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Primer on Laser Scattering Diagnostics

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Primer on Laser Scattering Diagnostics

Franz C. Jahoda

**Lecture Notes for
Monterey Mini-Course on Plasma Diagnostics
May 17-19, 1978**

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PRIMER ON LASER SCATTERING DIAGNOSTICS

by

Franz C. Jahoda

ABSTRACT

This report is a set of lecture notes as used for a presentation on "Light Scattering" at the minicourse on Modern Plasma Diagnostics held in Monterey, California, May 17 through 19, 1978, in conjunction with the 1978 IEEE Conference on Plasma Science.

The theory of laser scattering is presented in abbreviated format, with emphasis on physical interpretation, followed by sections on laser sources, practical considerations in designing experiments, and current developments in extending the techniques to multispace and multitime point measurements.

THEORY

1. Scattering from a single electron in an EM wave field:

a) Amplitude

Classical Electrodynamics:

$$P_e = \frac{2}{3} \frac{e^2 \ddot{r}^2}{c^3} \rightarrow P = \frac{8\pi}{3} \left(\frac{e^2}{mc^2} \right)^2 I$$

$P_e \equiv$ power scattered into 4π due to charge acceleration $\ddot{r}$

$$I = \text{incident intensity} \quad I = \frac{c}{4\pi} \vec{E} \times \vec{B} = \frac{cE^2}{4\pi}$$

$$\frac{8\pi}{3} \left(\frac{e^2}{mc^2} \right)^2 = \frac{8\pi}{3} r_e^2 = 6.65 \times 10^{-25} \text{ cm}^2.$$

Small value of Thomson cross section, requires high power lasers:

Total scattered signal $\sim 10^{-12}$ incident signal MW $\rightarrow \mu\text{W}$.

Have ignored angular dependence of scattering cross section. Due to transverse polarization nature of EM waves, this is not uniform into total solid angle.

a) Plane polarized

$$\frac{dP_e}{d\Omega} = r_e^2 \sin^2 \phi_E I$$

$$= r_e^2 [\hat{n} \times (\hat{n} \times \hat{e})]^2 I$$

b) Unpolarized

$$\frac{dP_e}{d\Omega} = r_e^2 \frac{(1 + \cos^2 \theta)}{2} I$$

(Both cases $\int d\Omega \rightarrow \frac{8\pi}{3}$ factor.)

$$E = E_i \cos(k_i \cdot r(t) - \omega_i t)$$

$$\ddot{r} = -\frac{e}{m} E_i \cos(k_i \cdot r(t) - \omega_i t).$$

Implicit time dependence of electron position means that Fourier analysis of scattered radiation will contain frequencies shifted from ω_i by amounts dependent on the characteristics of the electron motion.

i.e., scatterer does not remain at fixed spatial location; at fixed spatial location of detector the effective frequency ω_s includes the time-dependent parts originating from $r(t)$. Thus $\omega_s \neq \omega_i$.

Calculation is further complicated by the fact that for the wave disturbance at the detector at time t it is the position of the scatterer at the retarded time $t' = t - \frac{1}{c} [R - \hat{n} \cdot \vec{r}(t')]$ that enters computation.

Simplest case:

electron has only thermal motion

$$\vec{r}(t) = \vec{r}_0 + \vec{v} t.$$

Instead of pursuing formalism, $\rightarrow E_{sc} = E(\omega_s)$, can get result from simple physical argument:

- a) electron sees the incident wave at Doppler-shifted frequency corresponding to its velocity component, v_{i11} , along incident wave direction:

$$(1) \quad \omega' = \omega_i (1 - v_{i11}/c)$$

- b) electron re-radiates with a further Doppler-shifted frequency corresponding to its velocity component, v_{s11} , along the scattered wave direction:

(2) $w_s = w_i (1 + v_{s11}/c)$, the + sign appearing because we are considering the emitter.

Combining (1) and (2)

$$w_s = w_i (1 - v_{i11}/c) (1 + v_{s11}/c)$$

$$w_s \approx w_i + (\vec{k}_s - \vec{k}_i) \cdot \vec{v} \quad \text{where } \vec{k}_s = \hat{n} \frac{w_i}{c}$$

$$\vec{k}_i = \hat{i} \frac{w_i}{c}.$$

Small photon energy (no electron recoil) and energy conservation requires

$$|\vec{k}_s| \approx |\vec{k}_i|.$$

$$\text{Define } \vec{k} \equiv \vec{k}_s - \vec{k}_i$$

$$w = \Delta w = w_s - w_i = \vec{k} \cdot \vec{v}.$$

Then

$$|k| \approx 2|k_i| \sin \theta/2.$$

Theory is always cast in terms of scattering vector $\vec{k}$. Magnitude is function of incident wavelength + scattering angle. Direction is $\perp$ to bisector of incident, scattered directions

Choose a scattering geometry and given incident wavelength - fixes $\vec{k}$;
 scan λ_s : (scans $f(\vec{v})$).

Even at this point, have already made quite a number of explicit and implicit restrictive assumptions. (As some of the restrictions are dropped, theory becomes both more complicated and more generally applicable.)

- 1) $v/c \ll 1$
 $\therefore$ No force on electron due to EM wave magnetic field.
- 2) neglect ion scattering compared to electron scattering.
- 3) quantum effects neglected, $\frac{h\nu}{mc^2} \ll 1$; (e.g., no Compton effect).
- 4) scattering in far field, $R \gg v^{1/3}$.

In addition, in discussing a plasma will assume

- 1) $w \gg w_{pe}$, $\therefore$ negligible attenuation; single scattering.
- 2) Incident electric field small enough that it will not perturb the plasma

$$v_{\text{induced}} = \frac{eE_i}{m\omega} \ll v_{\text{th}} = \left(\frac{2kT}{m}\right)^{1/2}.$$

$\therefore$ unperturbed orbit. (Very weak constraint, easily satisfied.)

2. Sum over many scatterers:

$$\vec{E}_s = \sum_i \vec{E}_i$$

$$\begin{aligned} \frac{dP}{d\Omega} &= C \overline{E_s^2} = C (\sum_j \sum_k \vec{E}_j \cdot \vec{E}_k^*) \\ &= C (\sum_j \overline{E_j^2} + \sum_j \sum_{k \neq j} \vec{E}_j \cdot \vec{E}_k^*). \end{aligned}$$

Whether or not second summation term averages to zero depends on the phase correlations of scattering from different electrons. Instead of pursuing the mathematics - some physical insight:

- 1) Phase term in E_j has a position dependent part, $\vec{k} \cdot \vec{r}$.
- 2) Given scattering geometry, k is fixed.
- 3) Physical description of a plasma: each charge - due to Coulomb forces with all other charges - surrounds itself with a "cloud" of shielding charge. Its potential is screened out over a Debye sphere. (Yet many particles within a Debye sphere is definition of a plasma.)

$$\lambda_{De} = \left(\frac{kT_e}{4\pi e^2 n} \right)^{1/2}.$$

- 4) On a distance scale small compared to Debye length, positions are uncorrelated.

On a distance scale comparable to Debye length, there is a high degree of correlation.

On a distance scale large compared to Debye length, the clouds are uncorrelated, but their random motions are governed by the slower, heavier ions.

- 5) Examine phase factors $\vec{k} \cdot \vec{r}$ arising from collection of scatterers.

- a) If $k \lambda_D \ll 1$, all randomly spaced scatterers within a Debye sphere give nearly the same phase factor, i.e., high degree of correlation (2nd sum of importance) and full treatment will reflect characteristics of distribution measured on scale $\gg \lambda_D$.

- b) If $k\lambda_D \sim 1$, expect strong evidence of the correlations in positions that exist over scale length λ_D .
- c) If $k\lambda_D \gg 1$, large phase changes in the scattered power, even for electrons separated by much less than a Debye length. Since these are randomly distributed, expect them to cancel each other out.

$$\alpha \equiv (k\lambda_D)^{-1} \quad \text{Salpeter parameter}$$

$$\alpha = \frac{\lambda}{4\pi \lambda_D \sin \theta/2}$$

Case c) $\alpha \ll 1$ incoherent (independent) scatterers

Expect: total scattered power $\sim n_e$
spectral distribution $\sim f(v_e)$.

Summing over a Maxwellian thermal distribution, each different electron contributes a different scattered frequency component according to

$$\omega_s = \omega_i + \vec{k} \cdot \vec{v}_e \quad (\vec{k} \text{ is fixed}).$$

Changing from frequency scale to wavelength scale:

$$\frac{dP(\Delta\lambda)}{d\Omega} = C n_e \exp \left[\frac{-c^2 m_e}{8 k T_e \lambda_i^2 \sin^2(\theta/2)} \Delta\lambda^2 \right]$$

$$\text{for } \lambda = 6943 \text{ \AA} \quad \theta = 90^\circ \quad \text{HWHM} \approx 16 T_e^{1/2} \text{ \AA} \quad (T_e \text{ in eV})$$

$$\alpha \text{ is small for:} \quad \text{scales} \sim \frac{\lambda (n)^{1/2}}{(\sin \theta/2) (T_e)^{1/2}}$$

small λ , n
large θ , T

$$\text{e.g., } \alpha = 0.1 \text{ for } \lambda = 6943 \text{ \AA}, \quad \theta = 90^\circ, \quad n_e = 2 \times 10^{15} \text{ cm}^{-3}, \quad T_e = 20 \text{ eV.}$$

Formalism:

$$1) \quad \vec{E}(R, t|j) \approx - \left(\frac{e^2}{mc^2} \right) \frac{(\hat{n}_j \times \vec{E}_i) \times \hat{n}_j \cos[\vec{k}_i \cdot \vec{x}_j(t_j') - w_i t_j']}{|\vec{R} - \vec{x}(t_j')|}.$$

$$2) \quad \vec{E}_s(R, t) = \sum_j \vec{E}(R, t|j).$$

$$3) \quad n(\vec{x}, t) = \sum_j \delta(\vec{x} - \vec{x}_j(t)) \quad \text{to convert } \sum_j \vec{E} \text{ to } \int_V \vec{E} d^3x.$$

4) Neglect slight changes in amplitude and direction of E field from elementary volume elements d^3x (but retain phase differences!), so

$$\frac{(\hat{n}_j \times \vec{E}_i) \times \hat{n}_j}{|\vec{R} - \vec{x}|} \approx \frac{\vec{E}_i}{R}.$$

$$5) \quad r_0 = e^2/mc^2.$$

$$\rightarrow \vec{E}_s(R, t) \approx - \frac{r_0}{R} \vec{E}_i \int_V d^3x' n(\vec{x}', t') \cos(\vec{k}_i \cdot \vec{x}' - w_i t')$$

where the primes indicate retarded time and corresponding retarded spatial coordinates.

Using:

$$1) \quad t' = t'(t, x').$$

$$2) \quad \cos \theta = 1/2 (\exp i\theta + \exp -i\theta).$$

$$3) \quad n(\vec{x}, t) = \frac{1}{V} \sum_{\vec{k}} \int_{-\infty}^{\infty} \frac{dw}{2\pi} n(\vec{k}, w) \exp[i(\vec{k} \cdot \vec{x} - wt)].$$

4) Detector responds to a limited range Δw of central frequency $\bar{w}$.

5) Detected power

$$\approx \frac{c}{4\pi} \overline{[\delta \vec{E}_{sc}]^2} R^2 d\Omega$$

$$\text{where } \delta \vec{E}_{sc} \text{ is } \left. \frac{dE_{sc}}{dw} \right|_{\Delta w}$$

and time average is over a time interval $T \gg \bar{w}^{-1}$.

$$6) \quad I_0 = c E_i^2 / 8\pi \quad ; \quad E_0^\perp = \frac{(\hat{n} \times \vec{E}_i) \times \hat{n}}{|\vec{E}_i|}$$

$$\vec{k} = w_i \hat{n} / c - \vec{k}_i$$

$$\bar{w} = w - w_i$$

- 7) Make an ensemble average over phase space $\langle \rangle$, using an estimate of most probable electron velocity + position distribution (subject to known macro-state)

$$\rightarrow d^2P(\theta, w) \approx \Delta w \, d\Omega \, \frac{I_0}{2\pi} \, r_0^2 \, (E_0^\perp)^2 \, \lim_{T \rightarrow \infty} \frac{\langle |n(\vec{k}, \bar{w})|^2 \rangle}{T}$$

and integrating over frequency

$$\text{with} \quad n(\vec{k}, t) = \int_{-\infty}^{\infty} \frac{dw}{2\pi} \exp(-iwt) n(\vec{k}, w)$$

$$\rightarrow dP(\theta) \approx I_0 \, r_0^2 \, (E_0^\perp)^2 \, d\Omega \, \langle |n(\vec{k}, t)|^2 \rangle.$$

Total average power received in solid angle $d\Omega$.

Summarizing:

Given w_i , $\vec{k}_i$, (incident quantities) and scattering angle θ ,
the power received at frequency w is determined by

$n(\vec{k}, \omega)$ - the coefficient of the electron microdensity expressed as a superposition of plane waves; the wave with complex amplitude $n(\vec{k}, \omega)$ has phase velocity $\vec{v}_{ph} = (\bar{\omega}/k)\vec{k}$ and wavelength $(2\pi/k)$, - i.e., electrons contribute to the extent that the required velocity projections and spatial fluctuations of the required wavelength occur together. (If there were no spatial fluctuations, there would be complete phase cancellation.)

Can be given physical picture in terms of "Bragg reflections" off traveling density waves giving Doppler shifted frequencies.

Have formalism: Need to calculate a) power b) frequency distribution.

a) Power

$$n(\vec{k}, t) = \int_V d^3x \exp(-i \vec{k} \cdot \vec{x}) n(\vec{x}, t) = \sum_{j=1}^{N_e} \exp(-i \vec{k} \cdot \vec{x}_j(t))$$

$$\rightarrow |n(\vec{k}, t)|^2 = N_e + \sum_j^N \sum_{\ell}^N \cos \vec{k} \cdot (\vec{x}_\ell - \vec{x}_j)$$

< > must be same, no matter which pair is chosen and

$$\text{with } N_e(N_e - 1) \approx N_e^2$$

$$\langle |n(\vec{k}, t)|^2 \rangle = N_e + N_e^2 \langle \cos \vec{k} \cdot (\vec{x}_2 - \vec{x}_1) \rangle .$$

To compute $\langle \cos \vec{k} \cdot (\vec{x}_2 - \vec{x}_1) \rangle$

$$1) = \int_V dV \rho(r) \cos(\vec{k} \cdot \vec{r})$$

2) assume probability density $\rho(r)$ of other electrons around electron (1) is given by Boltzmann distribution

$$\langle n(r) \rangle \approx n_0 \exp\left(\frac{\phi(r)}{T}\right)$$

$$3) \quad \phi(r) \approx -\frac{e}{r} \exp(-r/D)$$

$D \equiv$ composite Debye length

$$\frac{1}{D^2} = \frac{1}{D_e^2} + \frac{1}{D_I^2} ; D_\sigma^2 = \frac{T_\sigma}{4\pi n Z_o e^2} , \sigma = e, I$$

$$4) \quad T_e = T_i , \quad \alpha \equiv (k D_e)^{-1}$$

$$\langle |n(\vec{k}, t)|^2 \rangle \approx N_e \left[\frac{1 + Z\alpha^2}{1 + \alpha^2(1+Z)} \right]$$

$$dP(\theta) = I_o r_o^2 (\hat{E}_o^\perp)^2 d\Omega N_e \left[\frac{1 + Z\alpha^2}{1 + \alpha^2(1+Z)} \right]$$

$\alpha \ll 1$, recover previous result: $\sim N_e$, and independent of electron charge. This means the relative frequency of appearance and magnitude of a fluctuation in electron density of wavelength much smaller than a Debye length is the same as if the electrons were a gas of non-interacting particles.

$\alpha \gg 1$, $Z = 1$, $\left[\quad \right] \rightarrow 1/2$, and average power scattered is reduced by a factor of 2. Long range forces reduce the large scale fluctuations.

b) Frequency Spectrum of Scattered Radiation.

$$\text{Start with } d^2 P(\theta, \omega) \approx \Delta\omega d\Omega \frac{I_o}{2\pi} r_o^2 (\hat{E}_o^\perp)^2 \lim_{T \rightarrow \infty} \frac{\langle |n(\vec{k}, \omega)|^2 \rangle}{T}$$

$$\lim_{T \rightarrow \infty} \frac{\langle |n(\vec{k}, \omega)|^2 \rangle}{T} \rightarrow \int_{-\infty}^{\infty} d\tau \exp(-i\omega\tau) \langle n(\vec{k}, t+\tau) n^*(\vec{k}, t) \rangle$$

now need $n(\vec{k}, t)$:

One way to proceed: concept of uncorrelated "dressed" quasi-particles that replace actual "bare" particles correlated by Coulomb forces. Each quasi-particle consists of a charged particle with its associated induced, shielding Debye cloud. Density + velocity distribution of quasi-particles is equivalent to that of actual particles - but being uncorrelated they move with constant velocity (straight line, fixed speed).

1) compute shielding "dressing" around test particle of charge

q and position $\vec{x}(t) = \vec{x}_0 + \vec{v}t$:

Using the usual EM relations

$$\text{div } \vec{E} = 4\pi(\rho + \sum_{\sigma} \rho_{\sigma}) \quad \sigma = e, I \quad \rho_{\sigma} \text{ is induced charge density}$$

$$\rho_{\sigma} = -\text{div} \vec{p}_{\sigma}$$

$$\vec{D} = \vec{E} + 4\pi \sum_{\sigma} \vec{p}_{\sigma}$$

$$\vec{p}_{\sigma} = \frac{\chi_{\sigma} \vec{E}}{4\pi}$$

$$\vec{D} = \epsilon \vec{E} \quad \text{where } \epsilon = 1 + \chi_e + \chi_I$$

with all the variables being functions of either $(\vec{k}, \omega)$ or $(\vec{x}, t)$ (or $(\vec{k}, t)$) and transforming between them with Fourier transforms

$$\rightarrow n_{je}(\vec{k}, t) = \frac{q}{e} \left\{ \frac{\chi_e(\vec{k}, w)}{\epsilon(\vec{k}, w)} \right\}_{w=\vec{k} \cdot \vec{v}_j} \exp(-i \vec{k} \cdot \vec{x}_j(t))$$

↓
↓

shielding charge +1 for ions
 around electron -1 for electrons

(leaves the real work of evaluating χ_e , χ_I yet to come)

$$\begin{aligned} n_j^{\text{core}}(\vec{k}, t) &= \int_V d^3x \exp(-i \vec{k} \cdot \vec{x}_j(t)) \delta[\vec{x} - \vec{x}_j(t)] \\ &= \exp(-i \vec{k} \cdot \vec{x}_j(t)) \end{aligned}$$

$$n = n(\vec{k}, t) = \sum_j (n_j^{\text{core}} + \sum_{\sigma} n_{j\sigma}^{\text{induced}})$$

$$\begin{aligned} &= \sum_{j=1}^{N_e} \exp[-i \vec{k} \cdot \vec{x}_j(t)] \left\{ 1 - \frac{\chi_e(\vec{k}, w)}{\epsilon(\vec{k}, w)} \right\}_{w=\vec{k} \cdot \vec{v}_j} \\ &+ \sum_{j=1}^{N_I} \exp[-i \vec{k} \cdot \vec{x}_j(t)] \left\{ \frac{Z \chi_e(\vec{k}, w)}{\epsilon(\vec{k}, w)} \right\}_{w=\vec{k} \cdot \vec{v}_j} . \end{aligned}$$

Use this expression to calculate:

$$\langle n(\vec{k}, t + \tau) n^*(\vec{k}, t) \rangle .$$

Formidable looking amount of algebra, but because of the assumed independence (no correlations) between the "dressed" quasi-particles, all cross-terms between electrons, ions, and electron-ion pairs drop out.

Using:

$$1) \int_I \vec{f}_e(\vec{v}) d^3v = \text{prob. initial velocity of dressed electrons, ions} \\ \text{in velocity element } \vec{v}, \vec{v} + d^3v$$

$$2) \int_V d^3x \exp(i \vec{k} \cdot \vec{x}) = V \delta_{\vec{k}} = V \text{ if } \vec{k} = 0 \\ 0 \text{ if } \vec{k} \neq 0$$

$$\int_{-\infty}^{\infty} d\tau \exp(-i\omega\tau) \exp(-i\vec{k} \cdot \vec{v}\tau) = 2\pi \delta(\omega + \vec{k} \cdot \vec{v})$$

$$3) \bar{f}_e(\hat{k}, v) \equiv \int d^3v \delta(v - \hat{k} \cdot \vec{v}) f_e(\vec{v})$$

$$4) \delta(\omega + \vec{k} \cdot \vec{v}) = \delta[k(\omega/k + \hat{k} \cdot \vec{v})]$$

$\hat{k}$ is unit vector in direction of $\vec{k}$

$\bar{f}$ is distribution fcn. in magnitude of $v (= |\vec{v}|)$ and $k = |\vec{k}|$

$$= \frac{1}{k} \delta(\omega/k + \hat{k} \cdot \vec{v})$$

$$d^2 P(\theta, \omega) \sim \frac{I_0 r_e^2 (\hat{E}_0 \cdot \hat{1})^2 d\Omega \Delta\omega N_e}{k |\epsilon(\vec{k}, -\omega)|^2} x$$

$$\left\{ \left| 1 + \chi_I(\vec{k}, -\omega) \right|^2 \bar{f}_e(\hat{k}, -\omega/k) + Z \left| \chi_e(\hat{k}, -\omega) \right|^2 \bar{f}_I(\hat{k}, -\omega/k) \right\}$$

$$\omega = \omega_s - \omega_i$$

$$\epsilon = 1 + \chi_e + \chi_I$$

Define form factor (= spectral density function) $S(k, \omega)$

$$d^2 P(\theta, \omega) \approx d\Omega \Delta\omega N_e I_0 \sigma_{Th} S(k, \omega)$$

$$\sigma_{Th} = r_e^2 (\hat{E}_0 \cdot \hat{1})^2 = \frac{6.65 \times 10^{-25}}{(8\pi/3)} \sin^2 \beta \quad (\text{cm}^2)$$

$S(k, \omega)$ is sum of 2 terms, one involving the electron velocity distribution function, the other involving the ion velocity distribution function, and both involving the electron and ion susceptibilities.

Calculate susceptibilities: χ_e, χ_I

This is where plasma physics enters: there exist different plasma models, different approximations, different analysis techniques.

We follow along 1 more step: Salpeter approximation for collisionless, unmagnetized, near thermal equilibrium plasma

Using linearized Vlasov eq., self consistent $\vec{E}$ fields, plasma neutrality, polarization response to test charge, Fourier transforms, Laplace transform ($f(w) = \lim_{\gamma \rightarrow 0} \int_0^\infty f(t) e^{-(i\omega + \gamma)t} dt$), isotropic velocity distribution

$$G_e = -\chi_e; \quad G_i = -\chi_i$$

$$G_e(w, \vec{k}) = - \lim_{\gamma \rightarrow 0} \frac{4\pi e^2 N_e}{m k^2} \int_{-\infty}^{\infty} \frac{k \partial f_e(v)/\partial v}{w - kv - i\gamma} dv$$

$$G_i(w, \vec{k}) = - \lim_{\gamma \rightarrow 0} \frac{4\pi e^2 Z^2 N_i}{M_i k^2} \int_{-\infty}^{\infty} \frac{k \partial f_i(v)/\partial v}{w - kv - i\gamma} dv$$

and if f_e, f_i are Maxwellian with T_e, T_i respectively

$$G_e(w, k) = -\alpha^2 \left\{ 1 - 2x e^{-x^2} \int_0^x e^{t^2} dt + i\pi^{1/2} x e^{-x^2} \right\}$$

$$G_I(w,k) = -\alpha^2 \frac{Z T_e}{T_I} \left\{ 1 - 2ye^{-y^2} \int_0^y e^{t^2} dt + \frac{1}{2} \pi^{1/2} y e^{-y^2} \right\}$$

$$\text{with } x \equiv \frac{w}{k} \left(\frac{m}{2 T_e} \right)^{1/2} \quad y \equiv \frac{w}{k} \left(\frac{M}{2 T_I} \right)^{1/2}$$

(T in energy units)

and

$$S(k,w) = \left| \frac{1 - G_I(y)}{1 - G_e(x) - G_I(y)} \right|^2 f_e(v) + Z \left| \frac{G_e(x)}{1 - G_e(x) - G_I(y)} \right|^2 f_I(v)$$

$v = -w/k$ and note different scale factors x and y .

Limit $\alpha \ll 1$

$$S(k,w) \rightarrow f_e(v)$$

(shield cloud plays no role, because in the Fourier expansion the range of wavenumbers Δk needed to describe the localization of the shielding cloud goes from $k=0$ to $k \approx 1/\lambda_D$ and does not include the scattering geometry k value ($\gg \frac{1}{\lambda_D}$) designated by $\alpha \ll 1$.)

Limit $\alpha \gg 1$

$S(k,w)$ determined almost entirely by 2nd term ($\sim f_I$).

(Physical interpretation: a) scattering geometry k value is very small. b) velocities of shielding clouds are clustered around origin of velocity space. c) required matching phase velocity ($\sim w/k$) to detect any plasma effect will be centered very closely on $w \approx 0$; and d) because of the large ion/electron mass ratio the "dressed" ions are more numerous near zero velocity in ratio $\sim (M/m)^{1/2}$)

Evolution of spectra as a function of

- 1) α (thermal equilibrium)
- *2) T_e/T_i
- *3) impurity content (large effect since $\sum_i G_i$ appears also in denominator)

Return to susceptibility calculations for

- *1) drift velocity
- 2) collisions
- 3) magnetic field
- 4) relativistic effects

We consider 3) and 4) only in the $\alpha \ll 1$ limit

*large effect on narrow central "ion feature"

Examples of scattered spectra for different α ($T_e = 5$ eV).

A. High temperature: $v/c \ll 1$

$$1) \quad \vec{F} \neq -e \vec{E}_i \quad \vec{F} = -e(\vec{E}_i + \vec{\beta} \times \vec{B}_i) \quad \vec{\beta} \equiv \vec{v}/c$$

$$\left| \vec{E}_s \right| \neq \left| \vec{E}_i \right| (\vec{E}_i, \phi) \quad \left| \vec{E}_s \right| = \left| \vec{E}_i \right| (\vec{E}_i, \phi, \theta, \beta_{||}, \beta_{\perp})$$

$\phi = \angle$ polarization dir. + scattering direction

$\theta = \angle$ incident dir. + scattering direction

$c\beta_{||}$ = electron velocity component parallel to wave B_i field

$c\beta_{\perp}$ = electron velocity component perpendicular to B_i field

2) finite transit time effect

during scattering observation time as one electron with given velocity component leaves scattering volume, another (statistically) enters scattering volume on opposite side with same velocity component. But these are not exactly equivalent because of retarded time effects;

i.e. if "new" electron is further away from detector, there will be a "dead" time during which its scattered power has not yet reached the detector. This affects both amplitude and phase (or frequency spectrum in Fourier transform space).

$$P_e^R(\theta) = P_e(\theta) \left(\frac{w_s}{w_i} \right)^2 \frac{1}{\left(1 - \frac{\vec{v}}{c} \cdot \vec{k}_s \right)}$$

(total power scattered by single electron)

and integrating over a non-relativistic Maxwellian velocity distribution (ok to first order correction in v/c) and converting to wavelength scale

$$\rightarrow \frac{dP^R(\Delta\lambda)}{d\Omega} = C^1 n_e \exp \left[- \frac{c^2 m_e}{k T_e \lambda_i^2 \sin^2(\theta/2)} \left(1 - \frac{\Delta\lambda}{\lambda_i} \right) \Delta\lambda^2 \right]$$

$C^1 = C(1 - 3.5 \frac{\Delta\lambda}{\lambda})$ ← assymetry factors

shifts Gaussian distribution to blue (slightly skewed)

T_e (true)	T_e (measured from blue side, ignoring v/c correction)
100 eV	108 eV
500	596
2000	2790

B. DC Magnetic Field

a) single electron

scattered power $E_s \approx E_i \cos(\vec{k} \cdot \vec{r}(t) - \omega_i t)$

$$= E_i \cos(\vec{k} \cdot \vec{r}(0) + \vec{k} \cdot \vec{v} t - \omega_i t)$$

$$\vec{k} \cdot \vec{v} t = \vec{k} \cdot \vec{v}_{||} t + \vec{k} \cdot \vec{v}_{\perp} t$$

($||$, $\perp$ w.r.t. $\vec{B}$)

and $\vec{v}_{||}$ is constant in time

$\vec{v}_{\perp}$ gyrates about $\vec{B}$ with frequency $\Omega = \frac{eB}{mc}$

$$\vec{k} \cdot \vec{v} t = \underbrace{(k \sin \beta) v_{\parallel} t}_{\text{effective frequency shift}} + \underbrace{(k \cos \beta) (v_{\perp} \sin \Omega t) t}_{\text{is itself time dependent.}}$$

effective frequency shift (coefficient multiplying t)
is itself time dependent.

Time-dependent frequency shift can be Fourier analyzed into a superposition of discrete frequency components

$$(e^{i x \sin \theta} = \sum_{-\infty}^{\infty} e^{i m \theta} J_m(x))$$

$$\rightarrow E_s \sim \sum_{n=-\infty}^{\infty} J_n \left(\frac{k v_{\perp} \cos \beta}{\Omega} \right) \cos \left[(\omega_i + k v_{\parallel} \sin \beta + n \Omega) t + \phi_n \right]$$

where J_n is n th Bessel function

and ϕ_n is time independent phase factor

∴ scattered spectrum from a single electron is a sum of discrete frequencies separated at frequency intervals Ω .

- b) For many uncorrelated ($\alpha \ll 1$) electrons, add intensities taking into account the velocity distribution of the electrons

Assuming Maxwellian, temperature T (energy units)

total intensity in n th line is

$$1/2 \exp \left(\frac{-k^2 \cos^2 \beta \cdot T}{m \Omega^2} \right) I_n \left(\frac{k^2 \cos^2 \beta T}{m \Omega^2} \right)$$

and it is Doppler broadened because of the distribution of $v_{\parallel}$ into a normalized Gaussian profile of form:

$$\exp\left(-\frac{m v_{||}^2}{2T}\right) = \exp\left(-\frac{m(\Delta w')^2}{k^2 \sin^2 \beta \cdot 2T}\right)$$

where I_n = modified Bessel function

$\Delta w'$ is frequency shift from line center at $w = w_i + n\Omega$
and, summing over all lines ($n = -\infty$ to $+\infty$)

$$\rightarrow S(k, w) = \frac{\exp\left(-\frac{k^2 \cos^2 \beta \cdot T}{m \Omega^2}\right)}{\left(\frac{\pi k^2 \sin^2 \beta \cdot 2T}{m \Omega^2}\right)^{1/2}} \times$$

$$\sum_{n=-\infty}^{\infty} I_n \left(\frac{k^2 \cos^2 \beta \cdot T}{m \Omega^2} \right) \exp \left(-\frac{m(w - (w_i + n\Omega))^2}{k^2 \sin^2 \beta \cdot 2T} \right)$$

Spectral form depends on

$$1) \quad a \equiv \frac{k^2 \cos^2 \beta}{m \Omega^2} T \quad (\sim \text{envelope})$$

$$2) \quad b \equiv \frac{k^2 \sin^2 \beta}{m \Omega^2} 2T \quad (\sim \text{modulation})$$

since

$$S(k, w) = \frac{\exp(-a)}{(\pi b)^{1/2}} \sum_{n=-\infty}^{\infty} I_n(a) \exp\left(-\frac{(w - w_i - n\Omega)^2}{b}\right)$$

Limiting cases

- 1) $a \ll 1$ Spectrum contains practically only the central line because amplitude of I_n negligibly small for $n \neq 0$. Width contracted by factor $\sin \beta$; for $\beta = \pi/2$, same as no magnetic field.

- 2) $a \gg 1$ Lines will overlap and smear out spectrum unless
 $b \ll 1$.

The only way to have $a \gg 1$ and $b \ll 1$ is for
 $\cos^2 \beta \approx 1$

i.e., $\vec{k}$ very nearly $\perp \vec{B}$ ($\vec{k} \cdot \vec{B} \approx 0$)

In consequence, very difficult experiment to detect
magnetic field modulation of the scattered spectrum,
but if such modulation is detected it is a good
measurement of $\vec{B}$ field direction just by virtue
seeing modulation at all.

SOURCE CONSIDERATIONS

- 1) Plasma parameter to be measured determines desirable α .
- 2) Given α and an estimate of n, T fixes requirement on $\lambda/\sin(\theta/2)$.
- 3) Small cross section requires both powerful laser and sensitive
detector - this limits options on λ .
- 4) For $\alpha \gtrsim 1$ $\theta \rightarrow 0^\circ$
 - Undesirable
 - a) limits spatial resolution
 - b) stray laser light problem most severe
 - c) large changes in α over small collection
solid angle.

$\therefore$ great interest in long wavelength lasers:

CO_2 or Far Infrared

- 5) Especially for $\alpha \ll 1$ (which gives T_e from spectral width and
 n from absolute intensity) the laser of choice is ruby
($\lambda = 6943 \text{ \AA}$).

photomultipliers available S-20 $\sim$ 5% QE

GaAs $\sim$ 10% QE

High quality ruby lasers are commercially available:

4 J, 20 ns, 200 MW, 3-5 mrad	\$18 K
12 J, 20 ns, 600 MW, 2 mrad	36 K
30 J, 1 ms, 30 kw, 3-5 mrad	10 K

Nevertheless, care and feeding (maintenance) of laser takes non-negligible time and effort, and is one of several elements that makes any scattering experiment a significant undertaking.

Just how much laser energy, power is required? Must exceed 2 minimal requirements:

a) Energy:

detect enough scattered photons to give adequate statistical accuracy:

$N(\# \text{ detected photons/channel})$

$$= \underbrace{\frac{\int I_o dt}{h\nu}}_1 \underbrace{\ell \cdot A}_2 \underbrace{\sigma_T}_{3} \underbrace{\frac{d\Omega}{4\pi}}_4 \underbrace{\frac{\Delta\omega}{\int d\omega}}_5 \underbrace{n_e \eta R}_6$$

where 1 = # of incident photons/unit area	I_o = power/unit area
	$h\nu$ = energy/photon
	$\int dt$ = time integral over laser pulse duration
2 = scattering volume	A = cross-sectional area
	ℓ = length along beam from which scatter signal is collected
3 = total cross-section x fractional solid angle	
4 = fractional spectral width of particular channel	
5 = scatterers (electrons)/unit volume	
6 = collection efficiency	η = detector quantum efficiency
	R = transmission factor of collection optics

[Note that $A \int I_o dt$ = total laser energy, and scattered signal is independent of cross-sectional area over which the laser is spread.]

If we require $N = 100$ (10% statistics)
and assume

$$f/5 \quad \text{collection optics} \quad \left(\frac{d\Omega}{4\pi} \approx 2.5 \times 10^{-3} \right)$$

$$\eta = 10\%$$

$$R = 10\%$$

$$\Delta\omega/\int d\omega = 0.1$$

$$\text{then for 1 Joule require } n_e \ell \approx 1.7 \times 10^{13} \text{ cm}^{-2}$$

- b) Power: scattered signal must compete with background plasma radiation. Bremsstrahlung is a lower limit on latter. (Often continuum radiation is measured to be an order of magnitude greater! Watch out for D_α line radiation at 6562 Å!)

Subdivides into 2 cases:

- 1) laser pulse duration is comparable to plasma duration. Then compare magnitude of scattered signal with magnitude of bremsstrahlung, remembering that plasma volume V_p from which bremsstrahlung reaches detector is larger than the laser scattering volume - by definition the intersection of V_p and laser beam.

Scattered power:

$$P_s (\text{watts}) = \sigma_T (\text{cm}^2) I_o \left(\frac{\text{watts}}{\text{cm}^2} \right) n_e (\text{cm}^{-3}) V_L (\text{cm}^3) S(o) \frac{d\Omega}{4\pi} \eta R \Delta\lambda (\text{\AA})$$

where $S(o)$ is maximum of spectral function/angstrom
normalized so $\int S(\lambda) d\lambda = 1$

$$S(o) = \frac{c}{2\pi^{1/2}} \frac{1}{\lambda(\text{\AA}) \sin\theta/2 \cdot \bar{v}_e (\text{cm/sec})}$$

$$= \frac{45}{[T_e (\text{eV})]^{1/2} \lambda(\text{\AA}) \sin\theta/2}$$

Bremsstrahlung:

$$P_B (\text{watts}) = 1.9 \times 10^{-28} n_e^2 \lambda^{-2} T_e^{-1/2} \exp\left(-\frac{hc}{\lambda T}\right) V_p \frac{d\Omega}{4\pi} \eta R \Delta\lambda$$

with $\lambda, \Delta\lambda$ in $\text{\AA}$

T_e in eV

for $\lambda = 6943$, $\theta = 90^\circ$, and $\frac{hc}{\lambda T} \ll 1$

$$R \equiv \frac{P_S}{P_B} \approx 1.5 \times 10^9 \frac{I_o V_L}{n_e V_p}$$

e.g. for $R \geq 1$ at $n_e = 1.5 \times 10^{15}$ and $V_p/V_L = 10$

requires $I_o \geq 10 \text{ MW/cm}^2$

or, more commonly,

- 2) laser pulse duration is short compared to any plasma changes.
Then, scattered signal need only be large compared to fluctuations of plasma background radiation on laser pulse time scale.

Relaxes power requirement and complicates calculation:

Go back to photon statistics consideration:

$$S/N = \frac{P_S \cdot N_o \Delta\tau}{\left[(P_S + P_B) \cdot N_o \Delta\tau \right]^{1/2}}$$

where N_o is #photo-electrons/joule

$$= 3.5 \times 10^{18}$$

and $\Delta\tau$ is pulse duration in seconds.

Approximating $P_S \ll P_B$

$$\longrightarrow S/N = 1.6 \frac{V_L}{V_p^{1/2}} \frac{I_o}{T^{1/4}} (d\Omega\eta R\Delta\lambda\Delta\tau)^{1/2}$$

I_o in watts/cm ²	$\eta = QE$
T in eV	$R = \text{transmission efficiency}$
$\Delta\lambda$ in Å	
$\Delta\tau$ in seconds	

Note that in this case density dependence drops out but geometry factors, collection efficiency, channel width, pulse duration, and weak temperature dependence all enter into optimizing signal-to-noise.

Numerical example:

$\eta = 10\%$	$T = 100 \text{ eV}$	$V_p/V_L = 10$	$f/5 \text{ optics}$
$R = 0.1$	$\Delta\lambda = 16 \text{ Å}$	$V_L = 1 \text{ cm}^3$	

Laser: 0.1 Joule in 20 ns into 1 cm^2 ($I_o = 5 \text{ MW/cm}^2$)

$\rightarrow S/N \sim 100$

Remember, however, that photon statistics on scatter signal itself may put the lower practical limit on attainable S/N ratio.

(from before, 1 joule input requires $n\lambda = 1.7 \times 10^{13} \text{ cm}^{-2}$
for 100 counts ($S/N = 10$))

$\therefore$ 0.1 joule input requires $n\lambda \approx 1.7 \times 10^{16} \text{ cm}^{-2}$
for 10,000 counts ($S/N = 100$)

PRACTICAL CONSIDERATIONS

1. Stray light

Biggest practical problem: prevent stray laser light on detector
swamping true plasma scatter signal.

- Problem:
- a) scattered signal 10^{-10} to 10^{-12} of incident laser power on plasma
 - b) even though signal is wavelength shifted, attenuation of simple wavelength selective device (filter, prism, grating, etalon) to unwanted wavelength is only $\sim 10^{-4}$
 - c) diffuse reflectivity off an optimized "black" surface is $> 10^{-5}$ per steradian, and very much greater for most surfaces.

Solutions:

- A.
 - a) Prevent detector from "seeing" any surface directly illuminated by the laser on its path into and out of the plasma, or even any path that has only 1 diffuse reflection to such a surface. Achieved by moving entrance and exit vacuum chamber windows off as far as possible on vacuum snouts and installing carefully designed baffles within the snouts.
 - b) Install laser beam exit "dump" and viewing "dump" i.e., minimize absolutely reflections from the surface that the laser eventually impinges on and the surface that the detection optics sees directly. (Rayleigh horns, blue glass, Brewster angle, etc.)
 - c) Use intermediate focal plane stops both in the laser delivery optics to reduce spatial distribution due to beam divergence and in the light path entering the detector to limit the plasma volume and surrounding wall area contributing to detector input.
- B. Build up the detector wavelength discrimination against unshifted laser light by using multiple stages in the dispersion device, arranged so each stage contributes its discrimination multiplicatively. Relaxes requirements of (A), but prevents Rayleigh calibration, can quickly run into material damage limits if vacuum windows are near focal point, and reduces collection transmission factor.

Either option requires careful design and quality craftsmanship.

2. Alignment

Requirements:

1. Ruby laser must traverse accurately the intended path between baffles, which on the one hand have been placed with tolerances to do their job, but which, on the other hand will prohibitively increase stray light if directly illuminated.

2. Accurate intersection of laser illuminated plasma scattering volume and light collection cone of detection optics, both held to tight tolerances by stray light problem.
3. Alignment must often be retained through severe thermal cycling.

Aids:

1. He-Ne CW laser beam repositionally mounted to accurately indicate ruby laser path.
2. Tracking of ruby beam with "burn" patterns.
3. Marking of scattering volume (within vacuum chamber) with removable strong scatterer.
4. He-Ne CW beam projected accurately backwards through detection optics.

3. Calibration

Generally use multi-channel detection to avoid need for shot-to-shot reproducibility to obtain spectral scan. Requires accurate calibration of wavelength position and relative intensity response to short duration pulses, perhaps in a hostile EM environment.

High gain PM tubes are subject to large drifts, so intensity calibrations must be continually repeated, as close as possible in time to data acquisition.

Density determination ($\alpha \ll 1$) requires absolute calibration. Most conveniently done by comparison in place (eliminates all geometry and attenuation factors) to known cross-section Rayleigh scattering from relatively high pressure cold gas replacing plasma. Since Rayleigh scattering does not shift the wavelength, depends on sufficiently low level of stray light and usually involves plotting pressure dependence to eliminate residual effects.

Since scattered photons are always at high premium, generally useful to increase wavelength pass-band with increasing wavelength separation from laser wavelength, in order to increase the low intensity scatter signals.

At thermonuclear temperatures, spectral widths are so broad (e.g., 2.5 KeV at 90° gives 800 Å half half-width) that calibration problems become harder and harder.

4. Data Acquisition

Multiple channel detection (generally there is fiber optic coupling between image plane of dispersed spectrum and bank of photomultipliers) is expensive in terms of requiring multiple fast oscilloscope traces.

Has been fertile field for implementing different time delays between channels and displaying on single scope trace. However requires gating as well as varying time delays to avoid summing cf (time displaced) background signals from all the channels.

Automated computer read-out and "instant" plotting and analysis extremely useful. Aided by availability of fan-out gate-and-hold plug-in integrators. By leading each signal to two such integrators separated by a cable length whose transit time is longer than the laser pulse the background level can be subtracted from the signal. Requires fast laser monitor and careful cable length adjustments to trigger gate circuits.

RECENT TRENDS

1. Simultaneous Multiple Space Point Measurements

Replace PM's with 2-D detectors:

SIT vidicon with micro-channel plate intensifier gating

stigmatic polychrometer → spatial coordinate

along 1 dimension, λ along 1 dimension

Princeton: Bretz et al., Applied Optics 17, 192 (1978)

LASL: Siemon, 1976 Annual Report

common elements: 10^3 dynamic linear range
 10^8 shutter ratio
 shot-noise-limited (single p.e.sensitivity)
 ~ 20 wavelength bands
 ≥ 30 spatial positions
 extensive, computer aided data collection +
 calibration procedures

Princeton: 2-4 J, 85% < 0.25 mrad divergence
 2 mm beam over 90 cm length
 Fiberoptic image dissection to match input slit
 n_e, T_e in $10^{13} - 10^{14} \text{ cm}^{-3}$, 50 eV-3 keV range

on PLT, routinely, every discharge, major diagnostic

LASL: Theta pinch, $n_e \sim 10^{16} \text{ cm}^{-3}$
 coupled to 3 stage polychrometer
 (energy limited by quartz damage threshold)
 proved out - not yet routine

Subcategory is idea of using laser pulses so short that time-of-flight gives spatial resolution (0.1 ns = 3 cm) - Eases stray light problem. Laser sources with enough energy, power seem attainable with state-of-the-art technology.

T_e profiles measured by Thomson
 scattering in the ST tokamak.
 (PPPL 733051)

2. Multiple Time Point Measurements

Brute force can achieve up to ~ 4 consecutive laser pulses during one flashlamp cycle with a) multiple Q-switching (1 μ s to 0.5 ms separation) or b) geometric division of ruby rod among separate Q-switches (20 ns to 0.5 ms separation)

Efficient cooling, engineering, etc. can produce repetitive flashlamp cycling on > 1 sec pulse separations.

Exists a need for continuous pulse trains over the discharge lifetime (i.e., anywhere from 100 μ sec to 1 sec). In general existing laser sources (e.g., CW) lack sufficient power to overcome plasma radiation.

In principle, by summing over many pulses and using lock-in synchronous detection can gain in signal/plasma background with square root of number of pulses summed over. Given laser energy is best distributed among shortest attainable high power pulses. Ultimate limit is photon statistics on total scattering signal (total energy or number of pulses summed over).

Döbele et al., (Stuttgart pre-print 1PF 77-5 + March 1978 Santa Fe Diagnostics Conference) report scattering from $n = 10^{15} \text{ cm}^{-3}$ DC arc using 6 W CW pumped Nd:YAG laser modified with acousto-optic Q-switch for 25 kHz repetition rate, 200 nsec pulse width and 10 sec integration time. They suggest feasibility for standard Tokamak parameters for several hundred watts average power Nd:YAG (commercially not available but envisaged) with a few ms resolution times.

Forman (LASL) calculates that if the 100 J available in 500 μ sec random spike ruby lasers can be maintained with active mode locking to achieve pulse regularity, 5 μ sec time resolution T_e scattering measurements could be obtained on $n \approx 10^{15} \text{ cm}^{-3}$ reverse field pinch plasma.

3. B Field Measurement

$\vec{k} \cdot \vec{B} \approx 0$ gives modulation period Ω in scattered frequency spectrum.

This periodicity is so small that there are too few photons/period to detect with conventional means.

Nonetheless can utilize the special properties of a Fabry-Perot interferometer and the fact that unknown poloidal field in Tokamaks is small compared to the accurately known toroidal field to measure poloidal field direction, and hence the radial current distribution.

- 1) Set free spectral range of F-P equal to Ω ($|B|$ known to about 1%). All peaks effectively add up.
- 2) Gather light scattered with scattering angle θ over a range of azimuth angles (i.e., figure of revolution of scattering direction around incident laser direction). Obtain Fabry-Perot ring pattern in which ring azimuth position corresponds to azimuth position of scattering angle - and find intensity modulation for the particular azimuths for which $\vec{k} \perp \vec{B}$.

Using image intensifiers, a successful measurement has only recently been reported on the Culham tokamak DITE (Forrest et al., Nature 271, 718 (1978)). For $\theta = 30^\circ$, $T_e = 350$ eV, B_θ/B_z measured to 0.3%. (Field line inclination to 0.15° , or a $\Delta q < 5\%$.)

CONCLUDING COMMENTS

There are many instances where accurate T_e scattering measurements have differed substantially (usually lower) from less direct T_e determinations.

A few cases are on record for $\alpha \ll 1$, where careful scattering measurements do not give a Gaussian distribution. The spectral

shapes remain unexplained, but strong turbulence is suspected.

This emphasizes importance of measuring spectral shape rather than, say, a two point measurement assuming Gaussian distribution.

A very large literature exists by now. The Garching Plasma Physics Index lists over 30 entries each year under "Thomson Scattering."

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