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Field Tests and Evaluations of the IAEA Active Well Coincidence Counter

Los Alamos

Los Alamos National Laboratory
Los Alamos, New Mexico 87545

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Field Tests and Evaluations of the IAEA Active-Well Coincidence Counter

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FIELD TESTS AND EVALUATIONS OF THE IAEA ACTIVE WELL COINCIDENCE COUNTER

by

M. S. Krick and P. M. Rinard

ABSTRACT

This report summarizes and evaluates field tests of the Active Well Coincidence Counter (AWCC) at Winfrith and Dounreay, United Kingdom. The applicability of the AWCC for assaying the uranium content of a wide variety of materials was demonstrated and calibration curves were generated. The AWCC was used in three modes (fast, thermal, and passive) while assaying powders, pellets, cartridges, plates, assorted residues, and materials-testing-reactor fuel assemblies.

I. INTRODUCTION

The Active Well Coincidence Counter (AWCC)¹ (Fig. 1) is designed primarily for assay of uranium fuel materials during field inspections and for quality control in a facility. The coincidence electronics unit counts events in ³He detectors that surround a cavity containing the sample (Fig. 2). Fissioning in the sample is induced by neutrons from two AmLi sources. To achieve higher count rates, some of the moderators, absorbers, and reflectors in the AWCC can be used or removed, depending on the type of material being assayed. In this way, the AWCC can be used for bulk UO₂ samples, highly enriched uranium metals, light-water-reactor (LWR) fuel pellets, ²³³U-thorium fuel materials, alloys, dross, and wastes.

A secondary application, assay of ²³⁸U and plutonium samples, is available by removing the AmLi sources and operating the AWCC in its passive neutron coincidence mode.

Fig. 1. The AWCC consists of coincidence electronics and a ring of detectors surrounding the sample cavity. A sample can is shown being placed into the cavity; the end plug containing an AmLi source will then be replaced. A preamplifier box is mounted on the side of the cart. The electronics box mounted near the cart's handle provides the high voltage to the ^3He detector tubes and also analyzes the detector pulses for coincidences.

Fig. 2. Top and side views of the inside of an AWCC are shown. Not indicated are removable sheets of cadmium that may be used to line the sample cavity.

Materials that are too long to fit into the normal cavity may be assayed by removing the end plugs. Special inserts are placed inside the AWCC to hold the AmLi sources and such samples as fuel pins and assemblies.

Field tests of the AWCC have been conducted at Winfrith, United Kingdom (February 1982), and at the Dounreay Nuclear Power Station, United Kingdom (May 1982). This report discusses the wide variety of materials studied and the results of the tests.

II. FIELD TESTS AND EVALUATIONS

A. MTR Fuel Assemblies

At Dounreay, the AWCC was used with materials-testing-reactor (MTR) fuel assemblies. With the AWCC positioned horizontally and its end plugs removed, a cylindrical polyethylene insert was centered in the sample cavity (Figs. 3 and 4). A notch at the top of the cylinder allows insertion of an assembly, and a hole near the bottom is for a tube containing the AmLi sources (Fig. 5) that were previously in the end plugs.

Fig. 3. The AWCC is shown on its side with the cylindrical polyethylene insert holding an MTR fuel assembly. When the AWCC is upright, the end shown is at the bottom.

Fig. 4. Shown is a front view of the AWCC, polyethylene insert, and MTR fuel assembly. The curved plates of the assembly are shown in a horizontal orientation. The source tube is below the assembly near the bottom of the insert.

Fig. 5. Two AmLi sources are shown inside a polyethylene sleeve that fits into the insert illustrated in Figs. 3 and 4. The geometries of the end caps on the sources are different, which allows the sources to fit into the sleeve with only one orientation. A polyethylene plug (shown on the right-hand side of the photograph) fills the sleeve.

The MTR assemblies at Dounreay were centered in the insert's notch so that equal amounts of fuel extended from the ends of the AWCC. This technique provides a linear fuel density measurement (in grams per centimeter).

An assembly was measured with its 16 plates positioned first horizontally and then vertically, with no difference observed other than a difference in counting statistics. However, it is recommended that the plates be positioned vertically to provide the best opportunity for uniform irradiation of the fuel plates throughout the assembly.

With 16 flat plates and a holder, a mockup of an assembly was measured for 4, 8, 12, and 16 vertical plates. A calibration curve for these data (Fig. 6) shows absorption effects, but not saturation. A normal assembly with 16 plates gave a response that was 4% greater than that from the 16 flat-plate mockup. Because the plates in the mockup assembly could not be placed as uniformly as with the normal assembly, geometry effects might have been responsible for the response difference.

The calibration curve of Fig. 6 has the shape fixed by the mockup measurements, but normalized to pass through the normal assembly's value. A power law and a parabola fit the data equally well. (The Appendix gives the details of the fitting procedure.) For $R = am^b$, with m in grams of ^{235}U and R in net coincidence counts per second, the parameters are $a = 9.6 \pm 1.5$ and $b = 0.752 \pm 0.032$.

With $R = am + bm^2$, $a = 3.83 \pm 0.12$, and $b = -0.00705 \pm 0.00086$. With one plate removed, the coincidence rate is reduced by 3.5%, which is 2.2 times the counting-statistics error of 1.6% (1σ) found with a 4-min measurement.

After a 4-min measurement, the counting-statistics error in the net totals counting rate (totals minus AmLi totals background) is only 0.3%. As long as the geometry and background do not change, this high precision could be useful for quality control during fabrication as well as for safeguards. But more experience is needed to determine if there are effects that can produce additional errors of this small magnitude.

It has been demonstrated that the AWCC can be used as a practical instrument for verification of MTR fuel assemblies. With the use of simple procedures and standard electronics, a missing plate can be detected with more than 95% confidence after a 4-min coincidence response. If the net totals response can be used, the time required can be greatly shortened or the size of the smallest detectable missing mass can be reduced.

Fig. 6. The response curve of an MTR mockup as a function of number of plates shows some curvature due to absorption. Saturation does not occur when a full complement of 16 plates is used. The plates did not have equal masses, so the four data points shown are for the slightly irregular masses of 43.5, 86.4, 128.9, and 171.8 g. Counting-statistics errors are the same sizes as the circles indicating the data points. A fit to a power law gave this expression:

$$R = (10.8 \pm 1.6) m^{(0.737 \pm 0.030)},$$

where m is the ^{235}U mass in grams and R is the response in net coincidence counts per second.

B. High-Enrichment Uranium-Oxide Powders

Cans containing uranium-oxide powder were measured at Winfrith. The AWCC was used in its normal fast mode, which means that it was in an upright position with the AmLi sources in the end plugs and the nickel reflector and cadmium liner surrounding the cavity.

The cans were about 10 to 11.5 cm in diameter and from 12.5 to 19.5 cm high. Each can was always placed directly on the bottom end plug (at the

bottom of the 20-cm-high cavity), even though this meant that a partially filled can would not have its powder centered in the cavity. The lack of symmetry was not important because the calibration curve should automatically contain the necessary asymmetry correction factor.

The Winfrith calibration curve was obtained for ^{235}U masses from 53 to 698 g and enrichments from 9 to 93%. One thousand seconds was the usual counting time. The data (Fig. 7) indicate that a calibration curve may need to take into account the enrichment of the powder. Higher enrichment samples tend to produce lower count rates than do lower enrichment samples of similar masses. This observation has been substantiated by a multivariant analysis and function fit where the terms involving enrichment were statistically significant.² The recommended fit to these data is:

$$R = 0.120 E^{0.287} m (1.07 - 0.0646 \ln E) ,$$

Fig. 7. Cans of high-enrichment uranium-oxide powders gave the normal fast mode responses shown here. The numbers beside the data points are the per cent enrichments.

where m is the ^{235}U mass in grams, E is the enrichment in per cent, and R is the AWCC net coincidence response in counts per second. Representative plots of this function are shown in Fig. 8.

Curves have also been fitted to three groups of samples, with all members of a group having similar enrichments: either nearly 15, 40, or 90%. Except for the 90% enrichment data point at 120 g, the data for all the members of each group can be fitted by a single curve (Fig. 9).

To investigate this enrichment dependence computationally, a set of Monte Carlo simulations was performed using the Los Alamos National Laboratory MCNP code.³ The AWCC was modeled in detail; the 10-cm-diam samples, however, were centered in the cavity rather than placed on the bottom of the cavity. Three enrichments were used, matching those listed in the previous paragraph. The results agree with those for the three groups of measured samples, including a

Fig. 8. The multivariant analysis with a power law fit gives these curves for three enrichments typical of the Winfrith samples. The short dashes at the end of each curve indicate the widths of the 95% confidence limits at that mass value. These limits go to zero smoothly as the mass goes to zero.

Fig. 9. The data in Fig. 7 have been separated into three groups according to enrichment. Representative values of the groups are 90%, 40%, and 15% enrichment. Parabolic fits of the form $R = am + bm^2$ were made for the three general regions of enrichments, where R is the net coincidence response in counts per second and m is the ^{235}U mass in grams:

near 90% enrichment, $a = 0.153 \pm 0.012$ and
 $b = -0.000087 \pm 0.000031$;
 near 40% enrichment, $a = 0.1707 \pm 0.0073$ and
 $b = -0.000115 \pm 0.000022$;
 near 15% enrichment, $a = 0.1750 \pm 0.0030$ and
 $b = -0.000132 \pm 0.000029$.

tendency of the 90% enrichment responses to rise less steeply for masses between 100 and 200 g than for masses below 100 or above 200 g (Fig. 10).

There appears to be a consistent deviation from the parabolic shape for masses between 75 and 250 g, but the estimated errors on the calculated values are large enough to include the fitted curve for all the data points.

Using the AWCC on uranium-oxide powders with the highest accuracy thus requires additional information about the enrichment. The geometry of the sample is also an important factor. The cans measured were about 10 cm in diameter and thus large enough for multiplication to occur; they were always placed on the bottom of the cavity.

Fig. 10. Monte Carlo simulations of the measurements in Fig. 7 show the same features, in particular the enrichment dependence. The parabolic fits (see caption to Fig. 9) have these parameters:

90% enrichment, $a = 0.1363 \pm 0.0042$, $b = -0.0000589 \pm 0.0000075$;
 40% enrichment, $a = 0.1679 \pm 0.0078$, $b = -0.0001011 \pm 0.000019$;
 15% enrichment, $a = 0.2029 \pm 0.0089$, $b = 0.000258 \pm 0.000046$.

There appears to be a consistent deviation from the parabolic shape for masses between 75 and 250 g, but the estimated errors on the calculated values are large enough to include the fitted curve for all the data points.

C. Low-Enrichment Canned Materials

Because self-shielding is not important for low-enrichment materials, the AWCC is used without the nickel reflector and the cadmium lining; this is its thermal mode. Smaller counting-statistics errors are a feature of this mode.

The cans of materials measured at Winfrith had various diameters and heights. When the height was greater than that of the cavity, the top end plug was simply placed on top of the can, rather than shortening both end plugs by removing some of their polyethylene. The characteristics of the contents of the cans are shown in Fig. 11. Despite the diverse natures of the contents and the shapes of the cans, a single curve can be fitted to nearly all the data. (With a more uniform collection of sample types, a calibration curve could be

Fig. 11. Various low-enrichment materials in cans were measured using the thermal mode. The legend lists their forms, and the numbers by the data points give their per cent enrichments. With all the data used for a parabolic fit (see caption to Fig. 9), $a = 9.98 \pm 0.69$ and $b = -0.081 \pm 0.017$ (solid line). Without the UO_2 powder and pellets, $a = 9.87 \pm 0.26$ and $b = -0.0598 \pm 0.0069$ (dashed line).

created easily.) After a 100-s count, the statistical error for a 25-g sample was about 1.4 g.

D. Very Low Enrichment Materials

Samples with enrichments from natural to 3.3% were measured at Winfrith using the thermal passive mode where the AmLi sources, the nickel reflector, and the cadmium sheets are removed. Large samples provide enough spontaneous fission neutrons from ^{238}U for coincidence counting over a 1000-s count time. The ^{235}U content can then be inferred from knowledge of the enrichment, or the enrichment can be deduced after a separate active measurement of the ^{235}U content.

The various samples measured are indicated in Fig. 12. With self-shielding and multiplication not important, a straight line is a logical choice for a calibration curve, as is shown in the figure. For a 1000-s count of a 4500-g sample of natural uranium, the counting-statistics error is equivalent to 120 g ^{238}U or 1 g ^{235}U .

III. MEASUREMENT PROCEDURES FOR TESTED MATERIALS

From the exercises at Dounreay and Winfrith, procedures can be recommended for measuring the types of materials encountered at these two facilities.

Fig. 12. The passive mode was used to measure the ^{238}U content. The sample types are given in the legend and the numbers by the data points give their per cent enrichments (nat = natural). A straight-line fit through the origin gave a slope of 0.000419 ± 0.000012 .

A. MTR Fuel Assemblies

For MTR fuel assemblies, the AWCC is used in the horizontal rather than the vertical position. The end plugs, the nickel reflector, and the cadmium lining are removed.

A special insert is placed in the cavity to hold the assemblies and AmLi sources. This insert is a polyethylene cylinder with a notch at the top for the assemblies and a hole near the bottom for the sources. The insert should be centered in the cavity and precautions taken to ensure that it does not become displaced during insertion or removal of the assemblies.

With no fuel or sources present, a background check should give very nearly zero coincidence counts. Instrumental noise and nearby unknown sources can be detected by this check.

The sources are placed into a special holder that then slides into the hole of the polyethylene cylinder. The geometrical shapes of the sources guide the user into finding the proper orientation. A second background count rate is taken with the sources present so that net counts can later be calculated and a normalization constant can be provided for modification of a previously established calibration curve. Background counts should also be performed as part of the procedures for materials described in Secs. III.B, C, and D.

The MTR assemblies at Dounreay were centered in the insert's notch so that equal amounts of fuel extended from the ends of the AWCC. This technique provides a linear fuel density measurement (in grams per centimeter).

To assay the total fuel content of an assembly, two measurements should be made and their sum taken. The sources can be spaced apart to provide nearly uniform irradiations along the length of the insert. Then the assembly can be measured with one of its ends aligned with an end of the insert, followed by a second measurement with the other ends of the assembly and insert aligned. This two-position measurement was not attempted in the exercise discussed here. Even though the direction of the plates made no difference in these measurements at Dounreay, it is recommended that the plates be positioned vertically to provide the best opportunity for the most uniform irradiation of the fuel plates in the assembly.

B. High-Enrichment Uranium-Oxide Powders

For cans of high-enrichment uranium-oxide powders, the AWCC should be used in the normal fast mode with all the nickel and cadmium in place. With all

cans placed directly on the bottom of the cavity, the material is not centered in the cavity; however, the calibration curve obtained at Winfrith includes the subsequent geometrical differences. All the cans studied were about the same diameter (10 to 11.5 cm). (With much narrower cans, multiplication effects would be reduced and a new calibration required.)

A minor but statistically significant effect in working with these materials is the enrichment dependence of the calibration curve. An enrichment measurement (with a stabilized assay meter or multichannel analyzer) should be performed if a value is uncertain or needs verification.

C. Low-Enrichment Canned Materials

The thermal mode of the AWCC is the preferred mode for low-enrichment materials because self-shielding is not significant and errors in counting statistics are smaller. The nickel reflector and the cadmium linings are therefore removed. The samples are placed on the bottom of the cavity in order to use the Winfrith calibration curve.

The experience with these materials at Winfrith shows that a single calibration curve can be used for a diverse collection of samples, but when possible it is preferable to develop a curve that is specific to a collection of samples with similar characteristics.

D. Very Low Enrichment Materials

When materials have enrichments below $\sim 3.5\%$, the passive mode of the AWCC can determine the ^{238}U content, and then either (1) the declared enrichment allows the calculation of the ^{235}U amount, or (2) an AWCC active mode measurement of the ^{235}U amount (see Sec. II.C) can be used to check the declared enrichment.

In the passive mode, the AmLi sources are removed to a position where they do not affect the count rates. The Winfrith calibration curve was obtained with the nickel and cadmium inserts removed from the AWCC.

IV. CONCLUSIONS

The field tests of the AWCC have demonstrated its applicability for assaying a variety of materials. Cans containing powders, pellets, cartridges,

plates, and assorted residues were examined over an enrichment range up to 93%. Three modes of AWCC operation were employed in the process: fast, thermal, and passive. MTR fuel was also examined, with the use of an insert to hold the assembly and to reposition the sources. It was shown that the response from a complete MTR assembly is not near a saturation value and that a missing plate from any position with the assembly is detectable with more than a 95% probability in 4 min.

A calibration curve was determined for each mode of operation. For the passive mode, it was found that the best assay precision is obtained when the calibration is done over a narrow range of enrichments near the enrichment of the sample of interest.

APPENDIX

CURVE FITTING BY DEMING'S METHOD

The parameters of curves fitted to data in this report were obtained by a least-squares procedure developed by Deming.^{4,5} The procedure allows for errors in both variables in a symmetric fashion, it can be used to fit any function to the data without the necessity of linearization, and it yields estimates of the true values of the coordinates near each data point.

Assume that an ith data point has coordinates given by X_i and Y_i . Errors in each coordinate are used to generate weighting factors w_{xi} and w_{yi} . A relationship between the two variables is assumed, usually in the form of $y = f(x)$. Estimates of the true values, x_i and y_i , are to be found for each data point. These are also called calculated or estimated values of the variables. The function to be minimized is

$$S = \sum_{i=1}^n [w_{xi}(X_i - x_i)^2 + w_{yi}(Y_i - y_i)^2] \quad .$$

If $y_i = f(x_i)$ has m parameters to be found and there are n data points, then a total of $(n + m)$ parameters and calculated coordinates must be found. References 4 and 5 show the details. The codes to implement this technique are not particularly long or involved. The code that follows is written for the Hewlett-Packard HP-85 computer (with 32 k of memory), so it is written in Hewlett-Packard's version of BASIC. With minor changes, and with the omission of the specialized video graphics section, it could be adapted to versions of BASIC for other microcomputers.

The user is led through the data input portion of the code upon execution. Note that initial estimates of the parameters in $f(x)$ must be given. To use the code with a computer that has additional memory, it is convenient to have simple curve-fitting routines automatically provide these estimates.

To change the fitting function, replace the contents within the parentheses of line 350, using X3 for x , Y3 for y , and A3(I) for the parameters. The correct number of parameters must be in line 340.

```

10 !           "DEM85"
20 !
30 !   Deming's method of curve
40 !       fitting with general
50 !       functions.
60 !
70 !   W.E. Deming, Statistical
80 !   Adjustment of Data (John
90 !   Wiley, NY, 1943).
100 !
110 !
120 !       Implementation by
130 !       Phillip M. Rinard
140 !       Safeguards Assay, D-1
150 !       Los Alamos National Lab
160 !       Los Alamos, New Mexico
170 !       September, 1982
180 !
190 !
200 INTEGER I,J,K,N,M,I1,I2,J1,K2,N1,M1
210 !
220 !   For more than 3 parameters
230 !   or more than 30 data
240 !   points, change these
250 !   dimension statements.
260 !
270 DIM X(30),Y(30),X1(30),Y1(30),W1(30),W2(30),A(3),A3(3),
B(3),S9(3),I7(3),J7(3)
280 DIM V1(30),V2(30),S2(30),S3(30),L(30),L9(30),F0(30),
F9(3,30)
290 DIM Q(3,4),Q1(3,4),T$(32)
300 D=.0001
310 ! *****
320 !   CHANGE THE NEXT TWO LINES
330 !   FOR A DIFFERENT FUNCTION.
340 M=2 ! Number of parameters.
350 DEF FNF = Y3-A3(1)*X3^A3(2)
360 ! *****
370 !
380 PRINTER IS 2
390 !
400 FOR I=1 TO M @ A(I)=0 @ NEXT I
410 GOTO 730
420 ! Numerical derivatives of
430 !   the function.
440 !
450 !   df/dx(adj.)
460 DEF FNF1
470 FOR K=1 TO M @ A3(K)=A(K) @ NEXT K @ Y3=Y(I)
480 D3=D*X(I)
490 IF D3=0 THEN D3=D

```



```

500 X3=X(I)*(1+D/2)
510 IF X3=0 THEN X3=D/2
520 F1=FNF
530 X3=X(I)*(1-D/2)
540 IF X3=0 THEN X3=-(D/2)
550 F1=(F1-FNF)/D3
560 FNF1=F1
570 FN END
580 END
590 ! df/da(1)
600 DEF FNF3
610 FOR K=1 TO M @ A3(K)=A(K) @ NEXT K
620 D3=D+A(J)
630 IF D3=0 THEN D3=D
640 A3(J)=A(J)*(1+D/2)
650 IF A3(J)=0 THEN A3(J)=D/2
660 F3=FNF
670 A3(J)=A(J)*(1-D/2)
680 IF A3(J)=0 THEN A3(J)=-(D/2)
690 F3=(F3-FNF)/D3
700 FNF3=F3
710 FN END
720 !
730 CLEAR
740 DISP "The desired fitting function"
750 DISP "      must be in line 350,"
760 DISP " and the number of paramters"
770 DISP "      must be in line 340."
780 DISP
790 DISP "For more than 30 data points"
800 DISP " or more than 3 parameters."
810 DISP "      change lines 270-290."
820 DISP
830 DISP
840 PRINT "There are ";M;" parameters."
850 PLIST 350,350
860 PRINT
870 DISP "Data entry is from..."
880 DISP "  1. Keyboard"
890 DISP "  2. Tape File"
900 DISP
910 INPUT I
920 ON I GOTO 930,1540
930 CLEAR
940 DISP "What type of data?"
950 DISP "  1. Errors in x & y."
960 DISP "  2. Errors in y only."
970 DISP "  3. Errors in x only."
980 DISP "  4. No errors in x & y."
990 INPUT I1

```

```

1000 CLEAR
1010 ON I1 GOTO 1020,1040,1060,1070
1020 DISP "Enter x, v, sigma x, sigma y."
1030 GOTO 1090
1040 DISP "Enter x, v, sigma y."
1050 GOTO 1090
1060 DISP "Enter x, y, sigma x."
1070 DISP "Enter x, y."
1080 !
1090 DISP @ DISP "      End with x=1E99."
1100 PRINT "      n      x      v"
1110 PRINT "      sigma x      sigma y"
1120 PRINT "-----"
1130 IF I=2 THEN 1580
1140 N=1
1150 ON I1 GOTO 1160,1180,1210,1240
1160 INPUT X(N),Y(N),W1(N),W2(N)
1170 GOTO 1260
1180 INPUT X(N),Y(N),W2(N)
1190 W1(N)=W2(N)/1000000
1200 GOTO 1260
1210 INPUT X(N),Y(N),W1(N)
1220 W2(N)=W1(N)/1000000
1230 GOTO 1260
1240 INPUT X(N),Y(N)
1250 W1(N)=.000001 @ W2(N)=.000001
1260 IF X(N)=1.E99 THEN 1350
1270 PRINT USING 1280 ; N,X(N),Y(N)
1280 IMAGE 2X,DDD,2X,2(D.DDDE,2X)
1290 PRINT USING 1300 ; W1(N),W2(N)
1300 IMAGE 7X,2(D.DDDE,2X)
1310 W1(N)=1/W1(N)^2
1320 W2(N)=1/W2(N)^2
1330 N=N+1 @ GOTO 1150
1340 ! End of keyboard data entry
1350 N=N-1 @ PRINT @ PRINT
1360 CLEAR
1370 DISP "Save data on tape? Y/N"
1380 INPUT I$
1390 IF I$<>"Y" THEN 1670
1400 ! Save data on tape
1410 DISP "File name? (6 chrs. max.)"
1420 INPUT F$ @ IF LEN(F$)>6 THEN 1410
1430 ON ERROR GOTO 1410
1440 PRINT "Data File is ";F$
1450 CREATE F$,1,32*N+100
1460 ASSIGN# 1 TO F$
1470 PRINT# 1 ; N
1480 FOR I=1 TO N
1490 PRINT# 1 ; X(I),Y(I),W1(I),W2(I)

```

```

1500 NEXT I
1510 ASSIGN# 1 TO *
1520 CLEAR @ GOTO 1670
1530 ! Load data from tape
1540 CLEAR @ DISP "Name of tape file?"
1550 INPUT F$
1560 PRINT "Data File is ";F$
1570 PRINT @ GOTO 1100
1580 ASSIGN# 1 TO F$
1590 READ# 1 : N
1600 FOR I=1 TO N
1610 READ# 1 : X(I),Y(I),W1(I),W2(I)
1620 PRINT USING 1280 : I,X(I),Y(I)
1630 PRINT USING 1300 : 1/SQR(W1(I)),1/SQR(W2(I))
1640 NEXT I
1550 ASSIGN# 1 TO *
1660 ! Input estimated parameters
1670 CLEAR @ DISP "Give parameter estimates."
1680 DISP
1690 FOR I=1 TO M
1700 DISP "Parameter #":I
1710 INPUT A(I)
1720 NEXT I
1730 CLEAR @ DISP "Be patient. I'm doing my best."
1740 PRINT
1750 PRINT @ PRINT "Parameters:"
1760 FOR J=1 TO M
1770 A3(J)=A(J)
1780 PRINT USING "2X,DD,2X,D.DDDDE" : J,A(J) @ NEXT J
1790 FOR I=1 TO N
1800 X3=X(I) @ Y3=Y(I)
1810 F0(I)=FNF
1820 NEXT I
1830 FOR I=1 TO N
1840 L(I)=FNF1+FNF1/W1(I)+1/W2(I)
1850 L(I)=SQR(L(I))
1860 NEXT I
1870 FOR J=1 TO M
1880 FOR I=1 TO N
1890 X3=X(I) @ Y3=Y(I)
1900 F9(J,I)=FNF3
1910 NEXT I @ NEXT J
1920 FOR I=1 TO N
1930 F0(I)=F0(I)/L(I)
1940 FOR J=1 TO M
1950 F9(J,I)=F9(J,I)/L(I)
1960 NEXT J @ NEXT I
1970 FOR K1=1 TO M
1980 Q(K1,M+1)=0
1990 FOR I=1 TO N

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2000 Q(K1,M+1)=Q(K1,M+1)+F0(I)*F9(K1,I)
2010 NEXT I
2020 FOR K2=1 TO M
2030 Q(K1,K2)=0
2040 FOR I=1 TO N
2050 Q(K1,K2)=Q(K1,K2)+F9(K1,I)*F9(K2,I)
2060 NEXT I
2070 Q1(K1,K2)=Q(K1,K2)
2080 NEXT K2 @ NEXT K1
2090 GOSUB 3150 ! Cholesky's solution of linear equations
2100 FOR I=1 TO M
2110 B(I)=A(I) @ A(I)=B(I)-G(I)
2120 NEXT I
2130 FOR J=1 TO M
2140 IF B(J)=0 THEN 2160
2150 IF ABS(G(J)/B(J))>.0001 THEN 1750
2160 NEXT J
2170 PRINT @ PRINT "*****" @
PRINT
2180 PRINT "Final Parameters:"
2190 FOR J=1 TO M
2200 PRINT USING "2X,DD,2X,D.DDDDE" ; J,A(J) @ NEXT J
2210 FOR I=1 TO N
2220 L(I)=FNF1*FNF1/W1(I)+1/W2(I)
2230 J=1
2240 L9(I)=FNF-FNF3*G(1)
2250 IF M=1 THEN 2290
2260 FOR J=2 TO M
2270 L9(I)=L9(I)-FNF3*G(J)
2280 NEXT J
2290 L9(I)=L9(I)/L(I)
2300 NEXT I
2310 S=0
2320 FOR I=1 TO N
2330 X3=X(I) @ Y3=Y(I)
2340 V1(I)=L9(I)*FNF1/W1(I)
2350 V2(I)=L9(I)/W2(I)
2360 X1(I)=X(I)-V1(I)
2370 Y1(I)=Y(I)-V2(I)
2380 S=S+W1(I)*V1(I)^2+W2(I)*V2(I)^2
2390 NEXT I
2400 IF N=M THEN 2430
2410 S2=S/(N-M)
2420 GOTO 2440
2430 S2=0
2440 FOR I=1 TO M
2450 FOR J=1 TO M
2460 Q(I,J)=Q1(I,J) ! Restore Q
2470 NEXT J @ NEXT I
2480 GOSUB 3560 ! Matrix Inversion for covariances
2490 FOR J=1 TO M

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2500 S9(J)=SQR(Q(J,J)*S2)
2510 NEXT J
2520 PRINT "Std. Errors in Parameters:"
2530 FOR J=1 TO M
2540 PRINT USING "2X,DD,2X,D.DDDE" ; J,S9(J) @ NEXT J
2550 !
2560 ! t-Table for 95% confidence
2570 DATA 12.706,4.303,3.182,2.776,2.571,2.447,2.365
2580 DATA 2.306,2.262,2.228,2.201,2.179,2.16,2.145,2.131,2.
2.012,2.11
2590 DATA 2.101,2.093,2.086,2.08,2.074,2.069,2.064,2.06,
56,2.052
2500 DATA 2.048,2.045,2.042,1.96
2610 T=1.67 @ RESTORE 2570
2620 IF N-2>30 THEN 2640
2630 FOR I=1 TO N-2 @ READ T@ NEXT I
2640 PRINT @ PRINT "Variances and Covariances:"
2650 FOR K=1 TO M @ FOR L=K TO M
2660 PRINT USING 2670 ; K,L,S2*Q(K,L)
2670 IMAGE 2X,DD,2X,DD,3X,D.DDDE
2680 NEXT L @ NEXT K
2690 ! Std. Errors in y
2700 FOR I1=1 TO N
2710 X3=X(I1) @ Y3=Y(I1) @ S3(I1)=0
2720 FOR J5=1 TO M
2730 J=J5 @ F4=FN F3
2740 S3(I1)=S3(I1)+Q(J5,J5)*F4*F4
2750 IF M=1 THEN 2800
2760 FOR K5=J5+1 TO M
2770 J=K5 @ F5=FN F3
2780 S3(I1)=S3(I1)+2*Q(J5,K5)*F4*F5
2790 NEXT K5 @ NEXT J5
2800 S3(I1)=T*SQR(S2*S3(I1))
2810 NEXT I1
2820 ! Calculate r^2
2830 Y3=0 @ S1=0 @ S4=0
2840 FOR I=1 TO N
2850 S1=S1+Y(I) @ NEXT I @ Y7=S1/N
2860 S1=0
2870 FOR I=1 TO N @ X3=X(I) @ S4=S4+(Y(I)-Y7)^2 @
S1=S1+(Y(I)+FN F)^2
2880 NEXT I
2890 R2=1-S1/S4
2900 PRINT @ PRINT @ PRINT "r^2 =";R2
2910 PRINT "S =";S
2920 PRINT @ PRINT "      n      x-adj.      x-resid."
2930 PRINT "      y-adj.      y-resid."
2940 PRINT "      y-std. error"
2950 PRINT "-----"
2960 FOR I=1 TO N
2970 PRINT USING 2980 ; I,X1(I),V1(I) @ PRINT USING 2990 ;
Y1(I),V2(I) @ PRINT USING 3000 ; S3(I)
2980 IMAGE 2X,DD,2(2X,D.DDDE)
2990 IMAGE 4X,2(2X,D.DDDE)

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3000 IMAGE 12X,D.DDDE
3010 NEXT I
3020 PRINT @ PRINT @ PRINT
3030 CLEAR @ DISP "Plot it? Y/N"
3040 INPUT F$@ IF F$<>"Y" THEN 3090
3050 GOSUB 4160
3060 CLEAR @ DISP "Replot it? [Y/N]"
3070 DISP @ INPUT F$@ IF F$<>"Y" THEN 3090
3080 PRINT @ GOTO 3050
3090 CLEAR @ DISP @ DISP "All done, finally!"
3100 END
3110 ! Cholesky's solution of n liner equations
3120 ! M.L. James, G.M. Smith & J.C. Wolford.
3130 ! "Applied Numerical Methods for Digital Computation",
3140 ! (Harper & Row, NY 1977), p. 194.
3150 IF M>1 THEN 3180
3160 G(1)=Q(1,2)/Q(1,1)
3170 RETURN
3180 FOR K=2 TO M+1
3190 Q(1,K)=Q(1,K)/Q(1,1)
3200 NEXT K
3210 FOR I=2 TO M
3220 K=I
3230 FOR I1=K TO M
3240 T=0
3250 K1=K-1
3260 FOR J=1 TO K1
3270 T=T+Q(I1,J)*Q(J,K)
3280 NEXT J
3290 Q(I1,K)=Q(I1,K)-T
3300 NEXT I1
3310 I1=I+1
3320 FOR J1=I1 TO M+1
3330 T=0
3340 I2=I-1
3350 FOR J=1 TO I2
3360 T=T+Q(I,J)*Q(J,J1)
3370 NEXT J
3380 Q(I,J1)=(Q(I,J1)-T)/Q(I,I)
3390 NEXT J1
3400 NEXT I
3410 G(M)=Q(M,M+1)
3420 M1=M-1
3430 FOR N1=1 TO M1
3440 T=0
3450 I=M-N1
3460 I1=I+1
3470 FOR K=I1 TO M
3480 T=T+Q(I,K)*G(K)
3490 NEXT K

```

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3500 G(I)=Q(I,M+1)-T
3510 NEXT N1
3520 RETURN
3530 ! Matrix Inversion, from P.R. Bevington,
3540 ! "Data Reduction and Analysis for the Physical
    Sciences",
3550 ! (McGraw-Hill, NY, 1969), p. 302.
3560 D1=1
3570 FOR K=1 TO M
3580 C=0
3590 FOR I=K TO M
3600 FOR J=K TO M
3610 C1=ABS(C)-ABS(Q(I,J))
3620 IF C1>0 THEN 3640
3630 C=Q(I,J) @ I7(K)=I @ J7(K)=J
3640 NEXT J
3650 NEXT I
3660 IF C<>0 THEN 3680
3670 D1=0 @ RETURN
3680 I=I7(K)
3690 IF I<K THEN 3590
3700 IF I=K THEN 3750
3710 FOR J=1 TO M
3720 T=Q(K,J)
3730 Q(K,J)=Q(I,J)
3740 Q(I,J)=-T @ NEXT J
3750 J=J7(K)
3760 IF J<K THEN 3590
3770 IF J=K THEN 3820
3780 FOR I=1 TO M
3790 T=Q(I,K)
3800 Q(I,K)=Q(I,J)
3810 Q(I,J)=-T @ NEXT I
3820 FOR I=1 TO M
3830 IF I=K THEN 3850
3840 Q(I,K)=-(Q(I,K)/C)
3850 NEXT I
3860 FOR I=1 TO M
3870 FOR J=1 TO M
3880 IF I=K THEN 3910
3890 IF J=K THEN 3910
3900 Q(I,J)=Q(I,J)+Q(I,K)*Q(K,J)
3910 NEXT J @ NEXT I
3920 FOR J=1 TO M
3930 IF J=K THEN 3950
3940 Q(K,J)=Q(K,J)/C
3950 NEXT J
3960 Q(K,K)=1/C
3970 D1=D1*C @ NEXT K
3980 FOR L1=1 TO M
3990 K=M-L1+1

```

```

4000 J=I7(K)
4010 IF J<=K THEN 4060
4020 FOR I=1 TO M
4030 T=Q(I,K)
4040 Q(I,K)=-Q(I,J)
4050 Q(I,J)=T @ NEXT I
4060 I=J7(K)
4070 IF I<=K THEN 4130
4080 FOR J=1 TO M
4090 T=Q(K,J)
4100 Q(K,J)=-Q(I,J)
4110 Q(I,J)=T
4120 NEXT J
4130 NEXT L1
4140 RETURN
4150 ! Plotting Section
4160 CLEAR @ DISP "Give comment lines about graph."
4170 DISP @ DISP "No more than 32 chrs. per line."
4180 DISP @ DISP "Terminate with empty input." @ DISP
4190 INPUT T$ @ IF T$="" THEN 4210
4200 PRINT T$ @ GOTO 4190
4210 PRINT @ CLEAR @ DISP "x-Axis information:"
4220 DISP @ DISP "x-min, x-max, x-inc."
4230 INPUT X1,X2,X3
4240 DISP "What units?" @ INPUT U$
4250 PRINT " (Axis along edge of paper)"
4260 PRINT X1;" <";X3;"> ";X2;" ";U$
4270 CLEAR @ DISP "y-Axis information:"
4280 DISP @ DISP "y-min, y-max, y-inc.?"
4290 INPUT Y1,Y2,Y3
4300 DISP "What units?" @ INPUT U$
4310 PRINT " (Axis across paper)"
4320 PRINT Y1;" <";Y3;"> ";Y2;" ";U$
4330 PRINT @ PRINT @ PRINT @ PRINT
4340 Y4=Y1-7*(Y2-Y1)/24
4350 GCLEAR ,
4360 SCALE X1,X2,Y1,Y2
4370 XAXIS Y1,X3,X1,X2
4380 YAXIS X2,Y3,Y1,Y2
4390 YAXIS X1,Y3,Y1,Y2
4400 XAXIS Y2,X3,X1,X2
4410 H=(X2-X1)*1/64 @ V=(Y2-Y1)*1/56
4420 ! Plot squares for data points & pluses for adjusted
data.
4430 FOR I=1 TO N
4440 MOVE X(I),Y(I)
4450 IMOVE -(H/2),V/2 @ IDRAW H,0 @ IDRAW 0,-V @ IDRAW -H,0
@ IDRAW 0,V
4460 MOVE X1(I),Y1(I)
4470 IMOVE -(H/2),0 @ IDRAW H,0 @ IMOVE -(H/2),-(V/2) @ IDR
AW 0,V
4480 NEXT I
4490 D9=(X2-X1)/100

```



```

4500 PENUP
4510 FOR X3=X1 TO X2 STEP D9
4520 Y3=0
4530 IF X3=0 THEN X3=.0001
4540 PLOT X3,-FNF
4550 NEXT X3
4560 ! Plot 95% confidence limits
4570 D9=(X2-X1)/32 @ Z1=1 @ Y3=0
4580 PENUP
4590 S4=0 @ IF N=M THEN 4840
4600 FOR I=1 TO N @ X3=X(I)
4610 S4=S4+(Y(I)+FNF)^2 @ NEXT I
4620 S4=SQR(S4/(N-M))
4630 X4=X1 @ X5=X4+D9
4640 ! For higher resolution on confidence limits, change
STEP D9 to STEP D9/5 in the next line.
4650 FOR X7=X4 TO X5 STEP D9
4660 X3=X7
4670 IF X3=0 THEN X3=.0001
4680 Z=0
4690 FOR J5=1 TO M
4700 J=J5 @ F4=FNF3
4710 Z=Z+Q(J5,J5)*F4*F4
4720 IF M=1 THEN 4770
4730 FOR K5=J5+1 TO M
4740 J=K5 @ F5=FNF3
4750 Z=Z+2*Q(J5,K5)*F4*F5
4760 NEXT K5 @ NEXT J5
4770 Z=T*SQR(Z*S2)
4780 PLOT X3,-FNF+Z1*Z
4790 NEXT X7
4800 PENUP @ X4=X4+2*D9 @ X5=X5+2*D9
4810 IF X4<X2 THEN 4650
4820 IF Z1<0 THEN 4840
4830 Z1=-1 @ GOTO 4630
4840 COPY @ PRINT @ PRINT @ PRINT @ PRINT @ PRINT @ PRINT @
PRINT @ PRINT
4850 RETURN

```

REFERENCES

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3. LASL Group X-6, "MCNP--A General Monte Carlo Code for Neutron and Photon Transport, Version 2B," Los Alamos Scientific Laboratory report LA-7396-M, Rev. (November 1979).
4. W. E. Deming, Statistical Adjustment of Data (John Wiley and Sons, Inc., New York, 1943).
5. J. R. Wolberg, Prediction Analysis (D. Van Nostrand, Inc., Princeton, New Jersey, 1967).