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NORMAL ZONE DETECTORS FOR A LARGE NUMBER
OF INDUCTIVELY COUPLED COILS

Earl W. Owen
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Lawrence
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Earle W. Owen
and
Daniel W. Shimer

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ABSTRACT

In order to protect a set of inductively coupled superconducting magnets, it is necessary to locate and measure normal zone voltages that are small compared with the mutual and self-induced voltages. The method described in this report uses two sets of voltage measurements to locate and measure one or more normal zones in any number of coupled coils. One set of voltages is the outputs of bridges that balance out the self-induced voltages. The other set of voltages can be the voltages across the coils, although alternatives are possible. The two sets of equations form a single combined set of equations. Each normal zone location or combination of normal zones has a set of these combined equations associated with it. It is demonstrated that the normal zone can be located and the correct set chosen, allowing determination of the size of the normal zone. Only a few operations take place in a working detector: multiplication of a constant, addition, and simple decision-making. In many cases the detector for each coil, although weakly linked to the other detectors, can be considered to be independent. An example of the detector design is given for four coils with realistic parameters. The effect on accuracy of changes in the system parameters is discussed.

1. INTRODUCTION

The progress of the controlled fusion program is linked to the development of large superconducting magnets.⁽¹⁾ The size of the capital outlay and the need for reliability justify the installation of comprehensive safety systems.⁽²⁾ In the past neglect or ignorance of safety requirements has sometimes led to magnet failure.⁽³⁾

A superconducting coil can fail if part of the winding loses its superconductivity or turns normal and an excessive amount of heat is produced in a localized region. A number of protective measures have been devised.⁽⁴⁾ This report considers the problem of initially detecting the normal zone, after which the means of protection that is selected can be applied.

The detection of a normal zone in a single superconducting coil is a relatively simple matter. A bridge can be used to balance out the self-induced voltages, allowing the normal zone voltage to be measured.⁽⁵⁾ Detection of a normal zone in inductively coupled coils is more difficult because, in general, it is not possible to balance out the mutually induced voltages as well as the self-induced voltages. A method of detecting the normal zone in two superconducting coils has been described.^(6,7) The method was put to practical use during the test of the two yin-yang MFTF coils.

The MFTF-B project contains a large number of inductively coupled coils. The normal zone detector must determine the location and measure the size of one or more normal zones in the set of coils. It must function when the currents in the coils are being adjusted from one level to another as well as when the currents are constant. When the currents are changing the mutual and self-induced voltages are very large compared with the normal zone voltages that must be detected.

The largest of the induced voltages, the self-induced voltages, can be cancelled out by means of bridges across the coils balanced with respect to the self-inductances. The mutual inductances, however, are not distributed in the same way as the self-inductances. Consequently, the mutually induced voltages are only partially eliminated by the bridges, leaving a component of the mutually induced voltage. Even this residual is large compared with the normal zone voltages.

A method of eliminating the mutually induced voltages that first comes to mind is to directly measure the rates of change of current and calculate the mutually induced voltages. The resolution required for this measurement is related to the rate of change of current caused by a normal zone. This change is minute and its measurement impractical using existing instrumentation.

The detectors described in this report use the outputs of balanced self-inductance bridges and a supplemental set of voltages to determine the size and location of the normal zone. The choice of the supplemental voltages and the formulation of the detector are described. The accuracy required and the effect of changes in the circuit parameters is discussed. Examples, based on realistic values are presented.

2. THE BRIDGE EQUATIONS

Figure 1 shows n coils each with a bridge connected across the entire coil. The bridges are balanced with respect to the self-inductances of each coil.

The coils are coupled with each other by mutual inductance. Figure 2 defines the nomenclature used for the mutual inductance between two arbitrarily chosen coils j and k . The mutual inductance between one entire coil and the other entire coil is M_{jk} . A property of coils is that

$$M_{jk} = M_{kj}.$$

The mutual inductance between part of one coil and whole of the other coil also enters into the bridge equations. The mutual inductance of the bottom part of coil j to the whole of coil k is called M_{bjk} . In general

$$M_{bjk} \neq M_{bkj}.$$

The voltage across the first coil; v_{c1} is

$$v_{c1} = L_1 \frac{di_1}{dt} + M_{12} \frac{di_2}{dt} + M_{13} \frac{di_3}{dt} + \dots + M_{1n} \frac{di_n}{dt} + r_1 i_1. \quad (1)$$

The last term $r_1 i_1$ is due to a normal zone which is assumed to be present in coil 1.

If the normal zone is in the lower part of the coil, the voltage across the bridge associated with the first coil is

$$v_1 = L_{b1} \frac{di_1}{dt} + M_{b12} \frac{di_2}{dt} + M_{b13} \frac{di_3}{dt} + \dots + M_{bin} \frac{di_n}{dt} + r_1 i_1 - \frac{R_{b1}}{R_{B1}} v_{c1}. \quad (2)$$

FIGURE 1.
The Coils and Their Bridges

NOTE:

$M_{jk} = M_{kj}$
 but $M_{bjk} \neq M_{bkj}$

FIGURE 2.
 Nomenclature for the Inductances

If the normal zone is in the upper part of the coil, the normal zone voltage $r_1 i_1$, the next to last term of Equation 2, is missing from the equation. However, since the normal zone voltage is a term in the expression for v_{c1} it remains represented in v_1 .

When v_{c1} is replaced by the right side of Equation 1, there is a term

$$\left(L_{b1} - \frac{R_{b1}}{R_{B1}} L_1 \right) \frac{di_1}{dt} .$$

This term is eliminated by balancing the bridge so that

$$\frac{L_{b1}}{L_1} = \frac{R_{b1}}{R_{B1}} = p_1 .$$

When this is done only the mutually induced and normal zone voltages remain.

For a normal zone in the lower part of the bridge the resultant equation, after collecting terms, is

$$v_1 = (M_{b1} - p_1 M_{12}) \frac{di_2}{dt} + (M_{b13} - p_1 M_{13}) \frac{di_3}{dt} + \dots + (M_{b1n} - p_1 M_{1n}) + (1 - p_1) r_1 i_1 .$$

For a normal zone in the upper part of the bridge the equation is the same except for the normal zone term which is $-p r_1 i_1$ instead of $(1-p) r_1 i_1$

$$v_1 = (M_{b1} - p_1 M_{12}) \frac{di_2}{dt} + (M_{b13} - p_1 M_{13}) \frac{di_3}{dt} + \dots + (M_{b1n} - p_1 M_{1n}) - p r_1 i_1 .$$

A similar equation can be written for each of the coils. Combining them results in the matrix equation, Equation 3.

EQUATION 3, THE BRIDGE MATRIX EQUATIONS

THE BRIDGES ARE BALANCED SO AS TO ELIMINATE THE SELF-INDUCED VOLTAGES

$$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} 0 & M_{b12} - p_1 M_{12} & \dots & M_{b1n} - p_1 M_{1n} \\ M_{b21} - p_2 M_{12} & 0 & \dots & M_{b2n} - p_2 M_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ M_{bn1} - p_n M_{1n} & \vdots & \dots & 0 \end{bmatrix} \begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \\ \vdots \\ \frac{di_n}{dt} \end{bmatrix} + \begin{bmatrix} b_{11} & 0 & \dots & 0 \\ 0 & b_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & b_{nn} \end{bmatrix} \begin{bmatrix} i_1 r_1 \\ i_2 r_2 \\ \vdots \\ i_n r_n \end{bmatrix}$$

THE MATRIX EQUATION: $V = A \frac{dI}{dt} + BV_n$, V_n = THE NORMAL ZONE VOLTAGES

V = THE BRIDGE VOLTAGES

WHERE V , V_n & I ARE COLUMN MATRICES

A IS A SQUARE MATRIX SUCH THAT

$$a_{ij} = \begin{cases} 0, & i = j \\ M_{bij} - p_i M_{ij}, & i \neq j \end{cases}$$

B IS A DIAGONAL MATRIX SUCH THAT

$$b_{ii} = \begin{cases} 0, & \text{IF THERE IS NO NORMAL ZONE} \\ 1-p_i, & \text{IF THE NORMAL ZONE IS IN THE BOTTOM OF THE COIL} \\ -p_i, & \text{IF THE NORMAL ZONE IS IN THE TOP OF THE COIL} \end{cases}$$

The A matrix, which is a square matrix, with a zero diagonal, contains the mutual inductances and the bridge ratios. The diagonal is zero, because the self-inductance terms have been cancelled out by balancing the bridges. In forming the matrix, advantage has been taken of the fact that $M_{jk} = M_{kj}$. However, as is pointed out earlier, the partial mutual inductances are all unique.

The B matrix is a diagonal matrix with diagonal elements that depend on the existence and location of a normal zone. If a coil has no normal zone the diagonal element is zero. If the normal zone is in the top portion of the coil the element is $-p$, if it is in the bottom the element is $1 - p$. Since p is a ratio less than one, $-p$ is negative and $1 - p$ is positive. Therefore, the location of the normal zone determines the sign of the element.

5. THE COMBINED EQUATIONS

The bridge equations alone are not sufficient to solve for the normal zone voltages. The rates of change of current are unknown, making more than n unknowns for the set of n equations.

Additional information in the form of a set of voltage measurements is needed. It is not immediately clear what voltages should be used, but there are several candidates: the voltages across the coils, part of the voltages across the coils, the output of a second bridge balanced with respect to mutual inductance, or possibly voltages between coils.

Whatever the choice, the set of voltages, which will designate the supplementary voltages, can be written as a matrix equation with one term due to the mutual and self-induced voltages and the other due to the normal zone voltages.

$$V_s = C \frac{dI}{dt} + D V_n \quad (4)$$

Where V_s = the supplementary voltages

V_s , I , and V_n are column matrices.

C is a square matrix with elements that depend on the bridge ratio and the mutual and self-inductances.

D is a square matrix.

The supplemental voltage equation and the bridge equation comprise a set of $2n$ equations. There are n unknown rates of change of current and anywhere from zero to n normal zones if there is only one normal zone per coil.

An expression for the normal zone voltage can be derived in two different ways. In the first method an expression for the derivatives of the current is obtained from the supplemental equations and this is substituted into the bridge equation.

From Equation 4,

$$\frac{dI}{dt} = C^{-1}V_s - C^{-1}D V_n .$$

Eliminating the derivatives from Equation 3 yields

$$V = A(C^{-1}V_s - C^{-1}D V_n) + B V_n .$$

Collecting the normal zone terms on the left side,

$$(B - AC^{-1}D) V_n = V - AC^{-1}V_s . \quad (5)$$

The alternative method is to use the bridge equation to eliminate the derivatives from the supplemental voltage equation. From Equation 3

$$\frac{dI}{dt} = A^{-1}V - A^{-1}B V_n .$$

Placing the normal zone terms on the left side

$$(D - CA^{-1}B) V_n = V_s - CA^{-1} V \quad (6)$$

The two matrix equations, Equations 5 and 6, which will be called the combined equations, are each sets of n equations. At the moment there is no way of telling whether or not either equation has an advantage over the other. In a later section it is shown that Equation 5 is to be preferred.

Solving for the normal zones is not a simple matter of multiplying both sides of the combined equations by the inverses of $B - AC^{-1}D$ or $D - CA^{-1}B$. The B matrix is a diagonal matrix with elements that depend not only on the existence of a normal zone in a coil, but also whether it is in the top or bottom. The exact form of the D matrix depends on the choice of supplementary voltages, but it also has elements that depend on the location of the normal zones. Therefore, instead of being a single set of n equations, the combined equations are actually a large number of sets of n equations, one set for each normal zone location or combination of normal zone locations. There are a large number of these sets, for example, if only two normal zones are allowed, the number of sets of n equations is

$$\sum_{j=0}^{n-1} n - j \quad .$$

The existence and location of the normal zones are not known at the onset of the measurement, but are unknowns to be determined. Therefore, determination of which set of equations is valid is part of the initial problem. In effect, the location and evaluation of the normal zones requires the solution of a problem with many degrees of freedom.

Fortunately, the values that occur in practical problems allow the problem to be solved in stages. It is possible by examining the measured bridge and

supplementary voltages to determine the location of the normal zones. Once this is done, the correct B and D matrices are chosen and the normal zones evaluated.

The measured voltages V and V_s appear on the right side of the combined equations. The two matrices on the right side, A and C, do not depend on the location of the normal zone, but only on the bridge ratios, and the mutual and self-inductances. Therefore, the right-hand side of the matrix equation can be instrumented in an unambiguous manner, to form the first stage of the detector. This stage is used to determine the location of the normal zone.

4. THE DETECTOR BASED ON THE VOLTAGES ACROSS THE COILS

As far as can be determined, there is no simple principle that governs the optimum choice of the supplementary voltage. It is necessary to proceed by trial, choosing a supplementary voltage and then examining the properties of the resulting detector.

The voltages across the coils are a prime choice because of their ready accessibility. It is shown in the following sections that they are as good a choice as any.

The voltage across coil 1 of Figure 1 is given by Equation 1. Each coil voltage can be expressed by a similar equation. Putting all of these equations together results in the matrix equation, Equation 7.

EQUATION 7, THE MATRIX EQUATION FOR THE VOLTAGES ACROSS THE COILS

$$\begin{bmatrix} v_{c1} \\ v_{c2} \\ v_{c3} \\ \vdots \\ v_{cn} \end{bmatrix} = \begin{bmatrix} L_1 & M_{12} & M_{13} & \dots & M_{1n} \\ M_{12} & L_2 & M_{23} & \dots & M_{2n} \\ M_{13} & M_{23} & L_3 & \dots & M_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ M_{1n} & M_{2n} & M_{3n} & \dots & L_n \end{bmatrix} \begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_1}{dt} \\ \frac{di_3}{dt} \\ \vdots \\ \frac{di_n}{dt} \end{bmatrix} + \begin{bmatrix} d_{11} & 0 & 0 & \dots & 0 \\ 0 & d_{22} & 0 & \dots & 0 \\ 0 & 0 & d_{33} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & d_{nn} \end{bmatrix} \begin{bmatrix} i_1 r_1 \\ i_2 r_2 \\ i_3 r_3 \\ \vdots \\ i_n r_n \end{bmatrix}$$

THE MATRIX EQUATION: $V_c = C \frac{dI}{dt} + D V_n$

WHERE V_c , V_n AND I ARE COLUMN MATRICES

C IS A SQUARE MATRIX SUCH THAT

$$c_{ij} = \begin{cases} L_i, & i = j \\ M_{ij}, & i \neq j \end{cases}$$

D IS A DIAGONAL MATRIX SUCH THAT

$$d_{ii} = \begin{cases} 1, & \text{IF THERE IS A NORMAL ZONE IN THE COIL.} \\ 0, & \text{IF THERE IS NO NORMAL ZONE IN THE COIL.} \end{cases}$$

The C matrix is a symmetric square matrix with the self-inductances as the diagonal elements and whole coil mutual inductances as the off-diagonal elements. The symmetry is due to the fact that $M_{jk} = M_{kj}$.

The D matrix is a diagonal matrix with diagonal elements that depend on the presence or absence of a normal zone. If a normal zone is present in a

coil, the diagonal element corresponding to the coil is a 1. If there is no normal zone the diagonal element is a 0.

The Combined Equations for a Normal Zone in the Top of Coil i

The bridge equations and the coil equations can be united to form a combined equation. These are many set of combined equations, one set for each normal zone and combination of normal zone locations. Something of the structure of these equations can be seen by writing the equation for a single normal zone, arbitrarily located in the top of the i^{th} coil.

Since only the left side of the combined equation depends on the normal zone location, only the left side is written in expanded form. The preferred combined equation, Equation 5, is used.

$$B - AC^{-1}D = \begin{bmatrix} 00..0...0 \\ 00..0...0 \\ \\ 00..-p_i...0 \\ \\ 00..0...0 \end{bmatrix} - \begin{bmatrix} 0 & a_{12}..a_{1i}..a_{1n} \\ a_{21}0 & ..a_{2i}..a_{2n} \\ . & . & . & . \\ a_{i1}a_{i2}..0 & ..a_{in} \\ . & . & . & . \\ a_{n1}a_{n2}..a_{ni}..0 \end{bmatrix} \begin{bmatrix} x_{11} & x_{12}..x_{1i}..x_{1n} \\ x_{21} & x_{22}..x_{2i}..x_{2n} \\ . & . & . & . \\ x_{i1} & x_{i2}..x_{ii}..x_{in} \\ . & . & . & . \\ x_{n1} & x_{n2}..x_{ni}..x_{nn} \end{bmatrix} \begin{bmatrix} 00..0..0 \\ 00..0..0 \\ \\ 00..1..0 \\ \\ 00..0..0 \end{bmatrix}$$

$$\text{where } C^{-1} = \begin{bmatrix} x_{11} & x_{12} & . & . & x_{1n} \\ x_{21} & x_{22} & . & . & x_{2n} \\ . & . & . & . & . \\ x_{n1} & x_{n2} & . & . & x_{nn} \end{bmatrix}$$

$$\text{and } \begin{aligned} a_{12} &= M_{b12} - p_1 M_{12} \\ a_{1i} &= M_{b1i} - p_1 M_{1i} \\ a_{21} &= M_{b21} - p_2 M_{12} \\ a_{2i} &= M_{b2i} - p_2 M_{2i}, \text{ etc.} \end{aligned}$$

Partitioning the A and C^{-1} matrices is useful.

$$\text{Let } AC^{-1} = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_i \\ \vdots \\ A_n \end{bmatrix} [X_1 \ X_2 \ \dots \ X_i \ \dots \ X_n] ,$$

where the A_j 's are row matrices consisting of the rows of A and the X_j 's are column matrices consisting of the columns of C^{-1} . The left side of the combined equation can now be written,

$$B - AC^{-1}D = \begin{bmatrix} 0 & 0 & \dots & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & -p_i & \dots & 0 \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 0 & \dots & 0 \end{bmatrix} - \begin{bmatrix} A_1 X_1 & A_1 X_2 & \dots & A_1 X_i & \dots & A_1 X_n \\ A_2 X_1 & A_2 X_2 & \dots & A_2 X_i & \dots & A_2 X_n \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ A_i X_1 & A_i X_2 & \dots & A_i X_i & \dots & A_i X_n \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ A_n X_1 & A_n X_2 & \dots & A_n X_i & \dots & A_n X_n \end{bmatrix} \begin{bmatrix} 0 & 0 & \dots & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 0 & \dots & 0 \end{bmatrix}$$

$$B - AC^{-1}D = \begin{bmatrix} 0 & 0 & \dots & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & -p_i & \dots & 0 \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 0 & \dots & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & \dots & A_1 X_i & \dots & 0 \\ 0 & 0 & \dots & A_2 X_i & \dots & 0 \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & A_i X_i & \dots & 0 \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & A_n X_i & \dots & 0 \end{bmatrix} .$$

The normal zone voltages are zero except for coil i.

$$(B - AC^{-1}D) v_n = \begin{bmatrix} 0 & 0 & \dots & -A_1 X_i & \dots & 0 \\ 0 & 0 & \dots & -A_2 X_i & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & -p_i - A_i X_i & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & -A_n X_i & \dots & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ v_{ni} \\ \vdots \\ 0 \end{bmatrix} .$$

And finally, writing the right was well as the left side,

$$(B - AC^{-1}D) v_n = \begin{bmatrix} -A_1 X_i \\ -A_2 X_i \\ \vdots \\ -p_i - A_i X_i \\ \vdots \\ -A_n X_i \end{bmatrix} \quad v_{ni} = V - AC^{-1} v_c . \quad (8)$$

If the normal zone is in the bottom of coil i, the appropriate equation is obtained by substituting $(p_i - 1)$ for p_i .

As Equation 8 shows, a normal zone in coil i affects all of the combined equations. If it is known that there is only one normal zone and it is in coil i, any one of the equations could be solved for the value of the normal zone voltage v_{ni} .

However, it is not known, a priori, how many normal zones there are and where they are located. The question then is: "Is there a pattern in the measured outputs, when combined in accordance with the right side of the equation $V - CA^{-1} v_c$, that shows the number and location of the normal zone?"

A key factor is the relative values of the term $-p_i - A_i X_i$ and the other terms. The size of the $A_k X_k$ terms is not at all apparent because X_k is a submatrix of the inverse of C. Therefore, this question is set aside until a later section.

5. THE COMBINED EQUATION FOR NORMAL ZONES IN TWO COILS

In this section it is shown that the effect of two or more normal zones in different coils is a linear combination of their individual effects. In other words, the effects are superimposed. The result is not related to the superposition principle that applies to linear circuits. The superposition principle relates to the superposition of source elements, the superposition in this case is of the effect of dissipative elements and is due to the simplicity of the detector equations.

Superposition of the effect of more than one normal zone greatly reduces the dimensionality of the problem that must be solved in order to locate the normal zones. Instead of each combination of normal zones having its own set of equations, unrelated to each other, the matrix equation for a combination of normal zones is related to the sets of equations for single normal zones.

The development of the matrix equation for two normal zones closely follows the development of the equation for one normal zone. The left side is first written in expanded form:

For normal zones in the tops of coils i and l

$$\begin{aligned}
 B - AC^{-1}D &= \begin{bmatrix} -p_1 & 0 & \dots & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & -p_i & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & \dots & 0 \end{bmatrix} - \begin{bmatrix} A_1X_1 & A_1X_2 & \dots & A_1X_i & \dots & A_1X_n \\ A_2X_1 & A_2X_2 & \dots & A_2X_i & \dots & A_2X_n \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ A_iX_1 & A_iX_2 & \dots & A_iX_i & \dots & A_iX_n \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ A_nX_1 & A_nX_2 & \dots & A_nX_i & \dots & A_nX_n \end{bmatrix} \begin{bmatrix} 1 & 0 & \dots & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & \dots & 0 \end{bmatrix} \\
 &= \begin{bmatrix} -p_1 & -A_1X_1 & 0 & \dots & \dots & -A_1X_i & \dots & \dots & 0 \\ & -A_2X_1 & 0 & \dots & \dots & -A_2X_i & \dots & \dots & 0 \\ & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ & -A_iX_1 & 0 & \dots & \dots & -p_i & -A_iX_i & \dots & 0 \\ & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ & -A_nX_1 & 0 & \dots & \dots & -A_nX_i & \dots & \dots & 0 \end{bmatrix}
 \end{aligned}$$

And finally, since the normal zone voltages are zero except for coils l and i,

$$(B - AC^{-1}D) v_n = \begin{bmatrix} -p_1 & -A_1X_1 \\ & -A_2X_1 \\ & \vdots \\ & -A_iX_1 \\ & -A_nX_1 \end{bmatrix} v_{nl} + \begin{bmatrix} -A_1X_i \\ -A_2X_i \\ \vdots \\ -p_i & -A_iX_i \\ \vdots \\ -A_nX_i \end{bmatrix} v_{ni} = v - AC^{-1} v_c$$

As this final equation shows, the output of a detector constructed by combining the measured quantities according to $v - CA^{-1} v_c$, is simply the sum of the outputs that are obtained when the normal zones exist alone. Although only two coils have been used, it's apparent that the principle holds for any number of normal zones.

6. A SYMMETRIC SET OF COILS

Some important insights are gained if the equations are simplified by making the coils similar and assuming certain symmetries among the mutual inductances.

First, all of the inductances are made equal:

$$L_1 = L_2 = L_3 = \dots = L_n = L ,$$

$$L_{b1} = L_{b2} = L_{b3} = \dots = L_{bn} = L_b .$$

Next, the mutual inductance between coils a similar distance apart are assumed to be the same. That is,

$$M_{12} = M_{23} = M_{34} \dots ,$$

$$M_{13} = M_{24} = M_{35} \dots ,$$

$$M_{14} = M_{25} = M_{36} \dots ,$$

and so on.

The C matrix can now be written in simplified form by defining the ratios,

$$k_{ij} = \frac{M_{ij}}{\sqrt{L_i L_j}} = \frac{M_{ij}}{L} = \text{the coupling coefficient.}$$

$$C = L \begin{bmatrix} 1 & k_{12} & k_{13} & \dots & k_{1n} \\ k_{12} & 1 & k_{12} & \dots & k_{1(n-1)} \\ k_{13} & k_{12} & 1 & \dots & k_{1(n-2)} \\ \cdot & \cdot & \cdot & \dots & \cdot \\ k_{1n} & k_{1(n-1)} & k_{1(n-2)} & \dots & 1 \end{bmatrix} \quad (9)$$

$$C = L C_s .$$

Since the mutual inductance between coils close to each other is greater than that of coils distant from each other

$$k_{12} > k_{13} > k_{14} > \dots > k_{in} .$$

The A matrix can be written in a similar form if a few additional simplifications are made. A typical term of the A matrix is

$$a_{ij} = M_{bij} - p_i M_{ij} = M_{ij} \left(\frac{M_{bij}}{M_{ij}} - p_i \right) .$$

For a similar set of coils and bridges,

$$p_1 = p_2 = p_3 = \dots = p_n = \frac{L_b}{L} = p .$$

A set of similar coils has the simplified properties assumed thus far. A further simplification is now made, based more on the desire to simplify the A matrix than on the properties of a similar set of coils. The assumption is that the ratio of the partial mutual inductance to the full mutual inductances is the same throughout the set of coils. Designating this ratio as q,

$$\frac{M_{b12}}{M_{12}} = \frac{M_{b13}}{M_{13}} = \dots = \frac{M_{bin}}{M_{in}} = q .$$

Also, it is assumed that $M_{bij} = M_{bji}$. These two assumptions are only approximately true, even for a set of similar coils. An unusual geometry, which may not even exist, is required to make the equations exact.

A typical term of the A matrix can now be written as,

$$a_{ij} = M_{ij} (q - p) .$$

But since,

$$k_{ij} = \frac{M_{ij}}{L} ,$$

$$a_{ij} = Lk_{ij} (q - p) .$$

The A matrix can now be written

$$A = L (q - p) \begin{bmatrix} 0 & k_{12} & k_{13} & \dots & k_{1n} \\ k_{12} & 0 & k_{12} & \dots & k_{1(n-1)} \\ k_{13} & k_{12} & 0 & \dots & k_{1(n-2)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ k_{1n} & k_{1(n-1)} & k_{1(n-2)} & \dots & 0 \end{bmatrix}$$

$$= L (q - p) A_s .$$

The matrix part of A is similar to the expression for C except that the diagonal consists of zeros instead of ones as it does in the C matrix. Therefore,

$$A = L (q - p)(C_s - U) . \quad (10)$$

where U is the identity matrix.

Comparison of the Two Combined Equations

There are two methods of formulating the combined equations. If the equations are solved in one way, the matrix equation, Equation 5 is obtained. For convenience it is written below:

$$(B - AC^{-1}D) V_n = V - AC^{-1} V_s .$$

Solving the equations an alternate way yields Equation 6:

$$(D - CA^{-1}B) V_n = V_s - CA^{-1}V .$$

The properties of these two alternative combined equations can be investigated using the simplified set of coils.

If $p = q$, then $A = 0$,

and the first equation degenerates into

$$B V_n = V .$$

For a normal zone in the top of coil i

$$\begin{bmatrix} 0 & 0 & . & . & 0 & . & . & 0 \\ 0 & 0 & . & . & 0 & . & . & 0 \\ . & . & . & . & . & . & . & . \\ 0 & 0 & . & . & -p & . & . & 0 \\ . & . & . & . & . & . & . & . \\ 0 & 0 & . & . & 0 & . & . & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ . \\ v_{ni} \\ . \\ 0 \end{bmatrix} = V ,$$

which reduces to

$$\begin{bmatrix} 0 \\ 0 \\ . \\ -p \\ . \\ 0 \end{bmatrix} v_{ni} = V .$$

This equation shows that, if the ratio of the self-inductances p is equal to the ratio of the mutual inductances q , a normal zone in coil i cause all but the i^{th} equation to be zero. Therefore, it is possible to immediately locate the coil containing the normal zone.

If the normal zone is in the bottom of coil i, the equations reduce to

$$\begin{bmatrix} 0 \\ 0 \\ \cdot \\ 1 - p \\ \cdot \\ 0 \end{bmatrix} v_{ni} = V .$$

Since p is smaller than one, $1 - p$ is positive, and $-p$ is negative. Therefore, if the normal zone is in the top of the coil, a negative output is obtained from the detector; if it is in the bottom, a positive output is obtained. Not only is the coil located, but so is its position in the coil.

The consequences of setting p and q equal has a simple physical explanation. Setting them equal implies that the ratio of the partial to total mutual inductance, $\frac{M_{bij}}{M_{ij}}$, is the same as the ratio of the partial to total self-inductance, $\frac{L_{bi}}{L_i}$. When this condition is true, the bridge balances out the mutually-induced as well as the self-induced voltages. This causes the A matrix to be zero. It is no longer necessary to measure a supplemental set of voltages; the bridge voltages themselves are sufficient to locate and measure the normal zone.

Now, consider the effect of letting p equal q on the other set of equation,

$$(D - CA^{-1}B) V_n = V_s - CA^{-1} V .$$

Since

$$A^{-1} = \frac{1}{L(q - p)} A_s^{-1} ,$$

each element of A^{-1} is infinite when $p = q$. this leads to an indeterminate set of equations for a normal zone in the i^{th} coil,

$$\begin{bmatrix} \infty \\ \infty \\ \cdot \\ \infty \\ \cdot \\ \infty \end{bmatrix} v_{ni} = V_s - AC^{-1} V .$$

If a detector is formed by combining the measured quantities according to the matrix expression $V_s - CA^{-1} V$, each output of the detector is infinitely large when p and q are equal. It is impossible to determine which coil contains the normal zone.

When p and q are not exactly equal, a detector based on $V_s - CA^{-1} V$ can be built. Numerical experimentation shows that the output associated with the normal zone is the smallest rather than the largest of the outputs. Nonetheless, the normal zone can be located and the sign of the output used to determine whether the normal zone is in the top or bottom of the coil. As p approaches q and each output becomes larger, it is more difficult to distinguish the smallest value. This is because the ratio of the output associated with the normal zone to the neighboring outputs approaches unity.

On the basis of these observations, it is concluded that the formulation of the combined equations

$$(B - AC^{-1}D) V_n = V - AC^{-1} V_s$$

is preferable. Although a practical set of coils will not have the conditions of perfect symmetry described in this section, many practical configurations tend to symmetry. This formulation ensures that there is the best possible discrimination between a coil with a normal zone and one without a normal zone.

7. THE DISCRIMINATION RATIO OF THE COMBINED EQUATIONS

A criteria for evaluating the effectiveness of the detector is the ease with which the coil containing the normal zone can be located. In order to facilitate this evaluation, a ratio called the discrimination ratio, is defined.

$$\text{discrimination} = d_{ij} \\ \text{ratio}$$

$$= \frac{\text{the left (or right) side of combined equation } i, \text{ when coil } i \text{ contains a normal zone}}{\text{the left (or right) side of combined equation } j, \text{ when coil } i \text{ contains a normal zone}}$$

In order to clarify this definition it's useful to review and reformulate the combined equations. The combined matrix equation is

$$(B-AC^{-1}D) V_n = V-AC^{-1}V_s .$$

The right side of the equation does not depend on the location of the normal zone. For convenience let

$$AC^{-1} = \begin{bmatrix} y_{11} & y_{12} & \cdot & \cdot & y_{1i} & \cdot & \cdot & y_{1n} \\ y_{21} & y_{22} & \cdot & \cdot & y_{2i} & \cdot & \cdot & y_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ y_{i1} & y_{i2} & \cdot & \cdot & y_{ii} & \cdot & \cdot & y_{in} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ y_{n1} & y_{n2} & \cdot & \cdot & y_{ni} & \cdot & \cdot & y_{nn} \end{bmatrix} .$$

The left side of the equation does depend on the location of the normal zone. The subscript ti is used to denote a normal zone in the top of coil i and the subscript bi to denote a normal zone in the bottom of coil i.

The D matrix is the same for a normal zone in either top or bottom

$$D_{ti} = D_{bi} = \begin{bmatrix} 0 & 0 & . & . & 0 & . & . & 0 \\ 0 & 0 & . & . & 0 & . & . & 0 \\ . & . & . & . & . & . & . & . \\ 0 & 0 & . & . & 1 & . & . & 0 \\ . & . & . & . & . & . & . & . \\ 0 & 0 & . & . & 0 & . & . & 0 \end{bmatrix} .$$

For a normal zone in the top of coil i

$$B_{ti} = \begin{bmatrix} 0 & 0 & . & . & 0 & . & . & 0 \\ 0 & 0 & . & . & 0 & . & . & 0 \\ . & . & . & . & . & . & . & . \\ 0 & 0 & . & . & -p_i & . & . & 0 \\ . & . & . & . & . & . & . & . \\ 0 & 0 & . & . & 0 & . & . & 0 \end{bmatrix} .$$

For a normal zone in the bottom of coil i

$$B_{bi} = \begin{bmatrix} 0 & 0 & . & . & 0 & . & . & 0 \\ 0 & 0 & . & . & 0 & . & . & 0 \\ . & . & . & . & . & . & . & . \\ 0 & 0 & . & . & 1-p_i & . & . & 0 \\ . & . & . & . & . & . & . & . \\ 0 & 0 & . & . & . & . & . & 0 \end{bmatrix} .$$

Using the suffix TI to denote a normal zone in the top of coil i and BI to denote a normal zone in the bottom, the left side can be abbreviated as follows:

$$B_{ti} - AC^{-1}D_i = LTI$$

$$B_{bi} - AC^{-1}D_i = LBI$$

If only coil i has a normal zone the left side of the detector equation, $(S - AC^{-1}D) V_n$, reduces to a column matrix. For a normal zone in the top of coil i the combined equations are

$$\begin{bmatrix} LTI_1 \\ LTI_2 \\ \vdots \\ LTI_i \\ \vdots \\ LTI_n \end{bmatrix} v_{ni} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_i \\ \vdots \\ v_n \end{bmatrix} + \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1i} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2i} & \cdots & y_{2n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ y_{i1} & y_{i2} & \cdots & y_{ii} & \cdots & y_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ y_{n1} & y_{n2} & \cdots & y_{ni} & \cdots & y_{nn} \end{bmatrix} \begin{bmatrix} v_{s1} \\ v_{s2} \\ \vdots \\ v_{si} \\ \vdots \\ v_{sn} \end{bmatrix}.$$

A similar equation, but with a different left side, results for a normal zone in the bottom of coil i .

The discrimination ratio can now be written

$$d_{ij} = \frac{LTI_i}{LTI_j} \frac{v_{ni}}{v_{nj}} = \frac{LTI_i}{LTI_j}$$

Paraphrasing the results of the previous section in terms of the discrimination ratio gives the ratio further meaning. For a detector based on combining the measured voltages in accordance with $V - AC^{-1}V_c$ the discrimination ratio is infinite when $p = q$. For the detector based on $V_c - CA^{-1}V$ the ratio is indeterminate. When $p \neq q$ a detector using $V_c - CA^{-1}V$ is feasible, but the discrimination ratio is smaller than when $V - AC^{-1}V_c$ is used. Therefore, it is preferable to base the detector on $V - CA^{-1}V_c$.

The Discrimination Ratio for Symmetric Coils

In the previous sections it was found that the discrimination ratio of the preferred combined equation is infinite when q and p are equal. That is, conditions are ideal for locating the normal zone. In this section the condition $p = q$ is removed and the ease with which a normal zone can be located is examined.

A general solution is not feasible. Therefore, only a demonstration is given using two cases: two symmetric coils and four symmetric coils.

Two Symmetric Coils

The combined equations for two coils with a normal zone in the top of coil 1 is,

$$\begin{bmatrix} L T l_1 \\ L T l_2 \end{bmatrix} v_{nlt} = V - A C^{-1} v_c, \quad (11)$$

where v_{nlt} = the voltage across the normal zone in the top of coil 1,

$$L T l_1 = -p + \frac{(p-q)k_{12}^2}{k_{12}^2 - 1},$$

$$L T l_2 = \frac{-(p-q)k_{12}}{k_{12}^2 - 1}.$$

The discrimination ratio is

$$d_{12} = \frac{L_{t1}(1)}{L_{t1}(2)} = \frac{+p(k_{12}^2 - 1)}{(p-q)k_{12}} - k_{12}.$$

$$\text{Since } k_{12} < 1, k_{12}^2 - 1 \doteq -1$$

$$d_{12} = \frac{p}{(p-q)k_{12}} - k_{12} \doteq \frac{p}{(p-q)k_{12}}$$

When $p = q$, the discrimination ratio is infinite, as was found previously. Since p and q are both less than 1, the difference $p - q$, is small even for practical coils. The coupling coefficient between two coils is also a small number. Therefore, under usual circumstances the discrimination ratio is large and the normal zone is easily located.

Four Symmetric Coils

The combined equations for four symmetric coils, with a normal zone in the top of coil 1, is

$$\begin{bmatrix} LTI_1 \\ LTI_2 \\ LTI_3 \\ LTI_4 \end{bmatrix} v_{1t} \approx V - AC^{-1} v_c .$$

All of the left side coefficients have a common denominator, D.

$$D = 1 - k_{14}^2 - 2 k_{13}^2 - 3 k_{12}^2 + 4 k_{12} k_{13} k_{14} + 4 k_{12}^2 k_{13} - 2 k_{12}^2 k_{14}$$

$$- 2 k_{12} k_{13}^2 k_{14} + k_{12}^2 k_{14}^2 - 2 k_{12}^2 k_{13}^2 + k_{13}^4 + k_{12}^4 .$$

$$D \times LTI_1 = p (-1 + 2 k_{12}^2 + k_{13}^2 - k_{12}^2 k_{13})$$

$$+ q (k_{12}^2 + k_{13}^2 + k_{14}^2 - 4 k_{12} k_{13} k_{14} - 2 k_{12}^2 k_{13} - k_{12}^4$$

$$- k_{13}^4 - k_{12}^2 k_{14}^2 + 2 k_{12}^2 k_{13}^2 + 2 k_{12}^3 k_{14} + 2 k_{12}^2 k_{13} k_{14}) .$$

$$D \times LTI_2 = (p - q) (k_{12} - k_{12} k_{13} - k_{13} k_{14} - k_{12}^3 + k_{12}^2 k_{14} + k_{12} k_{13}^2) .$$

$$D \times LTI_3 = (p - q) (k_{13} - k_{12} k_{14} - k_{12}^2 + k_{12} k_{13} k_{14} - k_{12}^2 k_{13} - k_{13}^3) .$$

$$D \times LTI_4 = (p - q) (k_{14} - 2 k_{12} k_{13} + k_{12}^2 - k_{12}^2 k_{14} + k_{12} k_{13}^2) .$$

The coefficient of coupling between two adjacent coils k_{12} is typically about .1, between coils separated by a single coil, k_{13} , it is half this. Therefore, the products of coupling coefficients in the above expressions are very small. Ignoring all products yields the simplified expressions

$$LTI_1 \doteq -p$$

$$LTI_2 \doteq (p - q) k_{12}$$

$$LTI_3 \doteq (p - q) k_{13}$$

$$LTI_4 \doteq (p - q) k_{14} .$$

The discrimination ratios are

$$d_{12} \doteq \frac{-p}{(p - q) k_{12}}$$

$$d_{13} \doteq \frac{-p}{(p - q) k_{13}}$$

$$d_{14} \doteq \frac{-p}{(p - q) k_{14}} .$$

This result shows that the discrimination ratios are, under normal circumstances, large. The difference, $p - q$ is small and the coupling coefficient are also small numbers. Since $k_{12} > k_{13} > k_{14}$, the discrimination ratio increases as the distance from the coil with the normal zone increases.

8. THE DETECTOR

An important property of the combined equations, which are essential to the design of a detector, is established in the preceding sections. Although a normal zone in one coil affects all of the combined equations, the equation for the coil containing the normal zone is affected more than the others. This enables the coil containing the normal zone to be identified. The sign of the combined equation indicates whether the normal zone is in the top or bottom of the coil. Using this information the correct choice of D and B matrices can be made. The D matrix depends on which coil contains the normal zone and the B matrix depends on the coil and the location of the normal zone within the coil.

If the effect of a normal zone is largely confined to the equation associated with it, the structure of the detector can be simplified. The detector can consist of a number of independent detectors, one for each coil; each based on the equation associated with the coil. It is not necessary for the detector to encompass the equations as an interconnected set. Whether this simplified structure is possible or not depends on the system parameters. Evidence has already been presented that for practical cases the simplified structure is sufficient. The question is further explored in the examples.

If a simplified structure is feasible, the independence of the detectors holds even if there is more than one coil with a normal zone. As is demonstrated in Section 5, the effect of more than one normal zone is simple superposition of the effects of each individual normal zone. Therefore, the location of each normal zone can be identified and the value of the normal zone determined independently by means of the appropriate equations.

The detector equation for an arbitrary coil, coil i , can be isolated from the set of combined equations as follows. The set of combined equations is

$$(B - AC^{-1}D) v_n = v - AC^{-1} v_s .$$

Following the notation of the preceding section, LTI is defined as the matrix $B-AC^{-1}D$ when the normal zone is in the top of coil i and LBI when it is in the bottom of coil i. For a normal zone in the top of coil i the combined equations are

$$\begin{bmatrix} LTI_1 \\ LTI_2 \\ \vdots \\ LTI_i \\ \vdots \\ LTI_n \end{bmatrix} v_{ni} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_i \\ \vdots \\ v_n \end{bmatrix} + \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1i} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2i} & \cdots & y_{2n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ y_{i1} & y_{i2} & \cdots & y_{ii} & \cdots & y_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ y_{n1} & y_{n2} & \cdots & y_{ni} & \cdots & y_{nn} \end{bmatrix} \begin{bmatrix} v_{s1} \\ v_{s2} \\ \vdots \\ v_{si} \\ \vdots \\ v_{sn} \end{bmatrix} .$$

A similar equation but with a different left side, results from a normal zone in the bottom of coil i.

When the normal zone is in coil i, LTI_i or LBI_i are larger than the remaining terms on the left side of the equation. Therefore, the detector for coil i can be based on the i^{th} equation, and the other equations ignored.

For a normal zone in the top of coil i, the equation used in the detector is

$$LTI_i v_{ni} = v_i + (y_{i1} v_{s1} + y_{i2} v_{s2} + \cdots + y_{ii} v_{si} + \cdots + y_{in} v_{sn}) .$$

For a normal zone in the bottom of coil i, the equation used in the detector is

$$LBI_i v_{ni} = v_i + (y_{i1} v_{s1} + y_{i2} v_{s2} + \cdots + y_{ii} v_{si} + \cdots + y_{in} v_{sn}) .$$

A set of n coils requires n similar pairs of equations. Several of these equations, which will be called the detector equations are listed below for clarity

Coil 1:

$$\begin{aligned} LTI_1 v_{n1} &= \\ LBI_1 v_{n1} &= \end{aligned} \left\{ \begin{aligned} v_1 + (y_{11}v_{s1} + y_{12}v_{s2} + \dots + y_{1n}v_{sn}) \\ v_1 + (y_{11}v_{s1} + y_{12}v_{s2} + \dots + y_{1n}v_{sn}) \end{aligned} \right.$$

Coil 2:

$$\begin{aligned} LTI_2 v_{n2} &= \\ LBI_2 v_{n2} &= \end{aligned} \left\{ \begin{aligned} v_2 + (y_{21}v_{s1} + y_{22}v_{s2} + \dots + y_{2n}v_{sn}) \\ v_2 + (y_{21}v_{s1} + y_{22}v_{s2} + \dots + y_{2n}v_{sn}) \end{aligned} \right.$$

Coil i :

$$\begin{aligned} LTI_i v_{ni} &= \\ LBI_i v_{ni} &= \end{aligned} \left\{ \begin{aligned} v_i + (y_{i1}v_{s1} + y_{i2}v_{s2} + \dots + y_{in}v_{sn}) \\ v_i + (y_{i1}v_{s1} + y_{i2}v_{s2} + \dots + y_{in}v_{sn}) \end{aligned} \right.$$

Coil n :

$$\begin{aligned} LTI_n v_{nn} &= \\ LBI_n v_{nn} &= \end{aligned} \left\{ \begin{aligned} v_n + (y_{n1}v_{s1} + y_{n2}v_{s2} + \dots + y_{nn}v_{sn}) \\ v_n + (y_{n1}v_{s1} + y_{n2}v_{s2} + \dots + y_{nn}v_{sn}) \end{aligned} \right.$$

It is important to note that the set of detector equations are derived from many sets of combined equations. In fact, for the existence of only one normal zone, there are $2n$ sets of combined equations and all of these sets are used in forming the detector equations. One equation, the one associated with the normal zone locations, is drawn from each set of combined equations to create the set of detector equations.

Implementing the equations in a physical detector requires a combination of arithmetic and decision making. Figure 3 shows a diagram of the detector for a single, coil i . The detector for a set of n coils consists of n similar detectors.

As the diagram shows, for each detector a sum is formed of the output of a single bridge, the bridge associated with the detector, and a weighted sum of the supplementary voltages. This sum is tested to determine whether it is positive or negative. If it is negative, the normal zone is in the top, and

FIGURE 3.

A Combination Block Diagram End Flow Chart for the
Detector Associated With a Single Coil, Coil i

the right side sum is divided by the left side coefficient associated with the top, LTI_i . If it is positive the sum is divided by the left side coefficient associated with the bottom, LBI_i . The result in either case is the voltage across a normal zone, if one exists. The calculated normal zone voltage is tested against a threshold. If the value is less than the threshold the decision is made that no normal zone exists in the coil. If it is more than the threshold the normal zone voltage is divided by the current through the coil to yield the normal zone resistance. The value of the normal zone resistance forms the raw material for the remainder of the detector in which a decision is made as to what corrective action should be taken.

The Discrimination Ratio of the Detector

Although the detector associated with the coil containing the normal zone will register the largest normal zone voltage, the normal zone voltages indicated by the other detectors are not zero. The sensitivity of a detector to a normal zone in its coil as compared with its sensitivity to a normal zone in other coils is important in determining the threshold and the smallest normal zone that can be detected. An appropriate measurement of this sensitivity, which will be called the detector discrimination ratio is:

$$\begin{array}{l} \text{detector} \\ \text{discrimination} \\ \text{ratio} \end{array} = D_{ij}$$

$$= \frac{\text{the normal zone voltage produced by detector } i, \text{ when} \\ \text{coil } i \text{ contains a normal zone}}{\text{the normal zone voltage produced by detector } j, \text{ when} \\ \text{coil } i \text{ contains a normal zone.}}$$

Since the detector equations depend on whether the normal zone is in the top or bottom of the coil the location of the normal zone must also be specified when citing the detector discrimination ratio.

The detector discrimination ratio is related to the discrimination ratio used earlier, but the two ratios are not exactly the same. The discrimination

ratio of the combined equations is the ratio of one side of two equations drawn from the same set of combined equations. For a normal zone in the top of coil i:

$$d_{ij} = \frac{LTI_i}{LTI_j}$$

The detector discrimination ratio is the ratio of one side of two equations from the detector set, each of which comes from a different set of combined equations. For a normal zone in the top of coil i:

$$D_{ij} = \frac{[v_i + (y_{i1}v_{s1} + y_{i2}v_{s2} + \dots + y_{in}v_{sn})] / LTI_i}{[v_j + (y_{j1}v_{s1} + y_{j2}v_{s2} + \dots + y_{jn}v_{sn})] / LTJ_j}$$

The voltage sums, which form the right sides of both the combined and detector equations, can be related to the left sides of the combined equations as follows

$$\begin{aligned} D_{ij} &= \frac{LTI_i v_{ni} / LTI_i}{LTI_j v_{ni} / LTJ_j} \\ &= \frac{LTJ_j}{LTI_j} \end{aligned}$$

Therefore,

$$\frac{D_{ij}}{d_{ij}} = \frac{LTJ_j}{LTI_j}$$

As this shows, the ratios D_{ij} and d_{ij} differ to the extent that the left side coefficient of equation j when a normal zone is in coil j is different from the left side coefficient of equation i when a normal zone is in coil i.

If coils i and j are identical coils and occupy similar positions in an array of coils the two coefficients, and the two discrimination ratios, are the same. In a chain of similar coils, the coefficients can be expected to change slightly from one coil to the next. Specific cases must be calculated.

The threshold level, the detector discrimination ratio, and the smallest normal zone that can be detected are all related. Since a normal zone in one coil affects all of the detector outputs, the threshold must be set to preclude false alarms. However, increasing the threshold decreases the sensitivity of the detector to small normal zones. For the yin-yang $M^{+}F$ coils, the discrimination ratio is about 100. This means that if the threshold is set at 1 mV, a normal zone of 100 mV would have to occur in the adjacent coil before a false indication of a normal zone is generated in the adjacent detector. False indications could be completely eliminated by increasing the complexity of the detector beyond that described here so as to calculate and compensate for the effect of a normal zone in an adjacent coil. However, as a practical matter, a range of 100 to 1 probably encompasses the range between earliest measurement of a normal zone and a fast dump. Therefore, it's likely that a fast dump would occur before the threshold of the unaffected coil is exceeded.

Similar comments can be made about the effect of more than one normal zone. The output of each detector is affected by the normal zone in nearby coils. However, the effect is small and in any case can be calculated in advance and taken into account in setting the critical values that call for a slow or fast dump.

9. NORMAL ZONES IN THE TOP AND BOTTOM OF THE SAME COIL

Normal zone voltages in the top and bottom of the same coil tend to be balanced by the bridge. In the extreme, when the normal zones have the same ratio as the self-inductances, the effect of the normal zones is completely cancelled and the normal zones are undetected by the bridge.

In order to examine this property it is best to return to the bridge equations. If r_{lb} and r_{lt} are normal zones in the bottom and top of coil 1 the equation for the coil voltages, Equation 1, is

$$v_{c1} = L_1 \frac{di_1}{dt} + M_{12} \frac{di_2}{dt} + \dots + M_{1n} \frac{di_n}{dt} + r_{lb} i_1 + r_{lt} i_1.$$

The bridge equation is

$$v_1 = L_{b1} \frac{di_1}{dt} + M_{b12} \frac{di_2}{dt} + \dots + M_{b1n} \frac{di_n}{dt} + r_{lb} i_1 - p_1 v_{c1}.$$

Isolating the normal zone terms in the bridge equation yields the normal zone terms of

$$\begin{aligned} v_1 &= r_{lb} i_1 - p_1 (r_{lb} i_1 + r_{lt} i_1) \\ &= i_1 [r_{lb} - p_1 (r_{lb} + r_{lt})]. \end{aligned}$$

As this result shows, the normal zone portion of the bridge voltage is reduced by the presence of a normal zone in both the top and bottom of the coil. If

$$\frac{r_{lb}}{r_{lb} + r_{lt}} = p_1,$$

the normal zone terms are zero, and $b_{11} = 0$. Partial cancellation results in a b_{11} somewhere between 0 and the larger of p_1 or $1 - p_1$.

Division of the normal zone between the top and bottom of the coil has no effect on the normal zone term in the voltage across the coil. This voltage continues to register the total normal zone voltage. Therefore, the detector outputs are not zero.

The usefulness of the detector under these circumstances can be assessed by examining the effect of normal zones that cancel on the two coil example in Section 7. In the combined equation, Equation 11, the term due to the B matrix is $-p$ in LTI_1 . If $b_{11} = 0$, this term becomes zero. Therefore, for completely cancelling normal zones the combined equation is

$$\begin{bmatrix} LTI_1 \\ LTI_2 \end{bmatrix} v_{nlt} = V - AC^{-1} V_s$$

where

$$LTI_1 = \frac{(p - q)k_{12}^2}{k_{12}^2 - 1}$$

$$LTI_2 = \frac{-(p - q)k_{12}}{k_{12}^2 - 1}$$

Both of these outputs are small. In fact, since $k_{12}^2 < k_{12}$, the output associated with the normal zone is smaller than the other output.

Since the outputs are below the threshold set for the detection of a normal zone, the detector fails to detect the normal zone.

It is unlikely that two normal zones would balance out exactly; partial balance and the consequent decrease in the detector output is more probable. A method of overcoming this deficiency of the detector is to use more than one bridge for each coil. A following section discusses detectors based on this concept.

10. DETECTORS USING ALTERNATIVE SUPPLEMENTARY VOLTAGES

Using a balanced self-inductance bridge as the principle building block of the detector has its roots in logic and experience. The bridge balances out the largest of the induced voltages. In addition, a bridge by itself is an adequate detector if there is only one coil and is even sufficient, as Section 6 shows, if a set of coils have perfect symmetry.

No simple principle has been found for the choice of the supplementary voltage. All that can be done is to investigate the practical alternatives. The voltages across the coil have already been shown to comprise a satisfactory set of supplementary voltages. In this section three alternative sources of the supplementary voltage are investigated: the voltage across part of the coil, the voltage across the whole coil when bridges across part of the coil are used, and the voltages from a second bridge.

Detectors Using Part of the Coil Voltage

A detector can be based on measuring part instead of all of the coil voltage. There is even a potential advantage to using only part of the voltage. In this section it is shown that the discrimination ratio of the combined equations is infinite if the normal zone occurs outside of the portion encompassed by the measurement. If the normal zone occurs within the portion the discrimination ratio is not as good as when the full coil voltage is measured.

Since the A and B matrices are related to the bridge measurements, these two matrices are unaffected by the change in the supplementary voltage.

The C and D matrices are similar to, but not exactly the same as, those obtained when the supplementary voltage is measured across the coil. If the

normal zone is outside the portion of the coil spanned by the measurement, the voltage equations do not contain a normal voltage term. Therefore, for a normal zone outside the voltage span

$$D = [0]$$

Placing this in

$$(B - AC^{-1}D) V_n = V - AC^{-1} V_s$$

yields

$$B V_n = V - AC^{-1} V_s .$$

If the normal zone is in the top of the i^{th} coil as well as outside the span of the supplementary voltage measurement the detector equations are

$$\begin{bmatrix} 0 \\ 0 \\ \vdots \\ -P_i \\ \vdots \\ 0 \end{bmatrix} V_{in} = V - AC^{-1} V_s .$$

The discrimination ratio of the combined equations is infinite, the best attainable.

The improvement in the discrimination ratio when the normal zone is outside the span of the supplementary voltage is paid for by a decrease in the ratio when the normal zone is inside the span. Demonstrating this property requires some additional work. The analysis is simplified if it is assumed that reducing the voltage measurement from the full coil to part of the coil decreases all of the elements of the C matrix by the same amount, f , where f is smaller than 1. Then,

$$C_p = fC,$$

where

C = the C matrix when the voltage is measured across the whole coil,
 C_p = the C matrix when the voltage is measured across part of the coil.

When the normal zone is within the span of the supplementary voltage the D matrix is the same for both the whole coil and partial coil measurements. Therefore, the combined equation is

$$(B - AC_p^{-1}D) V_n = V - AC_p^{-1} V_p$$

where V_p = the voltages across part of the coil

$$C_p^{-1} = (f C)^{-1} = \frac{C^{-1}}{f}$$

Rewriting the combined equation,

$$(B - \frac{AC^{-1}D}{f}) V_n = V - AC_p^{-1} V_p$$

For a normal zone in the top of the i^{th} coil, the left side of the combined equations when the voltages are measured across the entire coil can be written as,

$$(B - AC^{-1}D) V_n = \begin{bmatrix} R_{i1} \\ R_{i2} \\ \vdots \\ -p_i + R_{ii} \\ \vdots \\ R_{in} \end{bmatrix}$$

The R or residual terms are due entirely to $\frac{AC^{-1}D}{f}$. Therefore, when the supplementary voltage is part of the coil voltage, the left side of the combined equation is,

$$\left(B - \frac{AC^{-1}D}{f} \right) V_n = \begin{bmatrix} \frac{R_{i1}}{f} \\ \cdot \\ \frac{R_{i2}}{f} \\ \cdot \\ -p_i + \frac{R_{ii}}{f} \\ \cdot \\ \frac{R_{in}}{f} \end{bmatrix} V_n$$

The discrimination ratio for the full voltage measurement is

$$d_1 \approx - \frac{p_i}{R_{i(i-1)}}$$

For the partial voltage measurement it is

$$d_{p11} \approx - \frac{p_i f}{R_{i(i-1)}}$$

Since f is smaller than one, the discrimination ratio for the partial voltage detector is less than the ratio for the full voltage detector.

It has been shown that using a fraction of the voltage across the coil as the supplementary voltage improves the discrimination ratio when the normal zone is outside the portion of the coil spanned by the measurement and degrades it if the normal zone is within the span. Therefore, it appears as if there is no particular advantage to using part of the coil voltage rather than the entire coil voltage.

However, if the additional investment is justifiable, advantage can be taken of these properties by measuring two partial voltages and building two detectors for each coil. The appropriate choice of voltages is the two voltages from the bridge midpoints to the ends. Each of the combined equations have an infinite discrimination when the normal zone is outside the span of its voltage. The output of the detector with the voltage within its span can be ignored. This is easily done because the undesired output can be identified by its sign.

More Than One Bridge for Each Coil

In Section 9 it is shown that a detector fails to detect a normal zone if the normal zones in the upper and lower parts of the coil have a ratio similar to the ratio of inductances. The probability of missing a normal zone is reduced if more than one bridge and detector is used for each coil. Figure 4 shows a circuit in which two additional bridges are used. It's unlikely that a normal zone that is symmetrical to the full bridge would also be symmetrical to the two partial bridges. Therefore, the probability of a normal zone being undetected is greatly reduced.

Although three bridges are used, it does not follow that it is necessary to measure three supplementary voltages. In this section the feasibility of using the voltage across the coil for all detectors is investigated.

Although a solution for the general case is not possible, considerable insight is obtained if an analysis is made using the set of symmetrical coils described in Section 6. For the symmetrical set there are only four parameters: k_{ij} the coupling coefficient, L the inductance, p the ratio of self-inductances, and q the ratio of mutual inductances.

The objective of the analysis is to compare the combined equations for the partial coils when the partial voltages are measured with the combined equations when only the full coil voltage is measured. In making this comparison it is sufficient to consider only one of the partial detectors. Rather than define a new set of parameters it is convenient to express the parameters of the partial detector in terms of the full detector.

FIGURE 4.

The use of Three Bridges Across a Single Coil in Order
 to Reduce the Probability of not Detecting a Normal Zone

Since p and q are ratios it can be assumed that these parameters are the same for a partial bridge as they are for the full bridge. This implies that the partial bridges are similar to the full bridge and that the coils have uniform properties.

The top bridge is arbitrarily selected for study and the fraction of the coil encompassed by the bridge is assigned the value m . If the coils have uniform properties the inductance of the top portion is reduced by the same fraction:

$$\text{inductance encompassed by the top bridge} = m L .$$

The mutual inductance between the top portion and a neighboring coil is decreased by a similar fraction. This is because the number of turns in one coil is reduced by the fraction m , but the number of turns in the neighboring coil remains unchanged

$$M_{tjk} = m M_{jk} .$$

The coefficient of coupling is defined as

$$k_{ij} = \frac{M_{ij}}{\sqrt{L_i L_j}} .$$

The inductance of the portion of the coil and the mutual coupling between the portion and its neighbor are both reduced by m . The inductance of the neighboring coil is the full coil inductance. Therefore, the coupling coefficient of the top of coil j to a neighboring coil k is

$$\frac{M_{tjk}}{\sqrt{mL \times L}} = \frac{m M_{jk}}{\sqrt{mL \times L}} = \sqrt{m} k_{jk}$$

where k_{jk} is the full coil coupling coefficient of the symmetrical coils in Section 6.

An expression for the A and C matrices can now be written. This is done by substituting the values derived for the partial coil into the equations for the full coil, Equations 9 and 10. In these equations each element is a product of L, which is a common factor, and a coupling coefficient. For example, Equation 9 is

$$C = L \begin{bmatrix} 1 & k_{12} & . & . & k_{1n} \\ k_{12} & 1 & . & . & k_{1(n-1)} \\ . & . & . & . & . \\ k_{1n} & k_{1(n-1)} & . & . & 1 \end{bmatrix}$$

Since

the L of the portion of the coil = mL and the coupling coefficient of the portion of the coil = $\sqrt{m}$ k_{jk} .

Therefore,

the C matrix for the portion = $m\sqrt{m}$ C.

And similarly,

the A matrix for the portion = $m\sqrt{m}$ A.

The B and D matrices are similar to those for the full coil. If a normal zone occurs in the top of coil i the d_{ii} element of the D matrix is 1, and the b_{ii} element of the B matrix is either $\sim p$ or $! \sim p$ depending on the location of the normal zone. Therefore, for the case where the voltage across the bridge is used as the supplementary voltage, the combined equation is

$$[B - (m\sqrt{mA})(m\sqrt{mC})^{-1} D] v_n = V - (m\sqrt{mC})(m\sqrt{mA})^{-1} V_t$$

where V_t is the voltage across the top section of the coil. This reduces to

$$(B - AC^{-1}D) V_n = V - A C^{-1} V_t .$$

The resulting equation is the same as that for the full coil, indicating that the detector designed around the top bridge and top voltage is equally effective in detecting a normal zone in the top as a full coil detector.

If the normal zone is in the part of the coil encompassed by the lower bridge, the top detector is unaffected by it. Both the B and D matrices are 0, causing the detector to have no output.

The alternative to using the voltage across the bridge as the supplementary voltage is to measure only one voltage, the full coil voltage, and use it for both detectors. The A and C matrices remain unchanged, but the D and B matrices need to be re-evaluated. The important difference between only measuring the full coil voltage and measuring the partial voltages is that the D matrix is not affected by the location of the normal zone. It contains a 1 at d_{ii} when there is a normal zone in coil i , regardless of whether it is in the top or bottom.

If the normal zone is in the top, the B matrix contains a $-p$ or $1-p$ at b_{ii} . The D matrix will also respond to the normal zone, so that the combined equation is the same as if the voltage across the bridge is measured. Therefore, there is no disadvantage to measuring only the coil voltage when the normal zone is in the portion of the coil associated with the detector.

If the normal zone is in the bottom of the coil it is undetected by the top bridge. Therefore, the B matrix is zero. The D matrix still contains a non-zero element. Therefore, the combined equation is

$$[[0] - (m\sqrt{m}A)(m\sqrt{m}C)^{-1}D] V_n = V - (m\sqrt{m}C)(m\sqrt{m}A)^{-1} V_t$$

$$AC^{-1}D V_n = V - CA^{-1} V_t$$

The left side of the equation, lacking as it does the B matrix, contains only the residual terms. For a normal zone in coil i the equation has the form

$$\begin{bmatrix} R_{i1} \\ R_{i2} \\ \cdot \\ R_{in} \end{bmatrix} v_{ni} = V - AC^{-1} v_t .$$

This analysis shows that when the normal zone is in the portion of the coil encompassed by the bridge, basing the detector on the full coil voltage is just as effective as if the measurement of the partial voltage is made. The disadvantage of using the full coil voltage for both detectors is that when the normal zone is in the part of the coil outside of the detector's range, the detector still registers an output. However, since the output consists of only the residual terms it is small and may not be significant.

An example helps to summarize these observations and to assess the importance of the unwanted outputs. In Section 7 the combined equation is given for two coils with a normal zone in coil 1. From this result and from the observations made in the preceding paragraphs, the output can be written for two detectors based on two bridges and a measurement of the coil voltage. The coils are assumed to have the kind of symmetry described previously.

If the normal zone is in the top of the upper bridge, the output of the upper detector is exactly the same as in Section 7. The upper combined equation is

$$\begin{bmatrix} -p + \frac{(p-q) k_{12}^2}{k_{12}^2 - 1} \\ \frac{-(p-q) k_{12}}{k_{12}^2 - 1} \end{bmatrix} v_{it} = V - AC^{-1} v_s .$$

The lower detector equation is similar, but lacks the term $-p$ contributed by the B matrix. The lower combined equation is

$$\begin{bmatrix} \frac{(p-q) k_{12}^2}{k_{12}^2 - 1} \\ \frac{-(p-q) k_{12}^2}{k_{12}^2 - 1} \end{bmatrix} \quad v_{lt} = V - AC^{-1} V_s .$$

Since k_{12} is smaller than one, the residual with the k_{12}^2 term is the smaller of the two residuals. Therefore, a large normal zone is required before the lower detector registers a false indication of a normal zone. Although in an absolute sense, measuring only one voltage for the two detectors is not as good as measuring two, from a practical point of view it is adequate.

Detectors Using a Bridge for the Supplementary Voltage

The self-inductance bridge, which is the basis of all the detectors, balances out the self-inductance voltages making the normal zone voltages more visible. The question arises: "Can the normal zone voltages be even more enhanced if a second bridge, which balances out some of the mutually induced voltages, is used as the supplementary voltage?"

A partial answer to this question is obtained by examining the symmetrical set of coils. For the symmetrical coils the A matrix is given by Equation 10.

$$A = L(q - p_1)(C_s - U),$$

where p_1 = the balance ratio of the self-inductance bridge.

The supplementary voltages are also the output of a bridge, but a bridge with a different balance ratio. Therefore, the C matrix has exactly the same form as the A matrix.

$$C = L (q - p_2)(C_s - U),$$

where p_2 = the balance ratio of the supplementary bridge.

Since the two bridges have different midpoints, a normal zone between these midpoints is in the top of one bridge and the bottom of the other. For simplicity this possibility is initially put aside and it is assumed that the normal zone is in the same end of both bridges. Therefore, the D matrix is similar to the B matrix except that p_1 is replaced by p_2 .

The combined equation is

$$(B - AC^{-1}D) V_n = V - AC^{-1}D$$

$$\{B - L(q - p_1)(C_s - U) [L(q - p_2)(C_s - U)]^{-1} D\} V_n = V - AC^{-1}D .$$

The product of $C_s - I$ and its inverse yields the identity matrix

$$(B - \frac{q - p_1}{q - p_2} D) V_n = V - AC^{-1}D .$$

For a normal zone in the top of coil i,

$$\left(\begin{bmatrix} 0 & 0 & \dots & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & -p_1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & \dots & 0 \end{bmatrix} \frac{-q-p_1}{q-p_2} \begin{bmatrix} 0 & 0 & \dots & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & -p_2 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & \dots & 0 \end{bmatrix} \right) \begin{bmatrix} 0 \\ 0 \\ \dots \\ v_{nit} \\ \dots \\ 0 \end{bmatrix} = V - AC^{-1} V_s$$

$$\begin{bmatrix} 0 \\ 0 \\ \dots \\ -p_1 + \frac{q-p_1}{q-p_2} p_2 \\ \dots \\ 0 \end{bmatrix} v_{nit} = V - AC^{-1} V_s .$$

The only non-zero term on the left side can be simplified

$$-p_1 + \frac{q-p_1}{q-p_2} p_2 = \frac{p_2-p_1}{q-p_2} q$$

Only the detector associated with the normal zone has a non-zero output. This is a consequence of assuming a symmetrical set of coils and is not an exclusive property of a detector using two bridges.

The size of the non-zero term gives information about the detector. If p_1 and p_2 are equal the term is zero and the detector is completely ineffective. The equality of the two bridge ratios means that the two bridges are identical and the supplementary measurement fails to supply needed information to the detector.

If p_2 is equal to q the non-zero term becomes infinite, increasing the effectiveness of the detector. In this case, the second bridge completely balances out the mutually induced voltage leaving only the normal zone among the measured values.

The preceding results can be modified to show the result of a normal zone between the two bridge midpoints. Depending on the relative sizes of the ratios, the normal zone is either in the top of the self-inductance bridge and the bottom of the second bridge or visa versa. If it is in the top of the self-inductance bridge, the B matrix is unchanged, but the D matrix contains the single non-zero element $1-p_2$ instead of $-p_2$. The non-zero term in the final detector equation is

$$-p_1 - \frac{(q-p_1)}{q-p_2} (1-p_2) = \frac{q(p_2-p_1) + p_1-q}{q-p_2}$$

There are, then, three possible left sides of the combined equation: one for a normal zone in the top of both coils, one for the zone in the bottom, and

one for a zone in the midcoil region. A detector would have to determine the location. This requires additional hardware, for example: a bridge across the midcoil region and additional decision making.

In order to justify the additional complexity of a two bridge detector some compensating benefit is needed. A high discrimination ratio is a possible advantage. Although it is dangerous to generalize from the analysis of the symmetrical coils, it appears as if a large discrimination ratio is obtained when p_1 and p_2 are far apart and the second bridge cancels out the largest of the mutual induced voltages. This implies coils with the ratio of the partial to full self-inductance much different from the corresponding mutual inductance ratio. However, since the analysis of the symmetric coils tells nothing about the outputs of the detectors not associated with the normal zone, each case must be studied individually by numerical methods.

A numerical analysis of a pair of the MFTF yin-yang coils is described in Appendix A. The study showed that, for these coils at least, there is no advantage to using two bridges. The discrimination ratio increases monotonically as p_2 is increased from p_1 to 1. For $p_2 = 1$ the midpoint of the bridge is at the top of the coil--which is the equivalent to using some fraction of the whole coil voltage as the supplementary voltage. Therefore, for this pair of coils, using the full coil voltage is better than using a bridge with any arbitrary balance ratio.

11. ERRORS IN THE DETECTOR

Errors in the determination of the normal zone occur because the numbers used in the detector are never exactly the same as the actual values of the circuit parameters. The values used in the detector are obtained either by calculation or by measurement. Calculated parameters, which are based on idealized computer models of the coils, are never exact. Measurements are also subject to error. In addition, the circuit parameters are not constant with time and operating conditions. Changes in the magnetic field cause changes in the forces on the structure and the resultant deflections of the structure alter the mutual and self-inductances of the coils.

In order to distinguish between the values used in the detector and the actual or true values of the circuit parameters the combined equation is rewritten as

$$(B_d - A_d C_d^{-1} D_d) V_{nd} = V - A_d C_d^{-1} V_s$$

The subscript d on the matrices indicate that they contain the values used in the detector. V_{nd} is the normal zone voltage as it is determined by the detector. The voltage V and V_s are voltages derived from the actual circuit and, therefore, reflect the actual circuit values. These two voltages are related to the actual or true normal zone voltage V_n and the rate of change of the current by the two equations:

$$V = A \frac{dI}{dt} + B V_n$$

$$V_s = C \frac{dI}{dt} + D V_n$$

The matrices in these equations contain the true values of the circuit parameters.

If the equations for V and V_s are substituted into the combined equation and terms collected, the following equation results:

$$(B_d - A_d C_d^{-1} D_d) V_{nd} = (A - A_d C_d^{-1} C) \frac{dI}{dt} + (B - A_d C_d^{-1} D) V_n$$

The matrix $B_d - A_d C_d^{-1} D_d$ is singular. Therefore, it isn't possible to obtain an explicit expression for V_{nd} . However, it is sufficient to analyze the errors that occur when there is a single normal zone at a specified location. From the results of Section 4, the left side of the equation can be written as follows for a normal zone in coil i :

$$(B_d - A_d C_d^{-1} D_d) V_{nd} = LTI V_{nd} = \begin{bmatrix} LTI_1 \\ LTI_2 \\ \vdots \\ LTI_i \\ \vdots \\ LTI_n \end{bmatrix} v_{ndi}$$

The matrix on the right side, $B - A_d C_d^{-1} D$, is similar and can be expressed as a column matrix, RTI . The matrix $A - A_d C_d^{-1} C$ is a square matrix, which will be called E . The equation relating the measured normal zone to the actual normal zone and the current derivatives can now be rewritten for a normal zone in coil i

$$(B_d - A_d C_d^{-1} D_d) v_{ndi} = (A - A_d C_d^{-1} C) \frac{dI}{dt} + (B - A_d C_d^{-1} D) v_{ni}$$

$$\begin{bmatrix} LTI_1 \\ LTI_2 \\ \vdots \\ LTI_i \\ \vdots \\ LTI_n \end{bmatrix} v_{ndi} = \begin{bmatrix} e_{11} & e_{12} & \cdots & e_{1i} & \cdots & e_{1n} \\ e_{21} & e_{22} & \cdots & e_{2i} & \cdots & e_{2n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ e_{i1} & e_{i2} & \cdots & e_{ii} & \cdots & e_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ e_{n1} & e_{n2} & \cdots & e_{ni} & \cdots & e_{nn} \end{bmatrix} \begin{bmatrix} \frac{di}{dt} \\ \frac{di2}{dt} \\ \vdots \\ \frac{dii}{dt} \\ \vdots \\ \frac{din}{dt} \end{bmatrix} + \begin{bmatrix} RTI_1 \\ RTI_2 \\ \vdots \\ RTI_i \\ \vdots \\ LTI_n \end{bmatrix} v_{ni}$$

$$LTI v_{ndi} = E \frac{dI}{dt} + RTI v_{ni}$$

These equations show that the error in the normal zone voltage determined by the detector has two parts: an additive term and a scale factor error. The additive term, which is proportional to the rate of change of current, shrinks to zero when the currents are quiescent, but on the other hand, it can exist even when there is no normal zone. The scale factor error is due to the fact that LTI and RTI are not exactly the same. As a result, the normal zone determination is in error, even when the current is not changing.

At this point, a brief review of how the detector operates is in order. In the first stage of the detector, the detector outputs are tested against a threshold level. If the threshold is exceeded by one of the detector outputs, the normal zone in the associated coil is calculated using the equation of that detector; the remaining equations are not used. Therefore, if there is a normal zone in coil i, the equation used to calculate the normal zone is

$$v_{ndi} = \frac{1}{LTI_i} (e_{i1} \frac{di_1}{dt} + e_{i2} \frac{di_2}{dt} \dots + e_{ii} \frac{di_i}{dt} + \dots + e_{in} \frac{di_n}{dt}) + \frac{RTI_i}{LTI_i} v_{ni}$$

The error due to the changing currents contribute to v_{ndi} even when there is no normal zone present. In order to avoid a false indication of a normal zone this error must be smaller than the threshold for the maximum rates of current change encountered.

The scale factor error is present even when the currents are constant. Under these circumstances

$$v_{ndi} = \frac{RTI_i}{LTI_i} v_{ni} = K_{ei} v_{ni}$$

If the measured or calculated circuit parameters are the same as the actual parameters, K_{ei} is one. The difference between one and K_{ei} is the scale factor error.

The Scale Factor Error

The factors influencing the scale factor error can be determined by examining the two matrices, LTI which is equal to $B_d - A_d C_d^{-1} D_d$ and RTI, which is equal to $B - A_d D_d^{-1} D$. The scale factor error is zero when these two matrices are the same. In order for them to be the same, D must equal D_d and B must equal B_d .

The D matrix serves to locate the normal zone, it does not contain any of the system parameters. For the detector equation in which the coil voltages are the supplementary voltage, the D matrix is a diagonal matrix with a 1 in a location corresponding to a normal zone and a 0 elsewhere. The D_d matrix is exactly the same.

The B matrix is also a diagonal matrix that locates the normal zone but it contains diagonal terms related to the bridge balance. The detector is designed on the assumption that the bridges are perfectly balanced. If, in fact, the bridges are perfectly balanced the B and B_d matrices are exactly the same. Therefore, perfectly balancing the bridge completely eliminates the scale error.

When the bridge is not perfectly balanced there is a scale error. However, the nature of the error is complex and it is necessary to resort to specific cases in order to further describe it.

The B matrix contains the diagonal terms $-p_i$, $1-p_i$, or 0 depending on the location of the normal zone. The B_d matrix is similar but contains the values used in the design of the detector, $-p_{di}$ and $1-p_{di}$, instead of the true values. Therefore, the two matrices $B_d - A_d C_d^{-1} D_d$ and $B - A_d C_d^{-1} D$ are similar with one containing p_i and the other p_{di} . With this in mind, the results of Section 7

can be adapted. Taking errors into account, the combined equation for two symmetric coils with a normal zone in the top of coil 1 is:

$$\begin{bmatrix} -p_d + \frac{(q-p_d) k_{12}^2}{1-k_{12}^2} \\ \frac{(q-p_d) k_{12}}{1-k_{12}^2} \end{bmatrix} v_{nd1} = \mathcal{E} \frac{dI}{dt} + \begin{bmatrix} -p + \frac{(q-p) k_{12}^2}{1-k_{12}^2} \\ \frac{(q-p) k_{12}}{1-k_{12}^2} \end{bmatrix} v_{n1}$$

The normal zone voltage is calculated in the detector from the first of the above two equations. If the error term due to changing currents is zero, the calculated normal zone v_{ndi} is

$$v_{ndi} = \frac{\frac{-p + (q-p) k_{12}^2}{1-k_{12}^2}}{\frac{-p_d + (q-p_d) k_{12}^2}{1-k_{12}^2}} v_{ni}$$

But since $k_{12} < 1$,

$$v_{ndi} \approx \frac{p}{p_d} v_{ni}$$

The erroneous scale factor is approximately equal to the ratio of p to p_d , where p is the the actual bridge balance ratio and p_d is the ratio that perfectly balances the bridge. A similar result is obtained for four symmetric bridges when appropriate approximations are made.

Although the results are not general, they do provide a rule of thumb by which to judge the extent of the scale factor error. That is, the erroneous scale factor is approximately equal to the ratio of the ideal bridge ratio to the actual bridge ratio.

The Error Term Due to Changing Currents, Unbalanced Bridge

In this section the error due to changing current is derived for the condition that the bridges are unbalanced but all of the system parameters are known exactly. It is shown that error from the changing currents when the bridge is unbalanced is potentially the largest source of error.

The error term due to changing current is

$$E \frac{dI}{dt} = (A - A_d C_d^{-1} C) \frac{dI}{dt} .$$

When the coil voltage is used as the supplementary voltage the C matrix does not depend on the balance of the bridges. Therefore, if all of the circuit parameters are known exactly, and bridge balance is the only source of error, C and C_d are the same. Then,

$$C_d^{-1} C = U = \text{the identity matrix}$$

and

$$(A - A_d C_d^{-1} C) \frac{dI}{dt} = (A - A_d) \frac{dI}{dt} .$$

The A and A_d matrices are different because of the bridge unbalance. The detector is designed on the assumption that the bridge is perfectly balanced. Therefore, the A_d matrix has a zero diagonal similar to the matrix in Equation 3.

$$A_d = \begin{bmatrix} 0 & M_{b12} - p_{d1} M_{12} & \cdot & \cdot & M_{b1n} - p_{d1} M_{1n} \\ M_{b21} - p_{d2} M_{12} & 0 & \cdot & \cdot & M_{b2n} - p_{d2} M_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ M_{bn1} - p_{dn} M_{1n} & \cdot & \cdot & \cdot & 0 \end{bmatrix}$$

The actual bridge voltages are not perfectly balanced. Therefore, the self-induced voltages are not completely balanced and the A matrix is

$$A = \begin{bmatrix} L_{b1} - p_1 L_1 & M_{b12} - p_1 M_{12} & \cdot & \cdot & M_{b1n} - p_1 M_{1n} \\ M_{b2} - p_2 M_{12} & L_{b2} - p_2 L_2 & \cdot & \cdot & M_{b2} - p_2 M_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ M_{bn1} - p_{dn} M_{1n} & \cdot & \cdot & \cdot & L_{bn} - p_n L_n \end{bmatrix}$$

Therefore, the error term is

$$(A - A_d) \frac{di}{dt} = \begin{bmatrix} L_1 (p_{d1} - p_1) & M_{12} (p_{d1} - p_1) & \cdot & \cdot & M_{1n} (p_{d1} - p_1) \\ M_{12} (p_{d2} - p_2) & L_2 (p_{d2} - p_2) & \cdot & \cdot & M_{2n} (p_{d2} - p_2) \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ M_{1n} (p_{dn} - p_n) & \cdot & \cdot & \cdot & L_n (p_{dn} - p_n) \end{bmatrix} \begin{bmatrix} di_1/dt \\ di_2/dt \\ \cdot \\ \cdot \\ di_n/dt \end{bmatrix}$$

The matrix $A - A_d$ can be resolved into the product of two matrices,

$$(A - A_d) \frac{di}{dt} = \begin{bmatrix} (p_{d1} - p_1) & 0 & \cdot & \cdot & 0 \\ 0 & (p_{d2} - p_2) & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & (p_{dn} - p_n) \end{bmatrix} \begin{bmatrix} L_1 & M_{12} & \cdot & \cdot & M_{1n} \\ M_{12} & L_2 & \cdot & \cdot & M_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ M_{1n} & \cdot & \cdot & \cdot & L_n \end{bmatrix} \frac{di}{dt}$$

If there is a normal zone in coil i the detector equation used to evaluate the normal zone is

$$v_{ndi} = \frac{(p_{di} - p_i)}{LTI_i} (M_{1i} \frac{di_1}{dt} + M_{2i} \frac{di_2}{dt} + \cdot \cdot \cdot + L_i \frac{di_i}{dt} + \cdot \cdot \cdot + M_{in} \frac{di_n}{dt}) + \frac{RTI_i}{LTI_i} v_{ni}$$

This result shows how bridge balance affects the error due to changing currents. The error is proportional to the difference between the bridge ratio that balances the bridge and the actual bridge ratio. The size of this error can be substantial. At the maximum charging or discharging rate, $L \frac{di}{dt}$ is comparable in size to the maximum voltage of the power supply. When the coils are closely coupled the sum of the mutually induced voltages is also appreciable.

The Error Term Due to Changing Currents, Balanced Bridge

When the bridge is balanced, errors that contribute to the error due to changing currents tend to cancel. It is not possible to prove this in a general way. However, a number of specific cases can be cited to show that the error term due to changing currents tends to zero when the bridge is balanced.

For example, if all the circuit parameters have the same error the error term due to changing currents is zero. To be more specific, consider the case where the values used in the detector are f times the actual values such that

$$\begin{aligned} A_d &= fA, \\ C_d &= fC. \end{aligned}$$

Then,

$$\begin{aligned} E \frac{dI}{dt} &= (A - A_d C_d^{-1} C) \frac{dI}{dt} \\ &= (A - fAC \frac{1}{f} C) \frac{dI}{dt} \\ &= [0] . \end{aligned}$$

The error term due to changing currents is also zero when one or more columns of the A and C matrices are affected by error. Columns of a square

matrix are modified if the matrix is postmultiplied by a diagonal matrix. A single column is multiplied by f if the matrix is postmultiplied by a diagonal matrix with f at the column to be multiplied and ones elsewhere.

$$A_d = A \begin{bmatrix} 1 & 0 & . & . & 0 & . & . & 0 \\ 0 & 1 & . & . & 0 & . & . & 0 \\ . & . & . & . & . & . & . & . \\ 0 & 0 & . & . & f & . & . & 0 \\ . & . & . & . & . & . & . & . \\ 0 & 0 & . & . & 0 & . & . & 1 \end{bmatrix}$$

$$= AF$$

$$C_d = CF$$

Then,

$$E \frac{dI}{dt} = [A - AF (CF)^{-1} C] \frac{dI}{dt}$$

$$= [A - AF F^{-1} C^{-1} C] \frac{dI}{dt}$$

$$= [0]$$

Further examples of how the errors contributing to the error term due to changing current tend to cancel are provided by a symmetric set of coils. A symmetric set of coils, as described in Section 6 is characterized by L , q , p , and the coupling coefficients k_{12} , k_{13} , ..., k_{in} . If there is an error in measuring L the error term due to changing currents is zero. This is demonstrated as follows:

The inductance can be factored out of the A and C matrices so that

$$A = L(q-p)(C_s^{-1}U)$$

$$C = L C_s$$

and

$$\begin{aligned} A_d &= L_d(q-p)(C_s-U) \\ C_d &= L_d C_s \end{aligned}$$

where C_s is a matrix of the coupling coefficients defined by Equation 9. Placing these values in the error term yields,

$$\begin{aligned} E \frac{dI}{dt} &= L(q-p)(C_s-U) - L_d(q-p)(C_s-U) \frac{C_s^{-1}}{L_d} C_s \\ &= [0] \end{aligned}$$

The effect of an error in the coupling coefficients is not as easily understood. However, an analysis of the two coil case can be made. Using the nomenclature of Section 7.

$$E \frac{dI}{dt} = \frac{L(q-p)(k_{12}-k_{12d})}{(1-k_{12d}^2)} \begin{bmatrix} k_{12d} & 1 \\ 1 & k_{12d} \end{bmatrix} \begin{bmatrix} \frac{di}{dt}^1 \\ \frac{di}{dt}^2 \end{bmatrix}$$

This result shows that the error due to changing currents is proportional to the difference between the value of the coupling coefficient used in the detector and the actual value of the coupling coefficient

Concluding Remarks About Errors

There are two kinds of errors in the detector: errors due to changing currents and scale error. Error due to changing current is a zero error, it exists even when there is no normal zone. Scale error affects the normal zone value and modifies the size of the error due to changing current. The size of both errors depends on the accuracy of the measured or calculated circuit parameters and on the exactness with which the bridge is balanced. The most important factor is bridge balance.

The detector is designed with the assumption that the bridges are perfectly balanced. If they are in fact perfectly balanced, the scale factor error is zero, even when the other circuit parameters are not known exactly. When the bridge is unbalanced the accuracy of the other parameter values enters into the scale factor. However, the effect of bridge balance predominates, and as a rule of thumb the scale factor error is roughly proportional to the ratio of the two bridge ratios, the actual bridge ratio and the one that balances the bridges perfectly.

Bridge balance is also an important factor in determining the size of the error due to changing currents, the error which is potentially the largest source of error. When all other parameters are known exactly this error is proportional to the difference between the actual bridge ratio and the ratio that perfectly balances the bridge.

Balancing the bridge reduces but does not completely eliminate the error due to changing currents. However, these residual errors, which are due to inaccurate knowledge of the circuit parameters, are relatively small. It can be shown that, under certain circumstances, the errors in the parameters tend to cancel.

Bridge balance is not only more important than the other detector parameters, it is also uniquely different. The other parameters are obtained from calculations based on computer models or on measurements made on the coils. The approximate bridge ratio can be determined in advance but, in addition, the bridge balance can be fine tuned when the entire system, coils and detectors, is assembled. This is done by open-circuiting all of the coils except the one whose detector is to be tested. A constant voltage is then applied to the coil and the bridge balanced.

12. AN ILLUSTRATIVE EXAMPLE

In order to provide an example, a detector for four inductively coupled coils was designed and tested using a computer simulation of the coils, their related equipment, and the detector. Since the design of the detector is accomplished by steps quite apart from the simulation, the simulation provides an independent test of the detector similar to an actual hardware test.

At the time of writing the physical parameters for an actual set of four coils were not available. Therefore, a hypothetical set of values were generated based on the properties of the two yin-yang MFTF coils.

A number of simulations were run with the following objectives: to test the ability of the detector to reject the mutual and self-induced voltages and to test the ability to locate and measure the normal zone.

Similar coils are used. That is, the full self-inductances, partial self-inductances, and consequently the bridges are the same. Symmetry of the full mutual inductances is also assumed. However, the ratio of partial to full mutual inductance is not the same for coils a different distance apart. As a result, the set of coils lack the perfect symmetry assumed in Section 7.

The self-inductance values are

$$L = L_1 = L_2 = L_3 = L_4 = 11.113 \text{ H}$$

$$L_b = L_{b1} = L_{b2} = L_{b3} = L_{b4} = 5.53 \text{ H}$$

$$p = \frac{L_b}{L} = .4976154$$

The mutual inductances of the full coils are

$$M_{12} = M_{23} = M_{34} = 1.149 \text{ H}$$

$$M_{13} = M_{24} = .556 \text{ H}$$

$$M_{14} = .2 \text{ H}$$

Therefore,

$$K_{12} = K_{23} = K_{34} = .103$$

$$K_{13} = K_{24} = .05$$

$$K_{14} = .02$$

The partial mutual inductances are assumed to have the symmetry:

$$M_{bij} = M_{bji}$$

and

$$M_{b12} = M_{b23} = M_{b34} = .634 \text{ H}$$

$$M_{b13} = M_{b24} = .307 \text{ H}$$

$$M_{b14} = .11$$

The ratio of the partial to full mutual inductances are

$$\frac{M_{b12}}{M_{12}} = .5527$$

$$\frac{M_{b13}}{M_{13}} = .5522$$

$$\frac{M_{b14}}{M_{14}} = .5500$$

In Section 7 where "perfect" symmetry is employed these ratios are the same.

The detector equation is

$$(B - AC^{-1}D) V_n = V - AC^{-1} V_c$$

Design of the detector requires the calculation of the matrix combinations on both sides of the equation. On the left side, $B - AC^{-1}D$ must be calculated for all possible single normal zones. There are eight locations, one for the top and one for the bottom of each coil. The right side product AC^{-1} is the same for all normal zone locations.

For illustrative purposes it is sufficient to evaluate the left side for a normal zone in the top of coil 1 and the bottom of coil 2. The symbols $T1$ and $B2$ are used to denote these two cases; the symbol L to denote the left side factors and the matrix $[y_{ij}]$ to denote $-AC^{-1}$.

For a normal zone in the top of coil 1, the detector equation is

$$\begin{bmatrix} LT1_1 \\ LT1_2 \\ LT1_3 \\ LT1_4 \end{bmatrix} v_{nt1} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} + \begin{bmatrix} y_{11} & y_{12} & y_{13} & y_{14} \\ y_{12} & y_{22} & y_{23} & y_{24} \\ y_{13} & y_{23} & y_{33} & y_{34} \\ y_{14} & y_{24} & y_{34} & y_{44} \end{bmatrix} \begin{bmatrix} v_{c1} \\ v_{c2} \\ v_{c3} \\ v_{c4} \end{bmatrix} \quad (11)$$

The $-AC^{-1}$ matrix is symmetric because of the symmetry of A and C .

$$\begin{aligned} y_{11} &= 6.76959099 \times 10^{-4} \\ y_{12} &= -5.42484872 \times 10^{-3} \\ y_{13} &= -2.15419853 \times 10^{-3} \\ y_{14} &= -4.60804530 \times 10^{-4} \\ y_{22} &= 1.20947139 \times 10^{-3} \\ y_{23} &= -5.23188073 \times 10^{-3} \\ y_{24} &= -2.15079864 \times 10^{-3} \\ y_{33} &= 1.20947139 \times 10^{-3} \\ y_{34} &= -5.42522239 \times 10^{-3} \\ y_{44} &= 6.76959099 \times 10^{-4} \end{aligned}$$

$$\begin{aligned} LT1_1 &= -4.96938446 \times 10^{-1} \\ LT1_2 &= -5.42522239 \times 10^{-3} \\ LT1_3 &= -2.15079864 \times 10^{-3} \\ LT1_4 &= -4.60804530 \times 10^{-3} \end{aligned}$$

The right side of the equations are the same, regardless of where the normal zone is, but the left side coefficients depend on the location of the normal zone. For a normal zone in the bottom of coil 2 the left side coefficients are:

$$\begin{aligned} \text{LB2}_1 &= 5.42484872 \times 10^{-3} \\ \text{LB2}_2 &= 5.03594066 \times 10^{-1} \\ \text{LB2}_3 &= 5.23188073 \times 10^{-3} \\ \text{LB2}_4 &= 2.15419853 \times 10^{-3} \end{aligned}$$

All of these equations are not necessary for the design of the detector. They have been written so that the discrimination ratio can be calculated later in this section. The detector equations used to implement the detector are:

For a normal zone in the top of coil 1,

$$\text{LTI}_1^{v_{nt1}} = v_1 + y_{11}^{v_{c1}} + y_{12}^{v_{c2}} + y_{13}^{v_{c3}} + y_{14}^{v_{c4}}$$

For a normal zone in the bottom of coil 2,

$$\text{LB2}_2^{v_{nb2}} = v_2 + y_{12}^{v_{c1}} + y_{22}^{v_{c2}} + y_{23}^{v_{c3}} + y_{34}^{v_{c4}}$$

A full detector for a coil, such as that shown in Figure 3, requires an additional equation for each coil, one for the bottom of coil 1 and one for the top of coil 2. These equations have the same left side, but different right sides.

The Simulation

The four coils and the detector were simulated using the Sceptre code. The circuit for one of the coils is shown in Figure 5. The capacitance and resistance, shown in dotted lines, in parallel with the coil have been added to obtain stability of the numerical integration routines used in Sceptre. A listing, in Appendix B, gives the values used in the simulation.

The detector part of the simulation is a combination of arithmetic and decision making operations. The decision making part is a short Fortran program embedded in the Sceptre program.

E	power supply	<u>suffixes</u>
RCB	resistance of the circuit breaker	
RB	bridge resistance	T top
RO	bridge output resistance	B bottom
RN	normal zone resistance	I number of coil
RD	dump resistor	
RG	grounding resistor	
C&R	a RC circuit added to increase numerical stability	

FIGURE 5.
One of the Coil Circuits Used in the Sceptre Simulations

Rejection of the Mutual and Self-induced Voltages

In order to locate and measure the normal zone, the detectors must reject the large mutual and self-induced voltages that are present when the currents are changing. To test this property a simulation was made in which the power supply voltages of all four coils were increased to their maximum values, held there, and decreased again as shown in Figure 6. None of the coils contained a normal zone. Figure 7 shows the current in coil 1; the other currents are similar. As Figure 8 and 9 show, most of the power supply voltage, about nine and one half volts, is applied to the coils; the remainder is lost in the cable resistance. The bridges balance out the self-induced voltages and part of the mutually induced voltage, leaving an unbalanced voltage of about a hundredth of a volt. Figures 10 and 11 show the bridge voltages for coils 1 and 2. The bridge voltages for coils 3 and 4 are similar because of the symmetry of the coils.

Since there are no normal zones the normal zone resistance, as calculated from the detector equations, should be zero. Instead, small values in the order of 10^{-8} ohms are obtained as shown in Figures 12 and 13. These small values can be attributed to computational errors. The detector constants are computed with great accuracy; making the simulation which contains extraneous elements and which relies on numerical integration the main suspected source of error.

The significance of the small computational error can be determined by comparing it with the resistance of a normal zone in the yin-yang conductor. Table 1 shows the resistance of a normal zone one meter long at a number of temperatures. The computational errors are equivalent to only a couple of hundredths of a meter of normal zone at 4°K.

The simulation demonstrates that, except for small computational errors, the detectors reject the mutual and self-induced voltages, even when the voltages are at a maximum.

FIGURE 6.

The Power Supply Voltage for Each of the Coils
For the Section: Rejection of the Mutual and Self-induced Voltages

PLOT OF ILB1 VS TIME

FIGURE 7.

The Current Through Coil 1.

For the Section: Rejection at the Mutual and Self-induced voltages

FIGURE 8.

The Voltage Across Coil 1.

For the Section: Rejection of the Mutual and Self-induced Voltages.

PLOT OF PVC2 VS TIME

FIGURE 9.

The Voltage Across Coil 2.

For the Section: Rejection of the Mutual and Self-induced Voltages.

FIGURE 10.

The Voltage Across Bridge 1.

For the Section: Rejection of the Mutual and Self-induced Voltages.

PLOT OF VR02 VS TIME

FIGURE 11.

The Voltage Across Bridge 2.

For the Section: Rejection of the Mutual and Self-induced Voltages.

PLOT OF PRR11 VS TIME

FIGURE 12.

The Normal Zone Resistance Calculated From the Detector for the top of Coil 1.

For the Section: Rejection of the Mutual and Self-induced Voltages.

FIGURE 13.

The Normal Zone Resistance Calculated From the Detector for the Bottom of Coil 2.

For the Section: Rejection of the Mutual and Self-induced Voltages.

TABLE 1.
The Resistance of One Meter of Yin-yang Conductor
as a Function of Temperature

Temperature k	Resistance ohms x 10^{-6}
4	1.12
20	1.19
32	1.40
40	2.03
50	3.29
60	7.00
80	14.00

A Normal Zone in One Coil

The coils were simulated using the same power supply voltages as in the previous section, but with an increasing normal zone in the upper part of coil 1. The purpose was to test the detector's ability to locate and measure the normal zone and to determine the detector discrimination ratio.

The four power supply voltages are identical to the voltage shown in Figure 6. Since the effect of the normal zone is small the coil currents and coil voltages when plotted are indistinguishable from those of the previous section. Even the bridge voltage plots, which are shown in Figures 14 and 15, are only slightly affected by the normal zone. The normal zone, which grows steadily from zero to 10^{-5} ohms in 100 S, is shown in Figure 16. Figure 17 shows the output of the upper detector for coil 1. The sign of the output is negative, indicating that the normal zone is in the upper part of the coil. After the threshold is reached the detector tracks the normal zone closely. The threshold is set at 1 mV, which at 1000 A corresponds to 10^{-6} ohms. The output of the detector for the lower part of coil 2 never exceeds the threshold. However, the resistance has been calculated and plotted in Figure 18.

The normal zone in coil 1 affects all of the detectors, the detector for coil 2 as well as the detector for coil 1. Identifying the coil containing

FIGURE 14.
The Voltage Across Bridge 1.
For the Section: a Normal Zone in One Coil.

FIGURE 15.
The Voltage Across Bridge 2.
For the Section: a Normal Zone in One Coil.

PLOT OF RNT1 VS TIME

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FIGURE 16.

The Normal Zone in the Upper Part of Coil 1.
For the Section: a Normal Zone in One Coil.

FIGURE 17.

The Output of Detector for the Upper Part of Coil 1.
For the Section: a Normal Zone in One Coil.

FIGURE 18.

The Normal Zone Resistance Calculated From the Detector in the Lower Part of Coil 2.
 For the Section: a Normal Zone in One Coil.

the normal zone depends on the normal zone affecting the detector associated with the coil much more than it affects the other detectors. This property is measured by the discrimination ratio of the detector and the discrimination ratio of the combined equations. The discrimination ratio of the detector is defined as the ratio of the output of the detector associated with the normal zone to the output of the other detectors. At 100 s the resistance registered by the coil 1 detector is 4.90×10^{-5} ohms while the resistance registered by the coil 2 detector is 5.33×10^{-7} ohms. Therefore, the discrimination ratio of the detector for a normal zone in the top of coil 1 is

$$D_{12} = \frac{4.90 \times 10^{-5}}{5.33 \times 10^{-7}} = 91.9 .$$

This value compares closely with the output discrimination ratio as calculated from the left sides of the combined equations

$$D_{12} = \frac{LB2}{LT1_2} = \frac{5.0359 \times 10^{-1}}{5.4252 \times 10^{-3}} = 92.8$$

Since the coils are similar, the output discrimination ratio is almost the same as the discrimination ratio of the combined equations, d_{12} .

$$d_{12} = \frac{LT1_1}{LT1_2} = 91.5$$

Detectors were omitted from the third and fourth coils for simplicity. However, had they been included they would have been less sensitive to the normal zone in coil 1 than the detector in coil 2. For a normal zone in the top of coil 1 the discrimination ratios for the third and fourth coils are

$$d_{13} = \frac{LT1_1}{LT1_3} = 231 ,$$

$$d_{14} = \frac{LT1_1}{LT1_4} = 1078 .$$

13. SUMMARY AND CONCLUSIONS

The function of the normal zone detector is to locate and measure one or more normal zones in a large number of inductively coupled coils. The normal zone voltages are small compared with the mutual and self-induced voltages. Even though the induced voltages are large it is preferable to base the detector design on the measurement of voltages rather than current. Compared with the currents that flow in the coil, the changes in current caused by a normal zone are virtually unmeasurable.

A natural choice for the first voltage measurement is the output of bridges balanced so as to eliminate the self-induced voltages. However, the mutually induced voltages are not completely balanced out. Therefore, the bridge outputs depend on the rate of change of currents as well as on the normal zone voltages. Since the rates of change of current as well as the normal zone voltages are unknown, the bridge outputs in themselves, do not contain sufficient information to determine the normal zone. In addition, the form of the bridge equation depends on the location of the normal zones. Consequently there are many sets of bridge equations, one for every normal zone location or combination of locations. It is this fact that converts the design of the detector from a standard mathematical problem to an interesting one.

In order to determine the normal zones, additional voltage measurements are needed. The number depends on the number of normal zones--a fact that is not known a priori. An empirical approach is used to investigate this problem.

An additional set of voltages, the supplementary voltages, equal in number to the bridge voltages is assumed to be available. The bridge and supplementary voltages can be combined into a single equation with all of the normal zone terms on the left side. The left side coefficients of the normal zone voltages depend on the location of the normal zones. However, the right side does not depend on the location of the normal zones. There are two ways of combining the bridge and supplementary equations, each having this property.

The usefulness of this formulation is investigated, using the voltages across the coils as the supplementary voltages. For a single normal zone the equations are redundant, any one could be used to solve for the normal zone. However, for one of the ways of combining the bridge and supplementary equations, the equation associated with the coil containing the normal zone is much more sensitive to the presence of the normal zone than the other equations. This fact suggests a method of locating a single normal zone. It is also found that the effect of more than one normal zone is a simple linear superposition of the effects of single normal zones. Therefore, any number of normal zones can be located by examining the combined bridge and coil voltage equations.

The detector consists of both logic and arithmetic elements. The arithmetic part, the detector equations, consists of two equations for each coil, one for a normal zone in the top of the coil, the other for a normal zone in the bottom. The equations are drawn from the combined set of equations. For example, the detector equation for a normal zone in the top of coil i is the i^{th} equation of the combined equations that apply to this location of normal zone. In the detector a weighted sum of the bridge and supplementary voltages, which constitute the right side of both of the detector equations for the coil, is tested for sign to determine whether the normal zone is in the top or bottom of the coil. The correct left side factor is then applied to calculate the magnitude of the normal zone. If a threshold is exceeded a normal zone is reported, if not it is decided that there is no normal zone.

Two ratios are useful in determining the ease with which a normal zone can be located: the discrimination ratio of the combined equations and the detector discrimination ratio. The discrimination ratio of the combined equations, which measures the effect of a normal zone on the equation associated with the coil it is in, to its effect on other equations, is used for preliminary studies. The detector discrimination ratio, which measures the actual effectiveness of the detector and which determines the smallest normal zone voltage that can be detected, is closely related to the discrimination ratio for the output equations.

Normal zones that are symmetrical to the self-inductance bridges are nearly invisible to the detector. The probability of missing a normal zone is reduced if additional bridges are placed across part of the coil. When this is done it is possible to use the voltage across the coils for all of the detectors.

Two alternative supplementary voltages were investigated: voltages across part of the coil and voltages across a second set of bridges. Using the voltage across part of the bridge is superior to using the whole coil voltage when the normal zone is within the span of the voltage but is inferior when the normal zone is outside the span. Using a set of bridges that balances out some of the mutually induced voltages has an intuitive appeal. However, whether an advantage is obtained appears to depend on the specific case.

There are two kinds of errors in the detector: scale factor error and error due to the changing currents. Improving the balance of the self-inductance bridges reduces both kinds of errors. When the bridge is perfectly balanced, the scale factor error is zero. Potentially the largest source of error is the error due to changing currents when the bridges are unbalanced. Fortunately, the bridge balance can be fine-tuned after the coils and detector are in place.

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APPENDIX A

A Detector for Two Coils Using a Second Bridge for the Supplementary Voltage

In Section 10 it is shown that using a second bridge has the clear disadvantage of adding complexity to the detector and the potential advantage of improving the output discrimination ratio. Numerical studies were carried out to determine if the advantage is obtained for the MFTF coils. In the studies, the left side of the detector equation was calculated for different balance points of the second bridge. The output discrimination ratio was then computed.

The system parameters used are

$$L_1 = L_2 = 11.113 \text{ H}$$

$$L_{b1} = L_{b2} = 5.13 \text{ H}$$

Therefore

$$p = \frac{L_{b1}}{L_1} = .4616$$

$$M_{12} = 1.147 \text{ H}$$

$$M_{b12} = M_{b21} = .634 \text{ H}$$

$$q = \frac{M_{b12}}{M_{12}} = .5527$$

The calculations were made for a normal zone in the top of coil 1. The detector equation is

$$\begin{vmatrix} LT1_1 \\ LT1_2 \end{vmatrix} v_{nt1} = V - AC^{-1} V_s .$$

V_s = the output voltage of the second bridge.

The second bridge balance ratio is equal to the ratio of the resistance of the bottom part of the bridge to the total bridge resistance.

The results of these calculations are given in the following table.

<u>The Ratio of the Second Bridge</u>	$d_{12} = \frac{LT1_1}{LT1_2}$		
	<u>LT1₁</u>	<u>LT1₂</u>	
p (.4616)	0	0	-
$\frac{M_{b12}}{M_{12}}$ (.5527)	-.279283	-.034515	8.1
.8	-.4110109	-.0198843	20.6
1.0 (Full Coil)	-.429351	-.016422	26.1

APPENDIX B

Values Used in the Sceptre Simulation

The circuit diagram, which defines the symbols, is shown in Figure 5.

LTI	=	5.583 H
LBI	=	5.530 H
RBTI	=	5.583×10^3 ohms
RBTI	=	5.530×10^3 ohms
RCI	=	10^7 ohms
XCBI	=	.002 ohms
RDTI	=	.02 ohms
RDBI	=	.02 ohms
RCI	=	100 ohms
RNTI, RNBI	=	the normal zone resistances
RTI = RBI	=	10^7 ohms
CTI = CBI	=	10^{-14} F
EI	=	the power supply voltage

The mutual inductances M_{ij} between coil i and coil j are

$$\begin{aligned}M_{12} &= M_{23} = M_{34} = 1.149 \text{ H} \\M_{13} &= M_{24} = .556 \text{ H} \\M_{14} &= .200 \text{ H}\end{aligned}$$

The partial mutual inductances are given in Table B1.

The values used in the simulation of the detector are given in the introductory section of the example.

TABLE B1.

The mutual inductances in henrys between the partial coils.

The top half is denoted by t, the bottom half by b.

	1t	1b	2t	2b	3t	3b	4t	4b
1t			.233	.232	.112	.137	.040	.05
1b			.282	.352	.137	.170	.05	.05
2t					.233	.282	.112	.137
2b					.282	.352	.137	.170
3t							.233	.282
3b							.282	.352
4t								
4b								

END- 3-2-83