179
10-21-80
gsk

1859

ornl

ORNL/TM-7503

**OAK
RIDGE
NATIONAL
LABORATORY**

**UNION
CARBIDE**

**On Currents Induced in
Tokamaks by Electron
Cyclotron Heating**

O. C. Eldridge

MASTER

**OPERATED BY
UNION CARBIDE CORPORATION
FOR THE UNITED STATES
DEPARTMENT OF ENERGY**

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

Printed in the United States of America. Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road, Springfield, Virginia 22161
NTIS price codes—Printed Copy: A03; Microfiche A01

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

This book was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

ORNL/TM-7503
Dist. Category UC-20 g

Contract No. W-7405-eng-26

FUSION ENERGY DIVISION
ON CURRENTS INDUCED IN TOKAMAKS
BY ELECTRON CYCLOTRON HEATING

O. C. Eldridge*

Date Published - October 1980

NOTICE This document contains information of a preliminary nature.
It is subject to revision or correction and therefore does not represent a
final report.

*The University of Tennessee and Oak Ridge National Laboratory

Prepared by the
OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennessee 37830
operated by
UNION CARBIDE CORPORATION
for the
DEPARTMENT OF ENERGY

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

THIS PAGE
WAS INTENTIONALLY
LEFT BLANK

ABSTRACT

Generation of a plasma current is predicted in association with strong electron cyclotron heating in tokamaks or in any plasma with transverse magnetic field gradients. The current predicted in present-day tokamaks is of the order of one-quarter ampere per watt, which is large enough to be detected in heating experiments in progress. The current scales linearly with electron temperature and heating power and inversely with density and major radius. The mechanism depends on the Doppler shift for electrons streaming along magnetic field lines. Electrons streaming toward the source of radiation are resonant at a larger magnetic field. When the interaction is strong, radiation incident from the high field side is absorbed before reaching the cold electron resonant surface, and, so, a unidirectional population of electrons is heated. The anisotropic electron distribution gains momentum by collisions with ions. For small tokamaks the extraordinary wave should be launched for current drive, but for reactors the ordinary wave produces a sufficiently strong interaction.

1. INTRODUCTION

Plasma containment in tokamaks depends on a toroidal current, which produces a poloidal magnetic field and an associated rotational transform. In present-day tokamaks the plasma acts as the secondary of a transformer with the loop voltage and plasma current controlled by the changes in an externally produced magnetic flux. These tokamaks necessarily operate in a pulsed mode. However, significant structural problems are expected in the thermal cycling of a pulsed reactor.

To overcome these problems in the reactor, the induction of a steady current on the tokamak is needed. A practicable method of driving a steady current has been the object of several papers: theories of current drive by radio frequency waves have been published by Wort,¹ by Ohkawa,² by Fisch,³ and by Karney and Fisch.⁴ Other schemes for current drive include the use of relativistic electron beams⁵ and neutral beams.⁶ And, recently, currents driven by rf waves⁷ and by neutral beams⁸ have been observed.

Fisch and Boozer⁹ have pointed out that anisotropic electron heating can induce momentum in the electron population and, so, induce electron currents. An interaction that preferentially heats electrons streaming in one direction along a magnetic field induces an electron momentum in that direction, since the momentum transfer from the ions by collisions is smaller for the hotter electrons. Electron cyclotron heating, produced by externally launched waves, is such an interaction whenever the wave absorption is strong enough.

According to theory,¹⁰⁻¹² the cyclotron interaction is a resonant diffusion in velocity space, and in a spatially varying magnetic field, heating occurs on the surface where

$$\omega_c(\vec{r}) = \omega - k_{\parallel} v_{\parallel} . \quad (1)$$

The Doppler effect shifts the resonant surface for electrons streaming along the magnetic field. For electrons moving opposite to $k_{\parallel}$, the resonance is shifted to a larger magnetic field. For radiation incident from the high field side, electrons with $k_{\parallel} v_{\parallel} < 0$ are heated preferentially since the energy flux is attenuated. A diagram of the interaction is shown in Fig. 1(a). For small tokamaks the absorption of the extraordinary (X) mode is large enough¹⁰ for complete absorption by electrons with one sign of $v_{\parallel}$. For larger tokamaks the ordinary (O) mode has a strong enough interaction.

For radiation incident from the high field side, the ions should transfer momentum to the electron population in the direction opposite to $k_{\parallel}$ and induce a current in the direction of $k_{\parallel}$. For radiation incident from the low field side, a current is induced opposite to $k_{\parallel}$.

In Sect. 2 of this report, the microwave interaction is discussed, and the diffusion in velocity space is related to the absorption coefficient of the radiation. In Sect. 3, the electron-ion interaction is treated in the Fokker-Planck approximation. Electron-electron collisions are neglected although they may change the induced current by a factor of two or more. Results are presented in Sect. 4. Universal spatial form factors are shown for current and energy deposition. However, the current density and power density depend in an essential way on the

THE WAVE-PARTICLE INTERACTION IN A TOKAMAK

Fig. 1. (a) Diagram of the cyclotron interaction on the high field side of the resonant surface. (b) The coordinate system.

absorption coefficient and drift surfaces of the electrons. The total current induced is more simply parameterized. In practice, for sufficiently strong absorption, the total current is linearly dependent on power and on electron temperature and inversely proportional to the electron density and the major radius of the torus.

The time scales are very important in untangling the relevant physics. In this paragraph the numbers are calculated for the Impurity Study Experiment (ISX-B) tokamak, but the relative times are typical of all plasmas to which the theory may be applied. The shortest time (30 ps) is the cyclotron period of the electrons and the period of the incident wave. The quasilinear diffusion equation is valid when the electron distribution varies on a much slower time scale. The drift time (500 ns) of electrons around the torus is short enough so that the energy and current are rapidly spread over the entire drift surface. The current density and power density are found by averaging over this surface. The momentum transfer from ions to electrons occurs on a collisional time scale (20 μ s), whereas the velocity diffusion that produces electron heating occurs on the heating time scale (10 ms). Since electron-electron collisions have about the same time scale, the electron distribution remains almost Maxwellian. Also, heating terms are negligible when they occur additively with collision terms.

The current cannot be instantly injected into the center of a highly conductive plasma. The first effect is a back electromotive force (emf) opposing the change in magnetic flux produced by the change in current. The fields, a toroidal electric field and a poloidal magnetic field, must diffuse outward on a resistive time scale (200 ms).

This diffusion also depends on other energy sources and sinks and is not considered in this paper.

2. VELOCITY DIFFUSION FROM THE RESONANT INTERACTION WITH MICROWAVES

In the quasilinear limit^{10,13} on a time scale much slower than a cyclotron period, electron cyclotron heating has the form of a resonant diffusion in velocity space. For incident radiation in the x,z plane with the static magnetic field in the z direction, one finds

$$\frac{\partial f}{\partial t} = \sum_{\text{modes}} \left[\frac{1}{v_{\perp}} \frac{\partial}{\partial v_{\perp}} \left(N \omega_c H_N f \right) + \frac{\partial}{\partial v_{\parallel}} \left(k_{\parallel} H_N f \right) \right] \quad (2)$$

where the operator H_N is

$$H_N = |A_N^2| \delta(\omega - N\omega_c - k_{\parallel} v_{\parallel}) \left(\frac{N\omega_c}{v_{\perp}} \frac{\partial}{\partial v_{\perp}} + k_{\parallel} \frac{\partial}{\partial v_{\parallel}} \right),$$

and A_N is a linear combination of electric fields and Bessel functions of argument $k_{\perp} v_{\perp} / \omega_c$,

$$A_N = \frac{e}{2m\omega} \left(v_{\perp} J_{N-1} E^- + v_{\perp} J_{N+1} E^+ + 2v_{\parallel} E_{\parallel} J_N \right),$$

and

$$E^{\pm} = E_x \pm iE_y.$$

When the distribution function on the right is replaced by a Maxwellian, a valid approximation since electron isotropization proceeds on a faster time scale than the heating, one can compare the momentum and the energy transfer to the electrons. One finds

$$\frac{dp_{\parallel}}{dt} \cong \frac{k_{\parallel}}{\omega} \frac{dW}{dt} \cong \frac{\cos \psi}{c} \frac{dW}{dt} \quad (3)$$

for excitation at any harmonic and with any mode. The angle between $\vec{k}$ and the magnetic field at the edge of the plasma is ψ . This is just the radiation pressure and is rather small. It is best evaluated by expressing it as an effective electric field,

$$E_{\text{eff}} = - \frac{\cos \psi}{\bar{n} V c} \frac{dW}{dt}, \quad (4)$$

where V is the plasma volume and $\bar{n}$ the average density. For a power density of 500 kW/m^3 and $\bar{n} = 10^{14} \text{ cm}^{-3}$, one finds $E_{\text{eff}} \cong 10^{-11} \text{ V/m}$, which can be neglected.

For either current drive or heating, one makes the following simplifications and approximations:

(1) Plasma dispersion is treated in the WKB approximation. The index of refraction $\vec{n} = \vec{k}c/\omega$ is determined with ω and $n_{\parallel}$ real and $n_{\perp}$ complex.

(2) Since wavelengths are much shorter than the field curvature and other scale lengths, the plasma is treated as a plane with parameters varying only with x . By Snell's law $n_{\parallel} = \cos \psi$ is constant.

(3) The cyclotron frequency varies in space as

$$\omega_c(x) = \omega \frac{R_0}{R} = \omega \left(\frac{1-x}{R} \right), \quad (5)$$

where R_0 is the major radius of the tokamak and R the distance from the axis of symmetry. Radiation enters the plasma at $x = -a$ at angle ψ . From the Doppler shift one finds the characteristic spatial width of the resonance

$$\Delta = \cos \psi R \sqrt{\frac{T}{mc^2}}, \quad (6)$$

for thermal electrons.

(4) The cyclotron radius for thermal electrons is much smaller than a wavelength, so the small argument expansion is used for the Bessel functions. Similarly, terms involving derivatives by $v_{\parallel}$ are smaller by a factor $k_{\parallel} \sqrt{T/m}/\omega$ and are neglected.

(5) Ratios of the electric fields are found from Maxwell's equations, and their magnitudes are determined by the Poynting vector. The attenuation of the Poynting vector in space is $\exp [-\eta(x)]$ where the optical depth is

$$\eta(x) = -\eta(a) \frac{\sqrt{2}}{\pi} \int_{-a}^x \frac{dx}{\Delta} \operatorname{Im} \left[\frac{1}{Z} \left(\frac{x}{\sqrt{2}\Delta} \right) \right], \quad (7)$$

which applies to either mode. For approximately perpendicular propagation, one finds for the X mode,^{10,11}

$$\eta(a) = \pi^2 \frac{R}{\lambda} \frac{T}{mc^2} \cos^2 \psi \frac{(2 - \alpha)^{3/2} (1 + \alpha)^2}{\alpha}, \quad (8)$$

and for the O mode

$$\eta(a) = \pi^2 \frac{R}{\lambda} \frac{T}{mc^2} \frac{\alpha(1 - \alpha)^{1/2}}{1 + \cos^2 \psi (1 - \alpha)}, \quad (9)$$

where $\alpha = \omega_p^2/\omega_c^2$. (More general formulas are given in Ref. 10.)

With these approximations the wave-plasma interaction is described by

$$\frac{\partial F}{\partial t} = \frac{1}{v_{\perp}} \frac{\partial}{\partial v_{\perp}} \left[v_{\perp} D(x) \delta \left(\frac{v_{\parallel}}{c} - \frac{x}{R n_{\parallel}} \right) \frac{\partial f}{\partial v_{\perp}} \right], \quad (10)$$

where

$$D(x) = \frac{P_0 e^{-\eta(x)}}{m} \frac{\eta(a)}{2 \Delta n A} \left(\frac{T}{m c^2} \right)^{1/2} \frac{1}{|Z(\xi)|^2}. \quad (11)$$

Here, P_0 is the incident power, A is the area irradiated, and $\xi = x/\sqrt{2}\Delta$. The electrons circulate around the tokamak in a time much shorter than the time scale for heating or the current drive. One sees that the effective area irradiated is a band spread all around the device, $A = 4\pi R b$, where b is the half height of the radiation pattern. One should also take the rotational transform into account by an average over the drift orbit, which is almost circular. Neither the total current nor total heating rate depends on this average, but the current density and heating profile do.

3. COLLISIONS WITH IONS

The current is induced by momentum transfer from slow ions. Collisions with electrons occur at about the same rate but will be neglected since this interaction conserves the total electron momentum and energy. The electron distribution is to be expanded in Legendre polynomials as a function of the pitch angle χ ,

$$f(v, \chi, t) = \sum_{\ell=0}^{\infty} f_{\ell}(v, t) P_{\ell}(\cos \chi). \quad (12)$$

Only the first two terms are included in the present approximate treatment. The distribution f_0 determines density and energy, while f_1 determines momentum and current. To be consistent, electron-electron collisions need be included only with f_2 and higher terms.

The torus is treated as a straight cylinder of radius r with azimuthal angle θ and axial coordinate z , which is taken to be periodic. Spherical coordinates are used for velocity space with magnitude v and pitch angle χ . The rotation of the electron drift orbit about the magnetic axis with frequency $\omega_q = v_{\parallel}/Rq$ is included, where $q = rB_T/R_0 B_p$ is the safety factor.

The effect of the radiation on the population of trapped electrons will not be treated here although this interaction is the basis of a theory of current drive by Ohkawa.² Rough estimates predict that the velocity diffusion simply interchanges trapped and untrapped electrons without producing a current. The displacement of the electron drift orbit is usually small and amounts to choosing the magnetic axis appropriately. A toroidal electric field is also included. One must solve

$$\begin{aligned}
 \frac{\partial f}{\partial t} + v \cos \chi \frac{\partial f}{\partial z} - \omega_q \frac{\partial f}{\partial \theta} &= \frac{cE}{m} \left(\cos \chi \frac{\partial f}{\partial v} \right. \\
 &\quad \left. - \frac{\sin \chi}{v} \frac{\partial f}{\partial \chi} \right) + \frac{v_{ei}(v)}{2 \sin \chi} \frac{\partial}{\partial \chi} \left(\sin \chi \frac{\partial f}{\partial \chi} \right) \\
 &\quad + \left(\frac{\cos \chi}{v^2 \sin \chi} \frac{\partial}{\partial \chi} + \frac{1}{v} \frac{\partial}{\partial v} \right) \left[D \delta \left(\frac{v \cos \chi}{c} - \frac{r \cos \theta}{R n_{\parallel}} \right) \right. \\
 &\quad \left. \times \sin \chi \left(v \sin \chi \frac{\partial f}{\partial v} + \cos \chi \frac{\partial f}{\partial \chi} \right) \right] ,
 \end{aligned} \tag{13}$$

where

$$v_{ei}(v) = \frac{1}{v} \left(\frac{T}{m} \right)^{3/2} \frac{1}{v^3} = \omega_p^2 \frac{e^2}{m} \frac{\ln \Lambda}{v^3} . \quad (14)$$

The momentum transfer from ions is treated correctly; energy transfer is of higher order in the mass ratio and is neglected.

The electron drift occurs on a faster time scale than the collisions. The characteristics of these orbits are $\theta_o = \theta + \omega_q t$ and $z_o = z - v_{||} t$, so that the time enters explicitly only when defining the boundaries of the heating zone. Stated more simply, electrons receive an increment in perpendicular velocity when passing through the heating zone; they coast around the torus a few times and receive another impulse. Singular drift surfaces, on which electrons come back to the same spatial point after an integral number of traverses, pose a formal problem. That is, a completely periodic phase for electrons violates the random phase assumption implicit in the derivation of the quasilinear diffusion. Since the number of electrons in these singular orbits is small, they will be neglected.

By averaging Eq. (13) over the drift time scale, or equivalently averaged over θ and z , the stepwise increase in perpendicular energy is smoothed out. The integral over z results in an effective area of illumination, $A = 4\pi bR$. The integral over θ may be done with the delta function. The integral over the pitch angle χ is done first, and the average over θ indicated by angular brackets. The coupled equations for $\langle f_o \rangle$ and $\langle f_1 \rangle$ are

$$\frac{\partial \langle f_o \rangle}{\partial t} = \frac{eE}{3m} \frac{1}{v^2} \frac{\partial}{\partial v} (v^2 \langle f_1 \rangle) \quad (15)$$

$$+ \left\langle \frac{DC}{v^2} \frac{\partial}{\partial v} \left[\left(1 - \frac{u^2}{v^2} \right) \left(\frac{\partial f_o}{\partial y} + u \frac{\partial f_1}{\partial v} - u \frac{f_1}{v} \right) \right] \right\rangle ,$$

and

$$\begin{aligned} \frac{\partial \langle f_1 \rangle}{\partial t} = & \frac{eE}{m} \frac{\partial \langle f_o \rangle}{\partial v} - v_{ei} \langle f_1 \rangle \\ & + \frac{3}{2} \left\langle \frac{DCu}{v^3} \frac{\partial}{\partial v} \left[\left(1 - \frac{u^2}{v^2} \right) \left(\frac{\partial f_o}{\partial v} + u \frac{\partial f_1}{\partial v} - u \frac{f_1}{v} \right) \right] \right\rangle , \end{aligned} \quad (16)$$

where $u = xc/Rn_{||}$ and $v \geq |u|$ for the diffusion terms. The relaxation time for $\langle f_1 \rangle$ is now the collision time, whereas $\langle f_o \rangle$ changes much more slowly. The neglected electron-electron collisions would tend to keep $\langle f_o \rangle$ near Maxwellian, so in the roughest approximation one solves for the time-asymptotic $\langle f_1 \rangle$ with $\langle f_o \rangle = F$, a Maxwellian distribution:

$$\begin{aligned} v_{ei} v^3 \langle f_1 \rangle = & \frac{cE}{m} \frac{v^3 \partial F}{\partial v} \\ & + \frac{3}{2} \left\langle DCu \frac{\partial}{\partial v} \left[\left(1 - \frac{u^2}{v^2} \right) v \frac{\partial F}{\partial v} \right] \theta(v - u) \right\rangle . \end{aligned} \quad (17)$$

The current density is

$$J_{||}(r) = -e \left\langle \int d^3v v_{||} f \right\rangle = \frac{-4\pi e}{3} \int_0^\infty v^3 dv \langle f_1 \rangle$$

$$\begin{aligned}
= & \frac{-4\pi e}{3} \left\{ \frac{eE}{m} \int_0^\infty \frac{dv}{v_{ei}} v^3 \frac{\partial F}{\partial v} \right. \\
& \left. + \frac{3}{2} \left\langle \text{DCu} \int_{|u|}^\infty \frac{dv}{v_{ei}} \frac{\partial}{\partial v} \left[\left(1 - \frac{u^2}{v^2} \right) v \frac{\partial F}{\partial v} \right] \right\rangle \right\}.
\end{aligned} \tag{18}$$

4. CURRENTS AND HEATING

4.1 PROFILES

It is useful to express the current density in terms of a universal current profile $G_J(x)$, which shows the shape of the current that would be induced if there were no attenuation. One defines

$$G_J(x) = \frac{9\sqrt{2}}{16\pi} \xi \frac{\left[\frac{2}{\sqrt{\pi}} |\xi| e^{-\xi^2} + \left(1 - \frac{2\xi^2}{3} \right) \text{erfc}(\xi) \right]}{|Z(\xi)|^2}, \tag{19}$$

where $\xi = x/\sqrt{2}\Delta$ and erfc is the complementary error function.

The power profile,

$$G_P(x) = \frac{2}{\pi} \frac{e^{-\xi^2}}{|Z(\xi)|^2}, \tag{20}$$

shows the spatial power deposition with no attenuation. These two functions are shown in Fig. 2. One should note that with no attenuation the current integrates to zero. The optical depth of the incident radiation is

$$\eta(x) = \eta(a) \int_{-a}^x \frac{dx}{\Delta} G_P(x). \tag{21}$$

Fig. 2. Spatial dependence of electron heating and current with negligible attenuation.

The current density is

$$J_{\parallel}(r) = \frac{32}{\sqrt{2\pi}} \frac{e^2 n}{m \bar{v}} E + \frac{e P_o}{m b R \bar{v}} \sqrt{\frac{m}{T}} \left\langle \frac{\eta(a)}{\Delta} e^{-\eta(x)} G_J(x) \right\rangle, \quad (22)$$

and the kinetic energy density $U(r)$ is changing at the rate

$$\frac{\partial U}{\partial t} = J_{\parallel} E + \frac{P_o}{4\pi b R} \left\langle \frac{\eta(a)}{\Delta} e^{-\eta(x)} G_P(x) \right\rangle, \quad (23)$$

which includes the Joule heating. The electrical conductivity of Eq. (22) is the Lorentz conductivity. The electric field has been included in the calculation to show the effect of electron-ion collisions only on the conductivity and will be omitted in the following.

Figures 3-5 show the current density and power density when attenuation of the incident wave is included. The functions within the angular brackets of Eqs. (22) and (23) are plotted as functions of x , the distance from the cold electron resonant surface. They have not been averaged over the electron drift surface. The horizontal axis of these figures is $x/\sqrt{2\Delta} \cong v_{\parallel} \sqrt{m/2T}$, which shows directly the parallel velocity of the electrons being heated. One sees that these electrons have velocities not far from the thermal velocity. The magnitude of the interaction is parameterized by the optical depth $\eta(a)$. With the strongest absorption shown, $\eta(a) = 10$, the peak of the heating curve occurs at about 1.5 times the thermal velocity. Within a given interval of parallel velocity, centered at $v_{\parallel} = 1.5\sqrt{2T/m}$, there are 0.1 as many electrons as there are in the same interval centered at $v_{\parallel} = 0$. For even larger absorption, the velocity of the interacting electrons is not much greater.

ORNL-DWG 80-3029 FED

Fig. 3. The current density and power density with moderate absorption.

ORNL-DWG 80-3030 FED

Fig. 4. The current density and power density with essentially complete absorption.

ORNL-DWG 80-3031 FED

Fig. 5. The current density and power density with strong absorption.

This is an important point. In Sect. 1 the time scale for heating was found to be much longer than the time scale for collisions with ions, and this inequality was used in solving the Fokker-Planck equation. If a very small fraction of the electron population were heated, the heating time would be proportionally shorter, and heating and collisional diffusion could not be separated. In contrast, when electrons are heated by Landau damping near the lower hybrid frequency, the interaction occurs at velocities much greater than thermal. In such cases the time scales are comparable, and the analysis is more complicated.

When attenuation is included, a net current is induced. In Fig. 3 where $\eta(a) = 1$, or 63% of the incident power absorbed, the power profile is shifted toward $-x$, and there is a net current. In Fig. 4, in which $\eta(a) = 4$, or 98% absorption, there is very little negative current, and in Fig. 5, in which $\eta(a) = 10$, or $4.5(10)^{-5}$ transmission, there is essentially no negative current.

4.2 TOTAL CURRENT

The total current induced is obtained by integrating over the cross section:

$$I_{ECH} = - \frac{e P_0}{m R v} \sqrt{\frac{m}{T}} Q_J(\eta) , \quad (24)$$

where

$$Q_J(\eta) = \int_{-a}^a \frac{dx}{\Delta} \eta(a) e^{-\eta(x)} G_J(x) . \quad (25)$$

The total power absorbed by the plasma is

$$\frac{d}{dt} \int U \, dV = P_o Q_P(\eta) \quad (26)$$

where

$$Q_P(\eta) = \int_{-a}^a \frac{dx}{\Delta} \eta(a) e^{-\eta(x)} G_P(x) = 1 - e^{-\eta(a)} . \quad (27)$$

The dimensionless magnitudes $Q_P(\eta)$ and $Q_J(\eta)$ are plotted in Fig. 6. The current generated increases linearly for small η and then increases more slowly. An analytic expression for this function has not been found. The important point is that very strong absorption is needed for current generation, far stronger than is required for essentially complete absorption. The numerical value of the current is

$$I_{ECH}(A) = 3.83(10)^{13} \frac{P_o(w)T(keV)}{R(m)\bar{n}(cm^{-3})} \frac{Q_J(\eta)}{\ln \Lambda} \quad (28)$$

for collisions with protons, where $\ln \Lambda$ is the Coulomb logarithm.

4.3 DRIFT SURFACE AVERAGES

The electron drift may be accurately approximated as a circular helix centered on the magnetic axis. The location of the magnetic axis is close to the center of the minor cross section of the tokamak, but it is dependent on the plasma equilibrium and current, including the current induced by radiation. The drift surface is approximated here as

$$x = \delta + r \cos \theta , \quad (29)$$

Fig. 6. The normalized current and power absorbed as a function of absorption coefficient.

where δ is the displacement from cold cyclotron resonance as illustrated in Fig. 1(b). The average over the azimuthal angle is done numerically, and the resulting current and power densities are illustrated in Figs. 7 and 8. The parameters of the ISX-B tokamak are used for this example.

The radial dependence depends on the radiation pattern of the launching structure or its vertical projection on the resonant surface. This pattern is approximated as a Gaussian with spatial half-width b , given by

$$b = 1.22 \frac{a\lambda}{d \sin \psi} , \quad (30)$$

where λ is the free-space wavelength and d is the diameter of a circular waveguide aperture.

For $\delta = 0$ the current and power profiles are hollow since the absorption is maximum at 9 cm from the cold resonant surface. By moving the magnetic axis toward the source, one can produce a more narrow current profile peaked in the center. If one assumes that the magnetic axis is the geometric minor axis, the peaking of the current density is accomplished by increasing the magnetic field. For the given example, increasing the toroidal field from 10 kG to 11 kG should move the resonant surface 9 cm outward so that current is generated at the plasma center. One really expects the magnetic axis to be determined by the details of the equilibrium or principally by the current. A non-linear shift in the magnetic axis is expected.

ORNL-DWG 80-3033 FED

Fig. 7. The radial profile of the induced current density for ISX-B parameters.

ORNL-DWG 80-3034 FED

Fig. 8. The radial profile of the power density for ISX-B parameters.

5. DISCUSSION

5.1 PREDICTIONS FOR AN EXPERIMENT

An experiment is under way on the ISX-B tokamak using parameters that were given in the last section. With only ohmic heating the electron temperature should be about 1 keV. With 100 kW of 28-GHz microwave power in the X mode, $\eta(a) = 5.6$, and the induced current is predicted to be 15 kA. The gyrotron is capable of 100-ms pulses, and during the pulse, the electron temperature should rise to 1.5 keV and produce 30 kA of electric current.

5.2 THE NRL/ORNL EXPERIMENT

In the experiment by Gilgenbach et al.¹⁴ on the ISX-B tokamak, electrons were heated at a frequency of 35 GHz by unpolarized radiation launched from the high magnetic field side of the machine at an angle of $\psi = 45^\circ$. The absorption coefficient was $\eta(a) \cong 5$ for the X mode, which was sufficient for current drive, and $\eta(a) \cong 0.6$ for the O mode, which produced little current. The temperature increased from 850 eV to 1250 eV during the pulse; accordingly, a current of about $I_{ECH} \cong 10$ kA should have been produced. The radiation was directed so that the induced current should have opposed the ohmic current.

As discussed in Sect. 1, tokamaks are usually operated at constant current, so the loop voltage should have increased by ten percent. Actually, this effect was completely masked by the increase in plasma conductivity with increased temperature, which produced a decrease in loop voltage by a factor of two. To detect an induced current of this

magnitude, one should compare the changes in loop voltage as the direction of the radiation is changed.

Also, in this experiment the heating power was pulsed for only 15 ms, which was a much shorter time than the resistive diffusion time. The current could not have diffused out of the tokamak by classical diffusion.

The current produces macroscopic forces on the plasma and changes the plasma equilibrium. The changes in equilibrium can be drastic with large temperature and current gradients and with hollow current profiles, and such shifts in the equilibrium may very well have caused the sudden decrease in plasma density observed by Gilgenbach et al. The effects of induced currents of the magnetohydrodynamic equilibrium and stability are being studied numerically by John Hogan. The model used has been applied to the currents associated with neutral beam injection.¹⁵

5.3 OTHER EXPERIMENTS

No current should have been generated in early experiments by Alikaev et al.^{16,17} and by Bulyginsky et al.¹⁸ on the FT-1 and TM-3 tokamaks, since the initial electron temperature was too low for strong absorption. The experiment by Alikaev¹⁹ on T-10 at the second harmonic should not produce any current either, since the radiation is incident at $\psi \approx 90^\circ$, and there is no Doppler shift.

5.4 INCLUSION OF ELECTRON-ELECTRON COLLISIONS

Electron-ion collisions have been treated as if the ions were immobile; that is, the energy transfer to ions has been neglected. As a

result, the Lorentz electrical conductivity appears in Eq. (19). To obtain the more correct Spitzer-Härm conductivity, electron-electron collisions must be included. The correction is a factor of about two when the plasma has ions with $Z = 1$.

There is some evidence that the current drive interaction is more sensitive to the details of collisions than is the conductivity. Whenever one uses an electron-ion collision term of the form $\nabla_v \cdot (v_{ei} \cdot \vec{v} f)$ instead of the term in Eq. (13), one reduces the electrical conductivity by a factor of four, but one reduces the rf current drive by a factor of 12. For accurate predictions, one clearly must include electron-electron collisions. This calculation will be done in the near future.

5.5 APPLICATION TO FUSION REACTORS

Clearly the application of this means of driving currents will be of great importance in reactors. Corrections to the theory from more exact treatment of collisions are not expected to change the scaling; rather, only the numerical factor is expected to be changed. The induced current density, averaged over the minor cross section, scales as

$$\bar{J} = \frac{\text{const}}{\bar{n}} \left(\frac{P_o}{V} \right) T \quad (31)$$

where V is the plasma volume. The energy density of the plasma contained is proportional to the plasma pressure or to the square of the poloidal magnetic field. One expects that the energy density contained should scale as

$$\bar{U} = \text{const } \bar{J}^2 a^2 \quad (32)$$

where a is the minor radius. By holding the heating power density constant and by increasing the electron temperature in proportion to the electron density, $\bar{J}$ is held constant and the energy density contained should increase with the size of the tokamak.

ACKNOWLEDGMENTS

I have benefited greatly from discussions with John Hogan and with Steven Hirshman.

**THIS PAGE
WAS INTENTIONALLY
LEFT BLANK**

REFERENCES

1. D. J. H. Wort, Plasma Phys. 13, 258 (1971).
2. T. Ohkawa (General Atomic), GA-A13847, 23 Feb. 1976 (unpublished).
3. N. J. Fisch, Phys. Rev. Lett. 41, 873 (1978).
4. C. F. F. Karney and N. J. Fisch, Phys. Fluids 22, 1817 (1979).
5. A. Mohri et al., *Proc. Fifth Int. Conf. on Plasma Physics and Controlled Nuclear Fusion Research*, Vol. III, p. 311 (1979).
6. T. Ohkawa, Nucl. Fusion 10, 185 (1970).
7. K. Wong, R. Horton, and M. Ono, Phys. Rev. Lett. 45, 117 (1980).
8. K. B. Axon et al., *Proc. Sixth Int. Conf. on Plasma Physics and Controlled Nuclear Fusion Research*, Brussels, 1980 (to be published).
9. N. J. Fisch and A. H. Boozer (Princeton Plasma Physics Laboratory), PPPL-1657, May 1980 (unpublished).
10. The interaction is discussed by O. Eldridge, W. Namkung, and A. C. England, "Electron Cyclotron Heating in Tokamaks," submitted to Nucl. Fusion.
11. Wave damping is calculated by A. G. Litvak, G. V. Permitin, E. V. Suvorov, and A. A. Frajman, Nucl. Fusion 17, 659 (1977).
12. Relativistic effects are calculated by I. Fidone, G. Granata, G. Ramponi, and R. I. Meyer, Phys. Fluids 21, 645 (1978).
13. The quasilinear equation was derived by C. F. Kennel and F. Engelmann, Phys. Fluids 9, 2377 (1966).
14. R. M. Gilgenbach et al., Phys. Rev. Lett. 44, 647 (1980).
15. J. Hogan, "Neoclassical Current Effects in Neutral Beam-Heated Tokamak Discharges," to be published in Nucl. Fusion.

16. V. V. Alikaev, G. A. Bobrovskii, M. M. Ofitservo, V. I. Poznyak, and K. A. Razumova, *Zh. Eksp. Teor. Fiz. Pis'ma Red.* 15, 41 (1972) [*JETP Lett.* 15, 27 (1972)].
17. V. V. Alikaev, G. A. Bobrovskii, V. I. Poznyak, K. A. Razumova, V. V. Sanikov, Y. A. Sokolov, and A. A. Shmarin, *Fiz. Plazmy* 2, 390 (1976) [*Sov. J. Plasma Phys.* 2, 212 (1976)].
18. D. G. Bulyginsky, V. E. Golant, M. M. Larinov, L. S. Levin, and N. V. Shustova, *Proc. Ninth European Conf. on Controlled Fusion and Plasma Physics*, p. 547 (September 1979).
19. V. S. Strelkov, *Proc. Ninth European Conf. on Controlled Fusion and Plasma Physics*, Vol. II, p. 387 (1979).

INTERNAL DISTRIBUTION

1. F. W. Baity	23. M. Murakami
2. J. A. Cobble	24. Y-K. M. Peng
3. R. J. Colchin	25. J. A. Rome
4. R. A. Dory	26. J. Sheffield
5. J. L. Dunlap	27. D. J. Sigmar
6. H. O. Eason	28. W. L. Stirling
7. P. H. Edmonds	29. D. W. Swain
8-9. O. C. Eldridge	30. C. C. Tsai
10. A. C. England	31. K. T. Tsang
11. J. C. Glowienka	32. N. A. Uckan
12. H. H. Haselton	33. T. Uckan
13. S. P. Hirshman	34-35. Laboratory Records Department
14. J. T. Hogan	36. Laboratory Records ORNL-RC
15. W. A. Houlberg	37. Document Reference Section
16. E. F. Jaeger	38-39. Central Research Library
17. T. C. Jernigan	40-41. Fusion Energy Division
18. P. W. King	Library
19. A. G. Kulchar	42. Fusion Energy Division
20. C. M. Loring	Reports Office
21. J. F. Lyon	43. ORNL Patent Office
22. D. H. Metzler	

EXTERNAL DISTRIBUTION

44. D. J. Anthony, Manager, Advanced Energy Systems, Energy Systems Program Department, Bldg. 23, Room 290, General Electric Company, Schenectady, NY 12345
45. Bibliothek, Max-Planck Institute für Plasmaphysik, 8046 Garching bei München, Federal Republic of Germany
46. Bibliothèque, Service du Confinement des Plasmas, C.E.A., B.P. 6, 92, Fontenay-aux-Roses (Seine), France
47. D. Blackfield, Plasma Fusion Center, Massachusetts Institute of Technology, 167 Albany Street, Cambridge, MA 02139
48. J. D. Callen, Nuclear Engineering Department, University of Wisconsin, Engineering Research Building, 1500 Johnson Drive, Madison, WI 53706
49. L. Cheung, Department of Electronics, University Science Center, The Chinese University of Hong Kong, Shatin, N.T., Hong Kong, China
50. F. F. Cap, Innerebnerstr 40, Innsbruck, Austria, Europe
51. CTR Library, c/o A. F. Haight, United Technologies Research Laboratory, East Hartford, CT 06108
52. CTR Reading Room, c/o A. N. Kaufman, Physics Department, University of California, Berkeley, CA 94720

53. J. N. Davidson, School of Nuclear Engineering, Georgia Institute of Technology, Atlanta, GA 30332
54. S. O. Dean, Director, Fusion Energy Development, Science Applications, Inc., 2 Professional Drive, Suite 249, Gaithersburg, MD 20760
55. Documentation S.I.G.N., Department de la Physique du Plasma et de la Fusion Controlee, Association EURATOM-CEA sur la Fusion, Centre d'Etudes Nucleaires, B.P. 85, Centre du Tri, 38041 Cedex, Grenoble, France
56. W. R. Ellis, Office of Fusion Energy, Office of Energy Research, Mail Station G-256, Department of Energy, Washington, DC 20545
57. H. K. Forsen, Exxon Nuclear Co., Inc., 777 106th Avenue, NE, C-000777, Bellevue, WA 98009
58. H. P. Furth, Princeton Plasma Physics Laboratory, Princeton University, Forrestal Campus, P.O. Box 451, Princeton, NJ 08540
59. R. W. Gould, California Institute of Technology, Mail Stop 116-81, Pasadena, CA 91125
60. R. L. Hirsch, Exxon Research and Engineering, P.O. Box 101, Florham Park, NJ 07932
61. R. A. Huse, Manager, Research and Development, Public Service Gas and Electric Company, 80 Park Place, Newark, NJ 07101
62. V. E. Ivanov, Physical-Technical Institute of the Ukrainian S.S.R. Academy of Sciences, 310108 Kharkov, U.S.S.R.
63. A. Kadish, Office of Fusion Energy, Office of Energy Research, Mail Station G-256, Department of Energy, Washington, DC 20545
64. E. T. Karlson, Department of Technology, Box 534, S-75121 Uppsala, Sweden
65. Y. Kiwamoto, Research Institute for Energy Materials, Yokohama National University, Yokohama 232, Japan
66. R. N. Kostoff, Office of Fusion Energy, Office of Energy Research, Mail Station G-256, Department of Energy, Washington, DC 20545
67. L. M. Kovrizhnykh, Lebedev Institute of Physics, Academy of Sciences of the U.S.S.R., Leninsky Prospect 53, Moscow, U.S.S.R.
68. G. L. Kulcinski, Nuclear Engineering Department, 1500 Johnson Drive, University of Wisconsin, Madison, WI 53706
69. G. Laval, Groupe de Physique Theorique, Ecole Polytechnique, 91 Palaiseau, Paris, France
70. Library, Centre de Recherches en Physique des Plasma, 21 Avenue des Bains, 1007, Lausanne, Switzerland
71. Library, Culham Laboratory, United Kingdom Atomic Energy Authority, Abingdon, Oxon, OX14 3DB, United Kingdom
72. Library, FOM-Institut voor Plasma-Fysica, Rijnhuizen, Jutphaas, Netherlands
73. Library, Institute for Plasma Physics, Nagoya University, Nagoya, Japan 464
74. Library, International Centre for Theoretical Physics, Trieste, Italy
75. Library, Laboratorio Gas Ionizzati, Frascati, Italy
76. J. G. Lominadze, Academy of Sciences of the Georgian S.S.R., 8 Dzerzhinski Street, 38004, Tbilisi, U.S.S.R.
77. O. P. Manley, Office of Fusion Energy, Office of Energy Research, Mail Station G-256, Department of Energy, Washington, DC 20545

78. D. G. McAlees, Exxon Nuclear Company, Inc., 777 106th Avenue, NE, Bellevue, WA 98009
79. J. E. McCune, School of Engineering, Department of Aeronautics and Astronautics, Building 37-391, Massachusetts Institute of Technology, Cambridge, MA 02139
80. A. T. Mense, Subcommittee on Energy Research and Production, B-374, Rayburn House Office Building, Washington, DC 20515
81. C. Mercier, Service du Theorie des Plasmas, Centre d'Etudes Nucleaires, Fontenay-aux-Roses (Seine), France
82. K. Nishikawa, Institute for Fusion Theory, Hiroshima University, Higashisendamachi, Hiroshima, Japan
83. B. Outten, Jr., Western Metal Products Company, 1300 Weber Street, Orlando, FL 32803
84. R. E. Papsco, Grumman Aerospace Corporation, 101 College Road, Princeton, NJ 08540
85. O. Petrus, Supervisor, C.T.R. Theory Group, Plasma Physics Laboratory, AL. I. CUZA University, Iasi, Romania, 6600
86. D. Pfirsch, Institute for Plasma Physics, 8046 Garching bei München, Federal Republic of Germany
87. Plasma Physics Library, c/o Dr. P. Rosenau, Department of Mechanical Engineering, Technion-Israel, Institute of Technology, Haifa, Israel
88. Plasma Physics Group, Department of Engineering Physics, Australian National University, P.O. Box 4, Canberra A.C.T. 2600, Australia
89. A. Rogister, Institute for Plasma Physics, KFA, Postfach 1913, D-5170, Julich 1, Federal Republic of Germany
90. W. Sadowski, Office of Fusion Energy, Office of Energy Research, Mail Station G-256, Department of Energy, Washington, DC 20545
91. V. D. Shafranov, I. V. Kurchatov Institute of Atomic Energy, 46, Ulitsa Kurchatova, P.O. Box 3402, Moscow, U.S.S.R.
92. Y. S. Sigov, Institute of Applied Mathematics of the U.S.S.R. Academy of Sciences, Miuskaya, Sq. 4, Moscow A-47, U.S.S.R.
93. W. M. Stacey, Jr., School of Nuclear Engineering, Georgia Institute of Technology, Atlanta, GA 30332
94. V. I. Tereshin, Physico-Technical Institute of the Ukrainian S.S.R. Academy of Sciences, Kharkov, 310108 Kharkov, U.S.S.R.
95. J. B. Taylor, Culham Laboratory, United Kingdom Atomic Energy Authority, Abingdon, Oxon, OX14 3DB, United Kingdom
96. Thermonuclear Library, Japan Atomic Energy Research Institute, Tokai, Naka, Ibaraki, Japan
97. N. L. Tsintsadze, Institute of Physics, Georgian Academy of Sciences, Guramishvili 6, Tbilisi-FF, U.S.S.R.
98. K. Uo, Plasma Physics Laboratory, Kyoto University, Gokasho, Uji, Kyoto, Japan
99. F. Verdager, Director, Division of Fusion, Junta de Energia Nuclear, Madrid 3, Spain
100. Office of Assistant Manager for Energy Research and Development, Department of Energy, Oak Ridge Operations Office, Oak Ridge, TN 37830
- 101-276. Given distribution as shown in TID-4500, Magnetic Fusion Energy (Distribution Category UC-20 g, Theoretical Plasma Physics)