THRESHOLD CURRENT FOR THE TRANSVERSE SINGLE-BUNCH INSTABILITY IN PEP*

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Theory

For a bunch with charge q moving in a storage ring under the influence of a constant dipole wake potential of amplitude W_1 and constant transverse beta-function with amplitude β_t , the two-macroparticle model for the transverse single-bunch instability predicts^{1,2,3} a threshold current

$$I_{th} = \frac{16 E_0 f_s}{\beta_t W_1} \quad , \quad (1)$$

where eE_0 is the energy and f_s is the synchrotron frequency. Since both β_t and W_1 vary with position around the ring, the total transverse momentum kick per revolution will be proportional to

$$\int \beta(z) w_1(z) dz \quad , \quad (2)$$

where $w_1 = dW_1/dz$ is the dipole wake per unit length in units of $V/C\text{-m}^2$.

If the wake is not constant, but instead depends on the separation between the two macroparticles, then w_1 in the preceding expression must be replaced by an effective wake $\bar{w}_1$, obtained by a proper averaging over one-half a synchrotron oscillation period. This complication will not be given further consideration here; in any case, the detailed forms of the wake functions for each of the major PEP ring components are not known. A computation of the transverse wake for the PEP rf cavities using the

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programs TUCI and TRANSVERS is in progress. When this work has been completed, an absolute prediction of the threshold current using Eq. (1) will be possible, at least to the extent that the transverse impedance of the rf cavities dominates the total impedance of the ring. A preliminary estimate of the fraction of the total dipole impedance exterior to the rf cavities is, in fact, one of the purposes of this note.

The impedance of the rf cavities is expected to depend strongly on the radius a of the beam aperture. Let the dipole wake potential per cavity cell be characterized by

$$W_c = \frac{A_1}{a^m} \quad V/C-m \quad . \quad (3)$$

For the SLAC disk loaded structure, the peak amplitude of the dipole wake $w_1(\tau)$ varies⁴ as $a^{-2.25}$, and the time location of the peak is given approximately by $c\tau_m \approx 0.64 a$. The slope of $w_1(\tau)$ as τ approaches zero varies as $a^{-3.48}$. However for the PEP bunch length ($\sigma_z \approx 2$ cm), the maximum separation between the two macroparticles during a synchrotron oscillation is expected to be on the order of $c\tau \approx 2\sigma_z \approx 4$ cm. From Table I the average value of the cavity aperture is $a \approx 5$ cm, and the maximum of the dipole wake therefore occurs at about $c\tau_m \approx 3$ cm.

Synchrotron oscillations therefore carry the tail of the bunch somewhat past the peak of the deflecting wake set up by the head of the bunch, and this implies that the $a^{-2.25}$ variation is more appropriate. Longer bunches would, in fact, see an effective wake which decreases with increasing bunch length, in agreement with the dependence of threshold current on bunch length observed at PETRA.

From Table 1 it is seen that there are eight different beam apertures in the rf cavity system. Thus, from Eqs. (2) and (3)

$$\int_{\text{rf}} \beta w_1 dz \approx 15 A_1 \sum_{n=1}^8 \frac{\beta_n}{(a_n)^m} \quad (4)$$

since there are 15 cells of each aperture in the PEP rf system. The dipole wake for all components external to the rf system can be characterized by a parameter α such that

$$W_{\text{ext}} \equiv \alpha W_{\text{rf}} = \alpha \sum_{n=1}^{\infty} \frac{15 A_1}{(a_n)^m}$$

In the absence of a detailed knowledge of the impedance of these external components, and of the beta-function at the location of each component, we take

$$\int_{\text{ext}} \beta w dz \approx \bar{\beta}_{\text{ext}} W_{\text{ext}} \quad (5)$$

where

$$\bar{\beta}_{\text{ext}} \approx \bar{\beta} = \frac{\bar{R}}{\nu} \quad (6)$$

Here ν is the transverse tune and $\bar{R}$ is the average radius (350 m for PEP).

Combining Eqs. (1) through (6), we obtain

$$I_{\text{th}} = \frac{A}{\sum_{n=1}^8 \left[\frac{\beta_n}{(a_n)^m} \right] + \frac{\alpha \bar{R}}{\nu} \sum_{n=1}^8 \frac{1}{(a_n)^m}} \quad (7)$$

where

$$A = 16 E_0 f_s / 15 A_1$$

Comparison With Observed Threshold

Threshold current values have been measured in PEP for a number of different lattice configurations. These values are given in Table II, together with a few parameters that specify the lattice (horizontal and

vertical tune and both beta-functions at the interaction point). For each configuration, the beta-function at each cavity location is computed using MAGIC. The parameters A, α and m in Eq. (7) are then determined using the following fitting procedure. The parameter space was searched for a global minimum for the function

$$F = \sum_{i=1}^7 \frac{(I_i^{\text{meas}} - I_i^{\text{cal}})^2}{\sigma_i^2} \quad (8)$$

where I_i^{meas} is the measured threshold current (See Table II), I_i^{cal} is the threshold current for a given configuration calculated according to Eq. (7) and σ_i is the measurement error (taken to be 5%). The search was performed using the CERN minimization package MINUIT.

As the result of this fitting procedure, values of A, α and m were determined which give the best fit to the measured threshold currents with the minimum chi-square. These values are shown as Fit A in Table II. This fit results, however, in a value for m which seems too large ($m = 3.8$), according to the discussion in the preceding section. If we arbitrarily choose $m = 2.5$, which is a reasonable value extrapolating from previous wake potential calculations for the SLAC structure, we obtain the results shown as Fit B in Table II. The chi-square value is about double that for Fit A, but the fit is still quite reasonable. Thus the experimental measurements do not give a sharp prediction for the coefficient m. The parameter α , which gives the fraction of the impedance external to the rf cavities, is also not specified precisely. These fits suggest that it lies in the range 0.5 to 1.0.

Future Predictions

We can use these fits to predict the threshold current that will be obtained after Q2 is moved. The result is given in Table III for two

configurations of potential interest. Note first that for both configurations the predicted threshold current are above the expected operating current of 6-7 mA per bunch. Note also that the predicted currents depend only weakly on the value of m . The standard deviations shown assume a $\pm 5\%$ error in the measured current values given in Table II. In all cases the blow-up is predicted to be in the horizontal plane.

References

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Table I

Locations and Beam Aperture Radii for PEP Cavities

Cavity Number N	Beam Aperture (cm)	Distance from IR Before Moving 1 & 2 (m)	Distance from IR After Moving 1 & 2 (m)
1	6.39	27.58	51.35
2	5.92	30.56	54.32
3	5.46	33.53	33.53
4	5.01	36.50	36.50
5	4.58	39.47	39.47
6	4.25	42.44	44.44
7	4.02	45.41	45.41
8	3.80	48.38	48.38

Table II
Measured and Fitted Threshold Currents

Configuration (v_x/v_y)	β_y^* (m)	β_x^* (m)	$\langle \beta_y \rangle_{RF}$ (m)	$\langle \beta_x \rangle_{RF}$ (m)	Measured I_{th} (mA)	Fit A I_{th} (mA)	Fit B I_{th} (mA)
25/20	0.11	3.0	17	63	10.4 h	10.6	10.0
22/19	0.11	2.0	21	158	3.5 h	3.7	3.9
25/16	0.11	2.0	61	86	8.0 v	7.4	7.7
25/16	0.15	2.0	48	87	8.4 h	8.0	7.7
25/16	0.15	3.0	47	60	9.2 v	8.9	9.0
25/16	0.20	3.0	38	61	9.9 v	10.3	10.3
25/20 with cavities moved	0.11	3.0	14	33	12.5 h	13.5	13.4

Notes: h, v = horizontal, vertical blow-up

Fit A: $m = 3.8$, $\alpha = 0.51$, $A = 23.4$

Fit B: $m = 2.5$, $\alpha = 0.93$, $A = 197$

Table III
Predicted Threshold Currents After Moving Q_2

Configuration	$m = 2.5$	$m = 3.8$
21/18	9.0 ± 0.2 mA	8.7 ± 0.2 mA
25/20	16.9 ± 0.6	17.3 ± 0.7