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AN ORBIT AVERAGED RADIAL BUILDUP CODE
FOR TANDEM MIRROR GEOMETRY

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AN ORBIT AVERAGED RADIAL BUILDUP CODE FOR TANDEM MIRROR GEOMETRY

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INTRODUCTION

DISCLAIMER

The radial Fokker-Planck (RFP) model of A. Futch^{1,2} was modified to treat plasma buildup in the tandem mirror plug and center cell with a self-consistent model (TOARRUC). Two major changes have been made to the original version of this code. First, the center cell is treated as having separate electron and ion "confining" potentials with the ion potential having the opposite sign of that in a conventional mirror. Second, a two-electron-temperature treatment derived by R. Cohen³ was included in the present model to allow the plug and center cell to have different T_e 's as observed in the experiment. The following sections explain these changes in greater detail.

The TOARRUC (for Tandem Orbit-Averaged Radial Buildup Code) model is based on the finite-orbit Fokker-Planck model developed by A. Futch to study conventional mirrors. A modification has been made in the code to allow the plug and center cell plasma parameters of a tandem mirror to be calculated during alternate iterations. The plug ion parameters are generated first at each time step followed by those of the center-cell ions and then the self-consistent potentials, and electron particle, and energy transfer between the two regions (time step $> \tau_{ee}$) are determined. New values for the ion densities in the plug and center cell (n_p, n_c), the electron and ion temperatures in these regions ($T_{ep}, T_{ec}; T_{ip}, T_{ic}$, respectively), and the ion and electron confining potentials in the center cell (ϕ_i, ϕ_e) and plug ($\phi_p = \phi_e + \phi_i$) are found for use in the next time step (see Fig. 1).

The TOARRUC code differs from a particle buildup code for the plasma ion density in several important aspects. Table I lists several basic differences

Fig. 1. Schematic diagram.

Table I. Basic differences.

TOARBUC	TAMBAR
1. Rate equations used for calculating atomic physics includes secondary neutrals (Stallard model).	1. Three-dimensional Monte Carlo calculation of neutral re-actions.
2. Orbit averaged.	2. Finite-orbit effects generally neglected except for beam deposition.
3. Distribution function: a. Plug - calculated b. CC - calculated	3. Distribution function: a. Plug - assumed b. CC - assumed
4. Ion end loss: a. All Coulomb processes b. Q.L., ϕ is an input parameter which may be set = 0 for classical processes only.	4. Ion end loss: a. Spitzer drag with decreased plug lifetime to satisfy Q.L. requirements.
5. Ion energy - calculated.	5. Ion energy - assumed = E_{inj}

between TOARRUC and the particle buildup code TAMBAR.⁴ Many other differences exist and no attempt for completeness is made here. In an orbit average code such as TOARRUC, the orbit averaged distribution function is calculated. The particle deposition and energy exchange loss rates are calculated by averaging around an orbit. In a particle (or fluid) code such as TAMBAR, this is not possible since the particle orbit is unknown. Figure 2 illustrates this effect. Consider the loss of a particle due to a charge exchange collision at x . The charge exchange collision at x results in a loss in density over the range $R - \rho$ to $R + \rho$ which is correctly calculated in an orbit average code (Fig. 2a) since the guiding center is known. For a particle code the guiding center is unknown; therefore, the charge exchange collision could result in the loss of a particle having orbits a , b , or c , etc. (Fig. 2b) and a corresponding uncertainty in the charge exchange loss rate occurs. Radial transport due to charge exchange will be more uncertain since it is the difference between trapping and loss rates of approximately the same magnitude. Similar considerations apply to end-loss, electron drag etc. The importance of orbit effects depend upon the spatial gradients of parameters such as the plasma density, thermal gas density, etc. as compared with the orbit size.

In TAMBAR the plug ion energy is fixed at the injection energy whereas in TOARRUC the distribution function is calculated self-consistently resulting in a distribution of orbit sizes.

Three-dimensional transport of neutral gas and neutral beams is calculated by a Monte Carlo code⁵ in TAMBAR. In TOARRUC atomic physics processes are described by rate equations including secondary beams modeled after Stallard.⁶

At present both codes require approximately the same computer time for a run. The more detailed atomic physics in TAMBAR appears to compensate for the more complete plasma physics of TOARRUC in the use of computer time.

Fig. 2. (a) Orbit averaged code. (b) Particle code.

ALTERATIONS TO PREVIOUS MODEL

A. ION LOSS PROBABILITY

In the plug region the ion loss-cone probability depends on a slowly varying function of the mirror ratio (R_{eff}) given by:

$$R_{eff} = R \left/ \left(1 + \frac{|q\phi_p|}{W_i} \right) \right.$$

or

$$R_{eff} = 1, \quad (1)$$

whichever is larger. Here R is the input mirror ratio, ϕ_p is the plasma potential and W_i the ion energy. In the center cell the ions are confined by a negative potential ϕ_i and the loss probability is zero for all ions with $W_i < |q\phi_i|$ and affects R_{eff} to give:

$$R_{eff} \rightarrow \infty, \quad W_i \leq |q\phi_i|$$

$$R_{eff} = R \left/ \left(1 - \frac{|q\phi_i|}{W_i} \right) \right., \quad W_i \geq |q\phi_i|. \quad (2)$$

In addition to the effect of ϕ_i on the loss-cone probability, the turning-point mirror ratio is affected by ϕ_i . For $|q\phi_i| > W_i$ the turning point for potential-confined ions is near the minimum-B in the plug where the potential is largest, but magnetically confined ions are reflected at the inside plug mirror. In order to facilitate a smooth transition between these two modes, the mirror ratio R in Eq. (2) is replaced by the expression:

$$R = \frac{R_c}{R_p} \left\{ 1 - \left[1 - \frac{1}{R_p} \right] \exp \left(\frac{-2\phi_i}{T_{ic}(R_p - 1)} \right) \right\}^{-1}, \quad (3)$$

where R_c is the center-cell mirror ratio, R_p is the plug mirror ratio and T_{ic} is the center-cell ion temperature (see Fig. 1).

B. ELECTRON BALANCES

Because different electron temperatures have been observed in TMX, the two-electron-temperature formulas developed by R. Cohen were incorporated into the TOARBUS model. Cohen's formulas provide for a "trapped" group of electrons which are confined by the plug potential ϕ_i and reside in the plug and a "passing" group that are confined by ϕ_e and occupy the entire tandem system. The groups, designated n_p and n_c respectively, can have different mean temperatures (T_p, T_c , where the "e" subscript has been dropped). Coupling between these groups is provided through the fraction of "passing" electrons which occupy the plug region during a fraction of their bounce period (density designated by n_μ) and allow direct energy transfer and scattering of electrons between the two groups. The formula given by Cohen for the particle (density) balance in the plug is:

$$\frac{dn_p}{dt} = -\frac{n_p}{\tau_p} \left[1 - (T_p/T_c)^{1/2} \frac{n_c}{n_p} \exp\left(\frac{q\phi_i}{T_p}\right) \right] + J_p, \quad (4)$$

where the "e" subscript on the temperatures is again omitted and J_p is the total source of plug electrons. The symbol τ_p is the Pastukhov time (Ref. 3) given functionally as:

$$\tau_p = \frac{1}{2} K \tau_{ee} \cdot g(R) \cdot \left(\frac{\phi_i}{T_p}\right) \exp\left(\frac{\phi_i}{T_p}\right), \quad (5)$$

where τ_{ee} is the electron self-collision time, $g(R)$ is a slowly varying function of the mirror ratio and K contains the Coulomb terms.

The electron energy (power) balance in the plug is given by:

$$\frac{3}{2} \frac{d(n_p T_p)}{dt} = - \frac{n_p}{\tau_p} \left[(q\phi_i + T_p) - (T_p/T_c)^{1/2} \frac{n_c}{n_p} \cdot \exp\left(\frac{q\phi_i}{T_p}\right) \cdot (q\phi_i + T_c) \right] \quad [1]$$

$$- (nT)_{ee} + (nT)_{e-ip} + P_{aux} + J_p E_{IP} , \quad (6)$$

[2] [3] [4] [5]

where the terms are represented as follows:

- [1] The interchange term between trapped and untrapped groups times the energy carried by an electron transferring from one group to the other (measured in the plug);
- [2] The collisional (drag) energy transfer term between the trapped and passing groups in the plug;
- [3] The plug ion energy transfer to the trapped electrons (because n_μ is small, energy transfer from ions to passing electrons is negligible);
- [4] Auxiliary heating, if any;
- [5] The energy input from the trapped-electron source.

Other terms such as radiation losses and anomalous cooling can be added as desired.

The electron energy transfer rate of [2] is given by Cohen as:

$$(nT)_{ee} = \frac{n_c (T_p/T_c)^{1/2} (T_p - T_c)}{\tau_v} ,$$

where

$$\tau_v^{-1} = \frac{2T_p}{\sqrt{\pi} R \tau_{ee} (q\phi_i)} . \quad (7)$$

Here R is the plug mirror ratio and τ_{ee} is the electron collision time in the plug. Since the primary source of energy to the CC electrons comes from the plug ($T_{ep} > T_{ec}$), a negative sign is attached to this term in Eq. (6).

The electron particle and energy balances in the center cell (CC) are only slightly more complex and while not given by Cohen are readily derived. The particle (density) balance is:

$$\frac{dn_c}{dt} = V_R \left\{ \underbrace{J_c V_c}_{[1]} + \underbrace{\frac{2n_p V_p}{\tau_p}}_{[2]} \left[1 - (T_p/T_c)^{1/2} \frac{n_c}{n_p} \cdot \exp\left(\frac{q\phi_i}{T_p}\right) \right] - \underbrace{\frac{n_c V_c}{\tau_c}}_{[3]} - \underbrace{\frac{2n_p n_c V_p}{n_c \tau_c}}_{[4]} \right\}, \quad (8)$$

where τ_c is the Pastukhov time in the center cell with ϕ_e being the confining potential. The coefficient V_R is

$$V_R = \frac{n_c}{2n_p V_p + n_c V_c}, \quad (9)$$

and represents the weighting given to the portion of the passing fraction occupying only the CC, on the average. The particle book-keeping is done such that only the fraction of passing electrons occupying the center cell constitute the density n_c . The passing electron density in the plug is designated by n_p (on the average) and for a square-well approximation the total number of passing electrons (N_{cp}) is:

$$N_{cp} = n_c V_c + 2n_p V_p. \quad (10)$$

Thus V_R accounts for the fractional change in n_c due to a change in N_{cp} , the remainder affecting n_p . [It should be noted that a factor of V_c was divided through in the particle balance to obtain the "density" balance of Eq. (8), giving the expression for V_R in Eq. (9).]

$$n_{\mu} = n_c \exp(Y) \cdot \left\{ \operatorname{erfc}(\sqrt{Y}) - \left(1 - \frac{1}{R}\right)^{1/2} \right. \\ \left. \cdot \exp\left(\frac{Y}{R-1}\right) \cdot \operatorname{erfc}\left[(RY/R-1)^{1/2}\right] \right\}, \quad (12)$$

where $Y = q\phi_i/T_c$, R is the plug mirror ratio, and erfc is the complementary error function:

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} \exp(-x^2) dx. \quad (13)$$

SOLVING THE TIME DEPENDENT EQUATIONS

For times longer than τ_{ce} , the plasma may be assumed to satisfy conditions of quasi-neutrality, particularly in the rapidly varying plug (if not true, the potentials readily become unrealistically large). The tandem's least confined particles are plug electrons, but rapid changes in the plug ion density generally determine quasineutrality conditions in this region because the ion loss rate in the plug is not very sensitive to changes in ϕ_i (provided $\phi_i \ll \phi_p$), which largely determine the electron transport.

In the TOARBUG model, the plug ion rate-of-change is set equal to the electron rate to enforce quasi-neutrality over time intervals $\Delta t > \tau_{ce}$:

$$\frac{\Delta n_i}{\Delta t} = \frac{\Delta n_e}{\Delta t}, \quad (14)$$

the value of $\Delta n_i/\Delta t$ being determined by beam trapping and various loss rates present in the plug (using the previous time step ϕ_p). This gives, for Eq. (4), the formula:

$$\frac{dn_p}{dt} = \frac{\Delta n_i}{t} = -\frac{n_p}{\tau_p} \left[1 - (T_p/T_c)^{1/2} \frac{n_c}{n_p} \exp\left(\frac{q\phi_i}{T_p}\right) \right] + J_p. \quad (15)$$

This expression can be solved for $q\phi_i$:

Let $J_p' = J_p - \Delta n_i / \Delta t$, then

$$q\phi_i = T_p \ln \left[(T_c/T_p)^{1/2} \left(\frac{n_p - J_p' \tau_p}{n_c} \right) \right] . \quad (16)$$

This expression for $q\phi_i$ can be substituted into Eqs. (6), (8), and (11) to simplify them. The energy balance in the plug then becomes:

$$\begin{aligned} -J_p'(q\phi_i + T_c) + \frac{n_p}{\tau_p} (T_p - T_c) - (nT)_{ee} + (nT)_{e-ip} \\ + P_{aux} + J_p E_{IP} = 0 , \end{aligned} \quad (17)$$

which, using $n_p = n_i$, the ion density (local) can be solved approximately for T_p , the new plug electron temperature, in terms of T_c and ϕ_i . The center cell particle and energy balances become:

$$\frac{dn_c}{dt} = v_R \cdot \left(J_c v_c + 2J_p' v_p - \frac{n_c v_c}{\tau_c} - \frac{2n_p n_p v_p}{n_c \tau_c} \right) , \quad (18)$$

and

$$\begin{aligned} \frac{3}{2} \frac{dn_c T_c}{dt} = v_R \cdot \left\{ 2v_p \cdot \left[\frac{n_p}{\tau_p} (T_p - T_c) + J_p' T_c + (nT)_{ee} \right] \right. \\ \left. - v_c \cdot (nT)_{e-ic} + J_c v_c E_{IC} - \left(\frac{n_c v_c}{\tau_c} + \frac{2n_p n_p v_p}{n_c \tau_c} \right) \right. \\ \left. \cdot (q\phi_e + T_c) \right\} . \end{aligned} \quad (19)$$

Equations (18) and (19) can be solved iteratively with Eqs. (17), (12), and (7) to find T_p , T_c , and n_c (with n_p already determined by the plug ion density). Convergence of the solution is very rapid for $\Delta t (= \delta t) \ll$ least of $\{\tau_p, \tau_c, \tau_v\}$.

COMPARISON OF TOARBUC WITH EXPERIMENTAL TMX DATA OF SEPTEMBER 24, 1980

Paul Drake⁷ has made an extension analysis of the TMX data for September 24, 1980 including a comparison with results from the TAMBAR code. Figure 3 shows an extension of his analysis which includes TOARBUC code results. Except for the end-loss current, j , and the plug density, n_p , both codes match the experimental data reasonably well. TOARBUC is in good agreement with the plug density, but is too low by approximately a factor of 2 for the end-loss current, j ; whereas, TAMBAR agrees with the end-loss current but gives a value for the plug density which is too low by almost a factor of 3. In addition both codes give somewhat higher electron temperatures, especially for the center cell.

Some additional observations on the radial density profiles should be made. The line density, $\int n dl$, comparisons which are given in Fig. 3 obscure the large differences in the density profiles vs radius between the experimental data and TOARBUC as compared with TAMBAR. In the end plugs, the measured profiles are either flat or slightly hollow in the plasma core in agreement with the TOARBUC results (Fig. 4). The TAMBAR density profiles are very hollow with peak to valley density ratio of about 5 vs 1.2 or less for both TOARBUC and the experimental data. In the center cell, both the measured data and TOARBUC (Fig. 4) give relative flat profiles in the plasma core with a gaussian like fall-off in the plasma boundary. TAMBAR predicts a hollow plasma with a second density peak near the plasma boundary. This second peak is presumably due to the formation of molecular hydrogen ions. In the TOARBUC run it was assumed that these ions could be neglected due to their very short lifetime; the neutral gas in TOARBUC was modeled by Franck-Condon atoms which with their longer mean free path results in density profiles

Fig. 3. Comparison of buildup codes with experiment.

Fig. 4. Plug and center cell densities.

without a density peak. A small amount of lower energy atoms (17 A at 0.08 eV) was also included as a source impinging on the plasma.

The TOARBUC input parameters for the comparison of Fig. 3 are given in Table II. Full- and half-energy neutral beams were injected in the plug. The half energy component was reduced slightly to account for a small third energy component. In both TOARBUC and TAMBAR an enhanced loss rate was introduced. In TAMBAR the classical loss rate was increased by a factor of 10. In TOARBUC, the radial transport term was calculated by a term

$$\frac{1}{R} \frac{\partial}{\partial R} \left(R \frac{\partial D}{\partial R} \right),$$

where D is the guiding center distribution function and R was taken to have the constant value of 1×10^6 cm³/s.

Historically, cold ions resulting from background gas ionization have been neglected in many buildup codes modeling plug densities. It has been assumed that these ions would be expelled from the plasma immediately by the plasma potential since they are born within the loss cone as modified by the potential. It has also been suggested that if these cold ions were input as a source and given a lifetime determined by their transit time to the mirrors that a cold plasma or "halo" surrounding the hot plasma would be created. The "halo" would shield the hot plasma from the thermal gas enabling buildup to occur with less injected beams or with higher background gas pressure.

To test this thesis, TOARBUC has been modified as discussed in the preceding paragraph by including the cold ions as a source (the associated electrons were included previously) with a transit lifetime. TOARBUC was then run with this modification and using the same input data given in Table II. Table III shows a comparison of some important parameters for these two runs. The retention of the cold ions in TOARBUC was observed to modify the plasma parameters by only a small amount. Figures 4, 5, and 6 show the particle

Table II. Input parameters.

Plug	Center cell
$E_{inj} = 18, 8.6 \text{ keV}$	$E_{inj} = 3, 0.08 \text{ eV}$
$I_{i,j} = 122 \text{ A}$	$I_D = 180 \text{ A}, 17 \text{ A}$
$\tilde{\phi} = 34 \text{ eV (for } \Delta\omega = 0.1 \omega_{ci})$	$\tilde{\phi} = 0$
$n_0^{FC} = 1 \times 10^{10} J_{trapped}/J_{norm} \text{ (atoms/cc)}$ $= 7.0 \times 10^9 \text{ (atoms/cc)}$	$\beta = 1 \times 10^6 \text{ cm}^3/\text{s}$

Table III.

Parameter	Run without halo	Run with halo
<u>Plug</u> f_{nd1}	$z=1^* 2.35 \times 10^{14}$	2.5×10^{14}
	$z=10 2.14 \times 10^{14}$	2.29×10^{14}
	$z=20 0.617 \times 10^{14}$	0.663×10^{14}
$T_{eq} (r = 0)$	98 eV	104 eV
$E_i (r = 0)$	11.0 keV	11.1 keV
$\phi_p (r = 0)$	489 V	499 V
$\hat{n}_p$ (at peak density)	1.6×10^{13}	1.7×10^{13}

* z is an index measuring the vertical displacement of the chord through the plasma.

Fig. 5. Plug temperatures vs radius.

Fig. 6. Center cell temperatures vs radius.

densities and temperatures vs radius for both the center cell and plug when the cold ions are retained.

Time has permitted only a limited comparison of TMX data with code results. Better agreement is to be expected in the future as modifications are made to both TOARBUC and TAMBAR to include additional physics of plug and center cell confinement. The next section discusses some modifications in progress which should improve the physics modeling in TOARBUC.

FUTURE DIRECTIONS

Code results tend to give somewhat higher electron temperatures than that observed experimentally in either the plug or center cell, indicating that the electron physics is incomplete. Two additional processes which would lower the electron temperature have been identified. Energetic neutral beams have been found to include approximately 2% oxygen as an impurity. The oxygen impurity is a source of cold electrons as it becomes ionized. Each cold electron results in the loss of a hot electron carrying out energy $\phi + T_e$ as the oxygen ion leaves the plasma. The cooling effect of oxygen on the electron temperature is presently included in the TOARBUC code; however, the electron temperatures are still higher than those frequently observed.

Higher gas pressure will lower T_e ; however, the plasma density calculated by TOARBUC does not buildup to the observed density with higher gas pressures. Recent experimental evidence indicates that higher gas pressure is responsible for lower electron temperatures on TMX. We believe that this dilemma can be explained by the penetration of neutral gas in the fan regions which lowers the electron temperature with minimum effect on the hot ions which are confined magnetically closer to the plug midplane. The fan penetration of neutral gas is presently being added to TOARBUC by averaging

the product $n^+ n_0$ along a field line from the midplane to the magnetic mirror. This results in a higher rate for neutral gas ionization in the plasma core and a rise in T_e from the cold electrons introduced. The end-loss current in the plasma core should also increase improving the agreement between the measurements and TOARBUC.

An additional modification in progress is to determine $\tilde{\phi}$ self-consistently by increasing $\tilde{\phi}$ until the end-loss current agrees with the required quasi-linear current. The perturbed potential, $\tilde{\phi}(r)$, would be calculated for each field line with local parameters averaged radially over an orbit diameter.

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