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The Visual Shape and Multipole Moments of the Sun

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Solar diameter measurements made in 1983 at the Santa Catalina Laboratory for Experimental Relativity by Astrometry (SCLERA) have been analyzed for the Sun's quadrupole and hexadecapole shape terms. Expressing the Sun's apparent diameter as a Legendre series, these two terms represent the P_2 and P_4 coefficients, respectively. The theoretical framework used to provide a relationship between the observed shape of the Sun and the multipole moments of the solar gravitational potential field has been improved to include in general the effect of differential rotation in both latitude and radius. The gravitational potential multipole moments, expressed as the P_2 and P_4 coefficients of a Legendre series, have been found to be $J_2 = (3.4 \pm 1.0)E{-6}$ and $J_4 = (1.7 \pm 1.1)E{-6}$, respectively.

The total apparent oblateness ΔR (equator-polar radii) found from SCLERA observations is $\Delta R = 13.8 \pm 1.3$ milliarcseconds. The surface rotation contribution $\Delta R'$ to the apparent solar shape is $\Delta R' = 7.9$ milliarcseconds. The quoted uncertainties represent formal statistical 1σ errors only.

Evidence for periodic shape distortions near the equator have also been found.

I. HISTORICAL SIGNIFICANCE

a) Experimental Relativity

The gravitational potential field associated with the mass quadrupole moment of the Sun creates principally a Newtonian perturbation on the orbi of the planets such that their perihelia are precessed. Since the perturbing potential in lowest order is $\propto 1/r^3$ (see equation 2), and because of the large ellipticity of Mercury's orbit, the precession for Mercury's orbit is greatest and consequently most easily detected. Unfortunately, general relativistic precession of the perihelia has proven difficult to decouple observationally from the precession caused from the quadrupole moment.

The equation for the perihelion advance for Mercury including relativistic and solar quadrupole moment (J_2) terms is:

$$\Delta\omega = 42.95\ [\tfrac{1}{3}(2 + 2\gamma - \beta) + 0.029\ J_2 \times 10^5] \qquad (1)$$

where $\Delta\omega$ represents the observed perihelion advance in arcseconds per century from oblateness or relativistic contributions, and γ and β represent parameters describing the theory of gravitation assumed in the parameterized post-Newtonian formalism (Will 1981). For general relativity (GR) these parameters are both unity. J_2 is a dimensionless parameter related to the gravitational potential Φ by:

$$\Phi(r,\theta,\phi) = -\frac{GM_\odot}{r}\left[1 - \sum_{\ell=2}^{\infty} \left(\frac{R}{r}\right)^\ell J_\ell \, P_\ell(\cos\theta)\right], \quad r > R \qquad (2)$$

where r, θ, and ϕ represent the radial, polar, and azimuthal coordinates of a spherical coordinate system. R is the solar radius, G is the gravitational constant, M_0 is the mass of the Sun and P_ℓ is the Legendre polynomial of degree ℓ.

b) Variability of the Solar Oblateness

The first modern oblateness measurement was performed in 1966 by Dicke and Goldenberg (1974) followed by the work at SCLERA in 1973 (Hill and Stebbins 1975). An important result from oblateness work at SCLERA in the early 1970's was the demonstration that extreme care must be exercised in relating an observed visual solar oblateness to intrinsic solar oblateness. An edge definition at SCLERA was developed that had high sensitivity to changes in the limb darkening function. The technique could distinguish between an actual diameter change and an apparent diameter change resulting from a variation in limb darkening functions between different polar angles.

In the 1980's, oblateness work has continued using both Princeton and SCLERA-type observations. Princeton-type solar observations were made in 1983, 1984, and 1985 (Dicke, Kuhn, and Libbrecht 1985, 1986, 1987). These results, along with the analysis of the 1966 observations and the SCLERA results from 1973 (Hill and Stebbins 1975), led to the suggestion by Dicke, Kuhn, and Libbrecht (1985, 1986, and 1987) that the intrinsic oblateness may be varying with the 11 year solar sunspot cycle (see summary of results in table 3). There was also marginal evidence for the 12.38 day sidereal period oscillation in 1983 and 1985, but no evidence for it in 1984.

Hill and Beardsley (1987, 1988) re-examined the published observational results and demonstrated statistically that the limb darkening function was different along orthogonal diameters for the 1983, 1984, and 1985 data sets of Dicke, Kuhn and Libbrecht (1985, 1986, and 1987). Also the form of the variations in the limb darkening functions changed every year. This result along with several other tests led Hill and Beardsley (1987, 1988) to the conclusion that a significant false oblateness due to the limb darkening function differences could remain in the results of Dicke, Kuhn and Libbrecht (1985, 1986, and 1987). The possibility for systematic error introduced by these differences considerably weakens the basis of the suggestion that the intrinsic oblateness (or surfaces of constant potential) may be varying with the solar cycle.

II. RELATIONSHIP BETWEEN THE VISUAL OBLATENESS AND THE GRAVITATIONAL POTENTIAL

The Sun's visual oblateness and gravitational potential have been traditionally related by use of Von Zeipel's Theorem (Von Zeipel 1924, Dicke 1970, Libbrecht 1984). This theorem states that if there are no surface stresses, such as those which may be introduced by velocity or magnetic fields, then surfaces of constant pressure, density, temperature, and Φ (gravitational potential) all coincide. The visual oblateness would then represent a constant gravitational potential surface.

A more general relationship can be derived by perturbing the momentum equation. From Beardsley (1987) the relationship between the observed oblateness and J_2 can be expressed as

$$\Delta R/R = -\sum_{\ell=2}^{\infty} (J_\ell + J_\ell') \; \Delta P_\ell \quad , \qquad (3)$$

where ΔR represents the difference in solar radius at any two solar polar angles. The coefficients $J_\ell' = [a_\ell(R)]/(gR)$ represent the surface stress contributions to $\Delta R/R$, and the ΔP_ℓ are differences in Legendre polynomials associated with the two solar polar angles for a given ℓ. The coefficients $a_\ell(r)$ are found to be

$$a_\ell(r) = \frac{-(2\ell + 1)\, r}{2\, \ell(\ell + 1)} \int_0^\pi P_\ell \, \frac{\partial(\sin\theta\, f'/\rho)}{\partial\theta}\, d\theta \quad . \qquad (4)$$

where f' represents an Eulerian perturbation in the $\hat{\theta}$ component of the surface stress.

For the surface stress introduced by solar rotation Beardsley (1987) found:

$$J_2' = 5.82 \times 10^{-6} \qquad (5)$$

and

$$J_4' = -0.59 \times 10^{-6} \qquad (6)$$

These results indicate an equatorial to polar distortion of $[\Delta R_e - \Delta R_p]/R = 8.4 \times 10^{-6}$.

III. THE SCLERA TELESCOPE

The SCLERA telescope, located near the top of Mt. Bigelow in the Santa Catalina Mountains, has been described in numerous documents (Zanoni 1966, Clayton 1973, Oleson et al. 1974, Patz 1975, Stebbins 1975, and Beardsley 1987). In figure 1 the optical and mechanical components are shown. The telescope is a Schupmann medial elevation-azimuth design, with a 12.5 cm f/100 objective, and color-correcting, folded, Mangin optics.

a) Solar Edge Definition

The solar edge is said to be located (see Hill, Stebbins, and Oleson 1975) at a radial distance q from the center when the following integral transform is zero:

$$F(I;q,a) = \int_{-1/2}^{1/2} I(q + a \sin \pi s) \cos 2\pi s \, ds \quad , \tag{7}$$

where I is the observed solar intensity as a function of radius and the parameter "a" is the amplitude over which the limb of the Sun is scanned. This definition has a high sensitivity to changes in slope of the limb darkening function making it possible to distinguish between apparent diameter changes and changes associated with the equipotential surfaces describing the internal mass distribution of the Sun. Although very sensitive to changes in the slope of the limb darkening function it has low sensitivity to a light redistribution on the limb caused by terrestrial atmospheric turbulence (Hill _et al_. 1975, Hill and Stebbins 1975). Inspection of equation 7 also shows that the definition is not affected by changes in atmospheric transparency.

IV. LEGENDRE POLYNOMIAL DESCRIPTION OF THE SOLAR IMAGE

Most of the systematic oblateness signals, that are introduced by the telescope, can be expressed very effectively in terms of Legendre series. In this description, an angular diameter measurement, $D(-\theta)$, of the solar image, in the solar coordinate system as observed projected down onto the solar detectors, can be written as:

$$D(-\theta) = \sum_{m=0}^{\infty} \left[J_m^* P_m(\theta) + \sum_{n=1}^{p-1} K_{m,n} P_m(\theta, \eta, AZ, Z, PA) \right] + \sum_{n=p}^{p+2} C_n B(\theta, a_i, a_j) \tag{8}$$

where P_m is the Legendre polynomial of degree m and the J_m^* represent intrinsic solar structure. J_m^* is related to the J_m used in describing the gravitational potential field in section 2 by

$$J_m + J_m' = -J_m^* / \bar{\Theta} \quad , \tag{9}$$

where $\bar{\Theta}$ is the mean angular diameter of the Sun. AZ, Z, PA and η, represent solar azimuth angle, solar zenith angle, solar position angle and solar parallactic angle respectively (for definitions see Smart 1977, Beardsley

1987). $K_{m,n}$ represent amplitudes associated with the instrumental and atmospheric distortions.

The primary problem with solar shape investigations has been establishing the existence of a scaling law which relates the false oblateness introduced by changes in the limb-darkening function to observable properties of the measurements. The 1983 SCLERA observations have several observables which may be used to establish the scaling law for both J_2^* and J_4^*. These observables are represented by the $C_n B(\theta, a_i, a_j)$ in equation 8. The values $B(\theta, a_i, a_j)$ are related to the diameter difference using two different values of a in the FFTD edge definition for a particular value of θ. For a complete description see Beardsley (1987).

V. RESULTS

The data were obtained between June 19 and July 20, 1983. After removal of observations contaminated by clouds and solar active regions, there were 1728 diameters found at each of the following locations: the pole, equator, and ± 45°. The coefficients in equation 8 were then determined from a phase sensitive, Fourier transform-least squares technique (Beardsley 1987). All sources of statistical and systematic errors have been listed in tables 1 and 2 for J_4^* and J_2^*, respectively. The best estimates of J_4 and J_2 are also listed. The statistical and systematic errors resulting from the instrument and terrestrial atmosphere are probably small and not a major source of error.

The uncertainty in the multipole moments due to a false oblateness created by changes in the limb darkening function is a more serious problem. However, the scaling parameters used in this work should minimize the magnitude of an error created by this effect.

The possibility of surface distortions also introduce some uncertainty in the current work, if they exist. Evidence for statistically significant amplitudes have been found at synodic frequencies of ν = 0.44, 0.88, and 1.76 μHz. These frequencies are in good agreement with a harmonic structure found by Hill and Czarnowski (1986). The ν = 0.88 and 1.76 μHz frequencies are similar to frequencies found by Dicke (1981) who analyzed 1966 Princeton-type oblateness observations, and Dicke, Kuhn, and Libbrecht (1985 and 1987) who used 1983 and 1985 Princeton-type oblateness observations (the harmonic pattern was not apparent in 1984 Princeton-type observations). The frequencies at ν = 0.88 and 1.76 μHz are also consistent with the harmonic structure found by Claverie et al. (1982) who analyzed whole disk intensity observations.

Using the observed value of $\Delta\omega$ (Shapiro et al. 1976) and equation 1 with $J_2 = (3.4 \pm 1.0) \times 10^{-6}$,

$$0.993 \pm 0.006 = \frac{1}{3}(2 + 2\gamma - \beta) \quad . \qquad (10)$$

The error represents the combination of measurement uncertainty for $\Delta\omega$ and the statistical uncertainty associated with J_2. The result is about 1σ from the general relativistic value of unity.

In table 3, which has been updated from Hill and Rosenwald (1986), are included all values of J_2 found from published visual oblateness observations and rotational splitting studies.

a) Absolute Errors from J_4

This is the first published result for J_4 derived from the visual solar shape and it is nearly consistent with a null value within the uncertainties as shown in table 1. The implications of this observation for an estimate of the reliability of the derived J_2 is very important. Most of the uncertainty remaining in this work is associated with solar phenomenon occurring near the equator (Beardsley 1987). Since a systematic error associated with the equator changes J_4 relative to J_2 in a well determined relationship (Beardsley 1987), any deviation of J_4 from a null value will be a measure of the systematic error in J_2 if it can be shown from independent theoretical or observational evidence that J_4 should be either zero or nearly zero.

It has been shown that this value is extremely small for internal rotation curves that are constant and equal to the surface rotation rate ($J_4 \sim 10^{-9}$, Ulrich and Hawkins 1981). If other viable rotation curves can be shown to produce similar values of J_4, then an important estimate of the systematic error related to solar phenomena has been achieved that has not been available in other solar shape studies.

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TABLE 1

J_4^* and J_4 Results

Statistical Error in J_4^*	± 2.1 marcsec
Scaling error due to I'	See section 5.2
Error resulting from variable J_4^*	See section 5.2
Best estimate of J_4^*	-2.1 ± 2.1 marcsec
Best estimate of J_4'	-0.6×10^{-6}
Best estimate of J_4	$(1.7 \pm 1.1) \times 10^{-6}$

TABLE 2

J_2^* and J_2 Results

Statistical error in J_2^*	± 1.9 marcsec
Scaling error due to I'	See section 5.2
Error resulting from variable J_2^*	See section 5.2
Best estimate of ΔD	27.5 ± 2.6 marcsec
Best estimate of J_2^*	-17.4 ± 1.9 marcsec
Best estimate of J_2'	5.8×10^{-6}
Best estimate of J_2	$(3.4 \pm 1.0) \times 10^{-6}$

Definitions

$$\Delta D = D(-90) - D(0)$$

$D(\theta)$ = Observed relative diameter at polar angle θ

$$J_2^* = -\frac{2}{3}\left[(\Delta D) + \frac{2}{7} \{ D(-90)+D(0)-D(-45)-D(45) \} \right]$$

$$J_4^* = \frac{16}{35} \left[D(-90)+D(0)-D(-45)-D(45) \right]$$

J_m' = Surface stress contribution to J_m

$$J_m = -(J_m^*/\bar{\theta}) - J_m'$$

$$\bar{\theta} = 1890 \text{ arcseconds}$$

TABLE 3

Summary of Efforts to Determine J_2

Rotational Splitting of Fine Structure[a]			$J_2 \times 10^6$
Duvall et al. (1984)			0.17 ± 0.04
Hill, Bos and Goode (1982)			5.5 ± 1.3
Hill et al. (1984)[b]			4.5
Hill, Rabaey and Rosenwald (1986)[c]			5.1 ± 1.2
Visual Solar Oblateness	**T[d]**	**ΔR[e]**	
Dicke (1981)[f]	1966	42.0 ± 2.0	22.8 ± 2.0
Hill and Stebbins (1975)	1973	9.2 ± 6.3	1.0 ± 4.3
Dicke, Kuhn and Libbrecht[g]	1983	18.2 ± 1.4	7.1 ± 0.9
		14.4 ± 4.1	4.4 ± 2.7
Beardsley (1987)[h]	1983	13.8 ± 1.3	3.4 ± 1.0
Dicke, Kuhn, and Libbrecht (1986)	1984	5.6 ± 1.3	-1.3 ± 0.9
Dicke, Kuhn, and Libbrecht (1987)	1985	14.6 ± 2.2	4.7 ± 1.5

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a The value obtained by Gough (1982) is not included because it was based on a preliminary set of multiplet classifications which was in error (cf. Hill, 1984).

b Based on rotational curve of Hill et al. (1984).

c The value of 7.7 ± 1.8 for J_2 reported by Hill, Rabaey and Rosenwald (1986) has been corrected for a factor of 2/3 omitted in their analysis.

d Year of visual oblateness observations.

e Value of apparent equatorial - polar surface radius in milliarcseconds.

f The original value of $J_2 = (24.7 \pm 2.3) \times 10^{-6}$ was found by Dicke and Goldenberg (1974).

g Two values are given based on whether or not a certain type of systematic error is taken into account. The values listed have been corrected for a small systematic error discussed by Dicke, Kuhn, and Libbrecht (1986).

h This value has a small J_4 contribution removed that other oblateness studies have assumed to be zero.

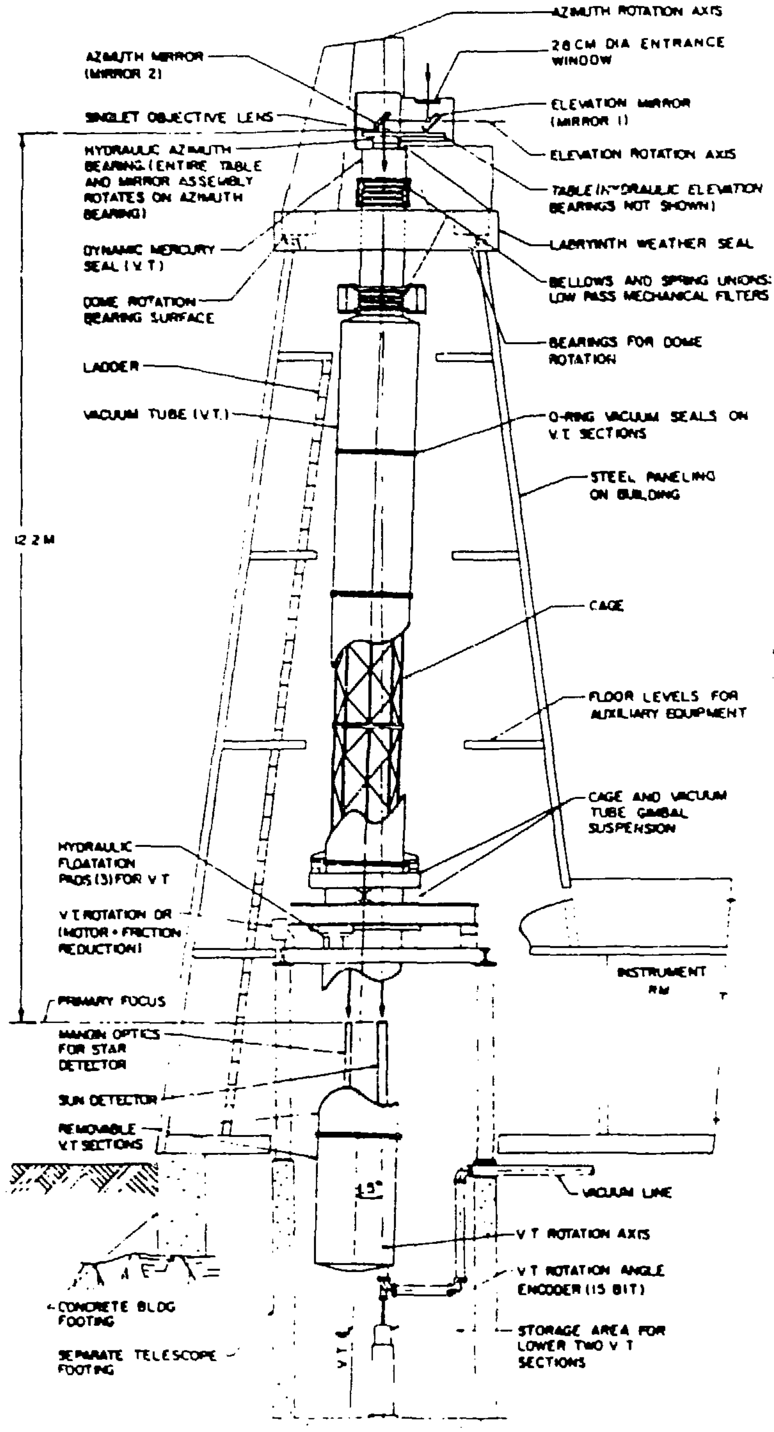

Figure 1. Cross Section of Telescope and Building.

This figure was reproduced from Stebbins (1975).