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**Estimation of Expected Value
and Coefficient of Variation for
Lognormal and Gamma Distributions**

University of California

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Gary C. White

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ESTIMATION OF EXPECTED VALUE AND COEFFICIENT OF VARIATION
FOR LOGNORMAL AND GAMMA DISTRIBUTIONS

by

Gary C. White

ABSTRACT

Concentrations of environmental pollutants tend to follow positively skewed frequency distributions. Two such density functions are the gamma and lognormal. Minimum variance unbiased estimators of the expected value for both densities are available. The small sample statistical properties of each of these estimators were compared for their own distributions, as well as for the other distribution, to check the robustness of the estimator. The arithmetic mean is known to provide an unbiased estimator of expected value when the underlying density of the sample is either lognormal or gamma, and results indicated the achieved coverage of the confidence interval is greater than 75 percent for coefficients of variation less than two. Further Monte Carlo simulations were conducted to study the robustness of the above estimators by simulating a lognormal or gamma distribution with the expected value of a particular observation selected from a uniform distribution before the lognormal or gamma observation is generated. Again, the arithmetic mean provides an unbiased estimate of expected value, and the achieved coverage of the confidence interval is greater than 75 percent for coefficients of variation less than two.

Additional Monte Carlo studies were conducted to study the estimation of the coefficient of variation, c . Three estimators of c and their associated confidence intervals were simulated: (1) the usual estimate of c , $s/\bar{x}$; (2) an estimator derived from the gamma distribution; and (3) an estimator derived from the lognormal distribution. Both the gamma and lognormal estimators of c performed satisfactorily when the underlying distribution was the one from which the estimator was derived. Neither of these estimators performed satisfactorily when the other distribution was substituted. The estimator $s/\bar{x}$ was negatively biased for all cases simulated.

I. INTRODUCTION

The concentrations of environmental contaminants have been suggested to follow positively skewed frequency distributions by numerous researchers. In particular, Pinder and Smith¹ investigated the goodness of fit of the lognormal, Weibull, exponential, and normal distributions to radiocesium concentrations in soil and biota. They found that the lognormal distribution fit the majority of the data sets. Giesy and Weiner² found that the lognormal also tended to fit the concentrations of trace metals in fish better than the Weibull, exponential, or normal distributions. Other investigators³ suggest the gamma distribution may be preferred to the lognormal distribution. Eberhardt and Gilbert⁴ made an extensive study of how to distinguish these two distributions, and concluded that this is difficult for fewer than 200 observations. Extensive Monte Carlo simulations were done to reach this conclusion. Forsythe et al.⁵ compared the fit of the gamma and lognormal distributions for the concentrations of DDT in earthworms, and concluded that both fit the data equally well. Figure 1 shows the similarity of the lognormal and gamma probability density functions for a variety of coefficients of variation and expected value equal to 1.

Assuming that both distributions appear to explain contaminant data equally well, I want to explore the implications of selecting one of these two distributions in estimating the expected value and coefficient of variation of concentration. Some investigators⁶ have suggested using the median of the observed data to measure central tendency when a number of sample values are reported as "below background" or "not detected." I believe that the median may be quite useful for answering some questions, but that often the expected value is the desired measure. This report presents the results of Monte Carlo simulation studies of estimating the expected value EX and coefficient of variation c for these two distributions.

II. ESTIMATORS

A. Expected Value

I will consider the estimation of EX for the lognormal distribution. The density function is given by⁷

$$f(x) = \frac{1}{\sigma_y x \sqrt{2\pi}} \exp \left[-\frac{1}{2\sigma_y^2} (\ln x - \mu_y)^2 \right] \quad (x > 0; \sigma_y > 0, -\infty < \mu_y < \infty) ,$$

where μ_y and σ_y^2 are the mean and variance of $y = \ln x$ respectively. Finney⁸ derived a minimum variance unbiased estimator (MVUE) for EX because of the large bias of the maximum likelihood estimator (MLE). Finney's estimator is

$$\hat{E}(x) = \exp(\bar{y}) g_n(s_y^2/2) ,$$

where $\bar{y}$ is the arithmetic mean of the log transformed x values, s_y^2 is the variance of the log transformed x values, and the function g is the infinite series

Coefficient of Variation = .25

Coefficient of Variation = .75

Coefficient of Variation = 1.00

Coefficient of Variation = 1.50

Fig. 1

Comparison of gamma and lognormal probability density functions with expected value of unity and four values of the coefficient of variation.

$$g_n(t) = 1 + \frac{(n-1)t}{n} + \frac{(n-1)^3 t^2}{n^2(n+1)2!} + \frac{(n-1)^5 t^3}{n^3(n+1)(n+3)3!} + \dots$$

A finite sample confidence interval can be constructed for EX by Cox's direct method.⁹ A confidence interval is calculated for the value $\bar{y} + 1/2 s_y^2$ and then exponentiated to achieve an interval on EX. This interval is asymmetric, but has the desirable property that the lower confidence bound cannot be negative. Cox's direct method was shown to be easily the best of the approximate methods considered by Land and was recommended by him when dealing with large sample sizes and moderate values of s_y^2 . Land's exact method is not used because of the computational difficulties involved.

Estimation of the EX for the gamma distribution is much simpler than for the lognormal distribution. The EX of the gamma distribution is the product of the parameters α and β , where the probability density function is

$$f_x(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} \exp(-x/\beta) x^{\alpha-1} \quad (x > 0; \alpha > 0, \beta > 0) \quad .$$

The maximum likelihood equations¹⁰ to estimate α and β are

$$\Psi(\hat{\alpha}) + \log \hat{\beta} = \bar{y} \quad , \quad \text{and} \quad \hat{\alpha} \hat{\beta} = \bar{x} \quad ,$$

where $\Psi(t)$ is the psi (digamma) function. Hence we see by the invariance property of MLE's that $\bar{x}$ is the MLE of EX for the gamma distribution. In addition, $\bar{x}$ is also a MVUE of EX. This result is known because (1) the gamma distribution is a member of the exponential family, (2) the set of minimal sufficient statistics (MSS) is

$$\left\{ \sum_{i=1}^n x_i \quad , \quad \sum_{i=1}^n \log x_i \right\} \quad ,$$

(3) this set of MSS is complete, and (4) any function of the MSS is a MVUE if the function is unbiased. An unbiased estimator of EX is known to be $\bar{x}$ since:

$$\begin{aligned} E(\bar{x}) &= E(x_1 + x_2 + \dots + x_n)/n \\ &= [E(x_1) + E(x_2) + \dots + E(x_n)]/n \\ &= (n \mu_x)/n \\ &= \mu_x \quad . \end{aligned}$$

Therefore $\bar{x}$ is a MVUE of EX. [The details of this type of proof can be obtained in Mood et al.¹³ The result that $\bar{x}$ is a MVUE is particularly fortuitous as the arithmetic mean of the sample has often been used to estimate EX for real

data. Depending upon the Central Limit Theorem and the asymptotic normality of a MLE, I assume $\bar{x}$ is approximately normal and use the confidence interval for the mean of a normal distribution as an interval estimate for EX. The variance estimate obtained in this fashion is not the same as the variance of $\bar{x}$ calculated by maximum likelihood estimation procedures. However, the calculations are much easier to perform, and the method is commonly used in the literature. Therefore, I want to determine whether confidence intervals based on this simple variance estimator are valid. Of particular concern is the validity of this approach for small sample sizes, say $n = 5$.

B. Coefficient of Variation

Estimation of the coefficient of variation has not been studied as extensively as estimation of EX. The coefficient of variation for the lognormal distribution is^{7, 14}

$$c = [\exp(\sigma_y^2) - 1]^{1/2},$$

and an estimate of c is given by replacing σ_y^2 with s_y^2 . Conveniently, $\hat{c}$ depends only on the variance of the log transformed values. Koopmans et al.¹⁵ provide a method of obtaining confidence intervals on c . The statistic

$$\chi^2 = (n - 1) s_y^2 / \sigma_y^2$$

possesses the chi-square distribution with $n - 1$ degrees of freedom. This fact can be used to obtain a confidence interval on σ_y^2 , which can be transformed to obtain a confidence interval on c . A probability statement on the above ratio is

$$P \left\{ \chi^2_{(1-\alpha/2; n-1)} \leq \frac{(n-1)s_y^2}{\sigma_y^2} \leq \chi^2_{(\alpha/2; n-1)} \right\} = 1 - \alpha$$

and it implies the probability that this ratio will be within the indicated boundary values of $\chi^2_{[n-1]}$ is $1 - \alpha$. Thus a 95% confidence interval on σ_y^2 is

$$\frac{(n-1)s_y^2}{\chi^2_{(0.025; n-1)}} \leq \sigma_y^2 \leq \frac{(n-1)s_y^2}{\chi^2_{(0.975; n-1)}}.$$

Because $[\exp(s_y^2) - 1]^{1/2}$ is a strictly increasing function of s_y^2 ,

$$\left\{ \exp \left[\frac{(n-1)s_y^2}{\chi^2(0.025; n-1)} \right] - 1 \right\}^{1/2} \leq c \leq \left\{ \exp \left[\frac{(n-1)s_y^2}{\chi^2(0.975; n-1)} \right] - 1 \right\}^{1/2}$$

is a 95% confidence interval for c .

The coefficient of variation for the gamma distribution is given by

$$c = (1/\alpha)^{1/2} ,$$

and an estimate of c is given by replacing α with $\hat{\alpha}$. A maximum likelihood estimator of α is obtained by solving for α in the likelihood equations. However an iterative solution is required. Therefore an approximate method given by Greenwood and Durand^{17,18} is much easier to use. Define

$$z = \log \frac{\text{arithmetic mean}}{\text{geometric mean}} .$$

An approximate estimate of α is given by

$$\hat{\alpha} \approx z^{-1}(0.5000876 + 0.1648852 z - 0.0544274 z^2) \quad (0 < z \leq 0.5772) ,$$

or

$$\hat{\alpha} \approx z^{-1}(17.79728 + 11.9684477 z + z^2)^{-1} \\ \cdot (8.898919 + 9.059950 z + 0.9775373 z^2) \quad (0.5772 \leq z \leq 17) .$$

The resulting estimate of α is used to estimate c . Approximate confidence intervals on c are given by Linhardt:¹⁶

$$P(\gamma_L \leq c \leq \gamma_U) = 1 - \alpha ,$$

where

$$\gamma_L = \left[2nz/C_L \chi^2(1 - \alpha/2; n-1) \right]^{1/2} , \\ \gamma_U = \left[2nz/C_U \chi^2(\alpha/2; n-1) \right]^{1/2} ,$$

and where

$$C_L = 1/2 \left\{ 1 + \left[1 + 4(n+1)z/3\chi^2(1 - \alpha/2; n-1) \right]^{1/2} \right\} ,$$

$$c_U = 1/2 \left\{ 1 + \left[1 + 4(n+1)z/3\chi^2_{(\alpha/2; n-1)} \right]^{1/2} \right\}.$$

In addition to the two estimators of c derived from the lognormal and gamma distributions, a third method was considered based on the normal distribution. This estimator is included because it is commonly used in practice, even when the underlying distribution is known not to be normally distributed. The coefficient of variation is defined as

$$c = \sigma_X / \mu_X,$$

and this equation can be used to estimate c when μ_X and σ_X are replaced by their respective estimates:

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}, \text{ and } s_X = \left[\frac{\sum_{i=1}^n x_i^2 - \left(\sum_{i=1}^n x_i \right)^2 / n}{n-1} \right]^{1/2}.$$

Approximate confidence intervals on c are derived from an estimate of the variance of $\hat{c}$ taken from Cramér:²⁰

$$\hat{\text{Var}}(\hat{c}) = \hat{c}^2(1 + 2\hat{c}^2)/2n.$$

Ninety-five percent confidence intervals can then be constructed for c as

$$\hat{c} \pm t_{(0.05; n-1)} [\hat{\text{Var}}(\hat{c})]^{1/2}.$$

Cramér's variance estimate assumes the sample is from a normal population truncated at zero and that c is fairly small. Neither of these assumptions is true for all the cases considered in this report, but I will consider this estimator of c because of the ease with which it can be calculated and its general usage in the literature.

III. ROBUSTNESS

An estimator is robust with respect to a particular assumption if the estimator is relatively insensitive to (slight) departures from the assumption. Both estimators of c are known to have optimal properties when used with data derived from their respective distributions. In addition, we are interested in how each estimator will perform when the data come from other distributions.

Neither the gamma nor lognormal distribution can be mixed with another gamma or lognormal distribution and the result still be gamma or lognormally distributed. Formally, let $f_1(x; \theta_1)$ and $f_2(x; \theta_2)$ be either gamma or lognormal

probability density functions with parameter vectors $\underline{\theta}_1$ and $\underline{\theta}_2$, respectively. Then assume we sample from a population with probability p that the variate is distributed as $f_1(x; \underline{\theta}_1)$ and with $1-p$ from $f_2(x; \underline{\theta}_2)$. The resulting variate is distributed as

$$p f_1(x; \underline{\theta}_1) + (1 - p) f_2(x; \underline{\theta}_2) .$$

The concept of mixing two distributions can be extended even farther. Suppose that the $EX = \mu_x$ of $f(x)$ is actually drawn from a second density, $g(\mu_x)$. Then the distribution of x is

$$f_{x|\mu_x}(x|\mu_x) = \frac{f_x(x, \mu_x)}{g(\mu_x)} ,$$

or

$$f_x(x, \mu_x) = f_{x|\mu_x}(x|\mu_x) g(\mu_x) ;$$

$f_x(x, \mu_x)$ is a family of distributions indexed by the parameter μ_x .¹³

This result can be applied to transuranic research by conceptualizing a distribution of radioactivity in a fallout pattern. Suppose we stratify the fallout area into n strata, each with mean concentration EX_i , $i = 1, 2, \dots, n$. If a random sample of n_i quadrats are taken from strata i , the expected concentration will be EX_i . However, quadrats closer to ground zero would be expected to have slightly larger concentrations on the average than quadrats farther away from ground zero. However, this process is random so one method of expressing this randomness is to assume the expected value of a quadrat is actually drawn from some distribution with density $g(\mu_x)$.

To simulate this process, $g(\mu_x)$ was assumed to be a uniform distribution with density function

$$g(\mu_x) = \frac{1}{b - a} .$$

Thus the expected value of an observation was first drawn from a uniform distribution, and then a variate generated from a gamma or lognormal distribution with this expected value.

The expected value of the resulting distribution must be evaluated to show that indeed the expected value is $(a + b)/2$:

$$\begin{aligned} EX &= E[E(x|\mu_x)] \\ &= E(\mu_x) \\ &= (a + b)/2 . \end{aligned}$$

Comparisons of the lognormal and gamma distributions and their respective compound distributions are given in Figs. 2 and 3, respectively.

IV. MONTE CARLO

The listings of the programs used to perform the Monte Carlo simulations reported on are provided in Appendix A. In addition some library routines were used, and references are given in the text below.

Random normal deviates were generated by the method suggested by Bell¹¹ and then transformed to a lognormal deviate by $x = \exp(y)$. Random gamma deviates with nonintegral shape parameters were generated with the method presented by Fishman.¹² Briefly the method involves summing k (= greatest integer of α) exponential variates, $\mathcal{E}(1)$, adding to this sum a product of a beta variate distributed as $\beta(\alpha - k, 1 - \alpha + k)$ and an exponential $\mathcal{E}(1)$, and multiplying the total by the parameter β .

Samples of size $n = 5, 10, 20, 30, 50$, and 100 were drawn from each of the lognormal and gamma distributions. All possible combinations of $EX = 1, 5, 10, 50$, and 100 , and coefficient of variation of $c = 0.25, 0.5, 0.75, 1.0, 1.25, 1.5$, and 2.0 were used for both distributions. These combinations give a total of 210 cases per distribution. Each case was replicated 1000 times to estimate the bias and achieved coverage (proportion of replications in which the constructed 95% confidence interval contained the true parameter value) for the estimators discussed above. In addition, the average length of the confidence interval was calculated for each estimator.

Parameters were calculated from EX and c for the lognormal distribution as:

$$\sigma_y^2 = \ln(c^2 + 1)$$

$$\mu_y = 1/2 \ln [(EX)^2 / (c^2 + 1)] .$$

Parameters were calculated from EX and c for the gamma distribution as:

$$\alpha = 1/c^2$$

$$\beta = EX c^2 .$$

In simulations where EX was selected from a uniform distribution, the parameters a and b are defined to be $\pm 50\%$ of the desired expected value of the distribution. For example, suppose EX is to be 10.0 . Then $a = (1 - 0.5)10$ and $b = (1 + 0.5)10$, or EX is selected from the interval $(5, 15)$. The parameter values for α , β , μ_y , and σ_y^2 were then calculated with the formulas given above to generate one realization of x .

The infinite series, $g_n(t)$, necessary to calculate $E(x)$ assuming lognormality was evaluated to a point where the ratio of an additional term to the summation was less than $1E-7$ (see Appendix A, FUNCTION $G(T,N)$). Values of the t -statistic were obtained from tabled values (FUNCTION $T(N)$ in Appendix A).

Fig. 2
Comparison of lognormal and compound uniform-lognormal probability density functions with expected value of unity and four values of the coefficient of variation.

Fig. 3
Comparison of gamma and compound uniform-gamma probability density functions with expected value of unity and four values of the coefficient of variation.

Chi-square values were obtained from an inverse chi-square probability distribution function (subroutine MDCHI of IMSL¹⁹).

V. RESULTS AND DISCUSSION

A. Estimation of Expected Value

Complete simulation results on estimating the expected value of x are given in Appendix B. In general, the expected value of the distribution had no effect on the results. Rather, the coefficient of variation tended to explain the observed phenomena. Hence in the following sections, the statements made will apply to all the expected values simulated.

1. Gamma Estimator with Gamma Distributed Data. The arithmetic mean is known to be an unbiased estimator for EX , and so the simulations show. Of course, individual point estimates may vary widely. Hence the main purpose of simulating this estimator is to check the achieved coverage against the predicted value. A gradual decline in achieved coverage was noted with an increase in the coefficient of variation (Fig. 4). However, for all cases simulated, the achieved coverage is greater than 70%.

A slight decline in achieved coverage is noted also for decreasing sample sizes. This trend is more apparent for $c = 2$ than any other case.

2. Lognormal Estimator for Lognormally Distributed Data. Because this estimator is MVUE for lognormally distributed data, the chief purpose for the simulation was to check the coverage of the confidence interval. The achieved coverage is always close to the predicted 95% for $n = 100$. However, the achieved coverage declines with decreasing sample sizes (Fig. 5). The minimum achieved coverage is greater than 80% in all cases. These results are consistent with the findings of Land.⁹

3. Lognormal Estimator for Gamma Distributed Data. A major finding is that the bias of the lognormal estimator becomes large for gamma distributed data as α becomes small (coefficient of variation becomes large). In particular, a relative bias of about 25% is present for $\alpha = 1$ (Fig. 6). The case $\alpha = 1$ corresponds to the exponential distribution. The relative bias of the lognormal estimator becomes much worse for $\alpha < 1$. This result would be expected because the shapes of the two distributions differ greatly for $\alpha < 1$. However, even for $c = 0.75$ ($\alpha = 1.78$), the relative bias of the lognormal estimator for gamma distributed data is about 6%. Also we see that the achieved coverage of the confidence interval begins to decrease for $c = 1.0$ and $n = 100$. Coverage becomes very poor for $c > 1$.

4. Gamma Estimator for Lognormally Distributed Data. In contrast to the lognormal estimator for gamma distributed data, the gamma estimator (arithmetic mean) for lognormal data does quite well. This estimator is unbiased, and so the simulations show. Also, the achieved coverage of this estimator is good for small sample sizes ($n = 5$), whereas the coverage of the lognormal estimator is usually significantly less than the predicted 95% for $n = 5$ (Fig. 7). However, the average confidence interval length is usually greater for the arithmetic mean. The achieved coverage of the arithmetic mean drops as c increases, but never below 75%, even for $n = 5$ and $c = 2$. Also for $c \geq 0.75$, the average confidence interval length becomes about the same for the two estimators.

Fig. 4

Monte Carlo simulation results on confidence interval coverage of the gamma estimator of EX (arithmetic mean) with gamma distributed data, EX = 1. Values at the intersections of rows and columns are the proportion of 1000 replications where the computed confidence interval included the true expected value.

Fig. 5

Monte Carlo simulation results on confidence interval coverage of the lognormal estimator of EX with lognormally distributed data, $EX = 1$. Values at the intersections of a row and column are the proportion of 1000 replications where the computed confidence interval included the true expected value.

Fig. 6
Monte Carlo simulation results on bias and confidence interval coverage of the lognormal estimator of EX with gamma distributed data, EX = 1. Values at the intersections of rows and columns are either 100[Ave($\hat{E}(x)$) - EX]/EX (left figure) or the proportion of 1000 replications where the computed confidence interval included the true expected value (right figure).

Fig. 7

Monte Carlo simulation results on confidence interval coverage of the gamma estimator of EX (arithmetic mean) with lognormally distributed data, EX = 1. Values at the intersections of rows and columns are the proportion of 1000 replications where the computed confidence interval included the true expected value.

B. Robustness

The same general conclusions discussed in the preceding four sections also hold when the lognormal and gamma estimators are applied to a compound lognormal or gamma distribution with the expected value selected from a uniform distribution. The arithmetic mean still provides an unbiased estimate of EX in all cases, while the lognormal estimator provides an unbiased estimate when the variate is uniform-lognormal distributed, but not for the case when the variate is uniform-gamma distributed.

Confidence interval coverage for both estimators is always greater than 70% when there is negligible bias. Generally the arithmetic mean had better coverage for the smaller sample sizes, while the lognormal estimator had better coverage for $n = 100$. Also, the lognormal estimator tended to have better coverage when $c \geq 1.0$. Of course, the average confidence interval was also greater for the lognormal estimator when the coverage was larger than the gamma estimator. For the case $EX = 1$ and uniform-lognormal data, Fig. 8 provides the reader with some feel for the relationship between sample size, c , and the coverage for the two estimators considered.

C. Estimation of Coefficient of Variation

Results of the Monte Carlo simulations of the estimation of the coefficient of variation are given in Appendix C. No simulations of estimating c when the underlying distribution was a compound probability density function were performed.

1. Gamma Estimator with Gamma Distributed Data. The gamma estimator tends to have a large negative bias for small samples (Fig. 9). In Appendix C, Tables C.1 - C.7, the decrease in the relative bias of the gamma estimator is consistent over all values of EX . A relative bias of approximately 15% for $n = 5$ is noticed for all values of c , tending to drop to less than 1% for $n = 100$.

The coverage of this estimator seems to be very close to the expected 95%, except possibly for $c = 2$ and $n = 100$. For these cases (Table C.7, Fig. 10) coverage appears closer to 85%.

The average confidence interval width is smaller than either of the other two estimators. Hence I conclude that the confidence interval for the gamma estimator with gamma data performs quite satisfactorily.

2. Lognormal Estimator with Lognormally Distributed Data. As for the gamma distributed data, the lognormal estimator with lognormally distributed data is biased (Fig. 11). However the bias changes directions with an increase in c . For $c < 0.75$, a negative bias is noted, decreasing with increased sample size. However, for $c > 1.25$, a positive bias is noted, with the bias again decreasing with increased sample size.

Coverage of the lognormal estimator for lognormally distributed data is close to the expected 95% for $n = 100$, with a slight decline for smaller n (Fig. 12). However, in all cases the coverage is greater than 90%. However the average confidence interval width is very large for $n = 5$, and usually larger than either of the other two methods simulated for other values of n . In order to obtain confidence interval widths of the magnitude observed for $n = 5$ in Tables C.12 - C.14, the upper confidence bound must occasionally become very large. Possibly, use of an average to evaluate the performance is not justified.

Fig. 8

Monte Carlo simulation results on confidence interval coverage of the two estimators of EX with the uniform-lognormal compound distributions, EX = 1. Values at the intersections of rows and columns are the proportion of 1000 replications where the computed confidence interval included the true value.

Fig. 9

Monte Carlo simulation results on the bias of the gamma estimator of c with gamma distributed data, $EX = 1$. Values at the intersections of rows and columns are $100 [\text{Ave}(\hat{c}) - c]/c$ for 1000 replications.

Fig. 10

Monte Carlo simulation results on confidence interval coverage of the gamma estimator of c with gamma distributed data, $EX = 1$. Values at the intersections of rows and columns are the proportion of 1000 replications where the computed confidence interval included the true coefficient of variation.

Fig. 11

Monte Carlo simulation results on the bias of the lognormal estimator of c with lognormally distributed data, $EX = 1$. Values at the intersections of rows and columns are $100[\text{Ave}(\hat{c}) - \hat{c}]/\hat{c}$ for 1000 replications.

Fig. 12

Monte Carlo simulation results on confidence interval coverage of the lognormal estimator of c with lognormally distributed data, $EX = 1$. Values at the intersections of rows and columns are the proportion of 1000 replications where the computed confidence interval included the true coefficient of variation.

3. Lognormal Estimator with Gamma Distributed Data. The only case where the lognormal estimator of the coefficient of variation performs adequately for gamma distributed data is $c = 0.25$. For $c = 0.5$, an approximately 10% positive bias is noted (depending on n), and this bias becomes very large for $c = 2$ (Fig. 13). A bias is noted even for $c = 0.25$.

Accordingly, the coverage of this estimator approaches zero as c increases (Fig. 14). For the values of c simulated, better coverage is noted for $n = 5$ than for $n = 100$. However, the average confidence interval width becomes very large as the bias of the estimator increases.

4. Gamma Estimator with Lognormally Distributed Data. The gamma estimator of c with lognormally distributed data performs almost as badly as the lognormal estimator with gamma data. The estimator is negatively biased for all values of c simulated, and approaches -50% relative bias for $c = 2.0$ (Fig. 15).

Accordingly, confidence interval coverage declines with increasing c (Fig. 16). Coverage is close to the expected 95% for $c = 0.25$, but falls to less than 70% for $c = 2.0$. Also the coverage is a function of n , with the coverage for $n = 5$ much better than for $n = 100$.

5. Normal Estimator. The normal estimator performs about the same for either gamma or lognormal data (Fig. 17). Generally the relative bias of the estimator is much larger for $n = 5$ than for $n = 100$. Also the relative bias increases with c . Thus for $c = 0.25$ and $n = 5$, a relative bias of about -6% would be expected, decreasing to about -0.5% for $n = 100$. However, for $c = 2.0$, relative biases of over 25% and 2% for $n = 5$ and 100, respectively, are observed for the gamma distribution, and 45% and 12% for $n = 5$ and 100, respectively, for the lognormal distribution. Thus the estimator tends to seriously underestimate c , especially for $c = 2.0$.

Coverage of the normal estimator is always close to 95% and usually over for the gamma distributed data (Fig. 18). However, the average confidence interval width is usually 2 - 3 times that of the gamma estimator. Coverage seems to increase slightly with increasing sample size.

Coverage for the normal estimator and lognormally distributed data is seriously affected by the value of c (Fig. 18). Thus for $c = 0.25$, coverage is generally greater than 90%, and increases slightly for $n = 100$. However, for $c = 2.0$, coverage is in the range 55 - 70%, with the larger values associated with sample sizes of $n = 5$.

VI. CONCLUSIONS AND RECOMMENDATIONS

For small values of c ($c < 1$), the differences between the two estimators of expected value are relatively small. Generally the confidence interval on the arithmetic mean provides better coverage at the expense of a wider confidence interval. For larger values of c ($1 < c < 2$), the lognormal estimator becomes very biased when applied to gamma distributed data and coverage tends to decline with increasing c . If no theoretical reasons are available for choosing between the lognormal or gamma distribution, then the arithmetic mean is to be preferred because it is unbiased for either of the distributions.

Both the gamma and lognormal estimators of the coefficient of variation seem to perform adequately when the data are distributed as gamma and lognormal, respectively. However, neither estimator performs satisfactorily when the other type of data is simulated.

Fig. 13

Monte Carlo simulation results on the bias of the lognormal estimator of c with gamma distributed data, $EX = 1$. Values at the intersections of rows and columns are $100[\text{Ave}(\hat{c}) - c]/c$ for 1000 replications.

Fig. 14

Monte Carlo simulation results on confidence interval coverage of the lognormal estimator of c with gamma distributed data, $EX = 1$. Values at the intersections of rows and columns are the proportion of 1000 replications where the computed confidence interval included the true coefficient of variation.

Fig. 15

Monte Carlo simulation results on the bias of the gamma estimator of c with lognormally distributed data, $EX = 1$. Values at the intersections of rows and columns are $100[\text{Ave}(\hat{c}) - c]/c$ for 1000 replications.

Fig. 16

Monte Carlo simulation results on confidence interval coverage of the gamma estimator of c with lognormally distributed data, $EX = 1$. Values at the intersections of the rows and columns are the proportion of 1000 replications where the computed confidence interval included the true coefficient of variation.

Fig. 17
Monte Carlo simulation results on the bias of the normal estimator of c with gamma and lognormally distributed data, $EX = 1$. Values at the intersection of rows and columns are $100[Ave(\hat{c}) - c]/c$ for 1000 replications.

Fig. 18
Monte Carlo simulation results on confidence interval coverage of the normal estimator of c with gamma and lognormally distributed data, $EX = 1$. Values at the intersections of rows and columns are the proportion of 1000 replications where the computed confidence interval included the true coefficient of variation.

The normal estimator is negatively biased in all the cases simulated, but is much more robust than either of the other two estimators. However, none of the methods perform well enough that they could be recommended for general use.

ACKNOWLEDGMENTS

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APPENDIX A

LISTINGS OF FORTRAN IV PROGRAMS USED TO PERFORM SIMULATIONS

<u>Program Name</u>	<u>Use</u>
GAMMASM	Estimation of EX for gamma distributed data
LOGNSIM	Estimation of EX for lognormally distributed data
CVGASM	Estimation of c for gamma distributed data
CVLOGSM	Estimation of c for lognormally distributed data
G	Sum the infinite series $g(t)$
T	Provide t-statistic for $\alpha = 0.05$

<u>Program Name</u>	<u>Use</u>
RNORM	Generation of normal random deviates
RGAMMA	Generation of gamma random deviates

```

PROGRAM GAMMASH(INPUT,OUTPUT,TAPE5,TAPE6)
REAL TEX(10),TVX(10),CV(10)
INTEGER N(6)
DATA CV/.25,.5,.75,1.,1.25,1.5,2.0/
DATA TEX/1.,5.,10.,50.,100./
DATA N/5,10,20,30,50,100/
DATA NREPS/1000/
DATA NTABLE/0/

C EX      - ACRONYM FOR EXPECTED VALUE OF X, WHERE X IS A RV.
C EXLN    - LOGNORMAL ESTIMATOR OF EX.
C EXGM    - GAMMA ESTIMATOR OF EX.
C S       - PREFIX TO INDICATE SUM OF.
C S2      - PREFIX TO INDICATE SUM OF SQUARES OF.
C X       - LOG TRANSFORMED VARIABLE.
C Y       - ORIGINAL VARIABLE, EITHER GAMMA OR LOGNORMALLY DIST.
C SD      - PREFIX TO INDICATE STANDARD DEVIATION OF.
C SE      - PREFIX TO INDICATE STANDARD ERROR OF.
C NN      - SAMPLE SIZE OF CURRENT CASE(REPLICATION).
C NREPS   - NUMBER OF REPLICATIONS.
C NCOVLN  - NUMBER OF TIMES CI COVERED TRUE EX FOR LOGNORMAL MLE.
C NCOVGM  - NUMBER OF TIMES CI COVERED TRUE EX FOR GAMMA ESTIMATOR.
C
C SET RANDOM NUMBER GENERATOR.
  X=TIME(X)
  CALL RANSET(X)
  DO 10 IMU=1,5
    NTABLE=NTABLE+1
    NDIG0=1
    IF(NTABLE.GT.9) NDIG0=2
    WRITE(6,90) NDIG0,NTABLE,TEX(IMU),NDIG0,FLOAT(NREPS)
90  FORMAT(1H1,10X,'TABLE B.',1=,*, SIMULATION OF THE GAMMA*,
1* DISTRIBUTION WITH EXPECTED VALUE *,F5.0/22X,=X,*THE NUMBER*,
2* OF REPLICATIONS PER CASE IS*,F6.0//1X,126(*=*)/
3* PARAMETERS*,38X,*LOGNORMAL ESTIMATOR*,29X,
4* GAMMA ESTIMATOR*/1X,12(*=*) ,16X,*SAMPLE*,2X,44(*=*) ,2X,44(*=*)/
51X,* ALPHA BETA C.V. VAR(X) SIZE *,
62(* E(X)-HAT REL. BIAS AVE(CI) COVERAGE*)/
71X,126(*=*)/)
    TRUEEX=TEX(IMU)
    DO 20 ISIG=1,7
      TVX(ISIG)=(TEX(IMU)*CV(ISIG))**2
      NDIG2=2
      IF(TVX(ISIG).GE.10000.) NDIG2=1
      IF(TVX(ISIG).GE.100000.) NDIG2=0
      ALPHA=1./CV(ISIG)**2
      BETA=TEX(IMU)*CV(ISIG)**2
      NDIG1=3

```

```

      IF (BETA.GE.10.) NDIG1=2
      IF (BETA.GE.100.) NDIG1=1
      IF (BETA.GE.1000.) NDIG1=0
      DO 30 IN=1,6
      NN=N(IN)
      SLNC1=0.
      SGMCI=0.
      NCOVLN=0
      NCOVGM=0
      SEXLN=0.
      SEXGM=0.
      S2EXLN=0.
      S2EXGM=0.
      DO 40 I=1,NREPS
      SUMX=0.
      SUMY=0.
      SUMSQX=0.

      SUMSQY=0.
      DO 50 II=1,NN
      Y=RGAMMA(ALPHA,BETA)
      X=ALOG(Y)
      SUMX=SUMX+X
      SUMY=SUMY+Y
      SUMSQX=SUMSQX+X*X
      SUMSQY=SUMSQY+Y*Y
50  CONTINUE
      XBAR=SUMX/FLOAT(NN)
      EXGM=SUMY/FLOAT(NN)
      XVAR=((SUMSQX-SUMX*SUMX/FLOAT(NN))/FLOAT(NN-1))
      EXLN=EXP(XBAR)*G(XVAR/2.,NN)
      SDEXGM=SQRT((SUMSQY-SUMY*SUMY/FLOAT(NN))/FLOAT(NN-1))
      TEMP1=SQRT(XVAR/FLOAT(NN)+XVAR*XVAR/FLOAT(2*(NN+1)))
      TEMP2=XBAR+XVAR/2.
      SEEXGM=SDEXGM/SQRT(FLOAT(NN))
      TEMP=T(NN-1)*SEEXGM
      SGMCI=SGMCI+2.*TEMP
      IF (EXGM-TEMP.LT.TRUEEX .AND. EXGM+TEMP.GT.TRUEEX) NCOVGM=NCOVGM
      I +1
      TEMP=1.96*TEMP1
      CIL=EXP(TEMP2-TEMP)
      CIU=EXP(TEMP2+TEMP)
      SLNC1=SLNC1+(CIU-CIL)
      IF (CIL.LT.TRUEEX .AND. CIU.GT.TRUEEX)
      I NCOVLN=NCOVLN+1
      SEXLN=SEXLN+EXLN
      SEXGM=SEXGM+EXGM
      S2EXLN=S2EXLN+EXLN*EXLN
      S2EXGM=S2EXGM+EXGM*EXGM
40  CONTINUE
      AEXLN=SEXLN/FLOAT(NREPS)
      AEXGM=SEXGM/FLOAT(NREPS)
      SDAEXLN=SQRT((S2EXLN-SEXLN*SEXLN/FLOAT(NREPS))/FLOAT(NREPS-1))
      SDAEXGM=SQRT((S2EXGM-SEXGM*SEXGM/FLOAT(NREPS))/FLOAT(NREPS-1))
      COVLN=FLOAT(NCOVLN)/FLOAT(NREPS)
      COVGM=FLOAT(NCOVGM)/FLOAT(NREPS)
      AC1GM=SGMCI/FLOAT(NREPS)
      AC1LN=SLNC1/FLOAT(NREPS)
      RBLN=(AEXLN-TRUEEX)*100./TRUEEX
      RBGM=(AEXGM-TRUEEX)*100./TRUEEX
      WRITE(6,101) ALPHA,NDIG1,BETA,CV(1SIG),NDIG2,TVX(1SIG),NN,
      I AEXLN,RBLN,AC1LN,COVLN,AEXGM,RBGM,AC1GM,COVGM
101  FORMAT(1X,F6.2,F6.2,F6.2,F6.2,F6.2,2(G10.4,2X,G10.4,2X,G10.4,

```

```

      1 2X,F7.4,3X))
30 CONTINUE
  WRITE(6,102)
20 CONTINUE
  WRITE(6,102)
102 FORMAT(1X)
  WRITE(6,104)
104 FORMAT(1X,126(*=*))
10 CONTINUE
  CALL SECOND(CP)
  WRITE(6,103) CP
103 FORMAT(*1CPU TIME REQUIRED WAS *,F10.2,* SECONDS.*)
  STOP
  END

```

```

PROGRAM LOGNSIM(INPUT,OUTPUT,TAPE5,TAPE6)
REAL TEX(10),TVX(10),CV(10)
INTEGER N(6)
DATA X/0.25,0.5,0.75,1.,1.25,1.5,2.0/
DATA TEX/1.,5.,10.,50.,100./
DATA N/5,10,20,30,50,100/
DATA NREPS/1000/
DATA NTABLE/5/
C  EX  - ACRONYM FOR EXPECTED VALUE OF X, WHERE X IS A RV.
C  EXLN - LOGNORMAL ESTIMATOR OF EX.
C  EXGM - GAMMA ESTIMATOR OF EX.
C  S    - PREFIX TO INDICATE SUM OF.
C  S2   - PREFIX TO INDICATE SUM OF SQUARES OF.
C  X    - LOG TRANSFORMED VARIABLE.
C  Y    - ORIGINAL VARIABLE, EITHER GAMMA OR LOGNORMALLY DIST.
C  SD   - PREFIX TO INDICATE STANDARD DEVIATION OF.
C  SE   - PREFIX TO INDICATE STANDARD ERROR OF.
C  NN   - SAMPLE SIZE OF CURRENT CASE(REPLICATION).
C  NREPS - NUMBER OF REPLICATIONS.
C  NCOVLN - NUMBER OF TIMES CI COVERED TRUE EX FOR LOGNORMAL MLE.
C  NCOVGM - NUMBER OF TIMES CI COVERED TRUE EX FOR GAMMA ESTIMATOR.
C
C  SET RANDOM NUMBER GENERATOR.
  X=TIME(X)
  CALL RANSET(X)
  DO 10 IMU=1,5
    NTABLE=NTABLE+1
    NDIGO=1
    IF(NTABLE.GT.9) NDIGO=2
    WRITE(6,90) NDIGO,NTABLE,TEX(IMU),NDIGO,FLOAT(NREPS)
90  FORMAT(1H1,10X,*TABLE 8.*,1=,*. SIMULATION OF THE LOGNORMAL*,
1* DISTRIBUTION WITH EXPECTED VALUE *,F5.0/22X,=X,*THE NUMBER*,
2* OF REPLICATIONS PER CASE IS*,F6.0//1X,126(*=*)/
3* PARAMETERS*,38X,*LOGNORMAL ESTIMATOR*,29X,
4*GAMMA ESTIMATOR*/1X,12(*=*),16X,*SAMPLE*,2X,44(*=*),2X,44(*=*)/
51X,* MU SIGMA C.V. VAR(X) SIZE *,
62(* E(X)-HAT REL. BIAS AVE(CI) COVERAGE*)/
71X,126(*=*)/)
    TRUEEX=TEX(IMU)
    DO 20 ISIG=1,7
      TVX(ISIG)=(TEX(IMU)*CV(ISIG))**2

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No. LP-0946

```

NDIG2=2
IF(TVX(ISIG).GE.10000.) NDIG2=1
IF(TVX(ISIG).GE.100000.) NDIG2=0
XSIG=SQRT(ALOG(CV(ISIG)**2+1.))
XMU=0.5*ALOG(TEX(IMU)**2/(CV(ISIG)**2+1.))
DO 30 IN=1,6
  NN=N(IN)
  SLNCI=0.
  SGMCI=0.
  NCOVLN=0
  NCOVGM=0
  SEXLN=0.
  SEXGM=0.
  S2EXLN=0.
  S2EXGM=0.
  DO 40 I=1,NREPS
    SUMX=0.
    SUMY=0.
    SUMSQX=0.
    SUMSQY=0.
    DO 50 II=1,NN
      X=RNORM(XMU,XSIG)
      Y=EXP(X)

      SUMX=SUMX+X
      SUMY=SUMY+Y
      SUMSQX=SUMSQX+X*X
      SUMSQY=SUMSQY+Y*Y
50  CONTINUE
    XBAR=SUMX/FLOAT(NN)
    EXGM=SUMY/FLOAT(NN)
    XVAR=((SUMSQX-SUMX*SUMX/FLOAT(NN))/FLOAT(NN-1))
    EXLN=EXP(XBAR)*G(XVAR/2.,NN)
    SDEXGM=SQRT((SUMSQY-SUMY*SUMY/FLOAT(NN))/FLOAT(NN-1))
    TEMP1=SQRT(XVAR/FLOAT(NN)+XVAR*XVAR/FLOAT(2*(NN+1)))
    TEMP2=XBAR+XVAR/2.
    SEEXGM=SDEXGM/SQRT(FLOAT(NN))
    TEMP=T(NN-1)*SEEXGM
    SGMCI=SGMCI+2.*TEMP
    IF(EXGM-TEMP.LT.TRUEEX .AND. EXGM+TEMP.GT.TRUEEX) NCOVGM=NCOVGM
  I +1
  TEMP=1.96*TEMP1
  CIL=EXP(TEMP2-TEMP)
  CIU=EXP(TEMP2+TEMP)
  SLNCI=SLNCI+(CIU-CIL)
  IF(CIL.LT.TRUEEX .AND. CIU.GT.TRUEEX)
  I NCOVLN=NCOVLN+1
  SEXLN=SEXLN+EXLN
  SEXGM=SEXGM+EXGM
  S2EXLN=S2EXLN+EXLN*EXLN
  S2EXGM=S2EXGM+EXGM*EXGM
40  CONTINUE
  AEXLN=SEXLN/FLOAT(NREPS)
  AEXGM=SEXGM/FLOAT(NREPS)
  SDAEXLN=SQRT((S2EXLN-SEXLN*SEXLN/FLOAT(NREPS))/FLOAT(NREPS-1))
  SDAEXGM=SQRT((S2EXGM-SEXGM*SEXGM/FLOAT(NREPS))/FLOAT(NREPS-1))
  COVLN=FLOAT(NCOVLN)/FLOAT(NREPS)
  COVGM=FLOAT(NCOVGM)/FLOAT(NREPS)
  ACIGM=SGMCI/FLOAT(NREPS)
  ACILN=SLNCI/FLOAT(NREPS)
  RBLN=(AEXLN-TRUEEX)*100./TRUEEX
  RBGM=(AEXGM-TRUEEX)*100./TRUEEX
  WRITE(6,101) XMU,XSIG,CV(ISIG),NDIG2,TVX(ISIG),NN,
1  AEXLN,RBLN,ACILN,COVLN,AEXGM,RBGM,ACIGM,COVGM

```

```

101 FORMAT(1X,2F6.3,F6.2,F8.=,17,4X,2(G10.4,2X,G10.4,2X,G10.4,
   1 2X,F7.4,3X))
30 CONTINUE
   WRITE(6,102)
20 CONTINUE
   WRITE(6,102)
102 FORMAT(1X)
   WRITE(6,104)
104 FORMAT(1X,126(==*))
10 CONTINUE
   CALL SECOND(CP)
   WRITE(6,103) CP
103 FORMAT(*1CPU TIME REQUIRED WAS *,F10.2,* SECONDS.*)
STOP
END

```

```

PROGRAM CVGANSIM(INPUT,OUTPUT,TAPE5,TAPE6)
REAL TEX(10),TVX(10),CV(10)
INTEGER N(6)
DATA CV/.25,.5,.75,1.,1.25,1.5,2.0/
DATA TEX/1.,5.,10.,50.,100./
DATA N/5,10,20,30,50,100/
DATA NREPS/1000/
DATA NTABLE/0/

```

LASL Identification
No. LP-0947

```

C  CV      - ACRONYM FOR COEFFICIENT OF VARIATION OF X, WHERE X IS A RV.
C  CVLN    - LOGNORMAL ESTIMATOR OF CV.
C  CVGM    - GAMMA ESTIMATOR OF CV.
C  CVCN    - NORMAL ESTIMATOR OF CV.
C  S       - PREFIX TO INDICATE SUM OF.
C  S2      - PREFIX TO INDICATE SUM OF SQUARES OF.
C  X       - LOG TRANSFORMED VARIABLE.
C  Y       - ORIGINAL VARIABLE, EITHER GAMMA OR LOGNORMALLY DIST.
C  SD      - PREFIX TO INDICATE STANDARD DEVIATION OF.
C  SE      - PREFIX TO INDICATE STANDARD ERROR OF.
C  NN      - SAMPLE SIZE OF CURRENT CASE(REPLICATION).
C  NREPS   - NUMBER OF REPLICATIONS.
C  NCOVLN  - NUMBER OF TIMES CI COVERED TRUE CV FOR LOGNORMAL MLE.
C  NCOVGM  - NUMBER OF TIMES CI COVERED TRUE CV FOR GAMMA ESTIMATOR.
C  NCOVCN  - NUMBER OF TIMES CI COVERED TRUE CV FOR NORMAL ESTIMATOR.
C  NDIGIT  - FUNCTION TO DETERMINE NUMBER OF DIGITS TO PRINT TO THE
C            RIGHT OF THE DECIMAL POINT. THIS METHOD IS USED TO
C            CRAM AS MUCH INFORMATION IN AS LITTLE SPACE AS POSSIBLE.
C  ICHAR   - FUNCTION TO RETURN A NUMBER IN CHARACTER REPRESENTATION
C            FOR PRINTING. THE PURPOSE IS TO SAVE SPACE ON THE
C            OUTPUT.
C  IC      - PREFIX TO INDICATE A CHARACTER REPRESENTATION OF A
C            VARIABLE FOR PRINTING.
C
C  START TIMER
C            CALL SECOND(START)
C  SET RANDOM NUMBER GENERATOR.
C            X=TIME(X)
C            CALL RANSET(X)
C
C  LOOP TO PRODUCE 7 TABLES EACH WITH A DIFFERENT CV
C

```

```

DO 10 ISIG=1,7
  TRUECV=CV(ISIG)
  NTABLE=NTABLE+1
  NDIGO=1
  IF (NTABLE.GT.9) NDIGO=2
  WRITE(6,90) NDIGO,NTABLE,CV(ISIG),NDIGO,Float(NREPS)
90 FORMAT(1H1,10X,'TABLE C.',I=,*, SIMULATION OF THE GAMMA*,
1* DISTRIBUTION WITH COEFFICIENT OF VARIATION *,F5.2*,*/22X,=X,
2*THE NUMBER OF REPLICATIONS PER CASE IS*,F6.0//1X,132(==*)//
3* PARAMETERS*,25X,*LOGNORMAL ESTIMATOR*,18X,
4 *GAMMA ESTIMATOR*,20X,*NORMAL ESTIMATOR*
4/1X,11(==*),17X,33(==*),2X,33(==*),2X,33(==*)/
51X,*ALPHA BETA EX VAR(X) N*,
6 3(* C-HAT REL. BIAS AVE(CI) COV. */
71X,132(==*))
C
C LOOP TO PRODUCE 5 BLOCKS IN EACH TABLE, EACH BLOCK A DIFFERENT EX
C
DO 20 IMU=1,5
  TRUEEX=TEX(IMU)
  TVX(ISIG)=(TEX(IMU)*CV(ISIG))**2
  ICTVX=ICHAR(TVX(ISIG),7,2)
  ALPHA=1./CV(ISIG)**2
  BETA=TEX(IMU)*CV(ISIG)**2

  ICBETA=ICHAR(BETA,5,3)
C
C LOOP TO PERFORM SIMULATION FOR 6 VALUES OF SAMPLE SIZE
C
DO 30 IN=1,6
  NN=N(IN)
  SLNCI=0.
  SMC1=0.
  SCMC1=0.
  NCOVLN=0
  NCOVGM=0
  NCOVCM=0
  SCVLN=0.
  SCVGM=0.
  SCVCM=0.
  S2CVLN=0.
  S2CVGM=0.
  S2CVCN=0.
  CALL MDCHI(0.025,Float(NN),CHINNL,IER)
  CALL MDCHI(0.975,Float(NN),CHINNU,IER)
  CALL MDCHI(0.025,Float(NN-1),CHINML,IER)
  CALL MDCHI(0.975,Float(NN-1),CHINMU,IER)
C
C LOOP TO DO EACH REPLICATION
C
DO 40 I=1,NREPS
  SUMX=0.
  SUMY=0.
  SUMSQX=0.
  SUMSQY=0.
C
C LOOP TO GENERATE THE MINIMAL SUFFICIENT STATISTICS FOR ONE REP.
C
DO 50 II=1,NN
  Y=RGAMMA(ALPHA,BETA)
  X=ALOG(Y)
  SUMX=SUMX+X
  SUMY=SUMY+Y

```

```

      SUMSQX=SUMSQX+X*X
      SUMSQY=SUMSQY+Y*Y
50  CONTINUE
C
C  GENERATE ESTIMATES AND CUMULATE THE STATISTICS FOR THIS REPLICATION.
C
      XBAR=SUMX/FLOAT(NN)
      YBAR=SUMY/FLOAT(NN)
      XVAR=((SUMSQX-SUMX*SUMX/FLOAT(NN))/FLOAT(NN-1))
      YVAR=((SUMSQY-SUMY*SUMY/FLOAT(NN))/FLOAT(NN-1))
C
C  CALCULATE THE LOGNORMAL ESTIMATOR OF C
C
      CVLN=SQRT(EXP(AMIN1(740.,XVAR))-1.)
      CIU=FLOAT(NN-1)*XVAR/CHINNL
      CIL=FLOAT(NN-1)*XVAR/CHINMU
      CIU=SQRT(EXP(AMIN1(740.,CIU))-1.)
      CIL=SQRT(EXP(AMIN1(740.,CIL))-1.)
      SLNCI=SLNCI+(CIU-CIL)
      IF(CIL.LT.CV(1SIG) .AND. CIU.GT.CV(1SIG)) NCOVLN=NCOVLN+1
C
C  CALCULATE THE MOMENT ESTIMATOR OF C
C
      CVCM=SQRT(YVAR)/YBAR
      CVSE=T(NN-1)*CVCM*SQRT((1.+2.*CVCM*CVCM)/FLOAT(2*NN))
      CIU=CVCM+CVSE
      CIL=CVCM-CVSE
      SCMCI=SCMCI+(CIU-CIL)
      IF(CIL.LT.CV(1SIG) .AND. CIU.GT.CV(1SIG)) NCOVCM=NCOVCM+1
C
C  CALCULATE THE GAMMA ESTIMATOR OF C
C
      Z=ALOG(YBAR)-XBAR
      CVGM=SQRT(1./ALPHAT(Z))
      CLPI=0.5*(1.+SQRT(1.+4.*FLOAT(NN+1)*Z/(3.*CHINMU)))
      CUP1=0.5*(1.+SQRT(1.+4.*FLOAT(NN+1)*Z/(3.*CHINML)))
      CIL=SQRT(2.*FLOAT(NN)*Z/(CLPI*CHINMU))
      CIU=SQRT(2.*FLOAT(NN)*Z/(CUP1*CHINML))
      IF(CIL.LT.CV(1SIG) .AND. CIU.GT.CV(1SIG)) NCOVGM=NCOVGM+1
      SGMCI=SGMCI+(CIU-CIL)
      SCVLN=SCVLN+CVLN
      SCVGM=SCVGM+CVGM
      SCVCM=SCVCM+CVCM
      S2CVLN=S2CVLN+CVLN*CVLN
      S2CVGM=S2CVGM+CVGM*CVGM
      S2CVCM=S2CVCM+CVCM*CVCM
40  CONTINUE
C
C  CALCULATE STATISTICS ON THE VARIOUS ESTIMATES
C
      ACVLN=SCVLN/FLOAT(NREPS)
      ACVGM=SCVGM/FLOAT(NREPS)
      ACVCM=SCVCM/FLOAT(NREPS)
      SDACVLN=SQRT((S2CVLN-SCVLN*SCVLN/FLOAT(NREPS))/FLOAT(NREPS-1))
      SDACVGM=SQRT((S2CVGM-SCVGM*SCVGM/FLOAT(NREPS))/FLOAT(NREPS-1))
      SDACVCM=SQRT((S2CVCM-SCVCM*SCVCM/FLOAT(NREPS))/FLOAT(NREPS-1))
      COVLN=FLOAT(NCOVLN)/FLOAT(NREPS)
      COVGM=FLOAT(NCOVGM)/FLOAT(NREPS)
      COVCM=FLOAT(NCOVCM)/FLOAT(NREPS)
      AC1GM=SGMCI/FLOAT(NREPS)
      AC1LN=SLNCI/FLOAT(NREPS)
      AC1CM=SCMCI/FLOAT(NREPS)
      RBLN=(ACVLN-TRUECV)*100./TRUECV

```



```

RSGM=(ACVGM-TRUECV)*100./TRUECV
RSCM=(ACVCM-TRUECV)*100./TRUECV
ICACVLN=ICAR(ACVLN,8,3)
ICRBLN=ICAR(RBLN,8,3)
ICACILN=ICAR(ACILN,8,3)
ICACVGM=ICAR(ACVGM,8,3)
ICRBGM=ICAR(RBGM,8,3)
ICACIGM=ICAR(ACIGM,8,3)
ICACVCM=ICAR(ACVCM,8,3)
ICRBCM=ICAR(RBCM,8,3)
ICACICM=ICAR(ACICM,8,3)
WRITE(6,101) ALPHA,ICBETA,INT(TRUEEX),ICTVX,NN,
1 ICACVLN,ICRBLN,ICACILN,COVLN,
2 ICACVGM,ICRBGM,ICACIGM,COVGM,
3 ICACVCM,ICRBCM,ICACICM,COVCM
101 FORMAT(1X,F5.2,1X,A5,I4,1X,A7,I4,3(1X,AB,1X,AB,1X,AB,
1 1X,F5.3,2X))
30 CONTINUE
WRITE(6,102)
20 CONTINUE
WRITE(6,102)
102 FORMAT(1X)
WRITE(6,104)
104 FORMAT(1X,132(==))
10 CONTINUE
CALL SECOND(STOP)
CP=STOP-START
WRITE(6,103) CP
103 FORMAT('CPU TIME REQUIRED WAS ',F10.2,' SECONDS. ')
STOP
END
REAL FUNCTION ALPHAT(Z)
C
C FUNCTION ALPHAT CALCULATES A MAXIMUM LIKELIHOOD ESTIMATE OF ALPHA
C WITH AN APPROXIMATION GIVEN BY
C GREENWOOD, J. A. AND D. DURAND. 1960. AIDS FOR FITTING THE
C GAMMA DISTRIBUTION BY MAXIMUM LIKELIHOOD. TECHNOMETRICS 20:55-65.
C
C THE ERROR FOR Z IN THE INTERVAL (0,0.5772) DOES NOT EXCEED 0.0088
C PERCENT, AND FOR Z IN THE INTERVAL (0.5772,17) DOES NOT EXCEED
C 0.0054 PERCENT. THIS IS QUITE SUFFICIENT FOR MOST USES, AND
C AVOIDS THE MUCH MORE COMPLICATED METHOD OF NUMERICALLY MINIMIZING
C THE LIKELIHOOD EQUATIONS FOR ESTIMATING ALPHA.
C
IF(Z.LT.0.) GO TO 20
IF(Z.GT.0.5772) GO TO 10
ALPHAT=(0.5000876+0.1648852*Z-0.0544274*Z**2)/Z
RETURN
10 ALPHAT=(8.898919+9.059950*Z+0.9775373*Z**2)
1 /((11.79728+11.968477*Z+Z**2)*Z)
IF(Z.LE.17.) RETURN
20 CALL LABRT(1,50,ESTIMATE OF ALPHA NOT VALID, Z =
1 ,Z)
RETURN
END
INTEGER FUNCTION ICHAR(VALUE,NCOLS,NDR)
C FUNCTION ICHAR RETURNS VALUE AS A CHARACTER STRING OF LENGTH
C NCOLS. AN EXPONENTIAL REPRESENTATION WILL BE USED IF NECESSARY,
C OTHERWISE THERE IS NDR DIGITS TO THE RIGHT OF THE DECIMAL POINT.
C
C INPUTS -
C VALUE - VALUE TO BE PRINTED
C NCOLS - NUMBER OF CHARACTERS TO BE USED IN OUTPUT (MAX OF 10)

```

```

C   NDR   - NUMBER OF DIGITS TO RIGHT OF DECIMAL POINT IF
C           EXPONENTIAL NOTATION IS NOT USED.
C
C   THIS FUNCTION CALLS NDIGIT, AND USED THE ERROR RETURN IER TO
C   DETERMINE IF EXPONENTIAL NOTATION MUST BE USED.
C
      NDIG=NDIGIT(VALUE,NCOLS,NDR,IER)
      IF(IER.GT.0) GO TO 10
      ENCODE(10,101,1CHAR) NCOLS,NDIG,VALUE
101  FORMAT(F=.,=)
      RETURN
10  NSIGN=0
      IF(VALUE.LT.0.) NSIGN=1
      IER=IER-NSIGN-1
      NDIGE=1
      IF(IER.GE.10) NDIGE=2
      IF(IER.GE.100) NDIGE=3
      NDIGF=NCOLS-NDIGE-1
      NDIGR=NDIGF-1-NSIGN
      ENCODE(10,102,1CHAR)NDIGF,NDIGR,VALUE/(10.**IER),NDIGE,IER
102  FORMAT(F=.,=,1HE,1=)
      RETURN
      END
      INTEGER FUNCTION NDIGIT(VALUE,NC,NDR,IER)
C   FUNCTION NDIGIT CALCULATES THE NUMBER OF COLUMNS IN A FIXED
C   FORMAT OUTPUT THAT SHOULD BE TO THE RIGHT OF THE DECIMAL AND
C   STILL BE ABLE TO PRINT THE NUMBER
C

```

```

C   INPUTS -
C   VALUE   - VALUE TO BE PRINTED
C   NC      - TOTAL NUMBER OF COLUMNS THAT CAN BE USED
C   NDR     - DEFAULT NUMBER OF DIGITS TO THE RIGHT OF THE DECIMAL
C   IER     - ERROR RETURN, 0 IS OKAY, POSITIVE VALUE IS NUMBER OF
C             COLUMNS NECESSARY TO HOLD VALUE IN F FORMAT.
C

```

```

      IER=0
      ND=NDR
      IF(ABS(VALUE).LT.1.E-15) GO TO 10
      NL=MAX0(INT(ALOG10(ABS(VALUE))+1.),0)
      IF(VALUE.LT.0.) NL=NL+1
      IF(NL.GE.NC) IER=NL+1
      ND=NC-NL-1
      IF(ND.LT.0) ND=0
      IF(ND.GT.NDR) ND=NDR
10  NDIGIT=ND
      RETURN
      END

```

```

PROGRAM CVLOGSM(INPUT,OUTPUT,TAPE5,TAPE6)
REAL TEX(10),TVX(10),CV(10)
INTEGER N(6)
DATA CV/.25,.5,.75,1.,1.25,1.5,2.0/
DATA TEX/1.,5.,10.,50.,100./
DATA N/5,10,20,30,50,100/
DATA NREPS/1000/
DATA NTABLE/7/

```

LASL Identification
No. LP-0948

```

C  CV      - ACRONYM FOR COEFFICIENT OF VARIATION OF X, WHERE X IS A RV.
C  CVLN    - LOGNORMAL ESTIMATOR OF CV.
C  CVGM    - GAMMA ESTIMATOR OF CV.
C  CVM    - NORMAL ESTIMATOR OF CV.
C  S       - PREFIX TO INDICATE SUM OF.
C  S2      - PREFIX TO INDICATE SUM OF SQUARES OF.
C  X       - LOG TRANSFORMED VARIABLE.
C  Y       - ORIGINAL VARIABLE, EITHER GAMMA OR LOGNORMALLY DIST.
C  SD      - PREFIX TO INDICATE STANDARD DEVIATION OF.
C  SE      - PREFIX TO INDICATE STANDARD ERROR OF.
C  NN      - SAMPLE SIZE OF CURRENT CASE (REPLICATION).
C  NREPS   - NUMBER OF REPLICATIONS.
C  NCOVLN  - NUMBER OF TIMES CI COVERED TRUE CV FOR LOGNORMAL MLE.
C  NCOVGM  - NUMBER OF TIMES CI COVERED TRUE CV FOR GAMMA ESTIMATOR.
C  NCOVCM  - NUMBER OF TIMES CI COVERED TRUE CV FOR NORMAL ESTIMATOR.
C  NDIGIT  - FUNCTION TO DETERMINE NUMBER OF DIGITS TO PRINT TO THE
C           RIGHT OF THE DECIMAL POINT. THIS METHOD IS USED TO
C           CRAM AS MUCH INFORMATION IN AS LITTLE SPACE AS POSSIBLE.
C  ICHAR   - FUNCTION TO RETURN A NUMBER IN CHARACTER REPRESENTATION
C           FOR PRINTING. THE PURPOSE IS TO SAVE SPACE ON THE
C           OUTPUT.
C  IC      - PREFIX TO INDICATE A CHARACTER REPRESENTATION OF A
C           VARIABLE FOR PRINTING.
C
C  START TIMER
C    CALL SECOND(START)
C  SET RANDOM NUMBER GENERATOR.
C    X=TIME(X)
C    CALL RANSET(X)
C
C  LOOP TO PRODUCE 7 TABLES EACH WITH A DIFFERENT CV
C
C    DO 10 ISIG=1,7
C      TRUECV=CV(ISIG)
C      NTABLE=NTABLE+1
C      NDIG0=1
C      IF(NTABLE.GT.9) NDIG0=2
C      WRITE(6,90) NDIG0,NTABLE,CV(ISIG),NDIG0,FLOAT(NREPS)
90  FORMAT(1H1,10X,*TABLE C.*,I=,*, SIMULATION OF THE LOGNORMAL*,
C        1* DISTRIBUTION WITH COEFFICIENT OF VARIATION *,F5.2*,*/22X,=X,
C        2*THE NUMBER OF REPLICATIONS PER CASE IS*,F6.0//1X,132(*=*)/
C        3* PARAMETERS*,25X,*LOGNORMAL ESTIMATOR*,18X,
C        4 *GAMMA ESTIMATOR*,20X,*NORMAL ESTIMATOR*
C        4//1X,11(*=*),17X,33(*=*),2X,33(*=*),2X,33(*=*)/
C        51X,* MU SIGMA EX VAR(X) N*,
C        6 3(* C-HAT REL. BIAS AVE(CI) COV. */
C        71X,132(*=*)/)
C
C  LOOP TO PRODUCE 5 BLOCKS IN EACH TABLE, EACH BLOCK A DIFFERENT EX
C
C    DO 20 IMU=1,5
C      TRUEEX=TEX(IMU)
C      TVX(ISIG)=(TEX(IMU)*CV(ISIG))**2
C      ICTVX=ICHAR(TVX(ISIG),7,2)
C      XMU=0.5*ALOG(TEX(IMU)**2/(CV(ISIG)**2+1.))
C      ICXMU=ICHAR(XMU,5,3)
C      XSIG=SQRT(ALOG(CV(ISIG)**2+1.))
C      ICXSIG=ICHAR(XSIG,5,3)
C
C  LOOP TO PERFORM SIMULATION FOR 6 VALUES OF SAMPLE SIZE
C
C    DO 30 IN=1,6
C      NN=N(IN)
C      SLNCI=0.
C      SGMCI=0.

```

```

      SCMC1=0.
      NCOVLN=0
      NCOVGM=0
      NCOVCM=0
      SCVLN=0.
      SCVGM=0.
      SCVCM=0.
      S2CVLN=0.
      S2CVGM=0.
      S2CVCM=0.
      CALL MDCHI(0.025,FLOAT(NN),CHINNL,IER)
      CALL MDCHI(0.975,FLOAT(NN),CHINNU,IER)
      CALL MDCHI(0.025,FLOAT(NN-1),CHINML,IER)
      CALL MDCHI(0.975,FLOAT(NN-1),CHINMU,IER)
C
C   LOOP TO DO EACH REPLICATION
C
      DO 40 I=1,NREPS
        SUMX=0.
        SUMY=0.
        SUMSQX=0.
        SUMSQY=0.
C
C   LOOP TO GENERATE THE MINIMAL SUFFICIENT STATISTICS FOR ONE REP.
C
        DO 50 II=1,NN
          X=RNORM(XMU,XSIG)
          Y=EXP(X)
          SUMX=SUMX+X
          SUMY=SUMY+Y
          SUMSQX=SUMSQX+X*X
          SUMSQY=SUMSQY+Y*Y
        50 CONTINUE
C
C   GENERATE ESTIMATES AND CUMULATE THE STATISTICS FOR THIS REPLICATION.
C
          XBAR=SUMX/FLOAT(NN)
          YBAR=SUMY/FLOAT(NN)
          XVAR=((SUMSQX-SUMX*SUMX/FLOAT(NN))/FLOAT(NN-1))
          YVAR=((SUMSQY-SUMY*SUMY/FLOAT(NN))/FLOAT(NN-1))
C
C   CALCULATE THE LOGNORMAL ESTIMATOR OF C
C
          CVLN=SQRT(EXP(AMINI(740.,XVAR))-1.)
          CIU=FLOAT(NN-1)*XVAR/CHINNL
          CIL=FLOAT(NN-1)*XVAR/CHINNU
          CIU=SQRT(EXP(AMINI(740.,CIU))-1.)
          CIL=SQRT(EXP(AMINI(740.,CIL))-1.)
          SLNCI=SLNCI+(CIU-CIL)
          IF(CIL.LT.CV(ISIG) .AND. CIU.GT.CV(ISIG)) NCOVLN=NCOVLN+1
C
C   CALCULATE THE MOMENT ESTIMATOR OF C
C
          CVCM=SQRT(YVAR)/YBAR
          CVSE=T(NN-1)*CVCM*SQRT((1.+2.*CVCM*CVCM)/FLOAT(2*NN))
          CIU=CVCM+CVSE
          CIL=CVCM-CVSE
          SCMC1=SCMC1+(CIU-CIL)
          IF(CIL.LT.CV(ISIG) .AND. CIU.GT.CV(ISIG)) NCOVCM=NCOVCM+1
C
C   CALCULATE THE GAMMA ESTIMATOR OF C
C
          Z=ALOG(YBAR)-XBAR
          CVGM=SQRT(1./ALPHAT(Z))
          CLP1=0.5*(1.+SQRT(1.+4.*FLOAT(NN+1)*Z/(3.*CHINMU)))
          CUP1=0.5*(1.+SQRT(1.+4.*FLOAT(NN+1)*Z/(3.*CHINML)))

```

```

      CIL=SQRT(2.*FLOAT(NN)*Z/(CLPI*CHINMU))
      CIU=SQRT(2.*FLOAT(NN)*Z/(CUPI*CHINML))
      IF(CIL.LT.CV(1SIG) .AND. CIU.GT.CV(1SIG)) NCOVGM=NCOVGM+1
      SGMCI=SGMCI+(CIU-CIL)
      SCVLN=SCVLN+CVLN
      SCVGM=SCVGM+CVGM
      SCVCM=SCVCM+CVCM
      S2CVLN=S2CVLN+CVLN*CVLN
      S2CVGM=S2CVGM+CVGM*CVGM
      S2CVCM=S2CVCM+CVCM*CVCM
40 CONTINUE
C
C   CALCULATE STATISTICS ON THE VARIOUS ESTIMATES
C
      ACVLN=SCVLN/FLOAT(NREPS)
      ACVGM=SCVGM/FLOAT(NREPS)
      ACVCM=SCVCM/FLOAT(NREPS)
      SDACVLN=SQRT((S2CVLN-SCVLN*SCVLN/FLOAT(NREPS))/FLOAT(NREPS-1))
      SDACVGM=SQRT((S2CVGM-SCVGM*SCVGM/FLOAT(NREPS))/FLOAT(NREPS-1))
      SDACVCM=SQRT((S2CVCM-SCVCM*SCVCM/FLOAT(NREPS))/FLOAT(NREPS-1))
      COVLN=FLOAT(NCOVLN)/FLOAT(NREPS)
      COVGM=FLOAT(NCOVGM)/FLOAT(NREPS)
      COVCM=FLOAT(NCOVCM)/FLOAT(NREPS)
      ACIGH=SGMCI/FLOAT(NREPS)
      ACILN=SLNCI/FLOAT(NREPS)
      ACICM=SCMCI/FLOAT(NREPS)
      RBLN=(ACVLN-TRUECV)*100./TRUECV
      RBGM=(ACVGM-TRUECV)*100./TRUECV
      RBGM=(ACVCM-TRUECV)*100./TRUECV
      ICACVLN=ICHAR(ACVLN,8,3)
      ICRBLN=ICHAR(RBLN,8,3)
      ICACILN=ICHAR(ACILN,8,3)
      ICACVGM=ICHAR(ACVGM,8,3)
      ICRBGM=ICHAR(RBGM,8,3)
      ICACIGH=ICHAR(ACIGH,8,3)
      ICACVCM=ICHAR(ACVCM,8,3)
      ICRBCM=ICHAR(RBCM,8,3)
      ICACICM=ICHAR(ACICM,8,3)
      WRITE(6,101) ICXMU,ICXSIG,INT(TRUEEX),ICTVX,NN,
1    ICACVLN,ICRBLN,ICACILN,COVLN,
2    ICACVGM,ICRBGM,ICACIGH,COVGM,
3    ICACVCM,ICRBCM,ICACICM,COVCM
101 FORMAT(1X,A5,1X,A5,14,1X,A7,14,3(1X,A8,1X,A8,1X,A8,
1    1X,F5.3,2X))
30 CONTINUE
      WRITE(6,102)
20 CONTINUE
      WRITE(6,102)
102 FORMAT(1X)
      WRITE(6,104)
104 FORMAT(1X,132(*=*))
10 CONTINUE
      CALL SECOND(STOP)

      CP=STOP-START
      WRITE(6,103) CP
103 FORMAT('CPU TIME REQUIRED WAS ',F10.2,' SECONDS.')
      STOP
      END
      REAL FUNCTION ALPHAT(Z)
C
C   FUNCTION ALPHAT CALCULATES A MAXIMUM LIKHOOD ESTIMATE OF ALPHA
C   WITH AN APPROXIMATION GIVEN BY
C   GREENWOOD, J. A. AND D. DURAND. 1960. AIDS FOR FITTING THE

```

```

C   GAMMA DISTRIBUTION BY MAXIMUM LIKELIHOOD.  TECHONOMETRICS 20055-65.
C
C   THE ERROR FOR Z IN THE INTERVAL (0,0.5772) DOES NOT EXCEED 0.0088
C   PERCENT, AND FOR Z IN THE INTERVAL (0.5772,17) DOES NOT EXCEED
C   0.0054 PERCENT.  THIS IS QUITE SUFFICIENT FOR MOST USES, AND
C   AVOIDS THE MUCH MORE COMPLICATED METHOD OF NUMERICALLY MINIMIZING
C   THE LIKELIHOOD EQUATIONS FOR ESTIMATING ALPHA.
C
      IF(Z.LT.0.) GO TO 20
      IF(Z.GT.0.5772) GO TO 10
      ALPHAT=(0.5000876+0.1648852*Z-0.0544274*Z*Z)/Z
      RETURN
10  ALPHAT=(0.898919+9.059950*Z+0.9775373*Z*Z)
      1 / ((17.79728+11.963477*Z+Z*Z)*Z)
      IF(Z.LE.17.) RETURN
20  CALL LABRT(1,50)ESTIMATE OF ALPHA NOT VALID, Z =
      1 ,Z)
      RETURN
END
      INTEGER FUNCTION ICHAR(VALUE,NCOLS,NDR)
C   FUNCTION ICHAR RETURNS VALUE AS A CHARACTER STRING OF LENGTH
C   NCOLS.  AN EXPONENTIAL REPRESENTATION WILL BE USED IF NECESSARY,
C   OTHERWISE THERE IS NDR DIGITS TO THE RIGHT OF THE DECIMAL POINT.
C
C   INPUTS -
C   VALUE - VALUE TO BE PRINTED
C   NCOLS - NUMBER OF CHARACTERS TO BE USED IN OUTPUT (MAX OF 10)
C   NDR - NUMBER OF DIGITS TO RIGHT OF DECIMAL POINT IF
C         EXPONENTIAL NOTATION IS NOT USED.
C
C   THIS FUNCTION CALLS NDIGIT, AND USED THE ERROR RETURN IER TO
C   DETERMINE IF EXPONENTIAL NOTATION MUST BE USED.
C
      NDIG=NDIGIT(VALUE,NCOLS,NDR,IER)
      IF(IER.GT.0) GO TO 10
      ENCODE(10,101,ICHAR) NCOLS,NDIG,VALUE
101  FORMAT(F=,=)
      RETURN
10  NSIGN=0
      IF(VALUE.LT.0.) NSIGN=1
      IER=IER-NSIGN-1
      NDIGE=1
      IF(IER.GE.10) NDIGE=2
      IF(IER.GE.100) NDIGE=3
      NDIGF=NCOLS-NDIGE-1
      NDIGR=NDIGF-1-NSIGN
      ENCODE(10,102,ICHAR)NDIGF,NDIGR,VALUE/(10.**IER),NDIGE,IER
102  FORMAT(F=,.,1HE,1=)
      RETURN
      END
      INTEGER FUNCTION NDIGIT(VALUE,NC,NDR,IER)
C   FUNCTION NDIGIT CALCULATES THE NUMBER OF COLUMNS IN A FIXED
C   FORMAT OUTPUT THAT SHOULD BE TO THE RIGHT OF THE DECIMAL AND
C   STILL BE ABLE TO PRINT THE NUMBER
C
C   INPUTS -
C   VALUE - VALUE TO BE PRINTED
C   NC - TOTAL NUMBER OF COLUMNS THAT CAN BE USED
C   NDR - DEFAULT NUMBER OF DIGITS TO THE RIGHT OF THE DECIMAL
C   IER - ERROR RETURN, 0 IS OKAY, POSITIVE VALUE IS NUMBER OF
C         COLUMNS NECESSARY TO HOLD VALUE IN F FORMAT.
C
      IER=0
      ND=NDR

```

```

      IF (ABS(VALUE).LT.1.E-15) GO TO 10
      NL=MAX0(INT(ALOG10(ABS(VALUE))+1.),0)
      IF (VALUE.LT.0.) NL=NL+1
      IF (NL.GE.NC) IER=NL+1
      ND=NC-NL-1
      IF (ND.LT.0) ND=0
      IF (ND.GT.NDR) ND=NDR
10  NDIGIT=ND
      RETURN
      END

```

```

      REAL FUNCTION G(T,N)
      REAL T,RN,SUM,PLUS
      INTEGER N,J
C     FUNCTION G CALCULATES THE INFINITE SERIES USED FOR MAXIMUM
C     LIKELIHOOD ESTIMATION OF THE LOGNORMAL DISTRIBUTION.
      RN=N
      PLUS=(RN-1.)*T/RN
      SUM=1.+PLUS
      DO 10 J=2,50
      PLUS=PLUS*FLOAT((N-1)*(N-1))*T/(RN*(RN+FLOAT(2*J-3))*FLOAT(J))
      SUM=SUM+PLUS
      IF (ABS(PLUS/SUM).LT.0.0000001) GO TO 11
10  CONTINUE
      CALL LABRT(1,50HFUNCTION G NOT CONVERGED AFTER 50 ITERATIONS.
      1 ,QB)
11  G=SUM
      RETURN
      END
      REAL FUNCTION T(N)
      REAL TVALUE(30)
      DATA TVALUE/12.706,4.303,3.182,2.776,2.571,2.447,2.365,2.306,
      2.262,2.228,2.201,2.179,2.160,2.145,2.131,2.120,2.110,2.101,
      2.093,2.086,2.080,2.074,2.069,2.064,2.060,2.056,2.052,2.048,
      2.045,1.96/
      NDF=MIN0(31,MAX0(N,2))-1
      T=TVALUE(NDF)
      RETURN
      END

```

```

      FUNCTION RNORM(XMEAN,XSD)
C     THIS PROCEDURE GENERATES PAIRS OF INDEPENDENT NORMAL RANDOM DEVIATES
C     WITH MEAN XMEAN AND STANDARD DEVIATION XSD. THE FUNCTION RETURNS
C     RNORM NORMALLY DISTRIBUTED ON THE INTERVAL (-INF, +INF). THE
C     METHOD IS EXACT EVEN IN THE TAILS.
C     REFERENCE:
C     BELL, J. R. 1965. ALGORITHM 334 NORMAL RANDOM DEVIATES (G5).
C     COMM. ACM 11(7)498.
C     RANF(L) IS ANY PROCEDURE RETURNING A RANDOM NUMBER UNIFORMLY
C     DISTRIBUTED ON THE INTERVAL FROM ZERO TO ONE. L IS THE RANDOM

```

```

C  SEED (INTEGER) SET BY THE CALLING PROGRAM.
C  BOOL IS A FLAG VARIABLE TO DEETERMINE WHETHER A NEW PAIR OF DEVIATES
C  ARE TO BE GENERATED ON THIS CALL, OR WHETHER ONE REMAINS FROM
C  THE PREVIOUS CALL.
      INTEGER BOOL
      DATA BOOL/0/
      IF(XSD.LE.0.) GO TO 30
      IF(BOOL.EQ.1) GO TO 20
C  VON NEUMANN REJECTION FOR CHOOSING A RANDOM ANGLE PHI = 2 * THETA
C  THETA = ARCTAN(Y/X)
10 X=RANF(L)
      Y=2.0*RANF(L)-1.0
      XX=X*X
      YY=Y*Y
      S=XX+YY
      IF(S.GT.1.0) GO TO 10
C  CHOOSES RL RANDOMLY FROM A CHI-SQUARED DISTRIBUTION AND NORMALIZED
C  WITH S
      RL=SQRT(-2.0*ALOG(RANF(L)))/S
C  COMPUTES DEVIATES AS RL * SIN(PHI) AND RL * COS(PHI)
      RNORM=XMEAN+XSD*2.0*X*Y*RL
      BOOL=1
      RETURN
20 RNORM=XMEAN+XSD*(XX-YY)*RL
      BOOL=0
      RETURN
30 CALL LABRT(1,50H)PARAMETER IS UNACCEPTABLE.
      I      ,0B)
      BOOL=0
      RNORM=XMEAN
      RETURN
      END

```

```

      REAL FUNCTION RGAMMA(ALPHA,BETA)
      IF(ALPHA.LE.0. .OR. BETA.LE.0.) GO TO 50
      K=INT(ALPHA)
      GAMMA=ALPHA-FLOAT(K)
      X=0.
      IF(K.LT.1) GO TO 20
      PROD=1.
      DO 10 I=1,K
10  PROD=PROD*RANF(0.0)
      X=-ALOG(PROD)
      IF(ABS(GAMMA).LT.1.E-7) GO TO 30
20  Z=-ALOG(RANF(0.0))
40  Y1=RANF(0.0)**(1./GAMMA)
      Y2=RANF(0.0)**(1./((1.-GAMMA)))
      IF((Y1+Y2).GT.1.0) GO TO 40
      Y=Y1/(Y1+Y2)
      RGAMMA=BETA*(X+Y*Z)
      RETURN
30  RGAMMA=BETA*X
      RETURN
50  CALL LABRT(1,50H)PARAMETERS FOR GAMMA GENERATION ARE UNACCEPTABLE.
      I      ,0B)
      RGAMMA=1.
      RETURN
      END

```


APPENDIX B

RESULTS FOR ESTIMATION OF THE EXPECTED VALUE

Tables B.1 through B.20 show results for the estimation of the expected value. Listed below are the column headings used in these tables and the corresponding text notations.

Listing of the table headings used in Appendix B and the corresponding text notation.

Table	Text
ALPHA	α -----parameter of gamma distribution
BETA	β -----parameter of gamma distribution
MU	μ_y -----parameter of lognormal distribution
SIGMA	σ_y -----parameter of lognormal distribution
C.V.	c -----coefficient of variation
VAR(X)	Var(x)-----variance of x
SAMPLE SIZE	n -----number of observations
E(X)-HAT	$\hat{E}(x)$ -----estimate of expected value
REL. BIAS	$100(\hat{E}(x)-EX)/EX$ -----relative bias
AVE(CI)	-----Average confidence interval width for the 1000 replications
COVERAGE	-----Proportion of the 1000 replications that the confidence interval included EX

TABLE B.1. SIMULATION OF THE GAMMA DISTRIBUTION WITH EXPECTED VALUE 1.
THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS				SAMPLE SIZE	LOGNORMAL ESTIMATOR				GAMMA ESTIMATOR			
ALPHA	BETA	C.V.	VAR(X)		E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE	E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE
16.00	.063	.25	.06	5	.9983	-.1674	.4294	.6040	.9980	-.1955	.6608	.9550
16.00	.063	.25	.06	10	.9937	-.6302	.3118	.9200	.9932	-.6806	.3505	.9470
16.00	.063	.25	.06	20	1.002	.2010	.2250	.9510	1.001	.1393	.2318	.9600
16.00	.063	.25	.06	30	1.002	.2397	.1844	.9560	1.002	.1847	.1864	.9550
16.00	.063	.25	.06	50	1.001	.7827E-01	.1424	.9600	1.000	.1734E-01	.1377	.9500
16.00	.063	.25	.06	100	1.001	.1014	.1006	.9470	1.000	.3787E-01	.9737E-01	.9460
4.00	.250	.50	.25	5	1.004	.4265	1.054	.8970	.9966	-.1386	1.308	.9520
4.00	.250	.50	.25	10	1.010	1.047	.7332	.9490	1.001	.1082	.7020	.9420
4.00	.250	.50	.25	20	1.013	1.250	.5169	.9570	1.002	.1763	.4658	.9460
4.00	.250	.50	.25	30	1.009	.8026	.4132	.9550	.9979	-.2136	.3669	.9340
4.00	.250	.50	.25	50	1.011	1.114	.3207	.9640	1.000	.3768E-01	.2767	.9500
4.00	.250	.50	.25	100	1.012	1.200	.2267	.9730	1.000	.4942E-01	.1956	.9460
1.78	.563	.75	.56	5	1.028	2.776	3.138	.9140	.9958	-.4227	1.908	.9340
1.78	.563	.75	.56	10	1.040	3.964	1.566	.9630	.9876	-.1245	1.024	.9160
1.78	.563	.75	.56	20	1.065	6.524	1.047	.9700	.9987	-.1321	.6876	.9340
1.78	.563	.75	.56	30	1.061	6.076	.8064	.9700	.9969	-.3071	.5481	.9330
1.78	.563	.75	.56	50	1.066	6.582	.6174	.9780	.9983	-.1686	.4092	.9410
1.78	.563	.75	.56	100	1.064	6.354	.4267	.9660	.9966	-.3401	.2904	.9460
1.00	1.000	1.00	1.00	5	1.181	18.05	.2288E+05	.9430	1.029	2.936	2.558	.9220
1.00	1.000	1.00	1.00	10	1.165	16.46	7.746	.9690	.9776	-2.245	1.312	.8830
1.00	1.000	1.00	1.00	20	1.262	26.16	2.932	.9800	1.006	.6373	.8920	.9170
1.00	1.000	1.00	1.00	30	1.262	26.21	1.975	.9670	.9920	-.7967	.7185	.9230
1.00	1.000	1.00	1.00	50	1.264	26.43	1.364	.9300	.9919	-.8123	.5408	.9030
1.00	1.000	1.00	1.00	100	1.278	27.81	.9157	.7860	.9999	-.5551E-02	.3866	.9490
.64	1.563	1.25	1.56	5	1.538	53.84	.3830E+14	.9450	.9826	-1.737	2.886	.8670
.64	1.563	1.25	1.56	10	1.988	98.85	.4051E+05	.9870	.9836	-1.642	1.594	.8880
.64	1.563	1.25	1.56	20	1.937	93.71	56.58	.9510	.9872	-1.276	1.098	.8970
.64	1.563	1.25	1.56	30	2.139	113.9	35.44	.8950	.9946	-.5413	.8995	.9160
.64	1.563	1.25	1.56	50	2.016	101.6	4.965	.7080	1.008	.8389	.6818	.9260
.64	1.563	1.25	1.56	100	1.992	99.24	2.746	.3200	.9949	-.5149	.4792	.9350
.44	2.250	1.50	2.25	5	2.906	190.6	.1408E+32	.9540	.9897	-1.030	3.299	.8330
.44	2.250	1.50	2.25	10	113.2	.1122E+05	.4910E+28	.9820	1.010	.9727	1.933	.8610
.44	2.250	1.50	2.25	20	9.395	839.5	.1397E+07	.8850	1.013	1.290	1.304	.8850
.44	2.250	1.50	2.25	30	4.997	399.7	300.3	.7030	.9959	-.4090	1.055	.8970
.44	2.250	1.50	2.25	50	5.688	468.8	76.00	.3700	.9909	-.9058	.7961	.9210
.44	2.250	1.50	2.25	100	5.589	458.9	26.81	.0470	1.002	.2149	.5802	.9240
.25	4.000	2.00	4.00	5	170.5	.1695E+05	.3013E+60	.9740	.9753	-2.475	3.954	.7640
.25	4.000	2.00	4.00	10	.5487E+08	.5487E+08	.5269E+63	.9670	1.001	.1310	2.394	.8150
.25	4.000	2.00	4.00	20	2560.	.2559E+06	.6325E+17	.5160	1.026	2.608	1.721	.8640
.25	4.000	2.00	4.00	30	.2006E+05	.2006E+07	.1769E+17	.2130	1.008	.7947	1.390	.8660
.25	4.000	2.00	4.00	50	.5148E+06	.5148E+08	.9804E+16	.0190	1.009	.9466	1.059	.8820
.25	4.000	2.00	4.00	100	3200.	.3199E+06	.3063E+08	0.0000	.9985	-.1501	.7550	.9040

TABLE B.2. SIMULATION OF THE GAMMA DISTRIBUTION WITH EXPECTED VALUE 5.
THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS					LOGNORMAL ESTIMATOR				GAMMA ESTIMATOR			
ALPHA	BETA	C.V.	VAR(X)	SAMPLE SIZE	E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE	E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE
16.00	.313	.25	1.56	5	4.994	-.1281	2.177	.8750	4.992	-.1564	3.351	.9630
16.00	.313	.25	1.56	10	4.993	-.1419	1.544	.9280	4.990	-.1921	1.737	.9490
16.00	.313	.25	1.56	20	5.008	.1699	1.120	.9380	5.005	.1100	1.156	.9460
16.00	.313	.25	1.56	30	5.007	.1392	.9182	.9490	5.004	.7824E-01	.9256	.9480
16.00	.313	.25	1.56	50	5.010	.2051	.7142	.9450	5.007	.1384	.6890	.9400
16.00	.313	.25	1.56	100	5.001	.1064E-01	.5068	.9510	4.997	-.5794E-01	.4895	.9450
4.00	1.250	.50	6.25	5	4.956	-.6836	5.085	.8950	4.943	-.1143	6.430	.9480
4.00	1.250	.50	6.25	10	5.036	.7266	3.642	.9360	4.991	-.1737	3.503	.9360
4.00	1.250	.50	6.25	20	5.057	1.136	2.516	.9550	5.007	.1347	2.283	.9460
4.00	1.250	.50	6.25	30	5.061	1.213	2.062	.9620	5.007	.1499	1.838	.9460
4.00	1.250	.50	6.25	50	5.069	1.388	1.586	.9670	5.016	.3134	1.377	.9410
4.00	1.250	.50	6.25	100	5.048	.9563	1.124	.9680	4.991	-.1804	.9710	.9500
1.78	2.813	.75	14.06	5	5.114	2.270	13.45	.9250	4.981	-.3731	9.448	.9410
1.78	2.813	.75	14.06	10	5.191	3.823	8.129	.9470	4.942	-.1156	5.066	.9050
1.78	2.813	.75	14.06	20	5.284	5.671	5.018	.9760	4.983	-.3490	3.397	.9290
1.78	2.813	.75	14.06	30	5.332	6.646	4.034	.9760	5.015	.2949	2.747	.9250
1.78	2.813	.75	14.06	50	5.318	6.369	3.047	.9700	4.996	-.8222E-01	2.048	.9240
1.78	2.813	.75	14.06	100	5.339	6.760	2.140	.9530	4.999	-.1145E-01	1.453	.9410
1.00	5.000	1.00	25.00	5	5.530	10.60	4569.	.9250	4.954	-.9208	12.31	.9020
1.00	5.000	1.00	25.00	10	6.223	24.46	61.27	.9760	5.031	.6237	6.811	.9080
1.00	5.000	1.00	25.00	20	6.212	24.23	12.62	.9870	5.006	.1216	4.506	.9320
1.00	5.000	1.00	25.00	30	6.214	24.28	9.230	.9750	4.984	-.3132	3.644	.9150
1.00	5.000	1.00	25.00	50	6.341	26.82	6.841	.9290	4.979	-.4117	2.709	.9240
1.00	5.000	1.00	25.00	100	6.338	26.76	4.525	.7970	4.986	-.2718	1.944	.9390
.64	7.813	1.25	39.06	5	7.461	49.22	.9026E+10	.9390	5.036	.7272	14.58	.8820
.64	7.813	1.25	39.06	10	11.05	121.0	.2409E+08	.9850	4.954	-.9138	8.136	.8750
.64	7.813	1.25	39.06	20	9.943	98.85	333.9	.9550	4.907	-.1855	5.373	.8910
.64	7.813	1.25	39.06	30	9.907	98.15	48.89	.8870	5.032	.6321	4.509	.9090
.64	7.813	1.25	39.06	50	9.932	98.64	25.09	.7250	5.044	.8829	3.396	.9170
.64	7.813	1.25	39.06	100	9.870	97.40	13.43	.3130	4.987	-.2642	2.409	.9310
.44	11.25	1.50	56.25	5	12.19	143.7	.1107E+22	.9420	5.015	.2970	16.75	.8390
.44	11.25	1.50	56.25	10	52.60	951.9	.1504E+17	.9880	4.922	-.1566	9.524	.8370
.44	11.25	1.50	56.25	20	30.42	508.4	.2220E+07	.9080	4.927	-.1461	6.491	.8710
.44	11.25	1.50	56.25	30	36.85	636.9	.5050E+05	.6970	5.021	.4212	5.361	.9020
.44	11.25	1.50	56.25	50	27.19	443.8	359.1	.3650	5.023	.4656	4.030	.9100
.44	11.25	1.50	56.25	100	27.29	445.8	111.5	.0440	5.008	.1514	2.900	.9310
.25	20.00	2.00	100.00	5	.6690E+05	.1338E+07	.1208+135	.9780	5.085	1.699	21.11	.7610
.25	20.00	2.00	100.00	10	.5126E+06	.1025E+08	.2514E+53	.9740	5.219	.4376	12.60	.8360
.25	20.00	2.00	100.00	20	.1583E+07	.3166E+08	.4460E+30	.5330	4.964	-.7216	8.255	.8550
.25	20.00	2.00	100.00	30	.6859E+06	.1372E+08	.5816E+21	.2230	5.037	.7405	6.916	.8820
.25	20.00	2.00	100.00	50	.3659E+08	.7317E+09	.4031E+20	.0190	4.979	-.4192	5.241	.8960
.25	20.00	2.00	100.00	100	.1351E+05	.2702E+06	.7462E+08	0.0000	4.931	-.1373	3.765	.9110

TABLE B.3. SIMULATION OF THE GAMMA DISTRIBUTION WITH EXPECTED VALUE 10.
THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS				SAMPLE SIZE	LOGNORMAL ESTIMATOR				GAMMA ESTIMATOR			
ALPHA	BETA	C.V.	VAR(X)		E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE	E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE
16.00	.625	.25	6.25	5	10.00	.3739E-01	4.339	.8790	10.00	.4758E-02	6.664	.9650
16.00	.625	.25	6.25	10	10.00	.4662E-01	3.111	.9180	10.00	.3116E-02	3.515	.9470
16.00	.625	.25	6.25	20	10.00	.4003E-01	2.248	.9290	9.998	-.2155E-01	2.316	.9330
16.00	.625	.25	6.25	30	9.998	-.2131E-01	1.821	.9520	9.992	-.8453E-01	1.833	.9540
16.00	.625	.25	6.25	50	10.01	.8057E-01	1.420	.9400	10.00	.1993E-01	1.373	.9390
16.00	.625	.25	6.25	100	10.00	.1829E-01	1.012	.9450	9.995	-.4572E-01	.9798	.9310
4.00	2.500	.50	25.00	5	10.08	.7600	10.37	.8990	10.02	.2019	13.02	.9510
4.00	2.500	.50	25.00	10	10.08	.7550	7.170	.9260	9.994	-.6466E-01	6.948	.9370
4.00	2.500	.50	25.00	20	10.09	.8946	5.113	.9600	9.989	-.1129	4.633	.9460
4.00	2.500	.50	25.00	30	10.10	.9970	4.106	.9600	9.999	-.1338E-01	3.679	.9490
4.00	2.500	.50	25.00	50	10.14	1.429	3.215	.9640	10.03	.3139	2.764	.9400
4.00	2.500	.50	25.00	100	10.11	1.110	2.259	.9590	9.999	-.1293E-01	1.956	.9400
1.78	5.625	.75	56.25	5	10.37	3.661	34.01	.9010	10.03	.2926	18.87	.9230
1.78	5.625	.75	56.25	10	10.62	6.174	16.53	.9570	10.07	.7416	10.54	.9320
1.78	5.625	.75	56.25	20	10.69	6.897	10.21	.9710	10.08	.7943	6.909	.9430
1.78	5.625	.75	56.25	30	10.60	5.972	8.031	.9740	9.971	-.2928	5.480	.9340
1.78	5.625	.75	56.25	50	10.59	5.902	6.040	.9810	9.951	-.4875	4.059	.9390
1.78	5.625	.75	56.25	100	10.71	7.109	4.293	.9440	10.05	.4591	2.939	.9390
1.00	10.00	1.00	100.00	5	11.21	12.10	.5719E+06	.9350	9.870	-1.297	24.20	.9000
1.00	10.00	1.00	100.00	10	12.12	21.24	79.27	.9770	10.02	.2371	13.60	.9150
1.00	10.00	1.00	100.00	20	12.52	25.15	27.77	.9780	10.01	.8961E-01	9.030	.9280
1.00	10.00	1.00	100.00	30	12.65	26.50	19.18	.9720	10.03	.3433	7.307	.9400
1.00	10.00	1.00	100.00	50	12.55	25.50	13.27	.9400	9.956	-.4395	5.433	.9270
1.00	10.00	1.00	100.00	100	12.63	26.27	8.950	.8040	9.965	-.3519	3.871	.9440
.64	15.63	1.25	156.25	5	13.70	37.02	.2356E+09	.9390	9.781	-2.191	28.73	.8610
.64	15.63	1.25	156.25	10	25.93	159.3	.2963E+11	.9760	9.961	-.3906	16.36	.8640
.64	15.63	1.25	156.25	20	20.35	103.5	501.5	.9620	10.00	.3775E-01	11.07	.9120
.64	15.63	1.25	156.25	30	19.37	93.72	88.36	.8860	10.01	.1476	8.976	.9070
.64	15.63	1.25	156.25	50	20.15	101.5	50.10	.7030	10.14	1.426	6.876	.9250
.64	15.63	1.25	156.25	100	19.55	95.53	26.19	.3260	10.02	.2417	4.841	.9230
.44	22.50	1.50	225.00	5	33.57	235.7	.4076E+28	.9580	10.02	.1975	33.77	.8270
.44	22.50	1.50	225.00	10	.1092E+05	.1091E+06	.6971E+31	.9810	10.09	.8775	19.25	.8650
.44	22.50	1.50	225.00	20	165.3	1553.	.4477E+10	.9020	10.09	.9003	13.19	.8030
.44	22.50	1.50	225.00	30	68.37	583.7	.2464E+05	.6740	10.08	.9068	10.70	.9070
.44	22.50	1.50	225.00	50	57.25	472.5	982.0	.3870	10.06	.5833	8.114	.9220
.44	22.50	1.50	225.00	100	52.31	423.1	191.4	.0420	10.06	.6110	5.808	.9480
.25	40.00	2.00	400.00	5	3582.	.3572E+05	.2484E+73	.9760	9.698	-3.019	39.51	.7470
.25	40.00	2.00	400.00	10	6735.	.6725E+05	.1404E+35	.9760	10.07	.7134	24.01	.8180
.25	40.00	2.00	400.00	20	.4862E+07	.4862E+08	.4222E+31	.5100	9.856	-1.444	16.36	.8390
.25	40.00	2.00	400.00	30	.3722E+05	.3721E+06	.9042E+15	.2260	9.889	-1.112	13.69	.8600
.25	40.00	2.00	400.00	50	.1593E+07	.1593E+08	.1708E+17	.0200	9.995	-.5271E-01	10.47	.8900
.25	40.00	2.00	400.00	100	.1006E+05	.1005E+06	.9904E+07	0.0000	10.10	1.033	7.670	.9290

TABLE B.4. SIMULATION OF THE GAMMA DISTRIBUTION WITH EXPECTED VALUE 50.
THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS					LOGNORMAL ESTIMATOR				GAMMA ESTIMATOR			
ALPHA	BETA	C.V.	VAR(X)	SAMPLE SIZE	E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE	E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE
16.00	3.125	.25	156.25	5	50.05	.9547E-01	21.60	.8800	50.03	.6605E-01	33.11	.9540
16.00	3.125	.25	156.25	10	50.23	.4595	15.74	.9120	50.21	.4118	17.74	.9410
16.00	3.125	.25	156.25	20	50.02	.4268E-01	11.25	.9470	49.99	-.1786E-01	11.59	.9510
16.00	3.125	.25	156.25	30	50.08	.1501	9.204	.9490	50.04	.8803E-01	9.271	.9510
16.00	3.125	.25	156.25	50	50.06	.1189	7.137	.9530	50.03	.6048E-01	6.909	.9400
16.00	3.125	.25	156.25	100	50.08	.1667	5.069	.9560	50.05	.1012	4.903	.9460
4.00	12.50	.50	625.00	5	50.11	.2145	52.10	.8940	49.85	-.3073	65.36	.9570
4.00	12.50	.50	625.00	10	50.67	1.340	36.27	.9350	50.27	.5416	35.32	.9400
4.00	12.50	.50	625.00	20	50.20	.4081	25.16	.9540	49.70	-.5937	22.79	.9380
4.00	12.50	.50	625.00	30	50.62	1.239	20.53	.9520	50.12	.2371	18.42	.9340
4.00	12.50	.50	625.00	50	50.73	1.458	15.93	.9690	50.18	.3615	13.70	.9450
4.00	12.50	.50	625.00	100	50.59	1.182	11.29	.9660	50.01	.2957E-01	9.743	.9470
1.78	28.13	.75	1406.25	5	51.34	2.678	537.4	.9010	49.51	-.9874	95.62	.9290
1.78	28.13	.75	1406.25	10	51.72	3.436	77.92	.9560	49.06	-.1.879	50.35	.9140
1.78	28.13	.75	1406.25	20	53.17	6.335	51.46	.9660	50.07	.1436	34.46	.9190
1.78	28.13	.75	1406.25	30	53.10	6.203	40.68	.9810	49.82	-.3557	27.49	.9480
1.78	28.13	.75	1406.25	50	53.05	6.095	30.40	.9720	49.90	-.2061	20.54	.9440
1.78	28.13	.75	1406.25	100	53.56	7.118	21.53	.9500	50.19	.3798	14.70	.9390
1.00	50.00	1.00	2500.00	5	59.63	19.26	.2598E+07	.9290	51.24	2.482	128.1	.9210
1.00	50.00	1.00	2500.00	10	62.04	24.08	588.5	.9750	50.34	.6842	67.57	.9040
1.00	50.00	1.00	2500.00	20	63.21	26.42	138.1	.9850	50.19	.3888	45.32	.9360
1.00	50.00	1.00	2500.00	30	62.93	25.86	98.17	.9670	50.11	.2194	35.94	.9200
1.00	50.00	1.00	2500.00	50	63.83	27.67	68.25	.9290	50.39	.7706	27.37	.9310
1.00	50.00	1.00	2500.00	100	63.85	27.70	45.61	.7740	50.08	.1631	19.45	.9390
.64	78.13	1.25	3906.25	5	76.51	53.02	.3471E+16	.9520	49.12	-.1.757	148.1	.8700
.64	78.13	1.25	3906.25	10	105.9	111.7	.5995E+10	.9800	50.81	1.626	84.21	.8870
.64	78.13	1.25	3906.25	20	95.10	90.20	1038.	.9610	49.82	-.3566	55.13	.8690
.64	78.13	1.25	3906.25	30	103.5	107.1	963.1	.8970	49.56	-.8868	44.35	.9150
.64	78.13	1.25	3906.25	50	96.34	92.69	220.2	.7260	50.18	.3635	33.84	.9320
.64	78.13	1.25	3906.25	100	99.37	98.73	136.5	.3320	49.92	-.1578	24.12	.9260
.44	112.5	1.50	5625.00	5	141.5	183.1	.5392E+26	.9510	50.00	-.5447E-02	166.2	.8330
.44	112.5	1.50	5625.00	10	258.4	416.7	.1312E+12	.9840	49.35	-.1.299	94.38	.8590
.44	112.5	1.50	5625.00	20	442.0	784.0	.5867E+08	.8790	50.56	1.114	65.49	.8900
.44	112.5	1.50	5625.00	30	5056.	1001E+05	.1215E+12	.7180	49.84	-.3209	53.00	.9050
.44	112.5	1.50	5625.00	50	260.4	460.8	3844.	.3660	49.69	-.6199	40.05	.9170
.44	112.5	1.50	5625.00	100	276.1	452.3	1380.	.0450	50.31	.6139	29.04	.9460
.25	200.0	2.00	10000.0	5	.6867E+05	.1372E+06	.4753E+93	.9770	49.97	-.6638E-01	203.0	.7420
.25	200.0	2.00	10000.0	10	.2188E+06	.4376E+06	.1659E+39	.9650	50.49	.9680	118.6	.7970
.25	200.0	2.00	10000.0	20	.7459E+06	.1492E+07	.1258E+23	.5130	49.13	-.1.733	80.93	.8480
.25	200.0	2.00	10000.0	30	.4769E+07	.9539E+07	.2052E+22	.2300	49.77	-.4699	68.80	.8680
.25	200.0	2.00	10000.0	50	.3314E+06	.6627E+06	.1952E+13	.0290	49.39	-.1.221	51.73	.8780
.25	200.0	2.00	10000.0	100	.3674E+06	.7348E+06	.4583E+10	0.0000	49.80	-.3992	37.89	.9190

TABLE F.5. SIMULATION OF THE GAMMA DISTRIBUTION WITH EXPECTED VALUE 100.
THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS					LOGNORMAL ESTIMATOR				GAMMA ESTIMATOR			
ALPHA	BETA	C.V.	VAR(X)	SAMPLE SIZE	E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE	E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE
16.00	6.250	.25	625.00	5	99.84	-.1616	42.86	.6790	99.80	-.1959	65.65	.9530
16.00	6.250	.25	625.00	10	100.4	.3529	31.93	.9310	100.3	.3015	35.88	.9590
16.00	6.250	.25	625.00	20	100.3	.2680	22.44	.9350	100.2	.2083	23.12	.9520
16.00	6.250	.25	625.00	30	100.1	.1083	18.32	.9450	100.0	.4918E-01	18.48	.9490
16.00	6.250	.25	625.00	50	99.79	-.2093	14.28	.9470	99.73	-.2710	13.81	.9400
16.00	6.250	.25	625.00	100	100.1	.5137E-01	10.09	.9640	99.99	-.1326E-01	9.763	.9560
4.00	25.00	.50	2500.00	5	101.4	1.385	111.6	.9000	100.6	.6493	134.2	.9610
4.00	25.00	.50	2500.00	10	101.0	1.034	71.88	.9340	100.2	.2351	69.77	.9340
4.00	25.00	.50	2500.00	20	100.9	.9202	50.66	.9550	99.87	-.1282	45.87	.9350
4.00	25.00	.50	2500.00	30	101.2	1.245	41.60	.9620	100.2	.2234	37.31	.9480
4.00	25.00	.50	2500.00	50	101.3	1.290	31.85	.9690	100.2	.2202	27.52	.9500
4.00	25.00	.50	2500.00	100	101.3	1.339	22.59	.9570	100.2	.1954	19.49	.9250
1.78	56.25	.75	5625.00	5	101.2	1.171	260.8	.9060	98.44	-1.1764	183.4	.9240
1.78	56.25	.75	5625.00	10	105.5	5.495	157.5	.9660	100.3	.2876	103.7	.9220
1.78	56.25	.75	5625.00	20	106.9	6.879	102.9	.9720	100.8	.7537	69.32	.9290
1.78	56.25	.75	5625.00	30	106.1	6.066	80.01	.9720	99.87	-.1348	54.68	.9390
1.78	56.25	.75	5625.00	50	106.6	6.630	61.72	.9730	100.1	.5222E-01	41.36	.9430
1.78	56.25	.75	5625.00	100	106.9	6.851	42.99	.9480	100.1	.1085	29.28	.9310
1.00	100.0	1.00	10000.0	5	114.5	14.45	.2393E+07	.9240	99.81	-.1882	247.3	.9020
1.00	100.0	1.00	10000.0	10	120.5	20.45	751.3	.9670	100.5	.5389	134.5	.9030
1.00	100.0	1.00	10000.0	20	126.0	26.02	283.0	.9800	100.1	.5050E-01	89.37	.9170
1.00	100.0	1.00	10000.0	30	129.1	29.09	207.7	.9560	101.5	1.468	73.54	.9340
1.00	100.0	1.00	10000.0	50	126.1	26.05	135.1	.9420	99.40	-.5990	54.55	.9260
1.00	100.0	1.00	10000.0	100	127.5	27.47	90.56	.7880	100.3	.2874	38.90	.9390
.64	156.3	1.25	15625.0	5	146.7	46.67	.2330E+14	.9540	100.7	.7067	299.8	.8780
.64	156.3	1.25	15625.0	10	196.9	96.93	.6323E+07	.9810	100.6	.6310	162.3	.8910
.64	156.3	1.25	15625.0	20	189.2	89.25	297.1	.9680	98.85	-1.154	100.8	.8960
.64	156.3	1.25	15625.0	30	197.3	97.34	1037.	.8900	99.42	-.5802	89.70	.9130
.64	156.3	1.25	15625.0	50	200.1	100.1	518.0	.7340	98.99	-1.015	67.24	.9150
.64	156.3	1.25	15625.0	100	200.8	100.8	278.3	.2950	100.0	.2587E-01	48.52	.9390
.44	225.0	1.50	22500.0	5	548.4	448.4	.7466E+32	.9590	103.5	3.482	348.1	.9380
.44	225.0	1.50	22500.0	10	685.5	505.5	.9127E+15	.9830	100.7	.6734	193.9	.8460
.44	225.0	1.50	22500.0	20	978.2	878.2	.6697E+08	.8790	100.6	.5512	130.7	.9010
.44	225.0	1.50	22500.0	30	711.0	611.0	.3548E+06	.6890	100.2	.1662	106.2	.8950
.44	225.0	1.50	22500.0	50	653.5	553.5	.3203E+05	.3630	99.82	-.1709	80.75	.9180
.44	225.0	1.50	22500.0	100	515.7	415.7	1707.	.0440	100.2	.1750	57.63	.9350
.25	400.0	2.00	40000.0	5	.8523E+05	.8513E+05	.2025E+79	.9750	105.0	4.969	433.2	.7580
.25	400.0	2.00	40000.0	10	.1156E+07	.1156E+07	.2038E+45	.9790	102.7	2.742	245.2	.8060
.25	400.0	2.00	40000.0	20	.1741E+07	.1741E+07	.3225E+26	.5520	103.1	3.132	174.2	.8770
.25	400.0	2.00	40000.0	30	.4828E+07	.4828E+07	.1245E+21	.1950	101.0	.9912	136.3	.8820
.25	400.0	2.00	40000.0	50	.4584E+06	.4583E+06	.2846E+13	.0220	99.05	-.9480	103.8	.8900
.25	400.0	2.00	40000.0	100	.1303E+06	.1302E+06	.1007E+09	0.0000	98.81	-1.192	75.30	.9130

TABLE B 6. SIMULATION OF THE LOGNORMAL DISTRIBUTION WITH EXPECTED VALUE 1.
THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS				SAMPLE SIZE	LOGNORMAL ESTIMATOR				GAMMA ESTIMATOR			
MU	SIGMA	C.V.	VAR(X)		E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE	E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE
-.030	.246	.25	.06	5	.9950	-.4985	.4303	.8670	.9951	-.4918	.6725	.9540
-.030	.246	.25	.06	10	1.005	.5274	.3058	.8980	1.005	.5259	.3523	.9350
-.030	.246	.25	.06	20	1.001	.1321	.2184	.9370	1.001	.1263	.2312	.9670
-.030	.246	.25	.06	30	1.000	.4701E-01	.1772	.9400	1.000	.4769E-01	.1839	.9370
-.030	.246	.25	.06	50	1.000	.3295E-01	.1378	.9440	1.000	.3657E-01	.1375	.9380
-.030	.246	.25	.06	100	1.002	.1635	.9816E-01	.9400	1.002	.1637	.9799E-01	.9400
-.112	.472	.50	.25	5	1.002	.1900	.9087	.8840	1.002	.2282	1.266	.9370
-.112	.472	.50	.25	10	1.003	.3108	.6336	.9070	1.003	.3320	.6696	.9220
-.112	.472	.50	.25	20	1.002	.2449	.4489	.9530	1.002	.2064	.4589	.9430
-.112	.472	.50	.25	30	1.006	.5940	.3610	.9420	1.006	.5968	.3678	.9370
-.112	.472	.50	.25	50	.9980	-.2031	.2765	.9390	.9981	-.1887	.2731	.9280
-.112	.472	.50	.25	100	1.000	.1387E-01	.1955	.9490	.9999	-.6286E-02	.1937	.9440
-.223	.668	.75	.56	5	1.007	.6558	1.627	.8820	1.005	.5374	1.836	.9180
-.223	.668	.75	.56	10	1.001	.1257	1.008	.8980	.9997	-.2978E-01	.9875	.8940
-.223	.668	.75	.56	20	.9956	-.4405	.9552	.9220	.9954	-.4597	.6479	.8990
-.223	.668	.75	.56	30	.9987	-.1300	.5386	.9340	.9992	-.7516E-01	.5368	.9210
-.223	.668	.75	.56	50	.9979	-.2147	.4145	.9370	.9971	-.2872	.4007	.9200
-.223	.668	.75	.56	100	1.004	.3682	.2934	.9500	1.004	.3784	.2913	.9300
-.347	.833	1.00	1.00	5	.9924	-.7573	2.562	.8690	.9947	-.5334	2.245	.8740
-.347	.833	1.00	1.00	10	.9917	-.8251	1.372	.9180	.9925	-.7537	1.230	.8950
-.347	.833	1.00	1.00	20	1.007	.6781	.9153	.9330	1.007	.7127	.8607	.9090
-.347	.833	1.00	1.00	30	.9903	-.9722	.7115	.9390	.9910	-.9007	.6048	.8990
-.347	.833	1.00	1.00	50	.9997	-.3273E-01	.5490	.9350	.9979	-.2113	.5202	.9050
-.347	.833	1.00	1.00	100	1.003	.2850	.3853	.9540	1.004	.3899	.3839	.9300
-.470	.970	1.25	1.56	5	1.001	-.5152E-01	3.615	.8580	.9969	-.3096	2.522	.8610
-.470	.970	1.25	1.56	10	.9950	-.4962	1.838	.8980	.9917	-.8294	1.430	.8350
-.470	.970	1.25	1.56	20	.9908	-.9186	1.156	.9420	.9921	-.7865	1.026	.8820
-.470	.970	1.25	1.56	30	1.007	.6755	.9081	.9470	1.007	.7261	.8456	.8980
-.470	.970	1.25	1.56	50	.9988	-.1169	.6772	.9500	.9974	-.2603	.6331	.9160
-.470	.970	1.25	1.56	100	1.005	.4819	.4749	.9420	1.004	.3845	.4672	.9150
-.589	1.086	1.50	2.25	5	1.045	.4528	13.16	.8630	1.051	5.115	3.103	.8330
-.589	1.086	1.50	2.25	10	.9833	-1.671	2.295	.8990	.9826	-1.740	1.614	.8150
-.589	1.086	1.50	2.25	20	.9993	-.7404E-01	1.368	.9210	1.003	.3014	1.180	.8500
-.589	1.086	1.50	2.25	30	.9956	-.4399	1.070	.9250	.9970	-.3016	.9640	.8640
-.589	1.086	1.50	2.25	50	.9971	-.2859	.7973	.9530	.9993	-.7084E-01	.7455	.8880
-.589	1.086	1.50	2.25	100	.9998	-.2309E-01	.5500	.9440	.9980	-.1977	.5389	.9120
-.805	1.269	2.00	4.00	5	1.020	1.999	20.88	.8450	1.028	2.823	3.492	.7650
-.805	1.269	2.00	4.00	10	1.014	1.373	3.328	.9120	1.009	.8609	1.969	.7930
-.805	1.269	2.00	4.00	20	.9979	-.2107	1.804	.9350	.9875	-.1251	1.364	.8160
-.805	1.269	2.00	4.00	30	1.002	.2344	1.430	.9360	1.001	.5999E-01	1.182	.8320
-.805	1.269	2.00	4.00	50	1.008	.7641	1.046	.9480	.9991	-.9347E-01	.9043	.8500
-.805	1.269	2.00	4.00	100	.9993	-.6042E-01	.6967	.9520	.9983	-.1734	.6919	.8940

TABLE B.7. SIMULATION OF THE LOGNORMAL DISTRIBUTION WITH EXPECTED VALUE 5.
THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS			LOGNORMAL ESTIMATOR					GAMMA ESTIMATOR				
MU	SIGMA	C.V.	VAR(X)	SAMPLE SIZE	E(X)-HAT	REL. BIAS	AVE(C1)	COVERAGE	E(X)-HAT	REL. BIAS	AVE(C1)	COVERAGE
1.579	.246	.25	1.56	5	4.998	-.4634E-01	2.114	.8700	4.997	-.5487E-01	3.285	.9590
1.579	.246	.25	1.56	10	4.992	-.1635	.1505	.9130	4.991	-.1716	1.730	.9420
1.579	.246	.25	1.56	20	5.003	-.5008E-01	1.082	.9200	5.003	-.5586E-01	1.152	.9410
1.579	.246	.25	1.56	30	4.999	-.4691E-01	.8875	.9330	4.998	-.4061E-01	.9198	.9430
1.579	.246	.25	1.56	50	4.998	-.3871E-01	.6826	.9430	4.998	-.3784E-01	.6903	.9410
1.579	.246	.25	1.56	100	5.006	.1142	.4908	.9470	5.006	.1183	.4908	.9480
1.498	.472	.50	6.25	5	5.015	.3052	4.653	.8810	5.017	.3305	6.404	.9430
1.498	.472	.50	6.25	10	5.003	.6789E-01	3.139	.9190	5.003	.6713E-01	3.414	.9320
1.498	.472	.50	6.25	20	5.007	.1308	2.190	.9320	5.006	.1114	2.251	.9230
1.498	.472	.50	6.25	30	5.006	.1140	1.779	.9440	5.007	.1476	1.821	.9370
1.498	.472	.50	6.25	50	4.986	-.2047	1.382	.9480	4.985	-.3000	1.359	.9410
1.498	.472	.50	6.25	100	4.995	-.1086	.9761	.9580	4.995	-.1057	.9715	.9540
1.386	.668	.75	14.06	5	5.005	.9007E-01	8.047	.8820	4.991	-.1791	8.860	.9160
1.386	.668	.75	14.06	10	5.071	1.4418	4.916	.9080	5.075	1.4486	4.959	.9040
1.386	.668	.75	14.06	20	5.024	.4820	3.366	.9350	5.026	.5103	3.331	.9190
1.386	.668	.75	14.06	30	4.995	-.2999	2.695	.9310	4.986	-.2870	2.668	.9220
1.386	.668	.75	14.06	50	4.985	-.2894	2.065	.9520	4.981	-.3741	1.992	.9360
1.386	.668	.75	14.06	100	4.995	-.9844E-01	1.453	.9550	5.000	.6459E-02	1.455	.9480
1.263	.833	1.00	25.00	5	4.950	-1.000	11.81	.8650	4.944	-1.119	10.92	.8920
1.263	.833	1.00	25.00	10	4.961	-.7791	6.823	.9110	4.968	-.6488	6.187	.8700
1.263	.833	1.00	25.00	20	4.980	-.4071	4.516	.9270	4.987	-.2556	4.262	.8940
1.263	.833	1.00	25.00	30	4.998	-.4535E-01	3.606	.9410	4.989	-.2219	3.428	.9100
1.263	.833	1.00	25.00	50	4.995	-.1047	2.755	.9390	4.997	-.5933E-01	2.649	.9160
1.263	.833	1.00	25.00	100	5.004	.8248E-01	1.921	.9450	5.000	.7640E-02	1.890	.9310
1.139	.970	1.25	39.06	5	5.060	1.180	20.63	.8620	5.081	1.611	13.35	.8480
1.139	.970	1.25	39.06	10	5.141	2.618	9.732	.9080	5.126	2.519	7.596	.8660
1.139	.970	1.25	39.06	20	4.544	-.3201	5.660	.9370	4.982	-.3527	5.041	.8760
1.139	.970	1.25	39.06	30	5.027	.5385	4.527	.9310	5.028	.5542	4.190	.8940
1.139	.970	1.25	39.06	50	5.027	.5317	3.403	.9490	5.007	.1485	3.147	.8930
1.139	.970	1.25	39.06	100	4.987	-.2504	2.332	.9420	4.985	-.3038	2.305	.9130
1.020	1.086	1.50	56.25	5	5.095	1.097	31.45	.8480	5.089	1.771	14.59	.8350
1.020	1.086	1.50	56.25	10	4.874	-.5294	12.30	.8870	5.001	.2927E-01	8.514	.8080
1.020	1.086	1.50	56.25	20	4.985	-.3083	6.927	.9170	4.993	-.1356	5.839	.8410
1.020	1.086	1.50	56.25	30	4.933	-.1370	5.379	.9320	4.986	-.2849	4.792	.8810
1.020	1.086	1.50	56.25	50	5.011	.2209	4.017	.9470	5.034	.6773	3.824	.8940
1.020	1.086	1.50	56.25	100	5.028	.5658	2.772	.9630	5.034	.6771	2.759	.9220
.805	1.269	2.00	100.00	5	4.659	-6.823	113.7	.8420	4.704	-5.911	15.63	.7620
.805	1.269	2.00	100.00	10	5.181	3.626	19.21	.9250	5.201	4.027	10.37	.8000
.805	1.269	2.00	100.00	20	4.920	-1.593	9.378	.9120	4.905	-1.904	6.876	.8100
.805	1.269	2.00	100.00	30	4.898	-.4730E-01	7.177	.9320	5.056	1.113	6.184	.8230
.805	1.269	2.00	100.00	50	4.948	-1.034	5.047	.9340	4.965	-.7014	4.568	.8330
.805	1.269	2.00	100.00	100	5.027	.5351	3.522	.9470	5.009	.1806	3.443	.8860

TABLE B.8. SIMULATION OF THE LOGNORMAL DISTRIBUTION WITH EXPECTED VALUE 10.
THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS					LOGNORMAL ESTIMATOR				GAMMA ESTIMATOR			
MU	SIGMA	C.V.	VAR(X)	SAMPLE SIZE	E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE	E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE
2.272	.246	.25	6.25	5	10.03	.3458	4.176	.8750	10.03	.3478	6.528	.9600
2.272	.246	.25	6.25	10	10.02	.2476	3.076	.9120	10.02	.2417	3.535	.9460
2.272	.246	.25	6.25	20	10.02	.1507	2.176	.9310	10.02	.1587	2.318	.9470
2.272	.246	.25	6.25	30	10.01	.7885E-01	1.784	.9400	10.01	.8006E-01	1.852	.9450
2.272	.246	.25	6.25	50	10.01	.5298E-01	1.387	.9580	10.01	.5269E-01	1.380	.9540
2.272	.246	.25	6.25	100	9.998	-.2231E-01	.9795	.9470	9.998	-.2308E-01	.9772	.9450
2.191	.472	.50	25.00	5	9.980	-.2021	9.007	.8620	9.987	-.1268	12.67	.9410
2.191	.472	.50	25.00	10	9.925	-.7535	6.225	.9160	9.924	-.7630	6.765	.9250
2.191	.472	.50	25.00	20	10.02	.1800	4.418	.9210	10.02	.1994	4.562	.9170
2.191	.472	.50	25.00	30	10.01	.1221	3.575	.9490	10.02	.1517	3.659	.9430
2.191	.472	.50	25.00	50	10.02	.1923	2.776	.9510	10.02	.1599	2.721	.9420
2.191	.472	.50	25.00	100	9.984	-.1574	1.958	.9490	9.982	-.1836	1.936	.9460
2.079	.668	.75	56.25	5	10.10	.9637	15.77	.8910	10.11	1.136	18.45	.9280
2.079	.668	.75	56.25	10	9.857	-1.425	9.652	.9110	9.844	-1.560	9.527	.8840
2.079	.668	.75	56.25	20	10.07	.7386	6.825	.9530	10.09	.9208	6.811	.9380
2.079	.668	.75	56.25	30	9.904	-.1564	5.420	.9320	9.991	-.9203E-01	5.394	.9040
2.079	.668	.75	56.25	50	10.01	.7200E-01	4.136	.9370	10.01	.6109E-01	4.012	.9110
2.079	.668	.75	56.25	100	10.05	.4942	2.938	.9340	10.05	.4712	2.908	.9340
1.956	.833	1.00	100.00	5	10.03	.3122	27.36	.8690	10.07	.6655	22.74	.8870
1.956	.833	1.00	100.00	10	9.987	-.1338	14.23	.9090	10.02	.1919	12.64	.8670
1.956	.833	1.00	100.00	20	10.02	.1723	9.063	.9240	10.05	.5490	8.673	.9010
1.956	.833	1.00	100.00	30	9.948	-.5248	7.165	.9330	9.952	-.4833	6.933	.8980
1.956	.833	1.00	100.00	50	9.997	-.2938E-01	5.477	.9440	9.992	-.8496E-01	5.216	.9220
1.956	.833	1.00	100.00	100	10.04	.3738	3.843	.9490	10.06	.5553	3.863	.9300
1.832	.970	1.25	156.25	5	9.597	-4.030	38.27	.8650	9.642	-3.577	25.05	.8450
1.832	.970	1.25	156.25	10	10.04	.4291	18.40	.9130	10.08	.7930	14.91	.8660
1.832	.970	1.25	156.25	20	9.996	-.4309E-01	11.45	.9300	9.990	-.9590E-01	10.09	.8850
1.832	.970	1.25	156.25	30	10.05	.5369	9.078	.9240	10.06	.6286	8.395	.8780
1.832	.970	1.25	156.25	50	9.946	-.5407	6.733	.9490	9.946	-.5439	6.350	.9010
1.832	.970	1.25	156.25	100	10.03	.2961	4.735	.9620	10.06	.5572	4.781	.9260
1.713	1.086	1.50	225.00	5	10.23	2.257	79.12	.8680	10.28	2.814	30.11	.8400
1.713	1.086	1.50	225.00	10	9.870	-1.295	23.36	.9010	9.905	-.9529	16.47	.8250
1.713	1.086	1.50	225.00	20	10.02	.2399	13.89	.9410	10.01	.8154E-01	11.50	.8570
1.713	1.086	1.50	225.00	30	9.903	-.9716	10.70	.9390	10.00	.3272E-01	9.915	.8610
1.713	1.086	1.50	225.00	50	10.11	1.065	8.135	.9460	10.02	.1613	7.221	.8880
1.713	1.086	1.50	225.00	100	10.03	.3344	5.531	.9430	10.03	.2637	5.435	.9160
1.498	1.269	2.00	400.00	5	10.01	.1441	244.7	.8410	9.932	-.6048	32.65	.7630
1.498	1.269	2.00	400.00	10	10.24	2.354	43.38	.9070	10.06	.6352	19.42	.7780
1.498	1.269	2.00	400.00	20	10.11	1.108	19.33	.9340	10.14	1.417	14.52	.8160
1.498	1.269	2.00	400.00	30	10.06	.6043	14.48	.9290	10.10	1.009	12.10	.8350
1.498	1.269	2.00	400.00	50	10.02	.1659	10.30	.9520	9.985	-.1493	9.124	.8570
1.498	1.269	2.00	400.00	100	10.02	.1894	7.002	.9280	10.02	.1706	6.936	.8870

TABLE B.9. SIMULATION OF THE LOGNORMAL DISTRIBUTION WITH EXPECTED VALUE 50.
THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS					LOGNORMAL ESTIMATOR				GAMMA ESTIMATOR			
MU	SIGMA	C.V.	VAR(X)	SAMPLE SIZE	E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE	E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE
3.882	.246	.25	156.25	5	50.17	.3458	21.28	.8600	50.17	.3412	33.09	.9520
3.882	.246	.25	156.25	10	50.13	.2670	15.13	.9180	50.13	.2644	17.43	.9430
3.882	.246	.25	156.25	20	50.02	.3981E-01	10.83	.9270	50.02	.3294E-01	11.45	.9400
3.882	.246	.25	156.25	30	49.97	-.5119E-01	8.889	.9410	49.97	-.5143E-01	9.215	.9500
3.882	.246	.25	156.25	50	50.01	.1715E-01	6.853	.9500	50.01	.1660E-01	6.825	.9470
3.882	.246	.25	156.25	100	50.05	.1026	4.927	.9440	50.05	.1026	4.916	.9510
3.800	.472	.50	625.00	5	50.38	.7668	46.03	.8790	50.40	.7993	63.93	.9380
3.800	.472	.50	625.00	10	49.66	-.6702	31.09	.9220	49.65	-.6929	33.86	.9350
3.800	.472	.50	625.00	20	49.96	-.7490E-01	21.90	.9320	49.94	-.1169	22.43	.9270
3.800	.472	.50	625.00	30	50.02	.3418E-01	18.02	.9370	50.02	.4496E-01	18.36	.9330
3.800	.472	.50	625.00	50	49.98	-.4368E-01	13.80	.9300	50.00	-.7368E-02	13.68	.9310
3.800	.472	.50	625.00	100	50.04	.7564E-01	9.799	.9500	50.05	.9023E-01	9.769	.9460
3.689	.668	.75	1406.25	5	49.48	-1.038	74.90	.8730	49.62	-.7587	88.68	.8970
3.689	.668	.75	1406.25	10	49.30	-1.404	48.06	.9170	49.35	-1.300	48.43	.9040
3.689	.668	.75	1406.25	20	50.11	.2251	33.65	.9390	50.10	.2009	33.09	.9240
3.689	.668	.75	1406.25	30	49.80	-.4001	26.95	.9230	49.78	-.4331	26.63	.9110
3.689	.668	.75	1406.25	50	49.95	-.1030	20.73	.9300	49.94	-.1195	20.08	.9200
3.689	.668	.75	1406.25	100	49.84	-.3201	14.54	.9520	49.82	-.3638	14.36	.9350
3.565	.833	1.00	2500.00	5	49.28	-1.433	117.3	.8640	49.30	-1.402	108.2	.8790
3.565	.833	1.00	2500.00	10	49.69	-.6260	68.82	.9170	49.55	-.8986	60.62	.8890
3.565	.833	1.00	2500.00	20	49.73	-.5414	45.02	.9200	49.69	-.6220	41.97	.8910
3.565	.833	1.00	2500.00	30	50.19	.3873	36.42	.9260	50.10	.2060	34.62	.8040
3.565	.833	1.00	2500.00	50	49.86	-.2719	27.40	.9210	49.81	-.3804	26.07	.8960
3.565	.833	1.00	2500.00	100	50.07	.1317	19.22	.9290	50.02	.4611E-01	18.86	.9200
3.442	.970	1.25	3906.25	5	49.47	-1.064	165.3	.8720	49.43	-1.146	125.8	.8650
3.442	.970	1.25	3906.25	10	50.26	.5167	93.52	.9180	50.44	.8843	74.40	.8790
3.442	.970	1.25	3906.25	20	50.12	.2467	57.18	.9310	50.05	.9253E-01	50.45	.8660
3.442	.970	1.25	3906.25	30	50.35	.7013	45.37	.9350	50.39	.7862	42.03	.9030
3.442	.970	1.25	3906.25	50	50.13	.2656	34.37	.9400	50.07	.1435	31.96	.9010
3.442	.970	1.25	3906.25	100	49.89	-.2214	23.36	.9440	49.92	-.1690	23.25	.9130
3.323	1.086	1.50	5625.00	5	48.96	-2.004	382.1	.8440	48.82	-2.359	139.1	.8100
3.323	1.086	1.50	5625.00	10	48.53	-2.938	115.9	.9020	48.40	-3.190	78.83	.8130
3.323	1.086	1.50	5625.00	20	50.65	1.298	71.53	.9440	50.84	1.679	60.19	.8620
3.323	1.086	1.50	5625.00	30	49.65	-.6928	53.92	.9220	49.46	-1.073	47.35	.8510
3.323	1.086	1.50	5625.00	50	50.56	1.118	41.02	.9510	50.60	1.201	38.29	.8900
3.323	1.086	1.50	5625.00	100	50.24	.4780	27.75	.9530	50.13	.2627	27.12	.9180
3.107	1.269	2.00	10000.0	5	51.98	3.953	1207.	.8510	52.23	4.459	179.4	.7770
3.107	1.269	2.00	10000.0	10	50.50	1.002	193.3	.8990	50.36	.7215	98.91	.7740
3.107	1.269	2.00	10000.0	20	49.02	-1.963	92.48	.9210	48.90	-2.195	68.21	.7830
3.107	1.269	2.00	10000.0	30	50.01	.1709E-01	71.54	.9350	49.90	-.2086	58.98	.8290
3.107	1.269	2.00	10000.0	50	50.56	1.130	52.76	.9320	50.70	1.409	47.87	.8440
3.107	1.269	2.00	10000.0	100	50.21	.4243	35.32	.9530	50.25	.4950	34.83	.8700

TABLE B.10. SIMULATION OF THE LOGNORMAL DISTRIBUTION WITH EXPECTED VALUE 100.
THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS					LOGNORMAL ESTIMATOR				GAMMA ESTIMATOR			
MU	SIGMA	C.V.	VAR(X)	SAMPLE SIZE	E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE	E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE
4.575	.246	.25	625.00	5	100.5	.4766	42.66	.8820	100.5	.4659	66.53	.9580
4.575	.246	.25	625.00	10	99.77	-.2324	30.73	.9200	99.76	-.2356	35.38	.9410
4.575	.246	.25	625.00	20	99.96	-.4086E-01	21.94	.9390	99.96	-.4228E-01	23.26	.9520
4.575	.246	.25	625.00	30	99.99	-.6259E-02	17.69	.9340	100.0	-.3942E-02	18.38	.9400
4.575	.246	.25	625.00	50	99.85	-.1501	13.83	.9420	99.85	-.1537	13.75	.9440
4.575	.246	.25	625.00	100	100.1	.5569E-01	9.791	.9530	100.1	.5441E-01	9.766	.9510
4.494	.472	.50	2500.00	5	99.66	-.3427	91.53	.8710	99.60	-.3976	126.1	.9360
4.494	.472	.50	2500.00	10	100.2	.1549	63.26	.9190	100.1	.1324	68.44	.9360
4.494	.472	.50	2500.00	20	99.33	-.6684	43.48	.9360	99.29	-.7135	44.59	.9280
4.494	.472	.50	2500.00	30	100.0	-.3286E-02	35.72	.9430	99.98	-.2083E-01	36.22	.9380
4.494	.472	.50	2500.00	50	100.3	.3091	28.01	.9410	100.3	.3237	27.62	.9300
4.494	.472	.50	2500.00	100	100.1	.1305	19.61	.9430	100.1	.1432	19.54	.9440
4.382	.668	.75	5625.00	5	100.5	.4804	158.3	.8580	100.4	.4014	179.6	.9100
4.382	.668	.75	5625.00	10	100.5	.4830	98.42	.9160	100.4	.4103	97.70	.9060
4.382	.668	.75	5625.00	20	98.76	-.1237	65.92	.9270	98.78	-.1222	65.33	.8940
4.382	.668	.75	5625.00	30	99.88	-.1249	53.96	.9280	99.94	-.5720E-01	53.75	.9220
4.382	.668	.75	5625.00	50	99.89	-.1074	41.30	.9490	99.97	-.3105E-01	40.38	.9350
4.382	.668	.75	5625.00	100	99.62	-.3832	29.05	.9380	99.61	-.3945	28.00	.9310
4.259	.833	1.00	10000.0	5	99.55	-.4512	313.5	.8580	99.96	-.4413E-01	227.3	.8030
4.259	.833	1.00	10000.0	10	99.10	-.8957	140.6	.9230	99.12	-.8791	124.9	.8850
4.259	.833	1.00	10000.0	20	101.1	1.115	91.73	.9380	101.2	1.228	86.85	.9080
4.259	.833	1.00	10000.0	30	100.3	.3156	72.81	.9450	100.5	.4623	70.36	.9130
4.259	.833	1.00	10000.0	50	99.02	-.9834	54.22	.9260	98.93	-.1074	51.65	.8930
4.259	.833	1.00	10000.0	100	99.31	-.6876	37.97	.9370	99.29	-.7074	37.51	.9090
4.135	.970	1.25	15625.0	5	100.2	.1541	481.1	.8850	100.4	.3984	254.2	.8700
4.135	.970	1.25	15625.0	10	103.4	3.418	194.3	.9160	103.6	3.599	154.4	.8600
4.135	.970	1.25	15625.0	20	100.5	.4921	115.2	.9340	100.5	.5362	102.4	.8840
4.135	.970	1.25	15625.0	30	101.0	1.005	90.80	.9390	101.3	1.325	85.67	.8990
4.135	.970	1.25	15625.0	50	100.1	.1407	67.97	.9290	100.2	.2111	64.35	.8860
4.135	.970	1.25	15625.0	100	99.87	-.1299	47.16	.9470	99.71	-.2887	46.30	.9160
4.016	1.086	1.50	22500.0	5	98.07	-.1930	763.2	.8600	97.33	-.2669	275.2	.8300
4.016	1.086	1.50	22500.0	10	100.5	.5218	240.8	.9270	100.9	.8760	169.7	.8380
4.016	1.086	1.50	22500.0	20	99.39	-.6144	137.0	.9320	99.96	-.4244E-01	117.9	.8500
4.016	1.086	1.50	22500.0	30	100.5	.4626	108.4	.9470	100.1	.8434E-01	95.20	.8840
4.016	1.086	1.50	22500.0	50	100.2	.1846	80.62	.9340	99.95	-.5020E-01	73.95	.8900
4.016	1.086	1.50	22500.0	100	100.7	.7128	55.91	.9470	100.8	.7919	55.62	.9110
3.800	1.269	2.00	40000.0	5	97.80	-.2195	1422.	.8640	97.39	-.2607	320.2	.7810
3.800	1.269	2.00	40000.0	10	99.80	-.1990	383.0	.9050	101.1	1.143	204.4	.7780
3.800	1.269	2.00	40000.0	20	99.51	-.4936	196.0	.9180	100.5	.4964	143.0	.7880
3.800	1.269	2.00	40000.0	30	99.57	-.4321	141.0	.9300	99.63	-.3689	117.6	.8380
3.800	1.269	2.00	40000.0	50	99.31	-.6941	101.5	.9420	99.60	-.3909	91.77	.8570
3.800	1.269	2.00	40000.0	100	99.56	-.4419	69.47	.9440	99.44	-.5601	68.14	.8740

TABLE B.11. SIMULATION OF THE GAMMA DISTRIBUTION WITH EXPECTED VALUE SELECTED FROM A UNIFORM DISTRIBUTION ON THE INTERVAL .5 TO 1.5. THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS				SAMPLE SIZE	LOGNORMAL ESTIMATOR				GAMMA ESTIMATOR			
ALPHA	BETA	C.V.	VAR(X)		E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE	E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE
16.00	.063	.25	.06	5	.9906	-.9412	.7219	.8930	.9890	-1.102	1.022	.9570
16.00	.063	.25	.06	10	1.003	.2881	.5112	.9230	1.001	.7441E-01	.5447	.9340
16.00	.063	.25	.06	20	1.007	.7085	.3638	.9410	1.005	.4598	.3589	.9390
16.00	.063	.25	.06	30	.9994	-.6298E-01	.2998	.9500	.9968	-.3163	.2903	.9390
16.00	.063	.25	.06	50	1.000	.1442E-01	.2297	.9630	.9977	-.2343	.2142	.9480
16.00	.063	.25	.06	100	1.006	.5749	.1641	.9600	1.003	.2970	.1529	.9530
4.00	.250	.50	.25	5	1.012	1.188	1.288	.0760	1.006	.5941	1.527	.9310
4.00	.250	.50	.25	10	1.010	1.005	.8810	.9310	1.001	.1444	.8378	.9330
4.00	.250	.50	.25	20	1.005	.5021	.6042	.9570	.9933	-.6692	.5424	.9380
4.00	.250	.50	.25	30	1.014	1.361	.4959	.9590	1.001	1.035	.4409	.9480
4.00	.250	.50	.25	50	1.011	1.096	.3816	.9640	.9977	-.2298	.3279	.9420
4.00	.250	.50	.25	100	1.012	1.179	.2669	.9610	.9901	-.1905	.2311	.9370
1.78	.563	.75	.56	5	1.024	2.432	3.566	.8920	.9924	-.7567	2.054	.9070
1.78	.563	.75	.56	10	1.055	5.489	1.758	.9400	1.003	.2763	1.139	.8940
1.78	.563	.75	.56	20	1.053	5.344	1.100	.9690	.9913	-.8740	.7431	.9150
1.78	.563	.75	.56	30	1.073	7.313	.8927	.9760	1.004	.4148	.6056	.9370
1.78	.563	.75	.56	50	1.062	6.183	.6619	.9790	.9961	-.3942	.4535	.9350
1.78	.563	.75	.56	100	1.067	6.688	.4631	.9620	.9980	-.1982	.3231	.9530
1.00	1.000	1.00	1.00	5	1.127	12.71	.1076E+09	.9260	1.002	.2451	2.587	.8940
1.00	1.000	1.00	1.00	10	1.220	22.04	.396.8	.9690	1.001	.1302	1.436	.8970
1.00	1.000	1.00	1.00	20	1.239	23.92	2.915	.9730	1.003	.3196	.9605	.9170
1.00	1.000	1.00	1.00	30	1.244	24.39	2.001	.9750	.9830	-1.702	.7559	.9160
1.00	1.000	1.00	1.00	50	1.267	26.67	1.402	.9390	1.003	.3414	.5639	.9160
1.00	1.000	1.00	1.00	100	1.280	27.97	.9562	.7920	1.002	.2184	.4203	.9410
.64	1.563	1.25	1.56	5	1.539	53.87	.1482E+11	.9400	1.039	3.919	3.214	.8760
.64	1.563	1.25	1.56	10	1.700	78.05	.1753E+05	.9860	.9787	-2.128	1.690	.8690
.64	1.563	1.25	1.56	20	2.158	115.8	189.2	.9540	.9955	-.4467	1.165	.8840
.64	1.563	1.25	1.56	30	2.100	110.0	18.06	.8860	.9986	-.1365	.9400	.9170
.64	1.563	1.25	1.56	50	2.023	102.3	5.662	.7340	1.006	.5597	.7301	.9190
.64	1.563	1.25	1.56	100	2.022	102.2	2.883	.3510	.9980	-.2011	.5075	.9210
.44	2.250	1.50	2.25	5	3.127	212.7	.7505E+29	.9540	.9835	-1.649	3.309	.8120
.44	2.250	1.50	2.25	10	11.60	1060.	.4030E+14	.9650	.9926	-.7391	1.940	.8320
.44	2.250	1.50	2.25	20	9.914	891.4	.1715E+06	.8750	1.026	2.589	1.386	.8870
.44	2.250	1.50	2.25	30	7.469	646.9	.1744E+05	.7230	.9963	-.3708	1.102	.8880
.44	2.250	1.50	2.25	50	6.664	566.4	227.4	.3670	1.003	.3411	.8405	.9110
.44	2.250	1.50	2.25	100	5.545	454.5	21.30	.0440	.9991	-.8540E-01	.6125	.9230
.25	4.000	2.00	4.00	5	1153.	.1152E+06	.5618E+82	.9810	1.019	1.926	4.240	.7340
.25	4.000	2.00	4.00	10	5936.	.5935E+06	.1555E+38	.9790	1.019	1.921	2.501	.8020
.25	4.000	2.00	4.00	20	.1599E+06	.1599E+08	.6704E+27	.5450	1.010	1.045	1.726	.8420
.25	4.000	2.00	4.00	30	.1966E+05	.1966E+07	.6177E+16	.2220	1.006	.6156	1.430	.8630
.25	4.000	2.00	4.00	50	7703.	.7702E+06	.3474E+11	.0210	1.001	.1399	1.088	.8900
.25	4.000	2.00	4.00	100	1134.	.1133E+06	.7915E+06	0.0000	.9924	-.7572	.7814	.9190

TABLE B.12. SIMULATION OF THE GAMMA DISTRIBUTION WITH EXPECTED VALUE SELECTED FROM A UNIFORM DISTRIBUTION ON THE INTERVAL 2.5 TO 7.5. THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS					LOGNORMAL ESTIMATOR				GAMMA ESTIMATOR			
ALPHA	BETA	C.V.	VAR(X)	SAMPLE SIZE	E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE	E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE
16.00	.313	.25	1.56	5	5.018	.3542	3.600	.8890	5.011	.2156	5.140	.9620
16.00	.313	.25	1.56	10	5.009	.1805	2.572	.9290	4.999	-.2089E-01	2.746	.9420
16.00	.313	.25	1.56	20	5.037	.7496	1.826	.9540	5.026	.5207	1.810	.9530
16.00	.313	.25	1.56	30	5.011	.2199	1.481	.9550	4.999	-.2348E-01	1.434	.9530
16.00	.313	.25	1.56	50	5.022	.4460	1.155	.9590	5.008	.1878	1.071	.9520
16.00	.313	.25	1.56	100	5.028	.5560	.8161	.9570	5.014	.2815	.7608	.9480
4.00	1.250	.50	6.25	5	5.007	.1419	6.512	.8970	4.975	-.4960	7.617	.9420
4.00	1.250	.50	6.25	10	5.048	.9606	4.351	.9270	5.003	.5903E-01	4.122	.9190
4.00	1.250	.50	6.25	20	5.084	1.689	3.067	.9530	5.021	.4288	2.739	.9350
4.00	1.250	.50	6.25	30	5.069	1.389	2.468	.9510	5.002	.4750E-01	2.175	.9320
4.00	1.250	.50	6.25	50	5.091	1.829	1.908	.9590	5.024	.4895	1.638	.9360
4.00	1.250	.50	6.25	100	5.068	1.368	1.340	.9690	5.001	.1381E-01	1.163	.9450
1.78	2.813	.75	14.06	5	5.124	2.480	16.64	.9100	4.965	-.7057	10.35	.9310
1.78	2.813	.75	14.06	10	5.223	4.461	8.486	.9530	4.983	-.3363	5.585	.9080
1.78	2.813	.75	14.06	20	5.306	6.112	5.505	.9680	5.006	.1108	3.767	.9130
1.78	2.813	.75	14.06	30	5.370	7.403	4.524	.9650	5.018	.3675	3.055	.9160
1.78	2.813	.75	14.06	50	5.320	6.391	3.372	.9710	4.970	-.5961	2.269	.9330
1.78	2.813	.75	14.06	100	5.356	7.127	2.329	.9490	5.007	.1314	1.617	.9480
1.00	5.000	1.00	25.00	5	5.748	14.96	.1927E+05	.9110	5.012	.2445	13.20	.8760
1.00	5.000	1.00	25.00	10	6.084	21.68	3533.	.9600	4.941	-.1.170	7.033	.8770
1.00	5.000	1.00	25.00	20	6.246	24.92	13.96	.9840	4.987	-.2566	4.826	.9080
1.00	5.000	1.00	25.00	30	6.292	25.83	9.764	.9660	5.039	.7890	3.915	.9200
1.00	5.000	1.00	25.00	50	6.381	27.61	7.211	.9260	5.011	.2173	2.918	.9230
1.00	5.000	1.00	25.00	100	6.411	28.21	4.836	.8000	4.990	-.2042	2.074	.9330
.64	7.813	1.25	39.06	5	8.711	74.23	.3086E+18	.9400	4.900	-.2.008	15.08	.8490
.64	7.813	1.25	39.06	10	8.566	71.33	.3997E+07	.9740	5.078	.1.552	8.649	.8600
.64	7.813	1.25	39.06	20	10.07	101.5	182.2	.9590	4.994	-.1.133	5.846	.8910
.64	7.813	1.25	39.06	30	9.693	93.85	49.46	.9040	5.031	.6120	4.768	.9120
.64	7.813	1.25	39.06	50	9.962	99.24	26.60	.7430	4.951	-.9900	3.537	.9090
.64	7.813	1.25	39.06	100	9.982	99.63	14.09	.3470	4.995	-.1.015	2.565	.9460
.44	11.25	1.50	56.25	5	16.92	238.4	.4973E+24	.9520	5.191	3.818	17.81	.8320
.44	11.25	1.50	56.25	10	27.27	445.3	.5177E+11	.9900	4.884	-.2.327	9.609	.8380
.44	11.25	1.50	56.25	20	38.14	662.7	.6641E+07	.8860	4.956	-.8778	6.713	.8620
.44	11.25	1.50	56.25	30	34.13	582.5	.1352E+05	.6980	5.016	.3274	5.605	.9130
.44	11.25	1.50	56.25	50	32.32	546.4	2688.	.4090	4.963	-.7357	4.184	.9010
.44	11.25	1.50	56.25	100	27.29	445.7	100.6	.0450	5.057	.1.150	3.091	.9300
.25	20.00	2.00	100.00	5	.1197E+05	.2393E+06	.1969E+85	.9790	4.980	-.4057	20.96	.7350
.25	20.00	2.00	100.00	10	.2635E+06	.5269E+07	.6501E+50	.9710	4.959	-.8124	12.06	.7790
.25	20.00	2.00	100.00	20	.1609E+07	.3218E+08	.6288E+30	.5540	4.991	-.1.780	8.562	.8530
.25	20.00	2.00	100.00	30	.2295E+08	.4590E+09	.7795E+26	.2410	4.897	-.2.058	7.004	.8770
.25	20.00	2.00	100.00	50	.8053E+05	.1610E+07	.1140E+13	.0270	5.076	.1.514	5.530	.8060
.25	20.00	2.00	100.00	100	.7627E+05	.1525E+07	.1432E+10	0.0000	4.991	-.1.072	3.974	.9200

TABLE B.13. SIMULATION OF THE GAMMA DISTRIBUTION WITH EXPECTED VALUE SELECTED FROM A UNIFORM DISTRIBUTION ON THE INTERVAL 5.0 TO 15.0. THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS					LOGNORMAL ESTIMATOR				GAMMA ESTIMATOR			
ALPHA	BETA	C.V.	VAR(X)	SAMPLE SIZE	E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE	E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE
16.00	.625	.25	6.25	5	10.04	.3675	7.112	.8610	10.02	.2385	10.17	.9490
16.00	.625	.25	6.25	10	9.928	-.7152	5.129	.9280	9.908	-.9211	5.475	.9340
16.00	.625	.25	6.25	20	10.04	.3587	3.644	.9520	10.01	.1098	3.594	.9490
16.00	.625	.25	6.25	30	10.04	.3937	2.968	.9520	10.01	.1235	2.862	.9450
16.00	.625	.25	6.25	50	10.03	.2958	2.306	.9520	10.00	.2589E-01	2.142	.9320
16.00	.625	.25	6.25	100	10.06	.5555	1.637	.9490	10.03	.2792	1.526	.9440
4.00	2.500	.50	25.00	5	9.951	-.4875	13.42	.8900	9.880	-1.201	15.29	.9300
4.00	2.500	.50	25.00	10	10.04	.4280	8.614	.9260	9.954	-.4632	8.203	.9190
4.00	2.500	.50	25.00	20	10.17	1.715	6.110	.9610	10.06	.5503	5.506	.9410
4.00	2.500	.50	25.00	30	10.04	.4209	4.817	.9600	9.926	-.7429	4.295	.9330
4.00	2.500	.50	25.00	50	10.08	.8181	3.787	.9660	9.944	-.5555	3.235	.9380
4.00	2.500	.50	25.00	100	10.11	1.084	2.671	.9700	9.971	-.2097	2.315	.9420
1.78	5.625	.75	56.25	5	10.23	2.294	37.22	.9250	9.899	-1.012	20.74	.9220
1.78	5.625	.75	56.25	10	10.51	5.065	17.41	.9520	9.999	-.9430E-02	11.15	.9150
1.78	5.625	.75	56.25	20	10.65	6.540	11.34	.9710	10.00	.1765E-01	7.563	.9170
1.78	5.625	.75	56.25	30	10.59	5.922	8.726	.9710	9.964	-.1105	6.053	.9170
1.78	5.625	.75	56.25	50	10.68	6.787	6.687	.9700	10.01	.1278E-01	4.550	.9320
1.78	5.625	.75	56.25	100	10.78	7.774	4.711	.9550	10.06	.13822	3.252	.9450
1.00	10.00	1.00	100.00	5	11.52	15.23	.1652E+08	.9320	10.18	1.754	27.05	.9030
1.00	10.00	1.00	100.00	10	12.19	21.88	149.1	.9660	9.964	-.3639	14.35	.8950
1.00	10.00	1.00	100.00	20	12.36	23.63	28.17	.9810	9.911	-.8883	9.522	.9100
1.00	10.00	1.00	100.00	30	12.67	26.71	20.55	.9770	9.980	-.2048	7.832	.9210
1.00	10.00	1.00	100.00	50	12.94	29.42	14.61	.9270	10.14	1.409	5.972	.9200
1.00	10.00	1.00	100.00	100	12.90	28.99	9.687	.7820	10.07	.6970	4.210	.9330
.64	15.63	1.25	156.25	5	15.41	54.08	.3971E+11	.9460	9.582	-4.185	28.93	.8410
.64	15.63	1.25	156.25	10	20.26	102.6	.6313E+07	.9830	10.25	2.538	17.86	.8670
.64	15.63	1.25	156.25	20	18.80	88.01	217.3	.9700	9.991	-.8644E-01	11.76	.9040
.64	15.63	1.25	156.25	30	19.05	90.45	95.71	.8980	10.06	.5891	9.515	.9030
.64	15.63	1.25	156.25	50	20.29	102.9	53.92	.7360	10.04	.3854	7.168	.9200
.64	15.63	1.25	156.25	100	19.91	99.13	28.30	.3610	10.01	.6447E-01	5.141	.9400
.44	22.50	1.50	225.00	5	46.80	368.0	.3163E+47	.9430	9.954	-.4622	35.06	.8220
.44	22.50	1.50	225.00	10	92.01	820.1	.2090E+17	.9870	10.11	1.133	19.91	.8470
.44	22.50	1.50	225.00	20	81.49	714.9	.4018E+07	.8790	10.08	.8077	13.70	.8910
.44	22.50	1.50	225.00	30	72.16	621.6	.7593E+05	.6910	10.24	2.437	11.40	.9010
.44	22.50	1.50	225.00	50	61.68	516.8	1578.	.3930	9.957	-.4269	8.404	.9060
.44	22.50	1.50	225.00	100	53.81	436.1	195.0	.0460	9.944	-.5604	6.079	.9330
.25	40.00	2.00	400.00	5	.5759E+05	.5758E+06	.4869+155	.9690	9.738	-2.618	40.31	.7320
.25	40.00	2.00	400.00	10	.2871E+08	.2871E+07	.2521E+51	.9690	9.771	-2.289	23.61	.7670
.25	40.00	2.00	400.00	20	.4111E+07	.4111E+08	.1218E+31	.5370	10.06	.5639	17.12	.8370
.25	40.00	2.00	400.00	30	.3456E+08	.3455E+07	.1613E+19	.2380	10.09	.9369	14.41	.8720
.25	40.00	2.00	400.00	50	.1702E+05	.1701E+06	.6730E+10	.0260	10.07	.6778	10.99	.8820
.25	40.00	2.00	400.00	100	.2140E+05	.2138E+06	.3675E+08	0.0000	9.968	-.3171	7.944	.9010

TABLE B.14. SIMULATION OF THE GAMMA DISTRIBUTION WITH EXPECTED VALUE SELECTED FROM A UNIFORM DISTRIBUTION ON THE INTERVAL 25.0 TO 75.0. THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS				SAMPLE SIZE	LOGNORMAL ESTIMATOR				GAMMA ESTIMATOR			
ALPHA	BETA	C.V.	VAR(X)		E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE	E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE
16.00	3.125	.25	156.25	5	49.93	-.1408	35.39	.8720	49.86	-.2827	50.53	.9500
16.00	3.125	.25	156.25	10	49.86	-.2807	25.70	.9260	49.77	-.4678	27.55	.9440
16.00	3.125	.25	156.25	20	50.05	.9211E-01	18.10	.9450	49.93	-.1384	17.92	.9420
16.00	3.125	.25	156.25	30	50.15	.2982	14.82	.9450	50.02	.4939E-01	14.35	.9420
16.00	3.125	.25	156.25	50	50.10	.2084	11.47	.9520	49.97	-.5460E-01	10.66	.9430
16.00	3.125	.25	156.25	100	50.04	.7926E-01	8.130	.9610	49.91	-.1872	7.589	.9440
4.00	12.50	.50	625.00	5	49.52	-.9523	63.90	.8970	49.21	-1.587	74.65	.9320
4.00	12.50	.50	625.00	10	50.51	1.026	43.50	.9340	50.02	.3452E-01	41.00	.9240
4.00	12.50	.50	625.00	20	50.78	1.552	30.39	.9580	50.18	.3516	27.28	.9440
4.00	12.50	.50	625.00	30	50.92	1.847	24.70	.9490	50.29	.5826	21.93	.9360
4.00	12.50	.50	625.00	50	50.71	1.426	18.95	.9640	50.08	.1602	16.36	.9440
4.00	12.50	.50	625.00	100	50.58	1.164	13.35	.9640	49.90	-.1975	11.57	.9430
1.78	28.13	.75	1406.25	5	53.09	6.171	706.0	.9040	51.20	2.408	106.5	.9180
1.78	28.13	.75	1406.25	10	52.88	5.753	87.80	.9520	50.22	.4362	56.92	.9170
1.78	28.13	.75	1406.25	20	52.70	5.400	55.50	.9730	49.55	-.9030	37.39	.9160
1.78	28.13	.75	1406.25	30	53.45	6.892	44.19	.9710	50.20	.3958	30.44	.9290
1.78	28.13	.75	1406.25	50	53.25	6.507	33.33	.9690	49.84	-.3138	22.60	.9280
1.78	28.13	.75	1406.25	100	53.50	7.010	23.29	.9470	50.02	.4635E-01	16.15	.9260
1.00	50.00	1.00	2500.00	5	56.53	13.05	.2222E+08	.9490	49.93	-.1407	120.4	.9120
1.00	50.00	1.00	2500.00	10	60.19	20.37	413.6	.9660	49.56	-.8056	70.66	.8830
1.00	50.00	1.00	2500.00	20	62.39	24.78	146.7	.9800	49.60	-.8056	48.17	.9130
1.00	50.00	1.00	2500.00	30	63.10	26.20	99.06	.9660	50.47	.9304	39.15	.9310
1.00	50.00	1.00	2500.00	50	63.79	27.58	71.23	.9350	50.24	.4853	29.26	.9310
1.00	50.00	1.00	2500.00	100	63.87	27.74	47.91	.8020	49.94	-.1284	20.97	.9420
.64	78.13	1.25	3906.25	5	74.17	48.34	.8622E+11	.9510	50.21	.4120	155.5	.8680
.64	78.13	1.25	3906.25	10	103.1	106.3	.4785E+08	.9840	49.29	-1.414	83.80	.8740
.64	78.13	1.25	3906.25	20	107.6	115.3	.4700E+05	.9580	50.04	.7851E-01	58.20	.9040
.64	78.13	1.25	3906.25	30	99.17	98.34	509.4	.8960	50.11	.2103	47.27	.9090
.64	78.13	1.25	3906.25	50	106.2	112.4	945.2	.7530	49.76	-.4714	35.42	.8980
.64	78.13	1.25	3906.25	100	98.71	97.42	138.2	.3620	50.04	.7887E-01	25.65	.9420
.44	112.5	1.50	5625.00	5	379.7	659.3	.3353E+43	.9530	51.34	2.687	180.0	.8290
.44	112.5	1.50	5625.00	10	268.3	436.5	.5109E+12	.9800	51.49	2.979	102.6	.8550
.44	112.5	1.50	5625.00	20	457.3	814.7	.2427E+09	.8990	49.65	-.7012	68.06	.8690
.44	112.5	1.50	5625.00	30	342.2	584.5	.2262E+06	.7040	49.95	-.1047	55.57	.8960
.44	112.5	1.50	5625.00	50	319.2	538.4	.1235E+05	.3990	49.87	-.2518	41.51	.8930
.44	112.5	1.50	5625.00	100	277.0	454.0	105.1	.0340	50.16	.3224	30.50	.9220
.25	200.0	2.00	10000.0	5	.2009E+07	.4018E+07	.5138+132	.9720	50.69	1.385	210.2	.7550
.25	200.0	2.00	10000.0	10	.3035E+08	.6071E+08	.1941E+65	.9740	51.11	2.216	123.8	.7950
.25	200.0	2.00	10000.0	20	.3886E+07	.7871E+07	.5720E+28	.5560	48.79	-2.429	83.94	.8360
.25	200.0	2.00	10000.0	30	.5155E+08	.1031E+09	.1282E+27	.2360	49.13	-1.741	69.79	.8440
.25	200.0	2.00	10000.0	50	.3678E+06	.7357E+06	.1616E+13	.0270	49.82	-.3507	54.24	.8680
.25	200.0	2.00	10000.0	100	.1158E+06	.2316E+06	.3895E+09	0.0000	50.64	1.271	40.35	.9310

TABLE B.15. SIMULATION OF THE GAMMA DISTRIBUTION WITH EXPECTED VALUE SELECTED FROM A UNIFORM DISTRIBUTION ON THE INTERVAL 50.0 TO 150.0. THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS					LOGNORMAL ESTIMATOR				GAMMA ESTIMATOR			
ALPHA	BETA	C.V.	VAR(X)	SAMPLE SIZE	E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE	E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE
16.00	6.250	.25	625.00	5	100.4	.3992	71.89	.0720	100.3	.2760	102.7	.9470
16.00	6.250	.25	625.00	10	100.4	.3928	51.74	.9300	100.2	.1734	55.05	.9510
16.00	6.250	.25	625.00	20	100.3	.2847	36.64	.9440	100.0	.2879E-01	38.07	.9440
16.00	6.250	.25	625.00	30	100.6	.5914	29.70	.9490	100.3	.3380	28.72	.9430
16.00	6.250	.25	625.00	50	100.2	.2293	23.03	.9480	99.96	-.3618E-01	21.41	.9440
16.00	6.250	.25	625.00	100	100.4	.4204	16.24	.9600	100.2	.1639	15.10	.9480
4.00	25.00	.50	2500.00	5	98.48	-1.516	127.6	.8920	97.79	-2.211	147.7	.9340
4.00	25.00	.50	2500.00	10	101.3	1.291	88.74	.9390	100.2	.1996	82.72	.9290
4.00	25.00	.50	2500.00	20	101.6	1.616	61.03	.9580	100.4	.3993	54.71	.9400
4.00	25.00	.50	2500.00	30	100.9	.8814	49.41	.9620	99.60	-.4013	43.87	.9430
4.00	25.00	.50	2500.00	50	101.3	1.328	37.98	.9500	100.0	.1544E-01	32.65	.9270
4.00	25.00	.50	2500.00	100	101.5	1.527	26.66	.9630	100.2	.1784	23.11	.9450
1.78	56.25	.75	5625.00	5	105.8	5.830	1638.	.9030	102.2	2.158	214.5	.9270
1.78	56.25	.75	5625.00	10	106.4	6.414	183.4	.9560	100.9	.8703	113.8	.9250
1.78	56.25	.75	5625.00	20	107.0	7.038	110.1	.9660	101.1	1.092	75.84	.9310
1.78	56.25	.75	5625.00	30	106.3	6.328	87.81	.9750	99.81	-.1910	60.61	.9270
1.78	56.25	.75	5625.00	50	107.0	6.953	67.39	.9680	100.0	.1649E-02	45.51	.9240
1.78	56.25	.75	5625.00	100	107.4	7.397	46.73	.9460	100.3	.3473	32.32	.9380
1.00	100.0	1.00	10000.0	5	110.1	10.06	.1533E+06	.9390	98.74	-1.258	255.0	.9000
1.00	100.0	1.00	10000.0	10	126.7	26.74	2962.	.9710	100.6	.6087	144.0	.8870
1.00	100.0	1.00	10000.0	20	124.6	24.56	272.5	.9750	100.5	.5024	96.03	.9140
1.00	100.0	1.00	10000.0	30	126.3	26.34	200.5	.9670	100.6	.6191	78.49	.9290
1.00	100.0	1.00	10000.0	50	126.8	26.78	141.5	.9370	99.93	-.7205E-01	58.33	.9260
1.00	100.0	1.00	10000.0	100	127.4	27.42	94.24	.8000	100.2	.2257	41.72	.9250
.64	156.3	1.25	15625.0	5	145.8	45.84	.1150E+15	.9410	97.89	-2.114	293.0	.8550
.64	156.3	1.25	15625.0	10	171.7	71.65	.1221E+07	.9700	99.42	-.5821	169.6	.8530
.64	156.3	1.25	15625.0	20	209.8	109.8	.1294E+05	.9550	101.3	1.262	117.4	.8950
.64	156.3	1.25	15625.0	30	206.5	106.5	194.0.	.9150	99.69	-.3080	94.68	.9160
.64	156.3	1.25	15625.0	50	200.2	100.2	512.3	.7230	100.1	.1295	72.14	.9110
.64	156.3	1.25	15625.0	100	198.4	98.36	278.5	.3600	99.51	-.4692	51.01	.9240
.44	225.0	1.50	22500.0	5	315.2	215.2	.4114E+23	.9560	99.14	-.8551	342.8	.8150
.44	225.0	1.50	22500.0	10	673.1	573.1	.1132E+14	.9880	98.59	-1.415	195.6	.8340
.44	225.0	1.50	22500.0	20	710.5	610.5	.6259E+06	.9000	100.2	.2238	137.6	.8950
.44	225.0	1.50	22500.0	30	804.0	704.0	.6077E+06	.7160	99.34	-.6634	109.5	.9050
.44	225.0	1.50	22500.0	50	626.2	526.2	.1415E+05	.3760	100.7	.7310	85.38	.9000
.44	225.0	1.50	22500.0	100	540.6	440.6	1985.	.0520	99.87	-.1321	60.94	.9210
.25	400.0	2.00	40000.0	5	.2216E+05	.2206E+05	.4538E+95	.9770	101.3	1.324	425.3	.7650
.25	400.0	2.00	40000.0	10	.1477E+07	.1477E+07	.1803E+53	.9740	100.8	.8468	246.3	.7940
.25	400.0	2.00	40000.0	20	.7865E+09	.7865E+09	.4666E+43	.5450	102.3	2.340	177.5	.8470
.25	400.0	2.00	40000.0	30	.5449E+07	.5448E+07	.3449E+21	.2490	100.3	.3269	143.1	.8450
.25	400.0	2.00	40000.0	50	.4652E+08	.4652E+08	.9120E+18	.0300	99.98	-.2249E-01	109.4	.8750
.25	400.0	2.00	40000.0	100	.3271E+06	.3270E+06	.1606E+10	0.0000	99.75	-.2474	79.89	.9070

TABLE B.16. SIMULATION OF THE LOGNORMAL DISTRIBUTION WITH EXPECTED VALUE SELECTED FROM A UNIFORM DISTRIBUTION ON THE INTERVAL .5 TO 1.5. THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS				SAMPLE SIZE	LOGNORMAL ESTIMATOR				GAMMA ESTIMATOR			
MU	SIGMA	C.V.	VAR(X)		E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE	E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE
-.030	.246	.25	.06	5	.9932	-.6821	.7122	.8750	.9922	-.7812	1.023	.9510
-.030	.246	.25	.06	10	.9988	-.1224	.5030	.9190	.9976	-.2448	.5468	.9420
-.030	.246	.25	.06	20	1.004	.3979	.3616	.9430	1.002	.2153	.3816	.9460
-.030	.246	.25	.06	30	1.001	.8353E-01	.2913	.9470	.9990	-.1042	.2858	.9440
-.030	.246	.25	.06	50	1.002	.1998	.2278	.9470	1.000	.3203E-02	.2147	.9300
-.030	.246	.25	.06	100	1.002	.2339	.1614	.9640	1.000	.2383E-01	.1524	.9590
-.112	.472	.50	.25	5	1.004	.4084	1.224	.8910	1.004	.3703	1.558	.9320
-.112	.472	.50	.25	10	.9932	-.6793	.7615	.9120	.9912	-.8779	.7863	.9110
-.112	.472	.50	.25	20	1.006	.5694	.5477	.9320	1.004	.3793	.5417	.9330
-.112	.472	.50	.25	30	.9995	-.1479	.4381	.9430	.9962	-.3842	.4291	.9300
-.112	.472	.50	.25	50	1.001	.1389	.3385	.9560	.9993	-.6855E-01	.3229	.9400
-.112	.472	.50	.25	100	.9985	-.1517	.2379	.9310	.9965	-.3473	.2305	.9260
-.223	.668	.75	.56	5	1.005	.5283	1.897	.8570	1.004	.4108	1.972	.8830
-.223	.668	.75	.56	10	1.001	.5521E-01	1.131	.9170	.9998	-.1599E-01	1.080	.8980
-.223	.668	.75	.56	20	.9965	-.3498	.7574	.9210	.9953	-.4675	.7281	.8970
-.223	.668	.75	.56	30	.9968	-.3219	.6094	.9360	.9935	-.6458	.5025	.9050
-.223	.668	.75	.56	50	1.003	.2665	.4688	.9280	1.001	.5012E-01	.4460	.8970
-.223	.668	.75	.56	100	1.001	.6972E-01	.3277	.9510	.9981	-.1912	.3178	.9380
-.347	.833	1.00	1.00	5	1.014	1.364	2.911	.8720	1.010	.9521	2.360	.8710
-.347	.833	1.00	1.00	10	1.023	2.279	1.613	.9210	1.022	2.162	1.369	.8610
-.347	.833	1.00	1.00	20	1.000	.4952E-02	.9897	.9400	.9974	-.2604	.8994	.8980
-.347	.833	1.00	1.00	30	.9981	-.1902	.7780	.9420	.9940	-.5985	.7225	.9090
-.347	.833	1.00	1.00	50	1.004	.3539	.6048	.9610	1.001	.1135	.5674	.9310
-.347	.833	1.00	1.00	100	1.003	.2719	.4193	.9510	1.000	.1542E-01	.4082	.9280
-.470	.970	1.25	1.56	5	.9741	-2.589	4.471	.8660	.9698	-3.025	2.562	.8470
-.470	.970	1.25	1.56	10	.9922	-.7791	1.984	.9130	.9926	-.7361	1.521	.8500
-.470	.970	1.25	1.56	20	1.016	1.580	1.272	.9500	1.014	1.357	1.094	.8680
-.470	.970	1.25	1.56	30	1.005	.4962	.9715	.9500	1.007	.6592	.8994	.8910
-.470	.970	1.25	1.56	50	1.014	1.441	.7446	.9470	1.010	.9639	.6784	.9060
-.470	.970	1.25	1.56	100	1.004	.3565	.5050	.9390	.9987	-.1300	.4882	.8940
-.589	1.086	1.50	2.25	5	1.025	2.487	7.279	.8500	1.023	2.265	3.008	.8190
-.589	1.086	1.50	2.25	10	1.003	.3373	2.645	.9020	.9863	-1.370	1.634	.8070
-.589	1.086	1.50	2.25	20	.9981	-.1948	1.497	.9280	.9982	-.1754	1.226	.8530
-.589	1.086	1.50	2.25	30	.9951	-.4867	1.138	.9390	.9945	-.5460	.9971	.8670
-.589	1.086	1.50	2.25	50	.9957	-.4274	.8428	.9440	.9973	-.2697	.7793	.8670
-.589	1.086	1.50	2.25	100	.9987	-.1257	.5811	.9330	.9939	-.6137	.5641	.8970
-.805	1.269	2.00	4.00	5	1.029	2.925	28.08	.8780	1.016	1.585	3.499	.7880
-.805	1.269	2.00	4.00	10	.9740	-2.596	4.085	.9000	.9771	-2.293	2.004	.7650
-.805	1.269	2.00	4.00	20	1.002	.1760	2.055	.9220	1.006	.6277	1.505	.8290
-.805	1.269	2.00	4.00	30	.9927	-.7329	1.480	.9350	.9937	-.6257	1.214	.8340
-.805	1.269	2.00	4.00	50	.9964	-.3601	1.076	.9390	.9836	-1.637	.9117	.8430
-.805	1.269	2.00	4.00	100	1.012	1.177	.7398	.9470	1.008	.8002	.7217	.8800

TABLE B.17. SIMULATION OF THE LOGNORMAL DISTRIBUTION WITH EXPECTED VALUE SELECTED FROM A UNIFORM DISTRIBUTION ON THE INTERVAL 2.5 TO 7.5. THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS					LOGNORMAL ESTIMATOR				GAMMA ESTIMATOR			
MU	SIGMA	C.V.	VAR(X)	SAMPLE SIZE	E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE	E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE
1.579	.246	.25	1.56	5	4.989	-.2160	3.486	.8830	4.984	-.3176	5.029	.9560
1.579	.246	.25	1.56	10	4.979	-.4169	2.563	.9370	4.972	-.5645	2.777	.9430
1.579	.246	.25	1.56	20	5.013	.2596	1.808	.9530	5.004	.8326E-01	1.809	.9480
1.579	.246	.25	1.56	30	5.005	.9858E-01	1.458	.9400	4.995	-.9578E-01	1.429	.9400
1.579	.246	.25	1.56	50	5.011	.2210	1.134	.9490	5.000	.6689E-02	1.064	.9360
1.579	.246	.25	1.56	100	5.009	.1872	.8030	.9460	4.999	-.1791E-01	.7591	.9390
1.498	.472	.50	6.25	5	5.028	.5635	5.799	.8880	5.022	.4395	7.477	.9390
1.498	.472	.50	6.25	10	4.960	-.8024	3.889	.9250	4.956	-.8899	4.052	.9300
1.498	.472	.50	6.25	20	5.011	.2170	2.741	.9460	5.003	.5396E-01	2.724	.9320
1.498	.472	.50	6.25	30	4.979	-.4280	2.195	.9380	4.968	-.6315	2.156	.9190
1.498	.472	.50	6.25	50	4.993	-.1357	1.689	.9450	4.983	-.3306	1.617	.9170
1.498	.472	.50	6.25	100	5.023	.4654	1.198	.9540	5.013	.2552	1.159	.9430
1.386	.668	.75	14.06	5	5.007	.1326	9.306	.8690	4.993	-.1429	9.666	.8960
1.386	.668	.75	14.06	10	5.071	1.425	5.709	.9200	5.050	1.010	5.366	.9040
1.386	.668	.75	14.06	20	5.002	.4689E-01	3.780	.9370	4.991	-.1710	3.612	.9030
1.386	.668	.75	14.06	30	5.023	.4638	3.073	.9290	5.008	.1697	2.947	.9160
1.386	.668	.75	14.06	50	5.048	.9632	2.365	.9370	5.031	.6113	2.226	.9170
1.386	.668	.75	14.06	100	5.000	-.4372E-02	1.639	.9410	4.986	-.2703	1.591	.9370
1.263	.833	1.00	25.00	5	5.088	1.757	15.68	.8740	5.066	1.310	11.94	.8860
1.263	.833	1.00	25.00	10	5.078	1.554	8.061	.9210	5.071	1.429	6.790	.8790
1.263	.833	1.00	25.00	20	5.008	.1610	4.939	.9450	4.989	-.2217	4.479	.8910
1.263	.833	1.00	25.00	30	5.044	.8801	3.962	.9450	5.028	.5650	3.696	.9210
1.263	.833	1.00	25.00	50	5.021	.4121	3.011	.9430	5.008	.1567	2.833	.9040
1.263	.833	1.00	25.00	100	5.028	.5667	2.117	.9410	5.024	.4701	2.090	.9330
1.139	.970	1.25	39.06	5	4.988	-.2456	22.68	.8820	4.998	-.3630E-01	13.61	.8630
1.139	.970	1.25	39.06	10	5.059	1.188	10.72	.9120	5.053	1.066	7.829	.8320
1.139	.970	1.25	39.06	20	4.980	-.3998	6.181	.9320	4.993	-.1412	5.410	.8610
1.139	.970	1.25	39.06	30	5.003	.5726E-01	4.848	.9340	4.979	-.4244	4.333	.8750
1.139	.970	1.25	39.06	50	5.025	.4909	3.634	.9360	5.001	.2466E-01	3.332	.8950
1.139	.970	1.25	39.06	100	4.972	-.5589	2.503	.9480	4.953	-.9305	2.431	.9100
1.020	1.086	1.50	56.25	5	4.898	-2.038	45.88	.8500	4.924	-1.528	14.64	.7980
1.020	1.086	1.50	56.25	10	4.966	-.6776	12.92	.9150	4.988	-.2433	8.595	.8160
1.020	1.086	1.50	56.25	20	5.032	.6409	7.523	.9260	5.035	.6910	6.130	.8600
1.020	1.086	1.50	56.25	30	5.044	.8858	5.912	.9390	5.023	.4668	5.085	.8590
1.020	1.086	1.50	56.25	50	5.052	1.035	4.309	.9410	5.024	.4847	3.878	.8820
1.020	1.086	1.50	56.25	100	5.016	.3258	2.941	.9370	4.988	-.2468	2.843	.8970
.805	1.269	2.00	100.00	5	5.201	4.012	432.7	.8520	5.107	2.137	17.54	.7660
.805	1.269	2.00	100.00	10	5.183	3.652	23.00	.9080	5.103	2.069	10.26	.7840
.805	1.269	2.00	100.00	20	4.949	-1.013	10.03	.9270	5.021	.4250	7.505	.7970
.805	1.269	2.00	100.00	30	5.012	.2415	7.534	.9240	5.049	.9881	6.330	.8330
.805	1.269	2.00	100.00	50	5.042	.8343	5.531	.9460	5.041	.8209	4.887	.8620
.805	1.269	2.00	100.00	100	4.970	-.6009	3.611	.9430	4.965	-.6994	3.574	.8540

TABLE B.18. SIMULATION OF THE LOGNORMAL DISTRIBUTION WITH EXPECTED VALUE SELECTED FROM A UNIFORM DISTRIBUTION ON THE INTERVAL 5.0 TO 15.0. THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS					LOGNORMAL ESTIMATOR				GAMMA ESTIMATOR			
MU	SIGMA	C.V.	VAR(X)	SAMPLE SIZE	E(X)-HAT	REL. BIAS	AVE(C1)	COVERAGE	E(X)-HAT	REL. BIAS	AVE(C1)	COVERAGE
2.272	.246	.25	6.25	5	10.03	.8444	7.144	.8680	10.07	.7006	10.21	.9550
2.272	.246	.25	6.25	10	10.04	.4079	5.130	.9320	10.02	.2352	5.504	.9430
2.272	.246	.25	6.25	20	9.989	-.1145	3.565	.9480	9.970	-.2976	3.562	.9470
2.272	.246	.25	6.25	30	10.02	.1825	2.945	.9430	9.998	-.2103E-01	2.878	.9380
2.272	.246	.25	6.25	50	10.05	.5349	2.289	.9550	10.03	.3460	2.163	.9470
2.272	.246	.25	6.25	100	10.02	.2299	1.613	.9600	10.00	.1413E-01	1.522	.9520
2.191	.472	.50	25.00	5	10.13	1.310	11.73	.8850	10.13	1.307	15.21	.9360
2.191	.472	.50	25.00	10	10.12	1.162	7.849	.9140	10.11	1.063	8.174	.9210
2.191	.472	.50	25.00	20	10.01	.8124E-01	5.384	.9470	9.988	-.1172	5.331	.9320
2.191	.472	.50	25.00	30	10.03	.2966	4.400	.9420	10.01	.1218	4.334	.9370
2.191	.472	.50	25.00	50	10.01	.1034	3.385	.9570	9.986	-.1419	3.219	.9450
2.191	.472	.50	25.00	100	9.997	-.2878E-01	2.392	.9500	9.976	-.2363	2.312	.9390
2.079	.668	.75	56.25	5	10.14	1.423	19.63	.8900	10.11	1.141	20.14	.9210
2.079	.668	.75	56.25	10	9.845	-1.546	10.99	.9170	9.832	-1.676	10.52	.8980
2.079	.668	.75	56.25	20	10.01	.1310	7.604	.9410	9.956	-.4165E-01	7.264	.9100
2.079	.668	.75	56.25	30	10.01	.5649E-01	6.122	.9320	9.988	-.1167	5.919	.9150
2.079	.668	.75	56.25	50	10.03	.2984	4.697	.9480	10.01	.7042E-01	4.455	.9270
2.079	.668	.75	56.25	100	10.01	.1152	3.285	.9500	9.993	-.7247E-01	3.207	.9420
1.956	.833	1.00	100.00	5	10.08	.8036	32.87	.8660	10.11	1.091	24.45	.8750
1.956	.833	1.00	100.00	10	10.00	.1226E-01	15.44	.9110	9.996	-.4081E-01	13.21	.8600
1.956	.833	1.00	100.00	20	9.936	-.6415	9.902	.9340	9.929	-.7101	9.059	.8940
1.956	.833	1.00	100.00	30	9.954	-.4562	7.837	.9430	9.912	-.8816	7.271	.8910
1.956	.833	1.00	100.00	50	9.991	-.9156E-01	5.979	.9470	9.963	-.3727	5.587	.9070
1.956	.833	1.00	100.00	100	10.03	.3020	4.189	.9500	10.00	.1083E-01	4.088	.9300
1.832	.970	1.25	156.25	5	9.912	-.8813	56.81	.8570	9.820	-1.803	25.89	.8290
1.832	.970	1.25	156.25	10	10.14	1.376	21.11	.9170	10.14	1.430	15.71	.8510
1.832	.970	1.25	156.25	20	10.07	.6517	12.51	.9300	10.04	.4389	10.78	.8700
1.832	.970	1.25	156.25	30	10.03	.2549	9.728	.9370	10.02	.1740	8.882	.8820
1.832	.970	1.25	156.25	50	10.05	.4555	7.317	.9500	10.02	.1563	6.778	.9020
1.832	.970	1.25	156.25	100	10.05	.4589	5.087	.9520	10.03	.2840	5.011	.9190
1.713	1.086	1.50	225.00	5	10.00	-.3557E-02	140.0	.8580	9.995	-.5473E-01	30.21	.8240
1.713	1.086	1.50	225.00	10	10.04	.4490	25.93	.9070	10.01	.1090	17.05	.8300
1.713	1.086	1.50	225.00	20	9.944	-.5629	15.08	.9200	9.923	-.7736	12.10	.8390
1.713	1.086	1.50	225.00	30	10.04	.4193	11.60	.9390	10.03	.3369	10.22	.8670
1.713	1.086	1.50	225.00	50	10.07	.7058	8.678	.9430	10.05	.5408	7.952	.8820
1.713	1.086	1.50	225.00	100	10.04	.3590	5.855	.9550	10.00	.4977E-01	5.726	.8990
1.498	1.269	2.00	400.00	5	10.40	3.669	547.5	.8590	10.67	6.657	38.21	.7750
1.498	1.269	2.00	400.00	10	9.938	-.6249	40.25	.9110	10.13	1.264	21.06	.7720
1.498	1.269	2.00	400.00	20	9.814	-1.660	19.43	.9210	9.711	-2.086	13.68	.7990
1.498	1.269	2.00	400.00	30	10.12	1.212	15.38	.9410	10.12	1.226	12.52	.8310
1.498	1.269	2.00	400.00	50	10.07	.7019	10.92	.9350	10.08	.7579	9.656	.8370
1.498	1.269	2.00	400.00	100	10.15	1.459	7.450	.9540	10.18	1.790	7.529	.8890

TABLE B.19. SIMULATION OF THE LOGNORMAL DISTRIBUTION WITH EXPECTED VALUE SELECTED FROM A UNIFORM DISTRIBUTION ON THE INTERVAL 25.0 TO 75.0. THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS					LOGNORMAL ESTIMATOR				GAMMA ESTIMATOR			
MU	SIGMA	C.V.	VAR(X)	SAMPLE SIZE	E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE	E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE
3.882	.246	.25	156.25	5	50.17	.3430	36.72	.8880	50.11	.2122	52.33	.9620
3.882	.246	.25	156.25	10	50.16	.3297	24.95	.9100	50.09	.1831	26.99	.9360
3.882	.246	.25	156.25	20	50.30	.5951	18.13	.9400	50.20	.4083	18.10	.9430
3.882	.246	.25	156.25	30	50.08	.1530	14.59	.9470	49.98	-.3578E-01	14.32	.9450
3.882	.246	.25	156.25	50	50.01	.1968E-01	11.28	.9620	49.92	-.1635	10.66	.9480
3.882	.246	.25	156.25	100	50.13	.2589	8.048	.9620	50.02	.4166E-01	7.588	.9500
3.800	.472	.50	625.00	5	50.28	.5563	58.37	.8690	50.25	.4935	74.96	.9360
3.800	.472	.50	625.00	10	50.38	.7546	39.50	.9190	50.28	.5586	40.59	.9220
3.800	.472	.50	625.00	20	50.36	.7255	27.21	.9370	50.26	.5224	26.94	.9290
3.800	.472	.50	625.00	30	50.03	.6091E-01	21.92	.9480	49.92	-.1607	21.45	.9350
3.800	.472	.50	625.00	50	50.10	.2067	16.93	.9520	50.01	.2377E-01	16.20	.9380
3.800	.472	.50	625.00	100	49.95	-.1091	11.91	.9530	49.86	-.2786	11.58	.9400
3.689	.668	.75	1406.25	5	48.78	-2.440	90.26	.8740	48.61	-2.773	93.57	.8980
3.689	.668	.75	1406.25	10	50.67	1.332	57.91	.9220	50.54	1.090	54.67	.9050
3.689	.668	.75	1406.25	20	49.71	-.5785	37.50	.9310	49.67	-.6604	36.16	.9000
3.689	.668	.75	1406.25	30	49.76	-.4730	30.48	.9470	49.72	-.5523	29.58	.9210
3.689	.668	.75	1406.25	50	50.27	.5309	23.60	.9470	50.12	.2341	22.22	.9170
3.689	.668	.75	1406.25	100	50.14	.2803	16.45	.9540	50.06	.1203	16.09	.9400
3.565	.833	1.00	2500.00	5	50.48	.9521	156.0	.8720	50.48	.9610	119.7	.8790
3.565	.833	1.00	2500.00	10	50.21	.4158	78.62	.9160	50.42	.0345	68.07	.8800
3.565	.833	1.00	2500.00	20	50.31	.6231	49.57	.9320	50.25	.4956	45.38	.8920
3.565	.833	1.00	2500.00	30	50.08	.1587	39.53	.9480	50.17	.3443	37.74	.9100
3.565	.833	1.00	2500.00	50	49.81	-.3766	29.85	.9490	49.68	-.6425	28.05	.9020
3.565	.833	1.00	2500.00	100	50.21	.4161	20.94	.9450	50.02	.4547E-01	20.26	.9140
3.442	.970	1.25	3906.25	5	48.60	-2.803	227.5	.8520	48.72	-2.565	131.4	.8410
3.442	.970	1.25	3906.25	10	50.08	.1678	103.8	.9120	49.89	-.2115	76.34	.8390
3.442	.970	1.25	3906.25	20	49.86	-.2897	61.69	.9190	49.55	-.8981	52.39	.8720
3.442	.970	1.25	3906.25	30	49.65	-.6970	48.20	.9340	49.44	-1.113	43.18	.8640
3.442	.970	1.25	3906.25	50	50.73	1.460	57.32	.9450	50.68	1.355	34.68	.9100
3.442	.970	1.25	3906.25	100	50.34	.6808	25.29	.9420	50.41	.8225	25.25	.9080
3.323	1.086	1.50	5625.00	5	49.62	-.7542	260.6	.8470	49.72	-.5542	140.8	.8050
3.323	1.086	1.50	5625.00	10	49.77	-.4536	131.6	.8860	49.59	-.8242	85.33	.8210
3.323	1.086	1.50	5625.00	20	51.02	2.047	76.79	.9360	50.93	1.858	61.94	.8670
3.323	1.086	1.50	5625.00	30	50.23	.4673	57.56	.9380	50.22	.4316	50.33	.8700
3.323	1.086	1.50	5625.00	50	50.08	.1933	42.96	.9430	50.14	.2716	39.77	.8790
3.323	1.086	1.50	5625.00	100	49.89	-.2171	28.93	.9470	49.91	-.1742	28.67	.8976
3.107	1.269	2.00	10000.0	5	50.04	.7870E-01	1317.	.8340	49.79	-.4272	168.1	.7600
3.107	1.269	2.00	10000.0	10	49.54	-.9243	214.0	.9060	49.32	-1.385	100.6	.7760
3.107	1.269	2.00	10000.0	20	50.04	.8044E-01	102.6	.9390	50.09	.1719	73.97	.8100
3.107	1.269	2.00	10000.0	30	49.83	-.3374	75.11	.9360	49.88	-.2466	61.59	.8370
3.107	1.269	2.00	10000.0	50	50.04	.7432E-01	53.88	.9400	49.71	-.5851	46.95	.8460
3.107	1.269	2.00	10000.0	100	50.34	.6701	36.81	.9450	50.45	.8959	36.65	.8820

TABLE B.20. SIMULATION OF THE LOGNORMAL DISTRIBUTION WITH EXPECTED VALUE SELECTED FROM A UNIFORM DISTRIBUTION ON THE INTERVAL 50.0 TO 150.0. THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS					LOGNORMAL ESTIMATOR				GAMMA ESTIMATOR			
MU	SIGMA	C.V.	VAR(X)	SAMPLE SIZE	E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE	E(X)-HAT	REL. BIAS	AVE(CI)	COVERAGE
4.575	.246	.25	625.00	5	99.73	-.2742	72.58	.9040	99.63	-.3732	104.1	.9540
4.575	.246	.25	625.00	10	99.41	-.5948	50.05	.9190	99.25	-.7478	54.02	.9360
4.575	.246	.25	625.00	20	100.4	.4335	36.18	.9420	100.2	.2289	36.03	.9470
4.575	.246	.25	625.00	30	99.93	-.7156E-01	29.14	.9550	99.75	-.2468	28.64	.9500
4.575	.246	.25	625.00	50	100.2	.1757	22.71	.9550	99.97	-.2807E-01	21.37	.9430
4.575	.246	.25	625.00	100	100.2	.1853	16.08	.9560	99.98	-.1781E-01	15.21	.9480
4.494	.472	.50	2500.00	5	98.70	-1.301	112.4	.8870	98.55	-1.452	143.9	.8350
4.494	.472	.50	2500.00	10	101.3	1.262	79.09	.9130	101.2	1.203	82.47	.9220
4.494	.472	.50	2500.00	20	100.8	.8080	54.44	.9220	100.7	.6881	54.33	.9230
4.494	.472	.50	2500.00	30	99.93	-.7013E-01	43.81	.9360	99.74	-.2627	43.04	.9290
4.494	.472	.50	2500.00	50	100.2	.2276	33.93	.9500	100.0	.1002E-01	32.35	.9250
4.494	.472	.50	2500.00	100	100.3	.2970	24.01	.9520	100.1	.1046	23.29	.9400
4.382	.668	.75	5625.00	5	99.14	-.8645	174.2	.8590	99.17	-.8326	190.9	.8890
4.382	.668	.75	5625.00	10	100.5	.4872	114.7	.9150	100.5	.4819	109.8	.8830
4.382	.668	.75	5625.00	20	100.6	.5974	77.21	.9330	100.6	.5660	74.75	.9210
4.382	.668	.75	5625.00	30	101.2	1.249	61.69	.9530	101.0	1.044	59.46	.9250
4.382	.668	.75	5625.00	50	100.3	.2818	46.93	.9480	100.1	.1239	44.84	.9240
4.382	.668	.75	5625.00	100	100.6	.5641	33.10	.9510	100.5	.5229	32.67	.9350
4.259	.833	1.00	10000.0	5	103.2	3.210	346.8	.8780	103.4	3.401	250.8	.8070
4.259	.833	1.00	10000.0	10	101.8	1.802	160.4	.9040	101.5	1.483	133.7	.8750
4.259	.833	1.00	10000.0	20	102.5	2.458	103.1	.9220	102.5	2.464	94.89	.8900
4.259	.833	1.00	10000.0	30	100.4	.3638	79.06	.9420	99.86	-.1368	72.77	.9070
4.259	.833	1.00	10000.0	50	100.5	.4824	60.55	.9420	100.3	.2580	56.78	.9040
4.259	.833	1.00	10000.0	100	99.87	-.1313	41.45	.9500	99.66	-.3434	40.50	.9270
4.135	.970	1.25	15625.0	5	100.9	.9319	455.2	.8620	100.9	.9187	267.8	.8660
4.135	.970	1.25	15625.0	10	99.29	-.7092	203.0	.9040	99.87	-.1328	156.4	.8350
4.135	.970	1.25	15625.0	20	99.39	-.6150	124.5	.9410	99.16	-.8372	106.8	.8750
4.135	.970	1.25	15625.0	30	100.1	.8311E-01	96.91	.9360	99.83	-.1690	87.52	.8810
4.135	.970	1.25	15625.0	50	101.2	1.186	74.28	.9390	101.1	1.109	69.06	.8940
4.135	.970	1.25	15625.0	100	99.76	-.2434	50.13	.9370	99.38	-.6240	48.67	.9140
4.016	1.086	1.50	22500.0	5	100.7	.7452	1813.	.8560	101.9	1.858	306.9	.8210
4.016	1.086	1.50	22500.0	10	98.50	-1.504	266.6	.8990	97.88	-2.116	168.9	.8100
4.016	1.086	1.50	22500.0	20	101.0	.9769	153.0	.9310	101.0	1.021	124.9	.8550
4.016	1.086	1.50	22500.0	30	100.1	.7970E-01	116.3	.9350	99.50	-.5008	99.93	.8660
4.016	1.086	1.50	22500.0	50	98.89	-1.112	83.75	.9480	98.73	-1.267	76.85	.8790
4.016	1.086	1.50	22500.0	100	100.3	.3037	58.57	.9510	100.5	.5309	59.16	.9060
3.800	1.269	2.00	40000.0	5	100.9	.9335	344.3	.8430	102.3	2.340	364.3	.7590
3.800	1.269	2.00	40000.0	10	99.62	-.3772	424.2	.9160	101.7	1.683	211.1	.7780
3.800	1.269	2.00	40000.0	20	100.3	.3349	202.1	.9470	100.2	.1893	146.1	.8160
3.800	1.269	2.00	40000.0	30	99.16	-.8399	148.4	.9190	99.79	-.2135	125.2	.8050
3.800	1.269	2.00	40000.0	50	100.6	.5593	109.0	.9430	100.1	.1055	95.20	.8620
3.800	1.269	2.00	40000.0	100	100.4	.4457	73.34	.9480	100.5	.4507	72.19	.8930

APPENDIX C

RESULTS FOR ESTIMATION OF THE COEFFICIENT OF VARIATION

Tables C.1 through C.7 show results for the estimation of the coefficient of variation. Listed below are the column headings used in these tables and the corresponding text notations.

Table	Text
ALPHA	α -----parameter of gamma distribution
BETA	β -----parameter of gamma distribution
MU	μ_y -----parameter of lognormal distribution
SIGMA	σ_y -----parameter of lognormal distribution
EX	EX-----expected value of x
VAR(X)	Var(x)-----variance of x
N	n-----number of observations
C-HAT	c-----estimate of coefficient of variation
REL. BIAS	$100(\hat{c}-c)/c$ -----relative bias of $\hat{c}$
AVE(CI)	-----average confidence interval width for the 1000 replications
COVERAGE	-----proportion of the 1000 relications that the confidence interval included c

TABLE C.1. SIMULATION OF THE GAMMA DISTRIBUTION WITH COEFFICIENT OF VARIATION .25.
THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS					LOGNORMAL ESTIMATOR					GAMMA ESTIMATOR					NORMAL ESTIMATOR				
ALPHA	BETA	EX	VAR(X)	N	C-HAT	REL.	BIAS	AVE(CI)	COV.	C-HAT	REL.	BIAS	AVE(CI)	COV.	C-HAT	REL.	BIAS	AVE(CI)	COV.
16.00	.063	1	.06	5	.245	-1.933	.451	.914	.212	-15.180	.507	.946	.236	-5.687	.510	.907			
16.00	.063	1	.06	10	.253	1.116	.270	.924	.232	-7.151	.272	.944	.244	-2.481	.268	.923			
16.00	.063	1	.06	20	.255	1.822	.176	.935	.240	-3.884	.170	.952	.246	-1.654	.174	.932			
16.00	.063	1	.06	30	.256	2.384	.141	.939	.244	-2.455	.134	.955	.248	-.858	.139	.954			
16.00	.063	1	.06	50	.257	2.867	.107	.926	.247	-1.356	.101	.953	.249	-.441	.104	.949			
16.00	.063	1	.06	100	.257	2.829	.075	.922	.248	-.876	.070	.952	.249	-.401	.073	.945			
16.00	.313	5	1.56	5	.245	-1.970	.449	.933	.212	-15.034	.509	.954	.237	-5.312	.511	.922			
16.00	.313	5	1.56	10	.252	.815	.269	.931	.232	-7.269	.271	.947	.244	-2.386	.269	.920			
16.00	.313	5	1.56	20	.255	2.164	.177	.947	.241	-3.522	.171	.957	.247	-1.146	.175	.948			
16.00	.313	5	1.56*	30	.255	2.046	.140	.951	.243	-2.799	.134	.952	.247	-1.306	.139	.928			
16.00	.313	5	1.56	50	.257	2.603	.107	.932	.246	-1.628	.101	.945	.248	-.788	.103	.940			
16.00	.313	5	1.56	100	.257	2.807	.075	.924	.248	-.920	.070	.956	.249	-.501	.073	.951			
16.00	.625	10	6.25	5	.249	-.460	.459	.912	.215	-13.953	.514	.940	.239	-4.306	.519	.909			
16.00	.625	10	6.25	10	.255	2.089	.272	.933	.234	-6.365	.274	.956	.245	-1.838	.270	.921			
16.00	.625	10	6.25	20	.255	1.811	.176	.935	.240	-3.902	.170	.952	.246	-1.692	.174	.932			
16.00	.625	10	6.25	30	.256	2.447	.141	.942	.244	-2.435	.134	.955	.248	-.950	.139	.951			
16.00	.625	10	6.25	50	.256	2.525	.107	.948	.246	-1.600	.101	.963	.249	-.563	.103	.952			
16.00	.625	10	6.25	100	.257	2.880	.075	.928	.248	-.852	.070	.955	.249	-.422	.073	.954			
16.00	3.125	50	156.25	5	.239	-4.455	.437	.917	.207	-17.220	.496	.940	.230	-7.959	.496	.905			
16.00	3.125	50	156.25	10	.253	1.180	.270	.931	.232	-7.224	.271	.950	.243	-2.836	.267	.925			
16.00	3.125	50	156.25	20	.253	1.232	.175	.934	.239	-4.374	.169	.950	.245	-2.019	.173	.937			
16.00	3.125	50	156.25	30	.256	2.540	.141	.930	.244	-2.370	.134	.945	.248	-.880	.139	.946			
16.00	3.125	50	156.25	50	.258	3.076	.108	.944	.247	-1.247	.101	.959	.249	-.482	.104	.945			
16.00	3.125	50	156.25	100	.257	2.867	.075	.912	.248	-.887	.070	.946	.249	-.502	.073	.949			
16.00	6.250	100	625.00	5	.242	-3.033	.444	.921	.210	-16.069	.503	.953	.233	-6.744	.503	.915			
16.00	6.250	100	625.00	10	.251	.497	.268	.938	.231	-7.565	.270	.945	.243	-2.673	.268	.923			
16.00	6.250	100	625.00	20	.254	1.785	.176	.934	.240	-3.822	.170	.946	.246	-1.404	.174	.932			
16.00	6.250	100	625.00	30	.256	2.404	.141	.932	.244	-2.571	.134	.939	.247	-1.248	.139	.936			
16.00	6.250	100	625.00	50	.259	3.403	.108	.927	.248	-.960	.102	.947	.250	-.180	.104	.948			
16.00	6.250	100	625.00	100	.257	2.781	.075	.935	.248	-.947	.070	.952	.249	-.526	.073	.942			

TABLE C.2. SIMULATION OF THE GAMMA DISTRIBUTION WITH COEFFICIENT OF VARIATION .50.
THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS					LOGNORMAL ESTIMATOR					GAMMA ESTIMATOR					NORMAL ESTIMATOR				
ALPHA	BETA	EX	VAR(X)	N	C-HAT	REL.	BIAS	AVE(CI)	COV.	C-HAT	REL.	BIAS	AVE(CI)	COV.	C-HAT	REL.	BIAS	AVE(CI)	COV.
4.00	.250	1	.25	5	.530	6.058	1.765	.910		.413	-17.394	.809	.952		.453	-9.339	1.140	.851	
4.00	.250	1	.25	10	.566	17.111	.764	.899		.465	-6.946	.512	.946		.483	-3.406	.618	.845	
4.00	.250	1	.25	20	.565	17.021	.456	.892		.479	-4.135	.325	.950		.485	-2.943	.396	.842	
4.00	.250	1	.25	30	.569	13.007	.358	.834		.488	-2.410	.258	.955		.493	-1.303	.321	.850	
4.00	.250	1	.25	50	.573	14.547	.271	.760		.495	-1.042	.196	.953		.497	-.511	.240	.958	
4.00	.250	1	.25	100	.570	10.039	.186	.594		.496	-.897	.135	.948		.497	-.614	.169	.950	
4.00	1.250	5	6.25	5	.560	12.035	1.920	.885		.429	-14.187	.915	.928		.468	-6.386	1.194	.918	
4.00	1.250	5	6.25	10	.565	13.080	.760	.893		.467	-6.667	.514	.955		.485	-2.903	.622	.831	
4.00	1.250	5	6.25	20	.567	13.443	.459	.869		.481	-3.805	.326	.951		.487	-2.616	.398	.940	
4.00	1.250	5	6.25	30	.568	13.506	.358	.836		.487	-2.606	.258	.948		.493	-1.445	.320	.956	
4.00	1.250	5	6.25	50	.568	13.518	.268	.767		.491	-1.869	.195	.948		.493	-1.326	.237	.949	
4.00	1.250	5	6.25	100	.575	15.049	.188	.587		.498	-.392	.136	.944		.498	-.395	.169	.960	
4.00	2.500	10	25.00	5	.560	12.089	1.925	.892		.428	-14.353	.914	.940		.465	-7.089	1.177	.918	
4.00	2.500	10	25.00	10	.573	14.637	.780	.878		.469	-6.289	.515	.945		.483	-3.477	.617	.831	
4.00	2.500	10	25.00	20	.569	13.806	.461	.851		.483	-3.483	.327	.954		.491	-1.868	.402	.946	
4.00	2.500	10	25.00	30	.566	13.271	.357	.831		.486	-2.709	.258	.942		.493	-1.457	.321	.942	
4.00	2.500	10	25.00	50	.568	13.609	.268	.769		.491	-1.704	.195	.956		.495	-1.019	.230	.957	
4.00	2.500	10	25.00	100	.575	15.031	.188	.587		.498	-.320	.136	.940		.499	-.217	.170	.961	
4.00	12.50	50	625.00	5	.543	8.524	1.794	.915		.420	-16.081	.901	.953		.459	-8.252	1.159	.931	
4.00	12.50	50	625.00	10	.558	11.581	.752	.876		.458	-8.362	.505	.939		.473	-5.473	.602	.813	
4.00	12.50	50	625.00	20	.567	13.407	.458	.863		.482	-3.622	.327	.951		.490	-1.942	.401	.942	
4.00	12.50	50	625.00	30	.568	13.567	.358	.834		.487	-2.600	.258	.948		.492	-1.630	.320	.948	
4.00	12.50	50	625.00	50	.568	13.627	.268	.772		.492	-1.659	.195	.957		.496	-.893	.238	.956	
4.00	12.50	50	625.00	100	.572	14.414	.187	.595		.496	-.805	.136	.955		.496	-.741	.169	.964	
4.00	25.00	100	2500.00	5	.567	13.418	3.704	.899		.430	-13.987	.917	.944		.467	-6.695	1.184	.928	
4.00	25.00	100	2500.00	10	.566	13.227	.765	.894		.465	-6.940	.512	.954		.482	-3.609	.616	.937	
4.00	25.00	100	2500.00	20	.566	13.141	.457	.854		.481	-3.827	.326	.953		.488	-2.373	.399	.944	
4.00	25.00	100	2500.00	30	.569	13.864	.359	.831		.488	-2.368	.258	.959		.493	-1.393	.321	.956	
4.00	25.00	100	2500.00	50	.566	13.291	.267	.772		.490	-2.013	.194	.948		.493	-1.375	.237	.951	
4.00	25.00	100	2500.00	100	.573	14.655	.187	.583		.498	-.496	.136	.952		.498	-.305	.170	.967	

TABLE C.3. SIMULATION OF THE GAMMA DISTRIBUTION WITH COEFFICIENT OF VARIATION .75.
THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS					LOGNORMAL ESTIMATOR					GAMMA ESTIMATOR					NORMAL ESTIMATOR				
ALPHA	BETA	EX	VAR(X)	N	C-HAT	REL.	BIAS	AVE(CI)	COV.	C-HAT	REL.	BIAS	AVE(CI)	COV.	C-HAT	REL.	BIAS	AVE(CI)	COV.
1.78	.563	1	.56	5	1.115	48.691	1282.098	.646		.646	-13.832	1.237	.962		.681	-9.176	2.037	.930	
1.78	.563	1	.56	10	1.077	43.553	3.128	.756		.702	-6.347	.717	.953		.720	-4.021	1.101	.940	
1.78	.563	1	.56	20	1.067	42.232	1.235	.619		.728	-2.999	.465	.954		.732	-2.447	.716	.946	
1.78	.563	1	.56	30	1.061	41.446	.906	.518		.733	-2.220	.368	.947		.734	-2.079	.568	.955	
1.78	.563	1	.56	50	1.070	42.675	.661	.309		.743	-.981	.280	.937		.741	-1.230	.425	.958	
1.78	.563	1	.56	100	1.055	40.658	.434	.095		.745	-.729	.194	.956		.745	-.690	.301	.959	
1.78	2.813	5	14.06	5	1.121	49.490	506.079	.824		.635	-15.354	1.219	.938		.670	-10.647	1.998	.925	
1.78	2.813	5	14.06	10	1.050	39.974	2.823	.786		.689	-8.183	.706	.955		.703	-6.242	1.062	.946	
1.78	2.813	5	14.06	20	1.046	39.491	1.196	.650		.719	-4.147	.460	.949		.721	-3.805	.700	.947	
1.78	2.813	5	14.06	30	1.055	40.674	.900	.526		.732	-2.443	.367	.945		.738	-1.653	.572	.956	
1.78	2.813	5	14.06	50	1.064	41.815	.655	.323		.740	-1.365	.279	.953		.736	-1.917	.420	.964	
1.78	2.813	5	14.06	100	1.054	40.592	.434	.099		.744	-.740	.194	.949		.745	-.726	.301	.971	
1.78	5.625	10	56.25	5	1.353	80.345	22396E8	.846		.653	-12.976	1.247	.958		.684	-8.842	2.041	.949	
1.78	5.625	10	56.25	10	1.079	43.852	3.444	.780		.698	-6.908	.713	.962		.711	-5.155	1.080	.948	
1.78	5.625	10	56.25	20	1.069	42.515	1.255	.655		.728	-2.976	.465	.959		.735	-1.953	.720	.961	
1.78	5.625	10	56.25	30	1.062	41.563	.909	.507		.733	-2.264	.368	.954		.736	-1.835	.571	.957	
1.78	5.625	10	56.25	50	1.065	42.001	.655	.321		.743	-.982	.280	.951		.745	-.720	.428	.964	
1.78	5.625	10	56.25	100	1.051	40.170	.433	.106		.744	-.856	.194	.950		.746	-.597	.302	.961	
1.78	28.13	50	1406.25	5	1.110	48.020	303.478	.851		.642	-14.416	1.233	.953		.675	-9.988	2.005	.946	
1.78	28.13	50	1406.25	10	1.101	46.819	3.462	.765		.704	-6.177	.718	.957		.713	-4.915	1.081	.936	
1.78	28.13	50	1406.25	20	1.061	41.496	1.233	.622		.725	-3.377	.463	.956		.730	-2.698	.711	.956	
1.78	28.13	50	1406.25	30	1.050	39.986	.887	.528		.732	-2.383	.368	.948		.738	-1.562	.573	.973	
1.78	28.13	50	1406.25	50	1.052	40.246	.644	.347		.739	-1.476	.279	.944		.743	-.924	.427	.961	
1.78	28.13	50	1406.25	100	1.055	40.630	.434	.097		.744	-.838	.194	.953		.744	-.810	.301	.964	
1.78	56.25	100	5625.00	5	1.346	79.516	34421E8	.821		.636	-15.257	1.220	.945		.660	-11.978	1.949	.927	
1.78	56.25	100	5625.00	10	1.082	44.238	2.961	.788		.700	-6.720	.715	.955		.707	-5.769	1.066	.944	
1.78	56.25	100	5625.00	20	1.069	42.558	1.255	.613		.726	-3.205	.464	.947		.731	-2.580	.714	.942	
1.78	56.25	100	5625.00	30	1.062	41.588	.905	.513		.734	-2.164	.368	.949		.734	-2.176	.567	.957	
1.78	56.25	100	5625.00	50	1.054	40.587	.645	.348		.737	-1.698	.278	.950		.737	-1.755	.421	.960	
1.78	56.25	100	5625.00	100	1.058	41.084	.436	.085		.745	-.607	.194	.945		.744	-.737	.301	.965	

TABLE C.4. SIMULATION OF THE GAMMA DISTRIBUTION WITH COEFFICIENT OF VARIATION 1.00.
THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS					LOGNORMAL ESTIMATOR					GAMMA ESTIMATOR					NORMAL ESTIMATOR				
ALPHA	BETA	EX	VAR(X)	N	C-HAT	REL.	BIAS	AVE(CI)	COV.	C-HAT	REL.	BIAS	AVE(CI)	COV.	C-HAT	REL.	BIAS	AVE(CI)	COV.
1.00	1.000	1	1.00	5	10.128	912.837	.2624E16	.740	.866	-13.414	1.505	.966	.866	-13.363	2.961	.947			
1.00	1.000	1	1.00	10	2.710	171.000	1071.108	.587	.932	-6.841	.862	.949	.925	-7.468	1.651	.938			
1.00	1.000	1	1.00	20	2.287	128.654	12.534	.371	.960	-4.038	.576	.958	.948	-5.151	1.082	.957			
1.00	1.000	1	1.00	30	2.152	115.245	3.552	.206	.975	-2.496	.461	.952	.968	-3.190	.887	.963			
1.00	1.000	1	1.00	50	2.140	113.967	2.213	.053	.990	-1.013	.352	.957	.982	-1.814	.667	.964			
1.00	1.000	1	1.00	100	2.077	107.683	1.308	.003	.994	-.622	.245	.953	.995	-.543	.479	.974			
1.00	5.000	5	25.00	5	43.594	4259.353	.6534E19	.726	.869	-13.081	1.508	.954	.870	-12.956	2.992	.942			
1.00	5.000	5	25.00	10	2.781	178.104	3227.651	.574	.931	-6.881	.883	.960	.925	-7.543	1.637	.950			
1.00	5.000	5	25.00	20	2.259	125.903	11.252	.329	.968	-3.209	.579	.956	.960	-4.005	1.108	.957			
1.00	5.000	5	25.00	30	2.152	115.190	3.567	.196	.977	-2.339	.461	.954	.970	-2.977	.889	.955			
1.00	5.000	5	25.00	50	2.101	110.118	2.104	.060	.987	-1.326	.351	.949	.981	-1.891	.667	.965			
1.00	5.000	5	25.00	100	2.049	104.941	1.279	.003	.991	-.939	.244	.955	.991	-.861	.477	.977			
1.00	10.00	10	100.00	5	7.346	634.571	.1070E14	.745	.849	-15.074	1.482	.951	.864	-13.578	2.973	.939			
1.00	10.00	10	100.00	10	3.011	201.073	2121.956	.584	.941	-5.920	.889	.959	.932	-6.799	1.663	.948			
1.00	10.00	10	100.00	20	2.386	138.589	22.431	.354	.965	-3.501	.718	.955	.956	-4.351	1.101	.949			
1.00	10.00	10	100.00	30	2.194	119.390	3.709	.183	.981	-1.879	.463	.953	.978	-2.154	.902	.968			
1.00	10.00	10	100.00	50	2.069	106.870	2.050	.053	.984	-1.583	.350	.963	.982	-1.808	.668	.966			
1.00	10.00	10	100.00	100	2.100	109.968	1.329	.002	.997	-.287	.246	.968	.995	-.523	.479	.981			
1.00	50.00	50	2500.00	5	4.257	325.724	.1269E12	.771	.845	-15.481	1.479	.952	.862	-13.842	2.963	.942			
1.00	50.00	50	2500.00	10	2.742	174.246	2678.747	.608	.923	-7.722	.877	.959	.917	-8.336	1.618	.951			
1.00	50.00	50	2500.00	20	2.309	130.882	8.401	.365	.964	-3.591	.578	.947	.956	-4.443	1.099	.961			
1.00	50.00	50	2500.00	30	2.118	111.825	3.375	.192	.976	-2.394	.461	.947	.973	-2.671	.894	.966			
1.00	50.00	50	2500.00	50	2.143	114.251	2.197	.053	.990	-.998	.352	.957	.982	-1.832	.667	.972			
1.00	50.00	50	2500.00	100	2.073	107.326	1.302	.002	.994	-.617	.245	.952	.991	-.873	.477	.983			
1.00	100.0	100	10000.0	5	3.614	261.374	.54257E9	.759	.852	-14.847	1.488	.957	.856	-14.363	2.922	.936			
1.00	100.0	100	10000.0	10	2.911	191.101	20674.74	.604	.934	-6.589	.885	.964	.928	-7.219	1.655	.952			
1.00	100.0	100	10000.0	20	2.410	141.019	10.481	.316	.976	-2.377	.583	.959	.965	-3.470	1.117	.962			
1.00	100.0	100	10000.0	30	2.103	110.325	3.321	.188	.974	-2.556	.461	.955	.971	-2.905	.891	.968			
1.00	100.0	100	10000.0	50	2.109	110.874	2.112	.061	.987	-1.252	.351	.965	.981	-1.935	.667	.964			
1.00	100.0	100	10000.0	100	2.085	108.549	1.320	.004	.993	-.665	.245	.954	.990	-.986	.476	.974			

TABLE C.5. SIMULATION OF THE GAMMA DISTRIBUTION WITH COEFFICIENT OF VARIATION 1.25.
THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS					LOGNORMAL ESTIMATOR					GAMMA ESTIMATOR					NORMAL ESTIMATOR				
ALPHA	BETA	EX	VAR(X)	N	C-HAT	REL.	BIAS	AVE(C1)	COV.	C-HAT	REL.	BIAS	AVE(C1)	COV.	C-HAT	REL.	BIAS	AVE(C1)	COV.
.64	1.563	1	1.56	5	.10193E9	.8154E10	.9740E50	.637		1.070	-13.725	1.722	.953		1.030	-16.958	4.015	.963	
.64	1.563	1	1.56	10	.19050E9	.1524E11	.1972E29	.390		1.154	-7.713	1.019	.948		1.099	-12.090	2.206	.931	
.64	1.563	1	1.56	20	11.640	831.212	5883.499	.134		1.211	-3.090	.678	.966		1.182	-5.413	1.584	.962	
.64	1.563	1	1.56	30	7.271	481.702	117.958	.045		1.219	-2.488	.540	.956		1.199	-4.044	1.282	.935	
.64	1.563	1	1.56	50	5.936	374.854	15.231	.002		1.236	-1.136	.413	.958		1.226	-1.956	.978	.973	
.64	1.563	1	1.56	100	5.583	346.644	6.975	0.000		1.246	-.299	.289	.952		1.234	-1.280	.694	.978	
.64	7.813	5	39.06	5	52582.39	4206491.	.6569E34	.619		1.087	-13.054	1.733	.958		1.042	-16.630	4.024	.959	
.64	7.813	5	39.06	10	78.755	6200.417	.1170E11	.365		1.179	-5.662	1.036	.968		1.119	-10.582	2.262	.958	
.64	7.813	5	39.06	20	40.297	3123.764	752291.2	.117		1.218	-2.591	.681	.958		1.189	-4.843	1.602	.963	
.64	7.813	5	39.06	30	7.552	504.171	127.626	.040		1.223	-2.123	.542	.968		1.189	-4.892	1.260	.964	
.64	7.813	5	39.06	50	6.017	381.366	15.463	.003		1.237	-1.079	.413	.957		1.219	-2.520	.970	.960	
.64	7.813	5	39.06	100	5.238	319.073	6.261	0.000		1.237	-1.023	.287	.954		1.235	-1.191	.695	.983	
.64	15.63	10	156.25	5	44292.23	3543279.	.6105E34	.618		1.081	-13.540	1.725	.949		1.043	-16.573	4.064	.939	
.64	15.63	10	156.25	10	152.346	12087.68	.2955E11	.404		1.167	-6.656	1.027	.959		1.122	-10.239	2.205	.919	
.64	15.63	10	156.25	20	9.705	676.374	884.398	.139		1.210	-3.189	.678	.952		1.176	-5.896	1.572	.959	
.64	15.63	10	156.25	30	10.469	737.533	1415.590	.035		1.226	-1.928	.542	.957		1.210	-3.208	1.301	.971	
.64	15.63	10	156.25	50	5.990	379.237	15.265	.005		1.241	-.750	.414	.962		1.220	-2.418	.968	.975	
.64	15.63	10	156.25	100	5.518	341.467	6.814	0.000		1.245	-.380	.289	.961		1.234	-1.243	.695	.975	
.64	78.13	50	3906.25	5	.36099E8	.2888E10	.6552E48	.600		1.095	-12.387	1.741	.957		1.045	-16.372	4.062	.955	
.64	78.13	50	3906.25	10	9360.119	748709.5	.2173E17	.389		1.163	-6.950	1.025	.957		1.132	-9.445	2.336	.948	
.64	78.13	50	3906.25	20	9.748	679.860	941.776	.144		1.208	-3.347	.677	.951		1.178	-5.767	1.573	.963	
.64	78.13	50	3906.25	30	6.546	423.697	39.859	.056		1.220	-2.399	.541	.957		1.192	-4.626	1.268	.953	
.64	78.13	50	3906.25	50	6.061	384.881	16.162	.007		1.230	-1.602	.412	.955		1.210	-3.177	.956	.974	
.64	78.13	50	3906.25	100	5.578	346.267	7.064	0.000		1.243	-.520	.288	.952		1.235	-1.199	.696	.975	
.64	156.3	100	15625.0	5	308343.7	.24667E8	.5405E38	.620		1.071	-14.304	1.715	.954		1.025	-18.008	3.935	.944	
.64	156.3	100	15625.0	10	347.988	27739.07	.1693E13	.377		1.154	-7.697	1.020	.957		1.109	-11.275	2.244	.941	
.64	156.3	100	15625.0	20	9.810	684.827	1248.707	.164		1.204	-3.662	.675	.951		1.176	-5.909	1.572	.947	
.64	156.3	100	15625.0	30	7.880	530.362	129.105	.038		1.232	-1.433	.544	.963		1.216	-2.739	1.311	.977	
.64	156.3	100	15625.0	50	6.289	403.107	18.443	.002		1.240	-.822	.414	.961		1.221	-2.305	.973	.970	
.64	156.3	100	15625.0	100	5.474	337.925	6.805	0.000		1.242	-.661	.288	.943		1.229	-1.660	.690	.974	

TABLE C.6. SIMULATION OF THE GAMMA DISTRIBUTION WITH COEFFICIENT OF VARIATION 1.50.
THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS				LOGNORMAL ESTIMATOR								GAMMA ESTIMATOR					NORMAL ESTIMATOR				
ALPHA	BETA	EX	VAR(X)	N	C-HAT	REL.	BIAS	AVE(CI)	COV.	C-HAT	REL.	BIAS	AVE(CI)	COV.	C-HAT	REL.	BIAS	AVE(CI)	COV.		
.44	2.250	1	2.25	5	.4546E11	.3031E13	.2709E62	.488		1.313	-12.465	1.939	.968		1.192	-20.502	5.074	.956			
.44	2.250	1	2.25	10	.78708E9	.5247E10	.1697E28	.222		1.402	-6.509	1.153	.962		1.301	-13.249	2.975	.946			
.44	2.250	1	2.25	20	6586.585	439035.6	.1684E11	.035		1.458	-2.816	.764	.963		1.381	-7.922	2.093	.971			
.44	2.250	1	2.25	30	165.380	10925.34	95219.46	.002		1.475	-1.650	.612	.968		1.437	-4.211	1.779	.966			
.44	2.250	1	2.25	50	52.340	3389.333	2053.630	0.000		1.480	-1.316	.465	.950		1.445	-3.691	1.316	.965			
.44	2.250	1	2.25	100	30.749	1949.953	106.222	0.000		1.496	-.275	.326	.946		1.474	-1.723	.955	.905			
.44	11.25	5	56.25	5	.1040E13	.6935E14	.1939E70	.470		1.326	-11.617	1.950	.966		1.212	-19.191	5.229	.955			
.44	11.25	5	56.25	10	1535182.	.10235E9	.2203E23	.229		1.402	-6.538	1.151	.950		1.298	-13.493	2.972	.939			
.44	11.25	5	56.25	20	646.091	42972.75	.47843E8	.037		1.450	-3.348	.761	.959		1.384	-7.721	2.102	.955			
.44	11.25	5	56.25	30	154.042	10169.44	209691.0	.005		1.474	-1.710	.612	.965		1.417	-5.550	1.726	.973			
.44	11.25	5	56.25	50	50.642	3276.113	1314.915	0.000		1.483	-1.154	.465	.959		1.453	-3.154	1.328	.972			
.44	11.25	5	56.25	100	30.222	1914.820	105.844	0.000		1.490	-.639	.325	.942		1.463	-2.460	.943	.975			
.44	22.50	10	225.00	5	.1356E12	.9042E13	.6451E65	.456		1.310	-12.673	.936	.955		1.184	-21.075	5.005	.943			
.44	22.50	10	225.00	10	6189669.	.41131E9	.1425E25	.234		1.395	-6.990	1.150	.964		1.308	-12.832	3.002	.950			
.44	22.50	10	225.00	20	1041.419	69327.94	.2513E9	.034		1.464	-2.409	.766	.966		1.402	-6.511	2.164	.956			
.44	22.50	10	225.00	30	15273.27	1018118.	.2471E10	.008		1.476	-1.571	.612	.947		1.430	-4.678	1.766	.966			
.44	22.50	10	225.00	50	55.328	3588.539	1859.463	0.000		1.478	-1.474	.464	.963		1.439	-4.038	1.305	.967			
.44	22.50	10	225.00	100	29.572	1871.483	100.405	0.000		1.495	-.327	.326	.945		1.476	-1.596	.959	.983			
.44	112.5	50	5625.00	5	.34590E8	.2306E10	.1186E48	.467		1.324	-11.752	1.949	.967		1.211	-19.295	5.203	.964			
.44	112.5	50	5625.00	10	.1339E11	.8926E12	.8397E33	.200		1.439	-4.056	1.171	.960		1.332	-11.219	3.105	.953			
.44	112.5	50	5625.00	20	7425.480	494932.0	.1807E11	.032		1.459	-2.752	.764	.959		1.395	-6.973	2.135	.955			
.44	112.5	50	5625.00	30	5977.759	398417.3	.35387E9	.010		1.465	-2.342	.609	.957		1.414	-5.763	1.722	.959			
.44	112.5	50	5625.00	50	68.372	4458.134	3531.025	0.000		1.493	-.442	.467	.960		1.460	-2.695	1.342	.969			
.44	112.5	50	5625.00	100	33.469	2131.235	143.501	0.000		1.492	-.554	.325	.948		1.470	-2.029	.951	.976			
.44	225.0	100	22500.0	5	.3374E21	.2249E23	.181E111	.486		1.308	-12.809	1.938	.977		1.188	-20.824	5.028	.963			
.44	225.0	100	22500.0	10	.1761E10	.1174E12	.9426E31	.194		1.424	-5.063	1.164	.964		1.319	-12.050	3.051	.948			
.44	225.0	100	22500.0	20	345794.4	.23053E8	.5254E14	.032		1.463	-2.468	.766	.958		1.387	-7.527	2.108	.962			
.44	225.0	100	22500.0	30	1379.997	91899.82	.34657E8	.006		1.469	-2.075	.610	.955		1.417	-5.522	1.727	.972			
.44	225.0	100	22500.0	50	93.792	6152.828	9180.315	0.000		1.483	-1.138	.465	.962		1.447	-3.538	1.318	.975			
.44	225.0	100	22500.0	100	30.630	1942.015	122.576	0.000		1.487	-.870	.324	.939		1.461	-2.632	.940	.980			

TABLE C.7. SIMULATION OF THE GAMMA DISTRIBUTION WITH COEFFICIENT OF VARIATION 2.00.
THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS				LOGNORMAL ESTIMATOR					GAMMA ESTIMATOR					NORMAL ESTIMATOR					
ALPHA	BETA	EX	VAR(X)	N	C-HAT	REL.	BIAS	AVE(CI)	COV.	C-HAT	REL.	BIAS	AVE(CI)	COV.	C-HAT	REL.	BIAS	AVE(CI)	COV.
.25	4.000	1	4.00	5	.2723E73	.1362E75	.147E159	.239		1.784	-10.793	2.305	.968		1.443	-27.855	7.043	.954	
.25	4.000	1	4.00	10	.2925E32	.1463E34	.4194E93	.062		1.873	-6.356	1.363	.970		1.628	-18.620	4.466	.959	
.25	4.000	1	4.00	20	.4141E14	.2070E16	.3504E30	.001		1.959	-2.060	.908	.954		1.791	-10.447	3.401	.964	
.25	4.000	1	4.00	30	.2972E14	.1486E16	.1958E26	0.000		1.967	-1.659	.724	.940		1.838	-8.081	2.816	.960	
.25	4.000	1	4.00	50	.74892E9	.3745E11	.9617E15	0.000		1.983	-.865	.552	.928		1.897	-5.129	2.187	.973	
.25	4.000	1	4.00	100	.18567E8	.92835E9	.4601E11	0.000		1.990	-.490	.386	.845		1.942	-2.904	1.601	.984	
.25	20.00	5	100.00	5	.5764E51	.2882E53	.293E159	.273		1.761	-11.946	2.288	.971		1.432	-28.413	6.935	.958	
.25	20.00	5	100.00	10	.7154E31	.3577E33	.8434E91	.044		1.893	-5.375	1.371	.974		1.653	-17.368	4.566	.962	
.25	20.00	5	100.00	20	.1177E12	.5886E13	.3559E25	.001		1.956	-2.204	.907	.954		1.786	-10.706	3.373	.971	
.25	20.00	5	100.00	30	.9327E11	.4663E13	.7992E21	0.000		1.963	-1.842	.724	.943		1.857	-7.126	2.872	.976	
.25	20.00	5	100.00	50	.11781E9	.5891E10	.3617E14	0.000		1.980	-.982	.552	.911		1.903	-4.843	2.210	.968	
.25	20.00	5	100.00	100	.5467356	.27337E9	.9156E10	0.000		1.990	-.500	.386	.841		1.952	-2.420	1.618	.981	
.25	40.00	10	400.00	5	.1200E39	.6000E40	.244E159	.263		1.769	-11.546	2.294	.976		1.436	-28.198	6.995	.960	
.25	40.00	10	400.00	10	.5460E33	.2730E35	.1410E97	.060		1.893	-5.363	1.370	.975		1.622	-18.876	4.436	.948	
.25	40.00	10	400.00	20	.3604E14	.1802E16	.4590E30	.002		1.938	-3.092	.903	.956		1.776	-11.216	3.343	.962	
.25	40.00	10	400.00	30	.7871E11	.3935E13	.6084E21	0.000		1.965	-1.736	.724	.938		1.845	-7.746	2.841	.957	
.25	40.00	10	400.00	50	.77150E8	.3857E10	.1700E14	0.000		1.981	-.940	.552	.924		1.885	-5.736	2.166	.966	
.25	40.00	10	400.00	100	.6652707	.33264E9	.1226E11	0.000		1.990	-.492	.386	.844		1.939	-3.067	1.595	.981	
.25	200.0	50	10000.0	5	.7527E48	.3763E50	.977E158	.283		1.735	-13.259	2.269	.976		1.418	-29.093	6.826	.956	
.25	200.0	50	10000.0	10	.1000E19	.5000E20	.1628E56	.051		1.887	-5.652	1.369	.962		1.638	-18.105	4.513	.951	
.25	200.0	50	10000.0	20	.3411E14	.1705E16	.1834E30	.002		1.946	-2.704	.905	.961		1.785	-10.772	3.360	.969	
.25	200.0	50	10000.0	30	.2072E13	.1036E15	.2606E24	0.000		1.961	-1.938	.723	.930		1.843	-7.867	2.837	.972	
.25	200.0	50	10000.0	50	.1894E10	.9472E11	.3841E16	0.000		1.973	-1.346	.551	.909		1.908	-4.607	2.216	.973	
.25	200.0	50	10000.0	100	.319199.9	.15960E8	.11901E9	0.000		1.987	-.675	.385	.828		1.939	-3.056	1.596	.980	
.25	400.0	100	40000.0	5	.2835E33	.1418E35	.489E158	.274		1.772	-11.398	2.296	.974		1.432	-28.420	6.945	.957	
.25	400.0	100	40000.0	10	.2025E24	.1013E26	.5783E70	.055		1.883	-5.862	1.367	.967		1.616	-19.178	4.414	.942	
.25	400.0	100	40000.0	20	.4826E16	.2413E18	.1021E35	.002		1.951	-2.428	.906	.966		1.770	-11.503	3.323	.957	
.25	400.0	100	40000.0	30	.2059E15	.1029E17	.8090E27	0.000		1.966	-1.678	.724	.925		1.842	-7.918	2.926	.965	
.25	400.0	100	40000.0	50	.1036E13	.5179E14	.5551E20	0.000		1.987	-.667	.553	.925		1.895	-5.242	2.188	.978	
.25	400.0	100	40000.0	100	.500804.1	.25040E8	.24236E9	0.000		1.987	-.647	.385	.849		1.936	-3.186	1.591	.980	

TABLE C.8. SIMULATION OF THE LOGNORMAL DISTRIBUTION WITH COEFFICIENT OF VARIATION .25.
THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS					LOGNORMAL ESTIMATOR					GAMMA ESTIMATOR					NORMAL ESTIMATOR				
MU	SIGMA	EX	VAR(X)	N	C-HAT	REL.	BIAS	AVE(CI)	COV.	C-HAT	REL.	BIAS	AVE(CI)	COV.	C-HAT	REL.	BIAS	AVE(CI)	COV.
-.030	.246	1	.06	5	.236	-5.404	.430	.930		.206	-17.816	.495	.954		.231	-7.708	.498	.918	
-.030	.246	1	.06	10	.243	-2.648	.258	.943		.225	-9.840	.264	.953		.240	-4.184	.263	.918	
-.030	.246	1	.06	20	.247	-1.111	.171	.955		.236	-5.649	.167	.961		.246	-1.693	.174	.922	
-.030	.246	1	.06	30	.249	-.295	.137	.959		.240	-4.116	.132	.956		.248	-.869	.139	.947	
-.030	.246	1	.06	50	.250	-.171	.104	.929		.242	-3.257	.099	.928		.249	-.568	.104	.922	
-.030	.246	1	.06	100	.250	-.118	.072	.949		.243	-2.867	.069	.941		.249	-.353	.073	.934	
1.579	.246	5	1.56	5	.233	-6.878	.423	.925		.203	-18.896	.487	.954		.227	-9.154	.489	.908	
1.579	.246	5	1.56	10	.242	-3.251	.257	.930		.224	-10.369	.263	.942		.238	-4.614	.262	.910	
1.579	.246	5	1.56	20	.246	-1.632	.170	.943		.234	-6.271	.166	.952		.243	-2.621	.172	.913	
1.579	.246	5	1.56	30	.248	-.638	.136	.953		.239	-4.281	.132	.952		.248	-.855	.139	.942	
1.579	.246	5	1.56	50	.248	-.604	.103	.957		.241	-3.632	.099	.960		.248	-.940	.103	.944	
1.579	.246	5	1.56	100	.249	-.277	.072	.946		.243	-2.794	.069	.941		.249	-.463	.073	.925	
2.272	.246	10	6.25	5	.235	-6.139	.428	.926		.204	-18.281	.490	.962		.229	-8.392	.494	.916	
2.272	.246	10	6.25	10	.245	-1.933	.260	.950		.227	-9.170	.266	.961		.241	-3.413	.266	.910	
2.272	.246	10	6.25	20	.246	-1.491	.170	.950		.235	-6.004	.156	.952		.245	-2.055	.173	.922	
2.272	.246	10	6.25	30	.249	-.500	.137	.937		.239	-4.280	.132	.943		.247	-1.147	.139	.928	
2.272	.246	10	6.25	50	.249	-.452	.104	.951		.241	-3.482	.099	.947		.248	-.713	.103	.924	
2.272	.246	10	6.25	100	.250	-.005	.073	.962		.244	-2.577	.069	.957		.249	-.311	.073	.943	
3.882	.246	50	156.25	5	.239	-4.465	.436	.920		.208	-16.875	.498	.940		.233	-6.816	.504	.907	
3.882	.246	50	156.25	10	.241	-3.636	.255	.939		.223	-10.693	.262	.956		.237	-5.033	.261	.896	
3.882	.246	50	156.25	20	.248	-.997	.171	.948		.236	-5.664	.167	.958		.245	-1.966	.173	.936	
3.882	.246	50	156.25	30	.249	-.566	.136	.947		.239	-4.342	.132	.951		.247	-1.192	.139	.940	
3.882	.246	50	156.25	50	.249	-.485	.104	.949		.241	-3.526	.099	.940		.248	-.795	.103	.922	
3.882	.246	50	156.25	100	.250	-.077	.073	.941		.244	-2.514	.069	.937		.249	-.225	.073	.932	
4.575	.246	100	625.00	5	.235	-6.131	.427	.920		.204	-18.229	.491	.953		.229	-8.270	.495	.899	
4.575	.246	100	625.00	10	.244	-2.583	.258	.949		.225	-9.816	.264	.959		.239	-4.303	.263	.914	
4.575	.246	100	625.00	20	.245	-1.876	.169	.955		.234	-5.519	.165	.963		.243	-2.939	.171	.922	
4.575	.246	100	625.00	30	.247	-1.261	.135	.946		.237	-5.029	.131	.945		.245	-1.996	.138	.915	
4.575	.246	100	625.00	50	.247	-1.145	.103	.956		.240	-4.151	.098	.953		.246	-1.498	.102	.926	
4.575	.246	100	625.00	100	.250	-.005	.073	.938		.244	-2.533	.069	.934		.250	-.179	.073	.924	

TABLE C.9. SIMULATION OF THE LOGNORMAL DISTRIBUTION WITH COEFFICIENT OF VARIATION .50.
THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS					LOGNORMAL ESTIMATOR					GAMMA ESTIMATOR					NORMAL ESTIMATOR				
MU	SIGMA	EX	VAR(X)	N	C-HAT	REL.	BIAS	AVE(CI)	COV.	C-HAT	REL.	BIAS	AVE(CI)	COV.	C-HAT	REL.	BIAS	AVE(CI)	COV.
-.112	.472	1	.25	5	.476	-4.805	1.267	.912	.394	-23.147	.639	.952	.432	-13.622	1.078	.930			
-.112	.472	1	.25	10	.491	-1.787	.612	.945	.427	-14.563	.475	.951	.465	-6.968	.593	.895			
-.112	.472	1	.25	20	.497	-.532	.395	.936	.446	-10.795	.304	.932	.479	-4.262	.391	.905			
-.112	.472	1	.25	30	.483	-1.321	.299	.946	.449	-10.140	.240	.912	.484	-3.254	.314	.895			
-.112	.472	1	.25	50	.498	-.478	.227	.949	.456	-8.842	.182	.895	.489	-2.220	.235	.915			
-.112	.472	1	.25	100	.499	-.197	.158	.944	.461	-7.864	.127	.823	.496	-.712	.169	.899			
1.498	.472	5	6.25	5	.469	-6.113	1.184	.921	.381	-23.789	.635	.958	.428	-14.412	1.059	.890			
1.498	.472	5	6.25	10	.494	-1.112	.616	.955	.429	-14.114	.478	.956	.466	-6.765	.594	.889			
1.498	.472	5	6.25	20	.492	-1.647	.379	.942	.442	-11.621	.302	.928	.475	-5.035	.387	.893			
1.498	.472	5	6.25	30	.498	-.401	.302	.936	.453	-9.401	.241	.914	.489	-2.206	.319	.897			
1.498	.472	5	6.25	50	.499	-.135	.228	.947	.458	-8.457	.182	.889	.493	-1.434	.238	.887			
1.498	.472	5	6.25	100	.497	-.512	.157	.945	.459	-8.205	.126	.795	.493	-1.380	.167	.914			
2.191	.472	10	25.00	5	.482	-3.563	1.266	.926	.390	-22.081	.649	.958	.439	-12.296	1.095	.897			
2.191	.472	10	25.00	10	.485	-2.993	.602	.935	.423	-15.352	.472	.954	.461	-7.700	.585	.880			
2.191	.472	10	25.00	20	.495	-.911	.383	.947	.446	-10.773	.304	.922	.483	-3.407	.396	.888			
2.191	.472	10	25.00	30	.493	-1.372	.298	.951	.449	-10.200	.239	.924	.483	-3.422	.313	.910			
2.191	.472	10	25.00	50	.498	-.359	.227	.947	.457	-8.574	.182	.853	.493	-1.331	.238	.867			
2.191	.472	10	25.00	100	.499	-.238	.157	.937	.480	-7.942	.126	.821	.495	-.981	.160	.898			
3.800	.472	50	625.00	5	.480	-4.009	1.239	.933	.388	-22.333	.648	.961	.437	-12.661	1.087	.905			
3.800	.472	50	625.00	10	.491	-1.818	.613	.934	.425	-14.970	.473	.948	.459	-8.130	.583	.884			
3.800	.472	50	625.00	20	.493	-1.355	.380	.952	.444	-11.258	.303	.930	.478	-4.313	.391	.893			
3.800	.472	50	625.00	30	.498	-.868	.300	.956	.451	-9.843	.240	.925	.485	-2.961	.315	.906			
3.800	.472	50	625.00	50	.499	-.223	.228	.950	.457	-8.668	.182	.887	.489	-2.164	.235	.907			
3.800	.472	50	625.00	100	.498	-.351	.157	.946	.459	-8.125	.126	.800	.493	-1.400	.167	.884			
4.494	.472	100	2500.00	5	.469	-6.277	1.202	.911	.380	-23.978	.633	.956	.427	-14.569	1.061	.889			
4.494	.472	100	2500.00	10	.492	-1.544	.615	.934	.428	-14.368	.476	.949	.466	-6.724	.595	.890			
4.494	.472	100	2500.00	20	.492	-1.531	.379	.953	.443	-11.448	.302	.942	.477	-4.652	.389	.911			
4.494	.472	100	2500.00	30	.497	-.832	.301	.950	.451	-9.701	.241	.925	.486	-2.890	.316	.911			
4.494	.472	100	2500.00	50	.497	-.819	.227	.941	.456	-8.869	.182	.886	.489	-2.277	.235	.897			
4.494	.472	100	2500.00	100	.498	-.750	.156	.949	.458	-8.322	.126	.806	.493	-1.377	.167	.896			

TABLE C.10. SIMULATION OF THE LOGNORMAL DISTRIBUTION WITH COEFFICIENT OF VARIATION .75.
THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS					LOGNORMAL ESTIMATOR					GAMMA ESTIMATOR					NORMAL ESTIMATOR				
MU	SIGMA	EX	VAR(X)	N	C-HAT	REL.	BIAS	AVE(C1)	COV.	C-HAT	REL.	BIAS	AVE(C1)	COV.	C-HAT	REL.	BIAS	AVE(C1)	COV.
-.223	.668	1	.56	5	.730	-2.624	3.776	.931	.534	-28.764	1.080	.953	.603	-19.579	1.730	.882			
-.223	.668	1	.56	10	.733	-2.203	1.152	.938	.584	-22.089	.620	.918	.646	-13.840	.949	.866			
-.223	.668	1	.56	20	.746	-.497	.675	.945	.618	-17.574	.406	.856	.693	-7.634	.667	.866			
-.223	.668	1	.56	30	.747	-.392	.520	.940	.627	-16.393	.322	.795	.709	-5.525	.546	.856			
-.223	.668	1	.56	50	.747	-.388	.385	.948	.634	-15.489	.244	.651	.721	-3.914	.412	.837			
-.223	.668	1	.56	100	.748	-.250	.265	.942	.640	-14.621	.171	.406	.737	-1.699	.299	.847			
1.386	.668	5	14.06	5	.727	-3.045	4.644	.921	.527	-29.754	1.068	.949	.590	-21.341	1.674	.880			
1.386	.668	5	14.06	10	.738	-1.592	1.166	.930	.586	-21.847	.621	.908	.647	-13.687	.953	.847			
1.386	.668	5	14.06	20	.743	-.996	.668	.956	.615	-17.960	.404	.858	.686	-8.568	.657	.846			
1.386	.668	5	14.06	30	.739	-1.463	.512	.941	.623	-16.918	.321	.760	.707	-5.706	.548	.838			
1.386	.668	5	14.06	50	.746	-.508	.385	.941	.633	-15.591	.244	.659	.720	-3.982	.413	.847			
1.386	.668	5	14.06	100	.748	-.307	.264	.956	.640	-14.613	.171	.401	.739	-1.527	.301	.851			
2.079	.668	10	56.25	5	.725	-3.344	3.556	.916	.532	-29.132	1.076	.949	.599	-20.188	1.704	.877			
2.079	.668	10	56.25	10	.742	-1.019	1.170	.939	.590	-21.373	.624	.919	.652	-13.095	.960	.849			
2.079	.668	10	56.25	20	.745	-.621	.673	.940	.618	-17.574	.406	.841	.696	-7.161	.678	.846			
2.079	.668	10	56.25	30	.749	-.081	.522	.945	.630	-16.008	.324	.812	.717	-4.464	.557	.868			
2.079	.668	10	56.25	50	.743	-.888	.383	.944	.631	-15.823	.244	.657	.718	-4.217	.410	.851			
2.079	.668	10	56.25	100	.744	-.774	.263	.947	.637	-15.041	.170	.401	.730	-2.710	.294	.832			
3.689	.668	50	1406.25	5	.735	-1.966	3.998	.916	.534	-28.851	1.078	.936	.599	-20.160	1.714	.869			
3.689	.668	50	1406.25	10	.739	-1.421	1.179	.933	.587	-21.747	.621	.908	.653	-12.959	.970	.832			
3.689	.668	50	1406.25	20	.748	-.216	.676	.940	.619	-17.428	.406	.863	.694	-7.472	.670	.866			
3.689	.668	50	1406.25	30	.747	-.356	.520	.945	.626	-16.520	.322	.801	.704	-6.142	.540	.860			
3.689	.668	50	1406.25	50	.746	-.497	.385	.940	.633	-15.601	.244	.659	.720	-3.952	.412	.848			
3.689	.668	50	1406.25	100	.747	-.449	.264	.942	.638	-14.901	.170	.396	.730	-2.673	.295	.831			
4.382	.668	100	5625.00	5	.736	-1.851	4.043	.943	.539	-28.198	1.089	.962	.608	-18.893	1.739	.893			
4.382	.668	100	5625.00	10	.735	-2.041	1.156	.925	.586	-21.879	.621	.912	.652	-13.084	.966	.843			
4.382	.668	100	5625.00	20	.751	.163	.681	.938	.619	-17.447	.406	.853	.690	-7.960	.665	.859			
4.382	.668	100	5625.00	30	.740	-1.320	.513	.941	.620	-17.289	.319	.768	.695	-7.393	.529	.847			
4.382	.668	100	5625.00	50	.744	-.778	.383	.934	.631	-15.932	.243	.648	.714	-4.765	.408	.825			
4.382	.668	100	5625.00	100	.746	-.545	.264	.940	.639	-14.824	.170	.382	.732	-2.455	.295	.835			

TABLE C.11. SIMULATION OF THE LOGNORMAL DISTRIBUTION WITH COEFFICIENT OF VARIATION 1.00.
THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS					LOGNORMAL ESTIMATOR					GAMMA ESTIMATOR					NORMAL ESTIMATOR				
MU	SIGMA	EX	VAR(X)	N	C-HAT	REL.	BIAS	AVE(CI)	COV.	C-HAT	REL.	BIAS	AVE(CI)	COV.	C-HAT	REL.	BIAS	AVE(CI)	COV.
-.347	.833	1	1.00	5	1.036	3.577	498.810	.924	.657	-34.299	1.253	.940	.736	-26.408	2.332	.860			
-.347	.833	1	1.00	10	1.014	1.409	2.184	.942	.723	-27.654	.733	.850	.815	-18.537	1.363	.817			
-.347	.833	1	1.00	20	.999	-.059	1.080	.933	.755	-24.534	.478	.668	.883	-11.703	1.001	.785			
-.347	.833	1	1.00	30	1.006	.623	.816	.956	.771	-22.888	.383	.527	.915	-8.491	.832	.815			
-.347	.833	1	1.00	50	.996	-.420	.586	.951	.776	-22.437	.290	.298	.933	-6.702	.625	.797			
-.347	.833	1	1.00	100	1.001	.114	.400	.953	.786	-21.388	.203	.060	.964	-3.564	.464	.796			
1.263	.833	5	25.00	5	1.018	1.839	16.077	.935	.654	-34.617	1.248	.941	.732	-26.766	2.308	.865			
1.263	.833	5	25.00	10	1.000	-.005	2.138	.947	.719	-28.058	.730	.849	.814	-18.552	1.376	.805			
1.263	.833	5	25.00	20	1.007	.720	1.091	.949	.759	-24.122	.480	.694	.883	-11.695	.988	.815			
1.263	.833	5	25.00	30	.999	-.056	.809	.943	.766	-23.356	.381	.514	.903	-9.873	.814	.812			
1.263	.833	5	25.00	50	1.000	-.026	.590	.947	.777	-22.344	.290	.282	.930	-6.958	.625	.798			
1.263	.833	5	25.00	100	1.004	.402	.402	.954	.787	-21.340	.203	.070	.962	-3.823	.461	.790			
1.956	.833	10	100.00	5	1.015	1.516	25.838	.924	.656	-34.371	1.253	.923	.733	-26.670	2.311	.871			
1.956	.833	10	100.00	10	.995	-.457	2.116	.937	.718	-28.243	.728	.856	.812	-18.762	1.374	.806			
1.956	.833	10	100.00	20	1.004	.398	1.083	.937	.758	-24.188	.480	.687	.886	-11.375	.996	.809			
1.956	.833	10	100.00	30	1.000	-.013	.810	.950	.769	-23.126	.382	.514	.910	-8.990	.822	.801			
1.956	.833	10	100.00	50	1.005	.517	.595	.939	.780	-21.953	.292	.307	.939	-6.135	.635	.805			
1.956	.833	10	100.00	100	.999	-.104	.399	.946	.786	-21.427	.203	.054	.963	-3.671	.462	.801			
3.565	.833	50	2500.00	5	1.037	3.688	22.508	.901	.658	-34.191	1.251	.917	.738	-26.172	2.371	.835			
3.565	.833	50	2500.00	10	1.004	.380	2.140	.946	.723	-27.661	.733	.860	.824	-17.550	1.408	.815			
3.565	.833	50	2500.00	20	.996	-.398	1.069	.940	.754	-24.554	.478	.696	.878	-12.158	.980	.813			
3.565	.833	50	2500.00	30	1.000	-.023	.809	.948	.766	-23.416	.381	.521	.897	-10.347	.798	.795			
3.565	.833	50	2500.00	50	.998	-.211	.588	.954	.775	-22.487	.290	.272	.926	-7.362	.618	.765			
3.565	.833	50	2500.00	100	1.000	.039	.400	.951	.785	-21.482	.203	.069	.950	-3.995	.461	.791			
4.259	.833	100	10000.0	5	1.005	.547	17.775	.935	.657	-34.317	1.254	.941	.745	-25.503	2.378	.874			
4.259	.833	100	10000.0	10	1.001	.073	2.143	.931	.718	-28.238	.729	.852	.812	-18.773	1.368	.806			
4.259	.833	100	10000.0	20	1.009	.889	1.095	.941	.759	-24.133	.480	.688	.884	-11.622	.996	.811			
4.259	.833	100	10000.0	30	1.008	.799	.820	.945	.770	-23.006	.383	.542	.906	-9.373	.816	.798			
4.259	.833	100	10000.0	50	1.003	.324	.593	.950	.779	-22.096	.291	.309	.934	-6.606	.627	.815			
4.259	.833	100	10000.0	100	.997	-.273	.398	.947	.785	-21.483	.203	.066	.967	-3.256	.468	.791			

TABLE C.12. SIMULATION OF THE LOGNORMAL DISTRIBUTION WITH COEFFICIENT OF VARIATION 1.25.
THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS				LOGNORMAL ESTIMATOR					GAMMA ESTIMATOR					NORMAL ESTIMATOR				
MU	SIGMA	EX	VAR(X)	N	C-HAT	REL. BIAS	AVE(CI)	COV.	C-HAT	REL. BIAS	AVE(CI)	COV.	C-HAT	REL. BIAS	AVE(CI)	COV.		
-4.70	.970	1	1.56	5	1.325	5.972	437.824	.928	.746	-40.353	1.364	.897	.839	-32.878	2.890	.809		
-4.70	.970	1	1.56	10	1.282	2.523	3.917	.938	.827	-33.831	.810	.735	.952	-23.845	1.781	.775		
-4.70	.970	1	1.56	20	1.254	.311	1.614	.945	.860	-31.163	.530	.418	1.023	-18.163	1.274	.735		
-4.70	.970	1	1.56	30	1.252	.164	1.175	.946	.879	-29.682	.425	.213	1.079	-13.684	1.108	.763		
-4.70	.970	1	1.56	50	1.245	-.406	.833	.955	.888	-28.931	.324	.055	1.116	-10.753	.857	.734		
-4.70	.970	1	1.56	100	1.251	.078	.553	.936	.901	-27.882	.227	.002	1.174	-6.052	.656	.745		
1.139	.970	5	39.06	5	1.330	6.439	394.222	.912	.743	-40.534	1.361	.888	.830	-33.588	2.850	.819		
1.139	.970	5	39.06	10	1.276	2.117	3.799	.936	.822	-34.259	.807	.740	.930	-25.584	1.701	.747		
1.139	.970	5	39.06	20	1.248	-.175	1.597	.936	.862	-31.078	.530	.427	1.033	-17.340	1.297	.751		
1.139	.970	5	39.06	30	1.254	.358	1.180	.946	.881	-29.508	.426	.230	1.091	-12.706	1.135	.753		
1.139	.970	5	39.06	50	1.256	.512	.847	.941	.894	-28.519	.325	.073	1.129	-9.709	.890	.744		
1.139	.970	5	39.06	100	1.241	-.743	.555	.962	.897	-28.249	.226	.003	1.159	-7.247	.646	.727		
1.832	.970	10	156.25	5	1.274	1.950	102.403	.919	.733	-41.373	1.349	.911	.820	-34.378	2.788	.819		
1.832	.970	10	156.25	10	1.258	.664	3.669	.930	.820	-34.400	.805	.715	.943	-24.562	1.761	.743		
1.832	.970	10	156.25	20	1.258	.647	1.621	.949	.864	-30.874	.532	.421	1.028	-17.732	1.287	.738		
1.832	.970	10	156.25	30	1.249	-.065	1.172	.949	.881	-29.519	.426	.225	1.092	-12.601	1.133	.757		
1.832	.970	10	156.25	50	1.259	.756	.849	.953	.892	-28.630	.325	.056	1.120	-10.382	.863	.751		
1.832	.970	10	156.25	100	1.248	-.128	.561	.937	.900	-28.001	.227	.001	1.164	-6.854	.643	.739		
3.442	.970	50	3906.25	5	1.298	3.858	78.786	.916	.742	-40.675	1.358	.886	.831	-33.509	2.855	.807		
3.442	.970	50	3906.25	10	1.289	3.083	3.890	.939	.828	-33.791	.810	.738	.948	-24.170	1.779	.754		
3.442	.970	50	3906.25	20	1.248	-.189	1.590	.949	.863	-30.961	.531	.415	1.037	-17.062	1.314	.748		
3.442	.970	50	3906.25	30	1.261	.894	1.180	.945	.879	-29.715	.425	.227	1.066	-14.727	1.081	.721		
3.442	.970	50	3906.25	50	1.252	.144	.841	.940	.892	-28.679	.325	.054	1.124	-10.050	.868	.751		
3.442	.970	50	3906.25	100	1.250	-.027	.562	.950	.900	-28.021	.227	.003	1.163	-6.942	.647	.712		
4.135	.970	100	15625.0	5	1.273	1.807	83.488	.918	.736	-41.156	1.353	.894	.824	-34.114	2.803	.826		
4.135	.970	100	15625.0	10	1.280	2.382	3.661	.943	.824	-34.055	.808	.725	.940	-24.809	1.751	.758		
4.135	.970	100	15625.0	20	1.249	-.084	1.614	.935	.859	-31.294	.529	.393	1.029	-17.661	1.299	.716		
4.135	.970	100	15625.0	30	1.254	.319	1.177	.955	.883	-29.386	.427	.244	1.096	-12.296	1.144	.770		
4.135	.970	100	15625.0	50	1.247	-.218	.836	.937	.890	-28.797	.324	.067	1.123	-10.121	.876	.734		
4.135	.970	100	15625.0	100	1.250	.018	.562	.948	.901	-27.935	.227	.005	1.170	-6.404	.653	.747		

TABLE C.13. SIMULATION OF THE LOGNORMAL DISTRIBUTION WITH COEFFICIENT OF VARIATION 1.50.
THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS					LOGNORMAL ESTIMATOR					GAMMA ESTIMATOR					NORMAL ESTIMATOR				
MU	SIGMA	EX	VAR(X)	N	C-HAT	REL.	BIAS	AVE(CI)	COV.	C-HAT	REL.	BIAS	AVE(CI)	COV.	C-HAT	REL.	BIAS	AVE(CI)	COV.
-1.589	1.086	1	2.25	5	1.857	10.435	672.658	.914		.818	-45.438	1.452	.859		.911	-39.235	3.293	.791	
-1.589	1.086	1	2.25	10	1.567	4.461	6.305	.937		.913	-39.130	.870	.576		1.063	-29.164	2.159	.718	
-1.589	1.086	1	2.25	20	1.530	2.013	2.334	.966		.963	-35.829	.577	.175		1.191	-20.631	1.670	.726	
-1.589	1.086	1	2.25	30	1.518	1.211	1.650	.945		.969	-35.423	.458	.058		1.213	-19.106	1.365	.654	
-1.589	1.086	1	2.25	50	1.516	1.094	1.156	.949		.989	-34.041	.352	.005		1.304	-13.082	1.142	.698	
-1.589	1.086	1	2.25	100	1.514	.943	.763	.953		1.001	-33.245	.246	0.000		1.383	-7.784	.895	.707	
1.020	1.086	5	56.25	5	1.617	7.829	437.776	.942		.820	-45.322	1.455	.862		.918	-38.824	3.338	.791	
1.020	1.086	5	56.25	10	1.558	3.834	6.514	.946		.906	-39.584	.866	.569		1.045	-30.360	2.087	.691	
1.020	1.086	5	56.25	20	1.518	1.180	2.332	.943		.952	-36.533	.572	.175		1.167	-22.193	1.631	.668	
1.020	1.086	5	56.25	30	1.491	-.628	1.594	.948		.964	-35.750	.456	.044		1.217	-18.837	1.384	.668	
1.020	1.086	5	56.25	50	1.484	-1.051	1.116	.941		.976	-34.909	.348	.007		1.274	-15.095	1.099	.650	
1.020	1.086	5	56.25	100	1.505	.343	.756	.951		.998	-33.483	.246	0.000		1.367	-8.893	.866	.695	
1.713	1.086	10	225.00	5	1.636	9.094	789.086	.925		.819	-45.413	1.454	.849		.910	-39.304	3.281	.789	
1.713	1.086	10	225.00	10	1.567	4.475	6.604	.941		.915	-39.031	.871	.586		1.068	-29.792	2.175	.729	
1.713	1.086	10	225.00	20	1.532	2.120	2.386	.931		.952	-36.542	.572	.177		1.163	-22.459	1.610	.681	
1.713	1.086	10	225.00	30	1.547	3.104	1.713	.934		.981	-34.601	.462	.074		1.253	-16.439	1.460	.708	
1.713	1.086	10	225.00	50	1.508	.538	1.143	.962		.986	-34.149	.351	.008		1.309	-12.706	1.156	.695	
1.713	1.086	10	225.00	100	1.504	.258	.754	.956		.998	-33.441	.246	0.000		1.371	-8.602	.873	.678	
3.323	1.086	50	5625.00	5	1.566	4.382	353.158	.930		.811	-45.943	1.445	.848		.906	-39.568	3.264	.782	
3.323	1.086	50	5625.00	10	1.566	4.392	7.116	.935		.907	-39.555	.865	.540		1.048	-30.105	2.115	.700	
3.323	1.086	50	5625.00	20	1.517	1.126	2.335	.935		.951	-36.589	.571	.190		1.165	-22.353	1.614	.674	
3.323	1.086	50	5625.00	30	1.517	1.162	1.640	.946		.976	-34.944	.461	.059		1.253	-16.474	1.456	.701	
3.323	1.086	50	5625.00	50	1.504	.238	1.140	.952		.982	-34.533	.350	.006		1.287	-14.215	1.118	.653	
3.323	1.086	50	5625.00	100	1.499	-.069	.751	.962		.996	-33.591	.245	0.000		1.370	-8.685	.876	.692	
4.016	1.086	100	22500.0	5	1.628	8.518	751.889	.908		.816	-45.603	1.447	.830		.911	-39.258	3.315	.767	
4.016	1.086	100	22500.0	10	1.570	4.691	7.054	.927		.909	-39.453	.867	.543		1.047	-30.229	2.096	.690	
4.016	1.086	100	22500.0	20	1.511	.741	2.317	.948		.951	-36.583	.571	.183		1.167	-22.204	1.629	.673	
4.016	1.086	100	22500.0	30	1.508	.547	1.632	.934		.972	-35.203	.459	.052		1.244	-17.059	1.424	.711	
4.016	1.086	100	22500.0	50	1.510	.646	1.147	.948		.987	-34.196	.351	.012		1.306	-12.947	1.151	.687	
4.016	1.086	100	22500.0	100	1.506	.415	.757	.958		.997	-33.562	.245	0.000		1.359	-9.430	.857	.688	

TABLE C.14. SIMULATION OF THE LOGNORMAL DISTRIBUTION WITH COEFFICIENT OF VARIATION 2.00.
THE NUMBER OF REPLICATIONS PER CASE IS 1000.

PARAMETERS				LOGNORMAL ESTIMATOR					GAMMA ESTIMATOR					NORMAL ESTIMATOR					
MU	SIGMA	EX	VAR(X)	N	C-HAT	REL.	BIAS	AVE(CI)	COV.	C-HAT	REL.	BIAS	AVE(CI)	COV.	C-HAT	REL.	BIAS	AVE(CI)	COV.
-1.805	1.269	1	4.00	5	2.323	16.143	17752.45	.919	.914	-54.299	1.557	.638	1.007	-49.657	3.915	.682			
-1.805	1.269	1	4.00	10	2.220	11.022	21.618	.944	1.047	-47.672	.956	.176	1.233	-38.352	2.804	.628			
-1.805	1.269	1	4.00	20	2.085	4.263	4.555	.944	1.090	-45.522	.630	.016	1.378	-31.092	2.209	.569			
-1.805	1.269	1	4.00	30	2.031	1.532	2.824	.955	1.104	-44.809	.504	0.000	1.452	-27.414	1.908	.553			
-1.805	1.269	1	4.00	50	2.038	1.887	1.926	.947	1.131	-43.436	.388	0.000	1.585	-20.758	1.661	.576			
-1.805	1.269	1	4.00	100	2.023	1.170	1.232	.944	1.144	-42.779	.272	0.000	1.704	-14.808	1.332	.594			
.805	1.269	5	100.00	5	2.433	21.654	71259.90	.919	.924	-53.802	1.567	.646	1.024	-48.788	4.026	.691			
.805	1.269	5	100.00	10	2.216	10.805	21.670	.943	1.039	-48.047	.951	.181	1.217	-39.167	2.744	.605			
.805	1.269	5	100.00	20	2.059	2.968	4.319	.952	1.088	-45.580	.630	.010	1.372	-31.410	2.157	.589			
.805	1.269	5	100.00	30	2.022	1.097	2.811	.949	1.104	-44.825	.504	0.000	1.458	-27.106	1.922	.579			
.805	1.269	5	100.00	50	2.029	1.429	1.911	.959	1.132	-43.417	.388	0.000	1.594	-20.312	1.680	.575			
.805	1.269	5	100.00	100	2.014	.677	1.223	.939	1.143	-42.853	.272	0.000	1.710	-14.518	1.336	.594			
1.498	1.269	10	400.00	5	2.507	25.384	671539.4	.919	.924	-53.797	1.570	.659	1.006	-49.706	3.879	.683			
1.498	1.269	10	400.00	10	2.150	7.508	19.574	.937	1.024	-48.804	.942	.163	1.186	-40.701	2.614	.588			
1.498	1.269	10	400.00	20	2.073	3.644	4.431	.934	1.092	-45.401	.631	.009	1.393	-30.354	2.231	.611			
1.498	1.269	10	400.00	30	2.051	2.559	2.878	.953	1.118	-44.121	.508	.001	1.402	-24.907	2.053	.575			
1.498	1.269	10	400.00	50	2.026	1.280	1.908	.946	1.133	-43.352	.388	.001	1.604	-19.806	1.700	.598			
1.498	1.269	10	400.00	100	2.014	.686	1.223	.945	1.146	-42.723	.272	0.000	1.726	-13.705	1.368	.596			
3.107	1.269	50	10000.0	5	2.390	19.477	110507.7	.939	.935	-53.232	1.583	.604	1.037	-48.132	4.076	.721			
3.107	1.269	50	10000.0	10	2.189	9.439	19.329	.941	1.034	-48.310	.948	.165	1.204	-39.797	2.688	.590			
3.107	1.269	50	10000.0	20	2.075	3.765	4.401	.945	1.088	-45.608	.629	.010	1.376	-31.223	2.198	.572			
3.107	1.269	50	10000.0	30	2.017	.848	2.800	.951	1.107	-44.667	.505	0.000	1.482	-25.912	2.003	.577			
3.107	1.269	50	10000.0	50	2.031	1.540	1.913	.953	1.128	-43.617	.387	.001	1.573	-21.343	1.647	.563			
3.107	1.269	50	10000.0	100	2.018	.822	1.226	.956	1.141	-42.839	.271	0.000	1.669	-16.540	1.260	.581			
3.800	1.269	100	40000.0	5	2.500	25.020	880987.4	.916	.923	-53.855	1.568	.660	1.019	-49.042	3.964	.696			
3.800	1.269	100	40000.0	10	2.202	10.116	20.010	.941	1.041	-47.841	.953	.178	1.224	-38.813	2.765	.628			
3.800	1.269	100	40000.0	20	2.075	3.733	4.438	.925	1.087	-45.628	.629	.011	1.382	-30.875	2.223	.568			
3.800	1.269	100	40000.0	30	2.051	2.565	2.890	.958	1.107	-44.638	.505	0.000	1.451	-27.433	1.888	.595			
3.800	1.269	100	40000.0	50	2.018	.777	1.889	.938	1.127	-43.637	.387	0.000	1.585	-20.763	1.668	.571			
3.800	1.269	100	40000.0	100	2.008	.392	1.217	.935	1.140	-42.994	.271	0.000	1.693	-15.348	1.329	.568			