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QUADRUPOLE VS SOLENOIDAL FOCUSING TO SUPPRESS THE IMAGE DISPLACEMENT
INSTABILITY IN LINEAR INDUCTION ELECTRON ACCELERATORS

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ABSTRACT

This work compares the effectiveness of quadrupole vs. solenoidal focusing in suppressing the image displacement instability in linear induction electron accelerators. A brief explanation of the instability is presented; a criterion is derived for the magnitude of the solenoidal magnetic field required to suppress the instability. Two quadrupole arrangements are treated, and criteria derived for the magnitude of the pole-face field required to suppress the instability. For one arrangement the required pole-face field is less than one-half the required solenoidal field, suggesting that quadrupole focusing may have a practical advantage.

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I. INTRODUCTION

In this work we compare the effectiveness of solenoidal focusing and quadrupole focusing in suppressing the image displacement instability. This instability, which is also known by the abominable acronym "aunt fanny" effect, arises from the displacement of the magnetic image of a beam of particles when the beam traverses an accelerating gap.¹

A beam of particles traveling along the axis of a conducting pipe gives rise to azimuthally symmetric charge and current distribution in the surrounding walls. If the beam is displaced transversely from the axis of the pipe, the charge and current distributions in the walls become asymmetric. The resulting transverse force on the beam particles may be found by considering the electric and magnetic images of the displaced beam. The electric image attracts the beam toward the wall and the magnetic image provides an opposite force. In a pipe with smooth walls, these two forces cancel to within a factor γ^{-2} , where γ is the particles' total energy in units of the rest energy.

At a discontinuity in the pipe, such as an accelerating gap, the charge distribution in the walls remains near the pipe radius because the electric self-field of the beam will not penetrate very far into the gap. The magnetic self-field, on the other hand, will fill the gap region out to some radius where there exists a conducting path for the asymmetric axial current distribution. Thus in the gap region the electric image is displaced only slightly from its position in a smooth pipe, but the magnetic image is displaced to such a large radius that the magnetic force on the beam particles is negligible compared to the electric force. The cancellation of these two forces that occurs in the smooth pipe is lost in the gap region, and for $\gamma \gg 1$ and

¹C. H. Woods, Rev. Sci. Instr., 41, 959 (1970).

high current the resulting transverse force on the beam particles becomes an important consideration in the design of the beam transport system.

Unfortunately, the transverse force resulting from image displacement at a gap is not amenable to an exact treatment, either analytically or computationally. For purposes of this work, an exact expression for the transverse force is not needed. We obtain an approximate expression for the change in transverse momentum of a particle in the displaced beam as it traverses the gap region.

We obtain this expression by calculating the force that arises from the magnetic image of a displaced beam. If the beam centroid is displaced a distance x from the axis of a smooth pipe, the magnetic image is located at $r_i = b^2/x$, where b is the pipe radius. If the beam carries current I , the image current is $-I$. The force on a beam particle is toward the axis, and given by

$$F_i = 2evI/c^2 r_i = 2evIx/(cb)^2, \quad (1)$$

in which e is the particles' charge, v the axial (z) particle speed, and c is the speed of light. Gaussian units are employed. We now neglect the displacement of the electric image in the gap region, and make the approximation that, over some distance w along the pipe (w may be thought of as an effective gap width) the magnetic force is completely absent. The absence of this force gives rise to an increase Δp_x in the transverse momentum of a particle as it traverses the gap. This momentum change is in the direction of the displacement x and is given by

$$\Delta p_x = 2eIw/(cb)^2. \quad (2)$$

It is conventional to describe the motion of single particles in the beam (incoherent motion) and the motion of the beam centroid (coherent motion) in terms of the variables x and x' , where $x' \equiv p_x/p_z$. In this treatment x and x' refer to the position and angle of the beam centroid. We assume that the displacement x does not change as particles traverse the gap. The motion across the gap may then be described by the matrix equation relating x_+ and x'_+ at the gap exit to x_- and x'_- at the gap entrance. We have

$$\begin{pmatrix} x_+ \\ x'_+ \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{1}{F} & 1 \end{pmatrix} \begin{pmatrix} x_- \\ x'_- \end{pmatrix}. \quad (3)$$

The quantity F is obtained from Eq. (2) and the relation $p_z = \gamma m_0 v = \gamma m_0 \beta c$ with m_0 particles' rest mass. We have

$$\frac{1}{F} = \frac{2eIw}{\gamma \beta m_0 c^3 b^2}. \quad (4)$$

In these units $m_0 c^3/e = 17$ kA for electrons. (All currents are in kA in this work, magnetic fields are in kG, and lengths in cm.)

We see that the matrix in Eq. (3) (which we designate M_g) is that of a defocusing lens. Part of the job the beam transport system must accomplish is to compensate for this defocusing.

II. SOLENOIDAL FOCUSING

We take the gap-to-gap spacing L to be fixed by the axial extent of the accelerating units. The focusing magnets, whether solenoids or quadrupoles, are placed inside the units, and extend over the entire

length L . We first treat a solenoidal transport system with axial magnetic field B_s . A matrix M_s may be used to relate the values of x and x' at the exit of the solenoid to the values at the entrance to the solenoid. This matrix is

$$M_s = \begin{pmatrix} \cos \theta & 2\rho \sin \theta \\ \frac{-\sin \theta}{2\rho} & \cos \theta \end{pmatrix}. \quad (5)$$

In Eq. (5), the quantities ρ and θ are defined as

$$\rho = \gamma \beta m_0 c^2 / e B_s, \quad (6)$$

and

$$\theta = L/2\rho. \quad (7)$$

We now derive a criterion on the magnitude of B_s necessary to overcome the defocusing effect of the image displacement as manifest in the matrix M_g . The method is not rigorously valid if, as in the situation here, γ changes from gap to gap. The method is rigorously valid only if the matrix elements do not change, but it will give a good criterion (perhaps a bit too stringent) if the matrix elements change slowly.

The criterion for stability of coherent transverse motion is that the absolute value of the trace of the matrix $M_s M_g$ be less than 2. With M_s from Eq. (5) and M_g from Eq. (3) we have the condition

$$|\cos \theta + (\rho/F) \sin \theta| < 1. \quad (8)$$

In practical situations $0 \leq \theta \leq \pi/2$ so that the trace is ≥ 0 and we have the condition

$$(\rho/F) < \tan(\theta/2).$$

A simple criterion that is a good approximation for most situations of interest is derived by setting $\tan(\theta/2) = \theta/2$. In this limit we obtain

$$F > 4\rho^2/L. \quad (9)$$

In this approximation the criterion states that the equivalent focal length f of the solenoid must be less than F . Inserting Eq. (4) for F and Eq. (6) for ρ into Eq. (9), we obtain the desired criterion for B_s ,

$$B_s(\text{kG}) \ b(\text{cm}) > [1.36 \ I(\text{kA}) \ \gamma\beta w/L]^{1/2}. \quad (10)$$

In deriving Eq. (10) we have used $m_0 c^3/e = 17 \text{ kA}$ and $\rho(\text{cm}) = 1.7\gamma\beta/B(\text{kG})$.

III. QUADRUPOLE FOCUSING

The matrix M_f relating the values x to x' at the exit of a focusing quadrupole of length s to the values at the entrance to the quadrupole is given by

$$M_f = \begin{pmatrix} \cos \psi & \frac{\sin \psi}{k} \\ -k \sin \psi & \cos \psi \end{pmatrix}, \quad (11)$$

in which

$$k^2 = \frac{e}{\gamma\beta m_0 c^2} \frac{\partial B_x}{\partial y}, \quad (12)$$

$$\psi = ks. \quad (13)$$

In a quadrupole field

$$\partial B_x / \partial y = \partial B_y / \partial x = B_p / r_p, \quad (14)$$

where r_p is the radius to the pole tip and B_p is the maximum magnetic field, which is the value at the pole face. Analogous to Eq. (11), the

matrix M_d for a defocusing quadrupole with the same field gradient is given by

$$M_d = \begin{pmatrix} \cosh \psi & \frac{\sinh \psi}{k} \\ k \sinh \psi & \cosh \psi \end{pmatrix} \quad (15)$$

A. One Quadrupole Arrangement

First we treat a configuration in which a focusing quadrupole and a defocusing quadrupole are placed between each gap. We consider only "symmetric doublets," that is, the two magnets have the same gradient. Each magnet has length $s = L/2$. A cell of this structure consists of a gap followed by a focusing quad followed by a defocusing quad for motion in one transverse plane. For motion in the other transverse plane, the quads are reversed. Although the matrix elements are not all the same, it can be shown that (Tr indicates trace)

$$\text{Tr}(M_d M_f M_g) = \text{Tr}(M_f M_d M_g) . \quad (16)$$

The stability criterion is

$$\cos \psi \cosh \psi + (1/2 kF)(\sin \psi \cosh \psi + \cos \psi \sinh \psi) < 1 . \quad (17)$$

We now seek a criterion on B_p with the condition $\psi \leq 1$. Expanding functions in Eq. (17) we have

$$\cos \psi \cosh \psi = 1 - \frac{\psi^4}{6} + O\psi^8 . \quad (18)$$

$$\sin \psi \cosh \psi + \cos \psi \sinh \psi = 2\psi + O\psi^5 . \quad (19)$$

Neglecting terms of order higher than fourth in Eq. (18) and higher than first in Eq. (19), and noting that $\psi = ks$, we write Eq. (17) in the form

$$\frac{s}{F} < \frac{\psi^4}{6} . \quad (20)$$

Recalling that for this configuration $s = L/2$, we have

$$k^2 L > (48/LF)^{1/2} . \quad (21)$$

From Eqs. (12 and (14) we have (in these units)

$$k^2 (\text{cm}^{-2}) = \frac{1}{1.7\gamma B} \frac{B_p (\text{kG})}{r_p (\text{cm})} . \quad (22)$$

A physically reasonable assumption is that $r_p \equiv b$. With this assumption the criterion becomes independent of b . Inserting Eqs. (4) and (22) into Eq. (21) we obtain

$$B_p (\text{kG}) L (\text{cm}) > [16.32 I (\text{kA}) \gamma B W / L]^{1/2} . \quad (23)$$

B. Another Quadrupole Arrangement

The second configuration we treat is one in which a focusing quad is placed between two gaps and a defocusing quad is placed between the next two gaps (each magnet now has length $s = L$). For motion in one transverse plane a cell of this structure consists of gap, focusing quad, gap, defocusing quad. For motion in the other transverse plane the quads are reversed. Again, the same stability criterion is obtained for both planes. This criterion obtained from the condition $\text{Tr}(M_d M_g M_f M_g) < 2$ is

$$\begin{aligned} & \cos \psi \cosh \psi + (1/kF)(\sin \psi \cosh \psi + \cos \psi \sinh \psi) \\ & + \sin \psi \sinh \psi / 2(kF)^2 < 1 . \end{aligned} \quad (24)$$

We use the expansions in Eqs. (18) and (19) as well as $\sin \psi \sinh \psi \approx \psi^2$ in Eq. (24) to obtain

$$\frac{\psi^4}{6} > \frac{2s}{F} + \frac{1}{2} \left(\frac{s}{F} \right)^2 . \quad (25)$$

Equation (25) is somewhat more complex than Eq. (20), but in practice $s/F (\equiv L/F) \ll 1$ for $I \approx 10$ kA and $\gamma \approx 50$ or higher. We neglect the second term in Eq. (25) and obtain

$$k^2 L > (12/FL)^{1/2} . \quad (26)$$

If we again take $r_p = b$, we may write the criterion in the form

$$B_p (\text{kG}) L (\text{cm}) > [4.08 I (\text{kA}) \gamma^2 w / L]^{1/2} . \quad (27)$$

We see that the second quadrupole arrangement requires a value of B_p that is one-half the value required by the first arrangement. The second arrangement has the further advantage of being more simple and thus easier and cheaper to construct.

For a numerical example, we take $I = 10$ kA, $\gamma = 100$, $w = 3$ cm, $L = 30$ cm and $b = 7$ cm. From Eq. (10) we find that the required solenoidal field $B_s = 1.67$ kG. From Eq. (27) we find the required pole-face field $B_p = 0.67$ kG. The condition on B_p is independent of b , and certainly the inner pipe radius would be reduced if quadrupole focusing were used. But the inner radius of the accelerating unit must be large enough to allow the insertion of the quadrupole and a return path for the flux. The flux cannot be allowed to penetrate into the unit or it will saturate the ferrite core.

Design and construction of conventional current-driven quadrupoles that are suitable for insertion into the accelerating units is more complex than that for solenoids, but the results indicate that quadrupole focusing certainly deserves more investigation.

During the past few years there has been considerable progress in the development of small, high-field, permanent-magnetic quadrupoles. This development will be of interest since such devices may well be suitable for focusing of high current relativistic electron beams, both in the accelerator and in the beam transport system. Permanent magnets have the advantages of simplicity and high reliability, but the disadvantage that they cannot be tuned once they are in place.

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