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TITLE EXPERIMENTAL SIGNATURES OF LOCALIZATION IN LANGMUIR WAVE TURBULENCE

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Experimental Signatures of Localization in Langmuir Wave Turbulence

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Abstract

Features in certain laser-plasma and ionospheric experiments are identified with the basic properties of Langmuir wave turbulence. Also, a model of caviton nucleation is presented which leads to certain novel scaling predictions.

1. Introduction

The hallmark of nonlinear Langmuir waves is self localization. Two well known examples are soliton formation and collapse¹. The mechanism is universal: electrostatic field concentrations expell plasma through the action of the ponderomotive force while the resulting "hole" in the plasma density hinders the dispersal of Langmuir waves (the carrier of electrostatic energy). In certain regimes, Zakharov's model¹ is a useful approximation to this physics,

$$\omega_1 \partial_t E + i \omega_{pe} E + \sqrt{2} E \nabla^2 E - n E = -E_0 \cdot \nabla n, \quad \nabla \times E = 0 \quad (1.1)$$

$$(\partial_t^2 + 2\omega_1 \partial_t - \sqrt{2}) n = \sqrt{2} |E + E_0|^2, \quad (1.2)$$

where $E(x, t)$ is the slowly varying temporal envelope of the electrostatic

field, n is the fluctuation in the plasma density, and standard dimensionless units ($3/2(m_i/m_e)\omega_{pe}^{-1}$ - time, $3/2(m_i/m_e)^{1/2}\lambda_D$ - space, $8(mn_0T_e m_e/3m_i)^{1/2}$ - electric field, $n_0(4m_e/3m_i)$ - density) are used. $\underline{E}_0$ is spatially uniform, $\underline{E}_0 \sim \exp(-i\omega_0 t)$, and may be viewed as a longwavelength electromagnetic wave with frequency near ω_{pe} . We consider only dissipation which is linear and local in $\underline{k}$ space (e.g., Landau damping), so that $v_e \circ$ and $v_i \circ$ denote convolution in $\underline{x}$ space.

The experiments of Wong and Cheung², provide direct evidence that Langmuir wave solitons evolve in the way predicted by Zakharov's model. In this paper we shall review the indirect evidence in the laser-plasma experiments of Walsh, Villeneuve and Baldis³ (WVB), Villeneuve, Baldis and Bernard⁴ (VBB), and in ionospheric experiments such as those of Djuth, Gonzales and Ierick⁵.

2. Coupling of SRS and SRS in Laser-Plasma Interaction

WVB reported on experiments in which they observed the interaction of electron density fluctuations associated with stimulated Raman scattering (SRS) and stimulated Brillouin scattering (SBS). There were several key observations:

(1) SRS fluctuations were observed to occur only in a narrow band of densities, $\omega_0/\omega_{pe} < n_0/n_c < \omega_0/\omega_b$, where n_c is the critical density for the CO_2 pump laser and n_b is the plasma density at the top of a roughly parabolic

profile; in the experiments n_0/n_c was always less than 0.25. While the lower density limit for SRS activity is consistent with the strong damping that an SRS Langmuir wave would be subject to, the upper limit is not consistent with the usual estimates of thresholds for SRS.

(ii) When SRS was observed, it was terminated at the time when SBS was first noticed.

(iii) The temporal duration of SRS decreased with increasing laser power.

(iv) At the upper density limit of SRS activity ($\approx 0.05 n_c$), there were large shot to shot variations in the level of SRS fluctuations.

In Rose DuBois and Bezzerides⁶, a natural generalization of Zakharov's model was used in which the right-hand sides of equations (1.1) and (1.2), with $E_0 = 0$, are supplemented by the usual source terms due to the ponderomotive forces arising from the interaction between the incident and backscattered light, and in addition there are enveloped versions of Maxwell's equations for the light waves with current sources arising from the interaction between the light and the Langmuir and ion-acoustic waves. Analysis of this experiment involves the processes of Langmuir turbulence - nucleation, collapse, burnout - and reveals how nucleation of "localized" SRS Langmuir waves in the density fluctuations of SRS results in the control of SRS by SRS.

Two experimental details were incorporated into the above model: the power in the pump laser increases linearly in time; the time between the

plasma formation and the pump laser turn on is long enough (for the observed densities and temperatures) to allow the plasma to come to local thermal equilibrium. The latter was modeled by adding noise sources for the Langmuir and ion-acoustic waves, and initial conditions, in a manner consistent with thermal equilibrium. The only free parameter was the size of the simulation plasma, L , which we took to be $\approx 15\lambda_0$, where λ_0 is the wavelength of the pump. Roughly speaking, boundary conditions were chosen in such a way that any waves generated in the plasma could escape, and the only incoming wave was that of the pump.

Each of the above experimental observations is consistent with the dynamical model. The novel physical mechanism which determines the high density cutoff for SRS ($\approx 0.05n_0$) is the detuning of the resonantly interacting Langmuir and light waves which make up the Raman instability, by the growing SRS fluctuations in the plasma density. At early times, the low frequency part of the ponderomotive force is dominated by SRS, not by the growing Langmuir waves, so that one may take the fluctuation of the plasma density to be sinusoidal, with a spatial envelope appropriate to SRS in a slab geometry. The Langmuir wave normal modes are no longer plane waves, but locally resemble "Bloch waves" with frequency dispersion $\omega_{k_F}(t)$ and the λ 's have an associated band structure. k_F is the Bloch wave number which assumes a value close to the SRS Langmuir wavenumber in a uniform plasma.

We note that time-stationary density ripples have relatively little effect on the SRS growth rate while significantly altering the spatial waveform. For example, the Langmuir wave envelope, that is, the Bloch wave, has definite spatial modulations which may significantly increase the Langmuir wave ponderomotive force (see fig. 1). For $n_0/n_c < 0.05$, this leads to collapse and burnout, which at the upper end of this density range, is strongly correlated with the SRS density fluctuations. This effect of pre-existing density fluctuations (i.e. density fluctuations not generated by an existing Langmuir wavepacket) is one example of what we mean by caviton nucleation.

Exponentially growing SRS density waves do have a significant effect on SRS growth rates⁷ when $\lambda_{k_E}(t)$ increases by a amount comparable to the SRS growth rate γ_R during an e-folding time γ_R^{-1} ; i.e., when $(d\lambda_{k_E}/dt)\gamma_R^{-1} \sim \gamma_R$. Depending on initial fluctuation levels either SRS is killed before reaching experimental observation thresholds, or the Langmuir wave ponderomotive force dominates that from SRS in determining the evolution of plasma density fluctuations, leading to collapse and burnout. In the latter case, SRS again dies, but now one could say it was a matter of "suicide instead of murder". These nonlinearly generated density fluctuations provide an enhanced seed from which SRS grows, consistent with (11). This latter conclusion seems quite general, even though the details may be incorrectly predicted by the dynamical

model because other nonlinearities and kinetic effects may become important if SRS is growing quickly. Point (iii) is simply a consequence of the more rapid onset of detuning, while point (iv) is a result of the temporal race between two exponentially growing fluctuations - Langmuir waves e-folding up to the observational threshold, and ion-acoustic waves e-folding up to the detuning threshold, in a regime where which wins depends on details of thermal fluctuations. In the transition regime about the upper density limit of SRS activity, at the time when SRS "wins" (or "loses") over SBS, the amplitudes of the Langmuir and ion acoustic waves are small enough for the dynamical model to be valid. Therefore the model's prediction of this transition regime is valid even though the evolution after the contest is decided may not be adequately described (see fig. 2).

If an external SBS seed could be introduced, so that the detuning threshold would be reached earlier in time, the effect would be to make SRS die having reached a smaller maximum amplitude, and the range of densities over which SRS is observable shrinks. This effect was observed in the subsequent experiment of VBB, by introducing a second pump beam 180° from the main pump beam, with an intensity about 1% of the pump's. Baldis[*per. comm.*] has mentioned the possibility that in a plasma with a critical surface, the reflection of the pump would act as an SBS seed, suppressing SRS, and that this may explain the appearance of SRS^B only after times such that the peak

density in an expanding plasma falls below critical.

3. Power Spectra of Fluctuations in Strong Langmuir Turbulence

Recently Russell DuBois and Rose⁹ presented simulations of the power spectrum $\langle |\underline{E}(\underline{k}, \omega)|^2 \rangle$, as determined by (1.1) and (1.2). Under conditions of strong to moderate ion sound wave damping ($\nu_i(k)/k \equiv \hat{\nu}_i > 0.1$), a major part of the spectral energy was contained in localized states. In ionospheric heating experiments⁵ the putative driving field intensities are in the strong turbulence regime. It is then essential to understand how the observed incoherent scatter radar (ISR) spectra are explained by strong turbulence theory.

Let the steady state turbulence be represented by collapse events occurring at the space time points $\underline{x}_1, t_1$, with local field $\underline{E}_1(\underline{x}-\underline{x}_1, t-t_1)$.

The spectrum is given by the sum over events in the observation time; $\underline{E}(\underline{k}, \omega) = \sum \exp i(\underline{k} \cdot \underline{x}_1 - \omega t_1) \underline{E}_1(\underline{k}, \omega)$. Even if $\langle |\underline{E}(\underline{k}, \omega)|^2 \rangle$ is known, the power spectrum of $\underline{E}$ depends in detail on the correlations between events. In Ref. 9, $|\underline{E}(\underline{k}, \omega)|^2$ from single collapse events in two spatial dimensions was observed. Spectra for two values of $\underline{k}$ are shown in Fig. 3 and have three qualitatively important features: 1) essentially all the spectral energy occurs for $\omega(\omega_0) \approx 0$. 2) There are well defined peaks in the spectrum and 3) for increasing k the spectrum broadens and the peaks in the spectrum for

more negative ω become relatively more important. These spectra are similar to the ISR spectra at early times in the low duty cycle experiments of Ref. 5.

We would like to understand what properties of the caviton evolution are reflected in the spectra of isolated events, and how these relate to a multicaviton system. In Ref. 10, $\underline{E}$ was resolved in terms of a complete orthonormal set of time dependent eigenfunctions $\underline{e}_a(t)$ with eigenvalues $\lambda_a(t)$, associated with the instantaneous density perturbation, n :

$$\nabla \cdot [\lambda_a(t) + \nabla^2 - n(\underline{x}, t)] \underline{e}_a(\underline{x}, t) = 0, \quad (3.1)$$

with $\nabla \times \underline{e}_a = 0$ and $\int d\underline{x} \underline{e}_a^* \cdot \underline{e}_b = \delta_{ab}$. $\underline{E}$ may then be expressed as

$$\underline{E}(\underline{x}, t) = \sum_a h_a(t) \underline{e}_a(\underline{x}, t), \quad (3.2)$$

where the sum may include an integral over continuum states. With a given collapse, terminating at, e.g., $t=0$, one may associate a particular localized eigenstate, say $a=0$, with $\underline{e}_0$ localized at $\underline{x} = 0$. The amplitude h_0 is coupled to other h 's because of the time dependence of n , and one obtains (suppressing the dissipative term)

$$i dh_0/dt - \lambda_0(t) h_0 = S_0 - i \sum_b h_b(t) \int d\underline{x} \underline{e}_0^*(\underline{x}, t) \cdot \partial \underline{e}_b(\underline{x}, t) / \partial t, \quad (3.3)$$

where

$$S_a = \int d\underline{x} \underline{E}_0 \cdot \underline{e}_a^*(\underline{x}, t) n(\underline{x}, t). \quad (3.4)$$

In general, the coupling between states cannot be ignored. Deep into collapse, however, $|\lambda_0|$ becomes sufficiently large so that the rhs of (3.3) is

ignorable. If $k\lambda_D$ is large enough, one expects $\underline{E}(\underline{k}, t)$ to be significant only during collapse. In this temporal regime $\underline{e}_0(t)$ should come closest to evolving in a self similar way (we now restrict to the physical case of three dimensions): $\underline{e}_0(\underline{x}, t) = \delta_0(t)^{-3/2} \underline{r}_0(\underline{x}/\delta_0(t))$, with the spatial Fourier transform $\underline{e}_0(\underline{k}, t) = \delta_0(t)^{3/2} \int d\underline{x} \underline{r}_0(\underline{x}) e^{-i\underline{k} \cdot \underline{x} \delta_0}$. with $\delta_0(t) \sim (t_c - t)^{2/3} \sim \lambda_0(t)^{-1/2}$. Since $\underline{r}_0$ is a smooth function of $\underline{x}$, its Fourier transform is an exponentially decreasing function of $k\delta_0$, or equivalently, it turns on rapidly in time. Once δ_0 becomes small enough, Langmuir wave damping turns on rapidly, and $|h_0|$ decreases rapidly. Except for the variation of $\phi_0(t)$, where ϕ_0 is the phase of h_0 , this rapid on and off of $\underline{\varepsilon}(\underline{k}, t) \equiv h_0(t) \underline{e}_0(\underline{k}, t)$ would yield a power spectrum modulated with an (angular) frequency period $\Delta\omega \approx 2\pi/\tau$, where τ is the on period of $\underline{\varepsilon}(\underline{k}, t)$ which is roughly a "caviton lifetime". The structure in the ISR ionospheric spectra⁵ have modulations which would imply a lifetime ≈ 0.1 ns. The envelope of the modulations in the power spectrum of $\underline{\varepsilon}$ depends on $\phi_0(t)$. For large negative ω , we can make asymptotic estimates based on a stationary phase evaluation of the time integral: $\underline{\varepsilon}(\underline{k}, \omega) = \int dt \exp i(\omega t + \phi_0(t)) |h_0(t)| \underline{e}_0(\underline{k}, t)$; the stationary phase points $t = t_s$ occur approximately where $\omega = -d\phi_0(t)/dt|_{t=t_s} \rightarrow \lambda_0(t) \approx t \rightarrow t_c$. One finds $|\underline{e}_0(\underline{k}, \omega)|^2 \propto |\omega|^{-13/4}$ as $\omega \rightarrow -\infty$. This estimate assumes $|h_0(t)|$ is constant because of decoupling from the pump during

collapse; during the burnout phase $|h_0(t)|$ is rapidly decreasing accounting for a more rapid decrease of the spectrum for very large ω . In order to calculate for finite ω , where most of the energy is observed, one must use (3.1) to (3.3)

4. A Nucleation Model

In a regime where most of the Langmuir energy is in localized states, it is a reasonable approximation to ignore all terms in the summation of equation (3.3) except where b corresponds to a localized state. Let us now restrict our attention to the case where $\hat{v}_i$ is large enough so that after burnout, the relaxing density fluctuation is essentially nonpropagating. Immediately after burnout, energy absorption is minimal because the eigenvalue is large and negative, implying a far from resonant coupling to S_0 . The ponderomotive force is negligible, and the density fluctuation evolves according to the acoustic Green's function. A simple model for this phase of the dynamics is obtained by replacing the rhs of (1.2) by $I\delta(t)\nabla^2\delta^3(\underline{x})$, where the "impulse" $I = \int |h_0(t)|^2 dt \approx (|h_0|^2)\tau$. In three dimensions, the response of n is

$$n(\underline{x}, t) = I \cdot \alpha(|\underline{x}|/t)/t^4 \quad (4.1)$$

where

$$\alpha(\rho) = \frac{v}{n^2 \rho} \frac{d^2}{d\rho^2} \left[\frac{\rho}{(1+\rho)^2 - 4\rho^2(1-v^2)} \right] \quad (4.2)$$

Even though n is evolving self similarly, e_0 is not. The figure of merit, μ , for the ground state is simply expressed in terms of the width w , $w(t) \sim t$, of n , and its depth, d , $d \sim I/t^4$, as $\mu \sim dw^2 \sim I/t^2$. If μ is too small, there is no localized state. Since 3D solitons are unstable, as t increases, either enough energy will be accumulated so that another collapse follows, or the bound state will be lost. In the latter case, the bound state will be localized in the immediate vicinity of the expanding density fluctuation, until just before the bound state is lost. So that during the time when energy is being injected, one may be able to ignore the coupling between states localized at different collapse sites. At a particular collapse site there is a lowest lying localized state which has a nonzero source. Excited states at the same site typically have a smaller source term because they are oscillatory while $F_0(\underline{x}, t)$ is essentially uniform in direction. Also at a particular site there may only be a small number of localized states. This motivates the study of a model for the evolution of a cavity in a previously existing density fluctuation - a process we call cavity nucleation - in which only one localized state is present. For a given ion fluctuation the lowest lying state, e_0 , with nonvanishing source, S_0 , (we shall call it the ground state) is calculated from (3.1), e_0 is evolved according to (3.3) without the coupling to other amplitudes,

$$i \left[\frac{d}{dt} + v_0 \right] h_0 - \lambda_0(t) h_0 = S_0, \quad (4.3)$$

where $v_0 = \int \underline{e}_0^* \cdot v_e \circ \underline{e}_0 \, d\underline{x}$, and $n(\underline{x}, t)$ is evolved by (1.2) where $\underline{E} = h_0 \underline{e}_0$.

We shall call this the local nucleation model (LNM).

5. Power Spectra of the Local Nucleation Model

Numerical integration of the LNM is nontrivial because (3.1) is a vector eigenvalue problem. Since for present purposes we are only interested in certain qualitative features of the model, a scalar version will be studied numerically. This is obtained by dropping the divergence operator in (3.1) and replacing vectors by scalars in (1.2) and (3.1). The scalar and vector models have several important properties in common: threshold and maximum growth rate for the modulational instability, collapse scaling exponents, no threshold energy for collapse, and the failure of a density well to always support a localized eigenstate. There is one important difference: the scalar model ground state can be spherically symmetric and therefore one may assume that all functions depend only on $r = |\underline{x}|$. Spherical symmetry is imposed by representing all fields in terms of the Fourier modes $\sin(k_l r)$, $k_l = n/r_0$, $l = 1, 2, \dots$, with r_0 chosen large compared to Δ , the maximum value of $b(t)$.

For given E_0 , zero initial F and initial n of the form (3.4), nucleation fails, i.e., the localized state is lost, unless E is large enough. For E_0 greater than some threshold value, E_c , it was possible to find an initial

impulse so that the long time evolution consisted of temporally cyclic collapses. As E_0 was lowered the basin of attraction for this limit cycle behavior seems to shrink as $E_0 \downarrow E_T$. The existence of a finite nucleation threshold, E_T means that a strictly isolated collapse site cannot sustain nucleation for $E_0 < E_T$. The presence of neighboring collapse sites may allow for nucleation even for $E_0 < E_T$. Presumably, the vector LNM has a numerically different E_T .

Cyclic collapses imply that E is phase locked with E_0 . Moreover, the numerical value of the phase difference appears to be an insensitive function of $|E_0|$, and does not vary much in the approach to the limit cycle. This "phase stability" may be significant in the establishment of correlations between one collapse and the next at a given nucleation site in a noisy environment.

In figures 4-6, the evolution of $|h_0|^2$, λ_0 , $d\psi/dt$, $|E|^2$ and n are shown for the case $E_0 = 5.0$, $m_i/m_e = 20,000$. Also shown is the evolution of δ , the radius of the ground state, defined as that value of r for which $r^2 e_0^2$ is maximized. One measure of damping is $\lambda_0/|\psi|_0$ which is shown in figure 7. (The erratic behavior of this curve for large values of the ordinate is the result of small numerical jitter in the determination of the ground state being amplified by the rapid variation of Landau damping). So long as v_0 is small compared to the plasma frequency ($\omega = m_i/m_e$), the Landau damping estimate

for v_0 is accurate. As argued in Ref. 9, even though the actual damping is different in a real plasma, this difference should not make any qualitative difference as long as the discrepancy is limited to short times at the end of collapse. In figures 8-9, single event power spectra are shown for this case. They also have the three qualitative features mentioned in connection with the 2d spectra. The peaks are separated by about 30 units of angular frequency, consistent with the estimate $\Delta\omega \approx 2\pi/\tau$, with $\tau \approx 0.2$. The k dependence in this model arises from the k dependence of $e_0(k,t)$; for $k\delta_0(t) \gg 1$ we expect $e_0(k,t) \sim \delta_0^{3/2}(t) \exp(-k\delta_0(t))$. For increasing k , smaller values of δ_0 are favored and these correspond to more tightly localized states with more negative frequencies.

Note, however, that there are differences between the LNM spectra and the 2d spectra: the former have longer negative frequency tails, while the latter have their peak shifted towards negative frequencies by roughly an ion acoustic frequency. If nonlocalized states are retained in the LNM, this shift is observed, but the other difference remains.

6. Scaling Properties of the Local Nucleation Model

There are certain qualitative features of the LNM which enable one to make certain scaling predictions. Consider a regime in which E_0 is time independent, and dissipative effects are significant during a time very short compared to τ , the cavity cycle time, with the cycle beginning at $t=0$, and

ending at $t = \tau$. We shall call this ideal nucleation. One dimensionless parameter of nucleation is the degree of localization, μ , at the turn-around time, t_1 , when δ begins to shrink. For $0 < t < t_1$, the ponderomotive force is negligible, and one obtains the estimates $\delta(t = t_1) \equiv \Delta \sim t_1$, and $\mu \sim 1/t_1^2$. Near turn-around, $|\lambda_0|$ attains its smallest value, Λ , (for an isolated nucleation site, localization requires $\lambda_0 < 0$), and $|h_0|^2$ attains its maximum value, $N_{\max}$. If one makes the hypothesis that μ does not depend sensitively on E_0 , then $\Lambda \sim 1/\Delta^2$. We now distinguish two regimes: $(\tau - t_1)\Lambda < 1$, and $(\tau - t_1)\Lambda \gg 1$. The former will be called sonic nucleation, and the latter supersonic nucleation.

6.A Sonic nucleation

The sonic regime is obtained for $E_0 \approx E_T$. At turn-around, the cavity is closest to resonance, and the oscillations in $|h_0|^2$ are potentially large. However there is not sufficient time between turn-around and burnout for these large oscillations to take place, so that $N_{\max} \approx N\tau$, the amount of energy taken into burnout, which is also the amount of energy dissipated, D . In this regime, one finds that $(\tau - t_1)/\tau \ll 1$, and therefore

$$\mu \sim 1/\tau^2, \quad I \sim N_{\max}\tau, \quad \Lambda \sim \tau \quad (6.1)$$

The evolution of the cavity parameters for $E_0 = 3.40$, and no damping until the very end of the cycle, are shown in figures 10-11. Note that the major variation in time of $|h_0|^2$ is on the time scale τ , which may be reinterpreted

as the sonic transit time over Δ , and that is why we call this sonic nucleation. Relations (6.1) do not depend explicitly on E_0 , and so we shall call them internal scaling relations. If one assumes that collapse sites are typically situated a distance Δ from each other, then the mean energy density $\langle |\underline{E}|^2 \rangle \equiv W \sim N_{\max}/\Delta^3 \sim 1/\Delta^2$, and the mean dissipation rate (per unit time per unit volume) $R \sim W/\tau \sim 1/\Delta^3$. Therefore

$$R \sim W^{3/2}. \quad (6.2)$$

Note that (6.2) is identical to the result obtained¹¹ by using an analysis based on the modulational instability.

Another set of scaling results is obtained by relating the above quantities, i.e. $N_{\max}$ and Δ , to E_0 . From dimensional analysis the source term in (3.4) can be estimated as $E_0 \lambda_0 \delta^{3/2}$. The maximum amplitude $|h_0|$ is then simply estimated as $E_0 \Delta^{3/2}$, because the oscillation amplitude of a nonresonantly driven oscillator $\sim$ (source/frequency mismatch). Therefore

$$N_{\max} \sim E_0^2 \Delta^3. \quad (6.3)$$

Combining (6.1) (6.3) one obtains

$$W \sim E_0^2, \quad R \sim E_0^3, \quad N_{\max} \sim 1/E_0, \quad \Delta \sim \tau \sim 1/E_0, \quad \Delta \sim 1/E_0^2 \sim I. \quad (6.4)$$

In words, the stronger the drive, the weaker, smaller, and briefer are individual collapses, in a way such that the overall dissipation rate is greater. As far as we can tell, the results (6.4) are quite different from those predicted from a modulational instability analysis.

6.B Supersonic Nucleation

As E_0 increases, and $(\tau - t_1)\Delta$ becomes large, there are large oscillations in N for $t_1 < t < \tau$, and D decreases faster than $N_{\max}$. The evolution of N for $E_0 = 8.0$ and 32.0 is shown in figures 12-13. In figures 14-19 the variations of μ , $N_{\max}$, D , Δ , τ , and Λ are shown for $3.4 < E_0 < 32.0$. Towards the end of this range, $D E_0^2 \approx \text{constant}$. The ansatz that the figure of merit, μ , is constant is not too badly violated away from the nucleation threshold, and the measured scaling exponents are close to the values in (6.4). An eyeball fit (the dashed lines in figures 15-19) yields $N_{\max} \sim 1/E_0^{5/4}$, $\Delta \sim 1/E_0^{7/6}$, $\tau \sim 1/E_0$, $\Lambda \sim E_0^2$. The scaling of R is roughly $R \sim E_0^2$.

In order to physically realize ideal nucleation, dissipative effects must be negligible when energy is being absorbed. Peak absorption rate occurs roughly at the same time that the caviton is fattest with width Δ . In dimensionless units the Debye wavenumber $\sim \left[M \equiv m_1/m_e \right]^{1/2}$. Therefore, if $\Delta M^{1/2} \sim M^{1/2}/E_0^{7/6} \gg 1$, nucleation will be essentially nondissipative. Ignoring the difference between $7/6$ and 1 , and recalling the units of electric field strength, this means the energy density of the external field must be small compared to the particle kinetic energy density

For example, if $E_0 = 8.0$, numerical studies indicate that $M > 40,000$, i.e. field energy/kinetic energy $< .020$, is required in order that D be within 20% of the ideal value. This estimate of a few tenths of one percent does not

depend sensitively on M .

For fixed *physical* value of the electric field, if M is quadrupled, E_0 doubles, Δ and τ are approximately halved, *physical* length scales are roughly invariant while the *physical* caviton cycle time doubles. Any fixed *physical* frequency, such as the electron-ion collision rate, or frequency mismatch between the external electric field and the average plasma frequency, becomes more important as M increases. It may be difficult to find a real plasma whose turbulent state is well described by supersonic ideal nucleation.

7. Conclusions

The validity of the LNM, as an approximation to Zakharov's dynamical model, depends on the degree of localization of a caviton cycle. If E_0 is below the nucleation threshold as estimated by the LNM, then sustained collapse in a given spatial region is only possible by interactions with neighboring collapse sites, and the LNM is not valid. Stronger drive favors localization, as does stronger ion damping. For fixed E_0 and ν_i , increasingly more negative ω_0 also favors localization. This is because after burnout the caviton needs a shorter time to come into resonance with E_0 the further is the drive frequency below the local plasma frequency. Note that small values of ion acoustic damping implies that the ion acoustic mode is a faithful representative of ion dynamical response. Conversely, strongly damped ion

acoustic waves represent only part of the ion response, and perhaps this implies a less faithful modeling of a real plasma by Zakharov's dynamical model. In order that the LNM more accurately model collapse in a real plasma, relatively weak ion damping, strong drive in an overdense plasma would be required.

Negative ω_0 increases the efficiency of nucleation. For example, the dissipation rate is a rapidly increasing function of the plasma density for fixed nearly resonant drive frequency because of the decreasing time (with increasing plasma density) between burnout and the time when λ_0 comes close to resonance with the drive. If there were a long wavelength (compared to a caviton size) fluctuation in the plasma density, such that its crests were overdense, one would find there a strong peak in the Langmuir wave turbulence. Therefore, the LNM might accurately describe the turbulence in those spatial regions where it is most intense, even if it is not uniformly accurate over the entire plasma. Such a spatial organization of the turbulence may play a role in the development of correlations amongst collapse events¹².

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Figure Captions

Figure 1. (a),(b) Spatial profiles of $|E|^2$, n , and $|A_R|^2$ (A_R is the envelope of the backscattered Raman light) for various times for $n_0/n_c = 0.045$ and $V_{osc}/c = 0.035$. The units are $4\pi n_0 T_e/459$ for $|E|^2$, $n_0/2754$ for n , arbitrary units for $|A|^2$, $91\lambda_{De}$ for the spatial unit, and 140 psec for the temporal unit. Only the leftmost portion of the slab, $0 \leq x \leq 3\lambda_0$, is shown. (c) Temporal history of total Langmuir energy, total ion wave energy, and SBS backscatter reflection coefficient (in arbitrary units) for the case above. (d) Same but for $V_{osc}/c = 0.07$, $n_0/n_c = 0.055$. Here the unit of time is 132 ps.

Figure 2. Probability that the Langmuir wave energy density exceeds thermal levels by a factor of 1400 (nominal observational threshold) during the coexisting evolution of SRS and SBS, as a function of the plasma density (in units of the critical density). Note: For $n/n_c < 0.01$ this probability drops to zero because of strong Langmuir damping. The curve to the left is for $V_{osc}/c = 0.035$ and the curve on the right for $V_{osc}/c = 0.070$.

Figure 3. Spectra, $|\psi(\underline{k}, \omega)|^2 = |\underline{E}(\underline{k}, \omega)|^2/k^2$, for two dimensional isolated collapse events. $E_0 = 0.8$, $v_1 = 0.9$, $m_i/m_e = 1836$, $\omega_0 = 0$; dashed curve, $\underline{k} = (4, 0)$, and solid curve $\underline{k} = (0, 0)$. Inset: (solid) negative frequency at maximum of spectrum versus $k = k_x$; (dashed) ion acoustic shift, $|\omega| = k$.

Figure 4. Local nucleation model for $E_0 = 5.0$, $m_i/m_e = 2 \times 10^4$, $v_1 = 0.9$, $\omega_0 = 0$. Evolution of ground state radius, b , and electrostatic energy, $|h_0|^2$.

Figure 5. Evolution of ground state eigenvalue, λ_0 , and phase velocity $d\phi/dt$.

Figure 6. Evolution of $|E|^2$ and n (at $r = 0$).

Figure 7. Evolution of v_0/λ_0 .

Figure 8. Power spectrum of the local nucleation model for the parameters of Fig. 4. $k = 12.0$

Figure 9. Power spectrum for $k = 40.0$.

Figure 10. Evolution of ground state radius and electrostatic energy for the nondissipative LNM with $E_0 = 3.4$.

Figure 11. For the case of Fig. 10, evolution of ground state eigenvalue and phase velocity.

Figure 12. Evolution of ground state radius and electrostatic energy for nondissipative LNM with $E_0 = 8.0$.

Figure 13. Evolution of ground state radius and electrostatic energy for nondissipative LNM with $E_0 = 32.0$.

Figure 14. Figure of merit of ground state at the transition from nucleation to collapse, as a function of E_0 .

Figure 15. Peak electrostatic energy versus E_0 .

Figure 16. Energy dissipated at the end of collapse versus E_0 .

Figure 17. Maximal width of Langmuir wavepacket, in the ground state, versus E_0 .

Figure 18. Duration of nucleation cycle versus E_0 .

Figure 19. Minimum value of $|\lambda|$ versus E_0 . Note the rapid fall off for E_0 near threshold.