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QCD CORRECTIONS TO QUARKONIA DECAYS

by

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QCD CORRECTIONS TO QUARKONIA DECAYS*†

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ABSTRACT

A summary is given of recent work done on QCD corrections to inclusive leptonic and hadronic decays of quarkonia. Particular attention is given to the decays $\psi \rightarrow e^+ e^-$ and $n_Q \rightarrow \text{hadrons}$.

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This is a review of some recent work done on QCD corrections to quarkonia decay widths. I will primarily discuss the decays $^3S_1 \rightarrow \ell^+ \ell^-$ and $^1S_0 \rightarrow \text{hadrons}$,³ but a few other results will be mentioned in passing.

$$\underline{^3S_1 \rightarrow e^+ e^-}$$

The calculation of decay rates of orthoquarkonium (Ψ, T) to lepton pairs follows by analogy with the well understood positronium system of QED^{4,5}. The procedure in QED is as follows:

- (1) Write the Bethe-Salpeter equation for positronium.

This is drawn schematically in Fig.1a

$$\chi = \chi G_0 K \quad (1)$$

where K is the two-particle irreducible (2PI) kernel. As discussed in detail in Caswell and Lepage⁵ this may be rewritten as

$$\chi = \chi \tilde{G}_0 \tilde{K} \quad (2)$$

where $\tilde{G}_0$ is a modified 2-particle propagator and $\tilde{K}$ is the correspondingly modified kernel (Fig.1b).

Fig.1a Bethe Salpeter equation for positronium.

Fig.1b Caswell-Lepage equation for positronium.

- (2) Write

$$\tilde{K} = K_0 + \Delta K \quad (3)$$

where K_0 is the Coulomb interaction (Fig.2a) and solve (Fig.2b)

$$\chi_0 = \chi_0 \tilde{G}_0 K_0 \quad (4)$$

This is just a Schrodinger equation with Coulombic potential.

Fig.2a The Coulomb Interaction

Fig.2b Lowest order Caswell-Lepage equation.

(3) Now do perturbation theory in ΔK . The decay width is given by

$$\Gamma = -2\text{Im}\Delta E \quad (5)$$

(4) The result of applying this formalism is that in leading order the leptonic width is just (Fig.3) the van Royen-Weisskopf Formula⁶ (vRW)

$$\Gamma_0 = \frac{16\pi\alpha^2 |\chi(0)|^2}{M^2} \quad (6)$$

where M is the positronium mass. The leading correction to this formula is found from the one-photon exchange diagram (Fig.4).

Fig.3 The leading order decay diagram. The 2-particle propagator is understood to be modified as in Fig.2b.

Fig.4 The correction to positronium decay.

Part of this diagram simply iterates the Coulomb interaction so one finds, in fact, that the $O(\alpha)$ correction arises from the region of k -space (Fig.4) where $k=O(m_e)$ (the ultraviolet region where momenta are much greater than the mean electron momentum.) It is also interesting to note that the correction is mostly from the longitudinal, not the transverse, photon.²

The difficulty in carrying this formalism over to QCD is that the Coulomb exchange is not a good approximation to K . Instead, K_0 is an unknown confining potential such as⁷ (in momentum space)

$$\bar{V}(q^2) = -\frac{4}{3} \left(\frac{12\pi}{33-2n_f} \right) \frac{1}{q^2} \frac{1}{\ln(1+q^2/\Lambda^2)} \quad (7)$$

ΔK is therefore not known and this makes it difficult to compute corrections to the leptonic widths.

One reasonable ansatz is to assume that the leading correction comes from the large- k region of the gluon exchange diagram (as in Fig.4). That leads to the formula^{1,2}

$$\Gamma(V \rightarrow e^+ e^-) = \frac{16\pi\alpha e_Q^2}{m_V^2} |\chi(0)|^2 \left[1 - \frac{16}{3} \frac{\alpha_s(m_Q)}{\pi} \right] \quad (8)$$

A number of authors⁸⁻¹¹ have observed that $O(v^2/c^2)$ corrections can be expected to be large for charmonium, since $\langle v^2/c^2 \rangle_\psi \sim 0.2$.¹² In the positronium system such corrections are $O(\alpha^2)$ and are therefore nonleading. However, for quarkonia, the numerical values of these relativistic corrections can be of the same order as the correction in Eq.(8). For instance, Michael and Payne¹¹ find with their light-plane analysis of the problem

$$\Gamma/\Gamma_0 = \frac{m^{3/2} \int p^2 dp \psi_s(p) \frac{[2(m^2 + p^2)^{1/2} + m]}{(m^2 + p^2)^{5/4}}}{3 \int p^2 dp \psi_s(p)} \quad (9)$$

In a harmonic oscillator model of the Ψ meson, the numerical value for Γ/Γ_0 is approximately 0.6, which is a considerable suppression.

All estimates of $O(v^2/c^2)$ corrections involve fairly specific guesses as to the nature of ΔK and the authors of Refs.8-11 have rather differing approaches to this problem. In all instances, a sizeable suppression is found. It has been suggested^{8,10} that the net effect of the higher order corrections is to "smear" the wavefunction at the origin so that in the van Royen-Weisskopf formula (Eq.6)

$$|\chi(0)|^2 \rightarrow |\chi(r = \frac{1}{m})|^2 \quad (10)$$

These large suppressions have some important consequences for phenomenology^{2,12-14}. Equation (8),

$$\Gamma(V \rightarrow e^+ e^-) = \frac{16\pi\alpha_s^2 e_Q^2}{m_V^2} |\chi(0)|^2 \left(1 - \frac{16}{3\pi} \alpha_s(m_Q)\right),$$

implies a suppression of 80% if $\alpha_s = 0.5$. Large higher-order corrections make the result suspect, but a considerable suppression might be expected. Because of this, $\Gamma(\Psi)$ and $\Gamma(T)$ should not be used as experimental input to fit potential model parameters. The possible exception to this rule is if the potential is designed to fit certain experimental constraints¹⁶ (as in the inverse scattering methods) rather than theoretical prejudice.^{12,15} It is noteworthy that in all of the potential models of Ref.15 it was found that $|\psi(0)|^2$ is indeed larger than the value expected

from the vRW formula. What is also interesting is that the ratios, $\Gamma(\Psi' \rightarrow e^+ e^-) / \Gamma(\Psi \rightarrow e^+ e^-)$, do fit the data. This suggests that the suppression factor cancels in the ratio as indeed it would if Eq.(8) were correct. In fact, this may be evidence that kinematic effects ($0(v^2/c^2)$ -corrections) may not be important-- if they were, one would expect them to give rather different corrections for the Ψ and the Ψ' . If it is true that relativistic corrections may be ignored then we might expect that $\Gamma(T \rightarrow e^+ e^-) / \Gamma(\Psi \rightarrow e^+ e^-)$ will provide interesting information. The suppression factors for this ratio differ due to the fact that $\alpha(m_c) \neq \alpha(m_b)$.

One further consequence of all of these considerations is that the hyperfine splitting between Ψ and η_c may be larger than was at first predicted. The reason for this is that in many models the hyperfine splitting is proportional to the wavefunction near the origin. As an example, the Fermi-Breit correction¹⁷ is proportional to $|\chi(0)|^2 s_1 \cdot s_2 / m_c^2$. If the vRW equation is correct, then the value of $|\chi(0)|^2$ extracted from the experimental Ψ decay width leads to a hyperfine splitting of 0(30 MeV). However, if Eq.(8) is used then $|\chi(0)|^2$ is much larger than previously thought, so it is not difficult to explain a Ψ - η_c mass difference of ~100 MeV.

Other leptonic decay rates

Following the methods described for $^3S_1 \rightarrow e^+ e^-$ we can also derive the QCD corrections to $^3S_1 \rightarrow 3\gamma$ and $^1S_0 \rightarrow 2\gamma$. To derive $\Gamma(^3S_1 \rightarrow 3\gamma)$ we can use the results of Caswell et al.¹⁸ (their result is modified by first subtracting the graph containing a single s-channel photon and then letting $\alpha \rightarrow \frac{4}{3} \alpha_s(m_Q)$) to obtain

$$\Gamma(^3S_1 \rightarrow 3\gamma) = \Gamma_0 \left(1 - 12.7 \frac{\alpha_s(m_Q)}{\pi} \right) \quad (11)$$

where

$$\Gamma_0 = \frac{\alpha^3}{M^3} \frac{64}{9} (\pi^2 - 9) |\chi(0)|^2 \quad (12)$$

$\Gamma(^1S_0 \rightarrow 2\gamma)$ is obtained directly from the QED result of Harris and

Brown¹⁹

$$\Gamma(^1S_0 \rightarrow 2\gamma) = \Gamma_0 \left[1 - \frac{4}{3} \left(5 - \frac{1}{4} \pi^2 \right) \frac{\alpha_s(m_Q)}{\pi} \right] \quad (13)$$

where

$$\Gamma_0 = \frac{48\pi\alpha_s^2 e_Q^4 |\chi(0)|^2}{M^2} \quad (14)$$

Hadronic decays

Some calculations^{20,21} have been done of QCD corrections to quarkonia hadronic decays. Specifically, the branching ratios $P = \Gamma(Q \rightarrow e^+ e^-) / \Gamma(Q \rightarrow \gamma\gamma)$ were computed beyond the leading order in α_s . These ratios may have the property of factorizing the binding effects³ described in the previous sections.²² In particular, the wavefunction factors out in the ratio.

As of yet there is little data on these ratios, but the calculations are of great interest--not only because they are high-order QCD corrections--but because they serve as prototypes for the calculation of corrections to more easily measured processes such as $\psi \rightarrow$ hadrons, etc.

I will mainly discuss the calculation of $P^{-+} = \Gamma(^1S_0 \rightarrow \text{hadrons}) / \Gamma(^1S_0 \rightarrow \gamma\gamma)$. (An example of the 1S_0 is the η_c state.) The quoted result of this calculation was,²⁰ for 4 flavors.

$$P^{-+} = C \alpha_s^2 \left(1 + 22.14 \frac{\alpha_s}{\pi} \right) \quad (15)$$

where

$$C = \frac{2}{9e_Q^4 \alpha_{\text{QED}}^2} \quad (16)$$

If $\alpha_s \sim 0.2$ then the size of the correction is ~140%! This has caused a great deal of consternation since the calculation seems to suggest that a perturbation expansion of P will be unreliable.

What I wish to point out here is that the quoted result may be misleading in that the largeness of the coefficient is simply a consequence of the particular expansion parameter, α_s , used by

Barbieri et al.²⁰ In fact, the definition of α_s depends on which method is used to renormalize the ultraviolet divergences encountered in the calculation of P^{-+} .²³⁻²⁵ The α_s used by Barbieri et al. is known technically as $\alpha_{MS}(2m)$, where m is the quark mass and "MS" denotes "minimal subtraction"--an elegant mathematical trick for removing the divergences. A perhaps more physical technique^{23,26} for eliminating divergences is "momentum space subtraction". This leads us to using the parameter $\alpha_{MOM}^{(m)}$ ²⁷ which I call α'_s and is related to α_s by the equation

$$\alpha'_s = \alpha_s \left(1 + 10.18 \frac{\alpha_s^2}{\pi} \right) \quad (17)$$

Substitution of Eq.(17) ($\alpha_s \sim \alpha'_s - 10.18 \frac{\alpha_s^2}{\pi}$) into Eq.(15) and Taylor expanding leads to

$$P^{-+} = C \alpha_s'^2 \left(1 + 1.78 \frac{\alpha_s'}{\pi} + \dots \right) \quad (18)$$

Eq.(18) is equivalent, to this order of perturbation theory, to Eq.(15). The correction has been decreased at the expense of increasing the size of the coupling. A natural question is "Is the total value of P^{-+} the same in both Eqs.(15) and (18)?" I will show that it is not. Suppose $\alpha_s = 0.20$. (This is consistent with deep inelastic data.) Then from Eq.(17), $\alpha'_s = 0.33$. Fig.5 shows P^{-+}/C for Eq.(15) and for Eq.(18). The leading order results are very different in the two cases and are still far apart after including the first QCD correction but the α'_s method appears to be converging better. What is clear is that to this order of perturbation theory, the physical prediction is very different in the two schemes. Therefore, if we have reason to trust the α'_s perturbation expansion, then it is advisable to use Eq.(18) for comparison with experiment. In fact, α'_s may be trustworthy, for it leads to small corrections in other processes such as deep inelastic scattering.

Fig.5

Buras²⁵ finds that the difference between $P_{(0)}^{-+}$ and $P_{(1)}^{-+}$ is not very large. Furthermore, this result is fairly insensitive to prescription. This analysis of Buras' agrees with our results expressed in terms of α'_s . That is to be expected simply because α_{MOM} leads to small corrections for both P^{-+} and for deep-inelastic scattering and therefore the correction must be small when comparing the two.

As a conclusion to this section on hadronic decays are the results of Barbieri et al.'s calculation of P-wave decays^{21,28}

$$\frac{\Gamma(0^{++} \rightarrow \text{hadrons})}{\Gamma(0^{++} \rightarrow 2\gamma)} = C\alpha_s'^2 \left(1 + 2.7 \frac{\alpha'_s}{\pi}\right) \quad (19)$$

and

$$\frac{\Gamma(2^{++} \rightarrow \text{hadrons})}{\Gamma(2^{++} \rightarrow 2\gamma)} = C\alpha_s'^2 \left(1 - 3.8 \frac{\alpha'_s}{\pi}\right). \quad (20)$$

They also predict

There is another argument due to Buras²⁵ which shows that the corrections to P^{-+} are probably small. He argues that in order to estimate the size of corrections to P^{-+} you can proceed as follows:

(1) Compute leading order P^{-+} and leading order deep inelastic scale violations (DIS).

(2) Use DIS data and the above leading order DIS result to obtain $\alpha_{(0)}$. Plug this into leading order P^{-+} to obtain $P_{(0)}^{-+}$.

(3) Repeat steps (1) and (2) but replacing "leading order" by "leading order + correction" and replacing $P_{(0)}^{-+}$ by $P_{(1)}^{-+}$.

(4) Finally, compare $P_{(1)}^{-+}$ and $P_{(0)}^{-+}$.

$$\frac{\Gamma(0^{++} \rightarrow \text{hadrons})}{\Gamma(2^{++} \rightarrow \text{hadrons})} = \frac{15}{4} \left(1 + 12 \frac{\alpha_s}{\pi} \right). \quad (21)$$

This correction is, apparently, the only large correction of those computed by Barbieri et al. It also seems to be prescription independent, but one might question the wisdom of taking this ratio. In any case, if Eq.(21) is approximately valid, then it may well explain the experimental value of ≥ 14 .²⁹

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