitution of m_T for m_e in the rates and by multiplying all the rates by $(m_T/m_e)^5$, as the cancellation of the m_e^5 from the numerator, with the m_e^5 from λ_0 is true only at zero temperature. (We did not explicitly include the factor of m_e^5 in the definitions of the rates and in the definition of λ_0 .)

The modification of the electron mass also has the effect of changing the relationship between the neutrino and photon temperatures. The neutrino temperature differs from the photon temperature because, to a good approximation, the neutrinos are decoupled when the entropy in the e^+e^- gas is released as e^+e^- annihilate. Therefore, the entropy released heats the photons, but not the neutrinos. Below, we discuss the validity of the

approximation that the neutrinos are not heated by e^+e^- annihilations, and the effect on Y of relaxing this assumption.)

An additional effect of the change in the electron mass that leads to an even smaller effect on ΔY , is that as the electron mass changes, the electron's contribution to the total energy density, and hence to the expansion rate, changes.

The changes in the primordial ${}^4\text{He}$ abundance due to finite temperature effects on the electron mass is given in Table I.

A final small effect we have considered is neutrino heating due to e^+e^- annihilation. Part of the "early Universe" lore is that neutrinos decouple at a temperature $T_d \cong 1$ MeV and therefore do not share in the entropy release from e^+e^- annihilations which occur at temperatures $\lesssim 0.5$ MeV. As a consequence, neutrinos are predicted to have a lower temperature today than the photons, $T_\nu = (4/11)^{1/3} T_\gamma$. The assumption of complete neutrino decoupling is explicitly incorporated into Wagoner's code. However, since the weak rates are extremely sensitive to the neutrino temperature, we have investigated the validity of this assumption. We have found that the usual approximation of $T_\nu = (4/H)^{1/3} T_\gamma$ is valid to 2×10^{-3} for ν_e , and 9×10^{-4} for ν_μ and ν_τ .

The slightly warmer neutrinos change the weak rates, and change the contribution to the total energy density due to neutrinos. The effects go in opposite directions, and tend to cancel.

The size of the effect is small. It is listed in Table I.

The changes in the predicted ${}^4\text{He}$ mass fraction due to all the effects discussed here are summarized in Table I. For a wide range of input parameters ($\tau_h = 10.1 - 11.1$ min, $N_\nu = 2 - 10$, and $\eta = 3 \times 10^{-11} - 3 \times 10^{-9}$), the sum of all these corrections results in an approximately constant, systematic decrease in Y , $\Delta Y \cong -0.0025$ ($\sim 1\%$ relative change). The changes in the predicted abundances of the other light elements (D, ${}^3\text{He}$, and ${}^7\text{Li}$) are also in the range of a few percent. However, their present abundances are known to much less precision than the present ${}^4\text{He}$ abundance, typically to only within a factor of 2. In Fig. 2 we show the predicted primordial ${}^4\text{He}$ abundance with and without the corrections we have discussed in this paper, as a function of η for $\tau_h = 10.6$ min and $N_\nu = 2, 3$, and 4.

The net ΔY we find is about the size of Wagoner's estimated uncertainty, and just slightly less than the uncertainty due to a 1σ change (± 0.16 min)

in $\tau_{1/2}$. Nevertheless, it is a systematic decrease in the predicted primordial abundance of ${}^4\text{He}$. Within a few years experiments with confined neutrons should significantly improve the determination of $\tau_{1/2}$. Recent studies of extragalactic, very metal-poor objects (in which the stellar contribution to the ${}^4\text{He}$ abundance should be small) have led to more accurate and reliable determination of the primordial mass fraction of ${}^4\text{He}$.¹⁰ A recent result typical of these studies is $Y_p = 0.24 \pm 0.01$.¹⁰ As more objects are studied the uncertainty in Y_p should continue to decrease. It has been argued on the basis of the abundances of D and ${}^3\text{He}$ that η must be greater than $(2 - 3) \times 10^{-10}$.¹¹ For $N_\nu = 3$ the standard model did predict $Y_p \gtrsim 0.240 - 0.246$ - leaving little room for concordance. It now predicts $Y_p \gtrsim 0.237 - 0.243$ - easing the situation a bit. In any case, since primordial nucleosynthesis is the most vigorous test of the standard, hot big bang model, and is also our most powerful probe of the early universe, it is important to continue to sharpen and to reexamine its predictions as the uncertainties in the input parameters decrease.

Fig. 1.

The primordial ${}^4\text{He}$ mass fraction as a function of the baryon-to-photon ratio with 2, 3, and 4 light neutrino families assuming a neutron half-life of 10.61 minutes. The upper curve in each set shows the result of Wagoner's calculation, and the lower curve shows the results of including all the corrections to the weak rates.

TABLE I

SENSITIVITY OF THE PRIMORDIAL ^4He ABUNDANCE TO THE
 CORRECTIONS DISCUSSED IN THE TEXT
 (The values quoted are for $n_{\text{B}}/n_{\text{Y}} = 3 \times 10^{-10}$, 3 light
 neutrinos, and $\tau_{1/2} = 10.6$ minutes.)

	Wagoner 1973	$Y_0 = 0.2456$		
1.	Wagoner 73 with numerical evaluation of rates	$Y_1 = 0.2443$	$Y_0 - Y_1 = 0.0013$	
2.	Above correc- tions plus Coulomb corrections	$Y_2 = 0.2434$	$Y_0 - Y_2 = 0.0022$	$Y_1 - Y_2 = 0.0009$
3.	Above cor- rections plus zero tempera- ture radiative corrections	$Y_3 = 0.2439$	$Y_0 - Y_3 = 0.0017$	$Y_2 - Y_3 = -0.0005$
4.	Above cor- rections plus finite tempera- ture radiative corrections	$Y_4 = 0.2435$	$Y_0 - Y_4 = 0.0021$	$Y_3 - Y_4 = 0.0004$
5.	Above cor- rections plus corrections to electron mass	$Y_5 = 0.2436$	$Y_0 - Y_5 = 0.0020$	$Y_4 - Y_5 = -0.0001$
6.	Above cor- rections plus e^+e^- heating of ν_e	$Y_6 = 0.2434$	$Y_0 - Y_6 = 0.0022$	$Y_5 - Y_6 = 0.0002$

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