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**Application of Robertson's Collective Focusing
to Final Focusing***

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Application of Robertson's Collective Focusing to Final Focusing

Introduction

Recently Robertson⁽¹⁾ suggested that a collective focusing scheme might be useful in the final focusing problem for Heavy Ion Fusion. He demonstrates that it is possible to focus a charge and current neutralized beam, and shows this scheme is applicable since HIF scenarios have current requirements smaller than the electron Alfvén current. In this note ion emittance is included in the analysis of such lenses. It is found that one can focus an acceptable fraction of the beam, provided the current density is large enough.

Having a large current density guarantees that an inconsequential fraction of the ions are in a sheath region where the focusing field is non-linear. Such ions would not be focused properly. By using this consideration, an expression is derived which describes when such a system is useful for final focus. It contains the kinetic energy of the ion, the ion transverse emittance, and the target radius. It is then shown that this type of collective focusing could be useful for final focus of the high temperature experiment.

Finally it is also noted that the Cs^{+1} transport experiment at LBL is suited for carrying out collective focusing experiments, and with some modifications the large Cs^{+1} source might also be used. Doing such

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experiments would demonstrate that extrapolation of Robertson's proton beam experiment to the heavy ion case is valid.

I Review of Principles

Robertson's collective focusing scheme is a means by which a charge and current neutralized electron-ion beam system may be focused using standard solenoidal lenses. The idea is that focusing is accomplished, not by using the direct action of the magnetic field on the ions, but by using a polarization electrostatic field which arises since electrons and ions respond differently to the magnetic field. The result is that the focusing strength K of the solenoid is increased by the ratio m_i/m_e over unneutralized ion focusing, where m_i is the ion mass and m_e the electron mass. This increase is considerable; whereas conventional solenoidal ion focusing is unpromising due to the large field required, with a Robertson collective focusing (RCF) system the field requirements are strikingly modest and consequently very interesting.

Suppose a very dense neutralized beam were shot axially into a solenoid. The electrons, due to their large charge to mass ratio will respond to the magnetic field much more than the ions and they will compress. A negative charge density is created within the beam as this compression takes place. Consequently a radially inward pointing electric field is produced which acts to halt the compression of the electrons. This same electric field focuses the ions.

To compute the radial electric field when the compression stops, note that by conservation of canonical angular momentum the electrons must rotate at the Larmor frequency inside the lens. Consequently the electric field

should produce a Brillouin flow and its strength may be computed using force balance. The result is:

$$E_r = -\frac{1}{4} \frac{m_e}{e} \Omega_e^2 r \quad (1)$$

where Ω_e is the electron cyclotron frequency $eB/m_e c$ within the focusing solenoid and e is the proton charge.

Neglecting the direct action of the magnetic field, the equation of motion for the ions in this field is

$$\ddot{x}_t + Kx_t = 0$$

where

$$K = \frac{1}{4} \frac{\Omega_e \Omega_i}{v_z^2} = \frac{r_{ce} \beta^2}{8W} \quad (2)$$

Ω_i is the ion cyclotron frequency in the lens, v_z is the velocity along the axis of the solenoid, W is the ion kinetic energy, and r_{ce} is the classical electron radius. Observe that this is larger than the K for conventional ion solenoidal focusing by a factor of m_i/m_e . The second expression applies since non-relativistic beams are involved and emphasizes that K depends on the ion chosen only through the kinetic energy.

Now current limitations for this scheme may be derived. One requirement is that the axial magnetic field be unperturbed by the electron motion. This yields the familiar electron Alfvén current limitation $I < I_0 \beta_z$ where $I_0 = 17000$ Amps. When this inequality holds the diamagnetic effect of the electrons remains small.

Robertson also gives a lower limit on the current. According to eqn. 1, the radial electric field depends only on the external magnetic

field. The excess electron density is given by the divergence of the field:

$$\frac{1}{2} \frac{m_e}{e} \Omega_e^2 = 4\pi e \Delta n \quad (3)$$

Robertson demands quasi neutrality, i.e. $\Delta n \ll n$ or

$$\frac{1}{2} \frac{\Omega_e^2}{\omega_e^2} \ll 1, \quad (4)$$

where ω_e is the usual electron plasma frequency. Eqn. 4 provides a lower limit on the current.

Gathering these two results there is a current window where RCF might be used. It is conveniently summarized in the following inequality⁽²⁾

$$1 > \frac{I}{I_0 \beta_z} > \frac{1}{8} \frac{r_b^2}{c^2} \Omega_e^2$$

where r_b is the beam radius.

II General Discussion

Focusing Efficiency

Robertson's demand of quasineutrality can be replaced with the idea of focusing efficiency. This idea is particularly relevant when a lens is being designed. Only the ions that see a linear radial electric field are focused properly. The remaining ions are less strongly focused and consequently a halo will be produced. In other words the ion radial distribution will become less sharp; a tail forms outside the focused core. A

clear design goal is to make the number of ions in this halo as small as possible.

To get a crude estimate, suppose the beam was precisely charge neutralized before the lens. When the beam is inside the lens assume that the particle density is radially uniform for each of the species out to radius r_s where it drops to zero. Using

$$n_i r_i^2 = n_e r_e^2$$

and eqn. 3 to give the excess electron density needed to produce the radial electric field, it is found that

$$\left(\frac{r_e}{r_i}\right)^2 = \frac{1}{1 + \frac{\Omega_e^2}{2\omega_{eo}^2}}$$

where ω_{eo} denotes the electron plasma frequency before the lens. For $r < r_e$ the radial field is linear and for $r > r_i$ the field is zero by Gauss's law. In the region $r_e < r < r_i$ the field strength decreases as r increases and so ions in this region are not focused as strongly as the ions in the linear region. The fraction of ions focused by the linear radial field is

$$\eta = 1/(1 + \Omega_e^2/2\omega_{eo}^2) \quad (5)$$

This quantity will be called the focusing efficiency of the lens. Note that Robertson's quasineutral limit is $\eta \gg 1$.

Another interpretation of eqn. 5 is possible. First compute the potential difference between the axis and beam edge when the beam is inside the lens. Using eqn. 1 the result is

$$V = - \int_0^{r_e} E_r dr = \frac{n_e}{8e} \Omega_e^2 r_e^2 \quad (6)$$

Now an unneutralized electron beam of radius r_e would have a potential difference of

$$V_{sc} = \pi n e r_e^2$$

Thus it follows that

$$\frac{\Omega_e^2}{2\omega_{e0}^2} \doteq \frac{V}{V_{sc}}$$

Consequently one way to determine n is to compare the collective potential needed for the focusing field with the space charge potential that is available in the electrons. The lens operates best when $V \ll V_{sc}$, which may be regarded as another way of defining quasineutrality.

Penetration of Electrons

In the previous section the potential difference between the beam edge and axis was computed. Typically the electron kinetic energy before reaching the lens is much less than this potential. The following question thus arises, how do the electrons get into the lens in the first place?

One way electron penetration can be ensured is by placing a positively biased conducting tube around the beamline inside the solenoid. The bias voltage is chosen to be consistent with conditions when the beam is inside the lens, thus $V_{bias} = V$. The electric field acts to draw the electrons

in. This choice assures that the potential on axis is zero. Comparing $V(r)$ inside the lens with the transverse energy of electrons inside the lens it is seen that energy conservation may be satisfied without the longitudinal velocity of the electrons changing. In other words, the mirror effect of the magnetic field may be eliminated. Instead the magnetic field acts to convert electrostatic potential energy into the Brillouin rotational energy of the electrons.

Final focusing with a collective lens

It is assumed that the ion beam radius can be described by an envelope equation using eqn. 2 for K . The standard hard edge model for the magnetic field is also used. The envelope equation is

$$\left[\frac{d^2}{ds^2} + K \right] r - \frac{\epsilon^2}{r^3} = 0 \quad (7)$$

where ϵ is the transverse phase space area/ π . It must be solved for two cases: $K = 0$ and $K > 0$. The solutions to eqn. 7 are

$$r^2(s) = r_0^2 + \frac{(s-s_0)^2 \epsilon^2}{r_0^2} \quad K = 0 \quad (8)$$

$$r^2(s) = r_e^2 [\cos^2(\sqrt{K}s + \delta) + \frac{\epsilon^2}{Kr_e^4} \sin^2(\sqrt{K}s + \delta)] \quad K > 0 \quad (9)$$

The pairs (r_0, s_0) and (r_e, δ) are the constants of integration for the solution in a given region. For a region of $K = 0$ where a minimum in the envelope occurs, r_0 is the value at the minimum and s_0 is its position. When a maximum occurs in a $K > 0$ region, its value may be chosen to be r_e . Then $\delta = -\sqrt{K}s_0$, where s_0 is the position of the maximum. Physically valid solutions have both r and r' continuous at magnet edges.

The geometry of a final focusing lens is given in Figure 1. A round neutralized ion beam enters a solenoidal lens with zero envelope slope. The beam is focused and hits the target with size r_t . Assume $s = 0$ at the entrance of the lens. The solution is given by matching at the back end of the lens $s = L$.

$$r_0^2 \cos^2 \sqrt{K}L + \frac{\epsilon^2}{Kr_0^2} \sin^2 \sqrt{K}L = r_t^2 \left(1 + \frac{\epsilon^2 L^2}{r_t^4}\right)$$

$$\sqrt{K} (-2 \cos \sqrt{K}L \sin \sqrt{K}L) \left(r_0^2 - \frac{\epsilon^2}{Kr_0^2}\right) = -2 \frac{\epsilon^2 L}{r_t^2}$$

The first equation is continuity of r and the second is continuity of r' . The standoff distance L , emittance ϵ , and the target radius r_t will be regarded as given, and may be combined into a dimensionless parameter:

$$\alpha = \frac{\epsilon^2 L^2}{r_t^4}$$

The other dimensionless parameters appearing are, for example, $\Psi \equiv \sqrt{K}L$, $\sqrt{K}L$,

and r_0^2/r_t^2 . Since there are two equations, two of these parameters can be solved for in terms of α and the other. Some algebra yields

$$\cot 2\psi = \frac{1}{2} \left[\frac{\alpha+1}{\alpha} \sqrt{KL} - \frac{1}{\sqrt{KL}} \right] \quad (10)$$

and

$$\frac{r_0^2}{r_t^2} = \alpha + 1 + \frac{\alpha}{\sqrt{KL}} \tan \psi \quad (11)$$

giving ψ and r_0^2/r_t^2 directly in terms of α and the field since $\sqrt{K} \approx B$. The results are shown in Figures 2 and 3. Typically $\alpha \gg 1$ for final focusing problems. More algebra yields the approximate equations:

$$\cot \psi = \sqrt{KL} \quad (12)$$

$$\left(\frac{r_0}{r_t} \right)^2 = \frac{\alpha}{\cos^2 \psi} \quad (13)$$

For $\alpha > 30$, eqns. 12 and 13 are good to better than 4%. Notice that they could be derived directly from eqns. 8 and 9 if the $\sin^2 \psi$ terms were deleted⁽³⁾. It becomes somewhat clearer what approximation is made in eqns. 12 and 13 since the $\sin^2 \psi$ terms are due to emittance pressure. Eqns. 12 and 13 apply in cases where the pressure developed by the focusing system dominates the thermal pressure of the beam⁽⁴⁾. Another way of stating the same thing is that $r_\ell \gg r_t$, r_ℓ being the beam size at $s = \ell$.

Using eqn. 5 it is now possible to show

$$\eta^{-1} = 1 + \frac{W}{m_e c^2} (K r_0^2) \frac{I_0 B_z}{I} \quad (14)$$

With eqns. 12 and 13 one finds

$$Kr_0^2 = \frac{\epsilon^2}{r_t^2} (1 + KL^2) \quad (15)$$

and eqns. 14 and 15 yield

$$\eta_I^{-1} = 1 + \eta_I \frac{I_0 \beta_z}{I} (1 + KL^2) \quad (16)$$

where

$$\eta_I = \frac{W}{n_e c^2} \frac{\epsilon^2}{r_t^2} = \frac{A m_p}{2 m_e} \frac{\epsilon_n^2}{r_t^2} \quad (17)$$

Recall that $I < I_0 \beta_z$ must hold, so for a current window to exist in the first place, $\eta_I \ll 1$. The maximum fraction of the ion beam that can be focused is found by taking the limit $KL^2 \ll 1$:

$$\eta_{\max}^{-1} = 1 + \eta_I \frac{I_0 \beta_z}{I} \quad (18)$$

For this type of the collective focusing system to be useful $\eta_{\max}$ should be reasonably large. Note that in eqn. 17 the emittance is normalized. Also it is true that eqn. 18 holds even for $\alpha \ll 30$.

In applying RCF there are two regimes of interest. If

$$\eta_I \frac{I_0 \beta_z}{I} < .01$$

there is little difficulty in coming up with acceptable solutions, i.e. solutions focusing 90% of the ion beam. Thin lenses ($\psi \equiv \sqrt{KL} \ll 1$) are possible since $\sqrt{KL}$ may be greater than 1, and yet a large percentage of the beam is still focused. Some explicit examples are presented later.

If the above inequality does not hold, it is more difficult to find adequate solutions. From eqn. 16 it is found that K should decrease for fixed L in order to increase n . Thus Ψ approaches $\pi/2$ by eqn. 12, and the lens becomes long. Since r_0 must increase by eqn. 13, one can proceed in this manner only to the point that transverse size limitations come into play. Also potential matching is necessary in this case, since a potential gradient along the lens will apparently be needed. This gradient would account for the fact the the beam potential (eqn. 6) decreases substantially as the radius decreases in the lens. This problem has not been examined in detail, and some high temperature experiment cases fall in this range.

III Application to High Temperature Experiment

The question to be addressed in this section is what all this implies about application of Robertson's idea to the high temperature experiment. Requirements for such an experiment are generally stated with a goal of 100 eV spot temperature⁽⁵⁾. In particular a power density of $P = 10 \text{ TW/cm}^2$ and a specific energy deposition of 10 MJ/gm are necessary. Now the current per beamlet is

$$I = \frac{e}{n_b W} \pi P r_t^2,$$

where n_b is the number of beamlets. The maximum percentage of the beam focused becomes

$$\eta_{\max}^{-1} = 1 + \frac{e^2 n_b}{r_t^4} \frac{I_0^2 Z W A n_D}{\pi e P 2 m_e} \quad (19)$$

Several conclusions may be drawn from this expression. Since the current density at the source does not depend on the number of beams, it is usually

safe to assume that $n_b \epsilon_n^2$ does not depend on the number of beams either. In this case eqn. 19 implies that there is no advantage in changing the number of beams. Most important is the dependence on energy. According to eqn. 19 the collective focusing scheme works best by going to lower energies and higher currents. To retain the same range lighter ions could be used, which improves matters still further. The r_t^4 in the denominator is also significant.

In order to get a feel for some representative numbers, results are tabulated for the case of 100MeV sodium. Figure 4 gives the maximum expected focused fraction (eqn. 19) vs. ϵ_n for $P = 10\text{TW}/\text{cm}^2$ and 64 beams. In addition the following table gives lens and beam parameters for various values of the emittance and magnetic field. The values $L = 1\text{m}$ and $r_t = 1\text{mm}$ are assumed. V is the axis to beam edge potential difference discussed previously at the entrance to the lens.

Table I: Results for High Temperature Experiment Parameters
 $B(\text{Gauss})$ $r_0(\text{cm})$ $q(\text{m})$ ψ $\eta(\%)$ $V(\text{volts})$

$\epsilon_n = 4 \times 10^{-7}$ mrad $\eta_{\text{max}} = 90\%$					
125	2.28	7.4	1.39	89	1790
250	1.20	3.3	1.21	88	1980
375	.86	1.90	1.06	87	2290
500	.70	1.24	.92	85	2700
625	.62	.87	.81	82	3310
750	.56	.64	.72	79	3890
875	.53	.49	.64	76	4740
1000	.51	.39	.57	73	5730
$\epsilon_n = 2 \times 10^{-7}$ mrad $\eta_{\text{max}} = 97\%$					
125	1.14	7.5	1.39	97	450
250	.60	3.3	1.21	97	500
375	.43	1.87	1.04	96	570
500	.35	1.20	.89	96	670
625	.31	.83	.77	95	830
750	.29	.60	.67	94	1040
875	.27	.45	.59	93	1230
1000	.26	.35	.52	91	1490
$\epsilon_n = 1 \times 10^{-7}$ mrad $\eta_{\text{max}} = 99\%$					
125	.57	7.5	1.39	99	110
250	.30	3.2	1.17	99	120
375	.22	1.73	.96	99	150
500	.19	1.03	.76	99	200
625	.17	.66	.61	98	250
750	.16	.45	.51	98	320
875	.16	.33	.43	97	430
1000	.15	.25	.37	97	500

This calculation shows that focusing is possible at very modest magnetic fields and voltages. Note that n approaches the value calculated with eqn. 18 ($n_{\max}$) for weak magnetic fields and that $r_0 \rightarrow \sqrt{6} r_t$ for strong magnetic fields. Also for $\epsilon_n = 1 \times 10^{-7}$ mmrad/beamlet, acceptable thin lens solutions are possible.

IV Experimental Tests of Robertson Collective Focusing

In this section, experiments using both the transport experiment Cs^{+1} source and the large Cs^{+1} source are suggested. The principal goal of such experiments is apparent; one would like to demonstrate that heavy ions may be collectively focused using co-moving electrons. Two types of experiments are suggested. The first type involves a single lens and is much like Robertson's proton experiment. The second is a transport experiment, much like the present one.

Focusing Experiments

At present, the transport experiment Cesium source parameters are $W = 160\text{keV}$, $\beta_z = .00157$, $r_b \approx 1.1\text{cm}$, and $i = 14\text{mA}^{(6)}$. Using eqn. 14 it is then possible to estimate the percentage of ions focused as a function of B . The results appear in Figure 5. If a .7 meter solenoid is used and emittance of $\epsilon_n = 1.1 \times 10^{-7}$ mmrad is assumed, the following table is calculated:

B(Gauss)	$r_t(\text{mm})$	L(m)	ψ	$n(\%)$	V(volts)
25	8.7	.68	.65	94	17
30	7.1	.57	.78	92	24
35	5.8	.46	.91	89	33
40	4.8	.35	1.04	86	43
45	4.0	.25	1.17	83	54

Note that the magnetic fields are very small and hence fairly easy to produce. Note also that the focusing force and emittance defocusing are not very different inside the lens, and eqns. 10 and 11 must be used to compute the table. A smaller spot could be achieved if the emittance were smaller.

It seems that more impressive experiments could be done if the large source were used. It is presently envisaged that the ions will be neutralized soon after leaving the source. They drift to the second drift tube, where they are focused by a solenoid wrapped on the outside of the vacuum tank. The following tables are generated using the previous formulae and the parameter values $W = 200$ keV, $r_0 = 15$ cm, and $I = 335$ mA. The magnetic coil is assumed to be 3.1 m long (the length of the second drift tube) and the magnetic fields are chosen so that focus is achieved inside the diagnostic tank, 5.5 meters from the second drift tube:

B(Gauss)	r_t (cm)	L(m)	ψ	$n(\%)$	V(volts)
<hr/>					
$\epsilon_\eta = 1 \times 10^{-6}$ m rad					
<hr/>					
6.2	3.0	6.3	.64	97	190
6.4	2.8	5.8	.66	96	200
6.6	2.7	5.5	.68	96	220
6.8	2.5	5.1	.70	96	230
7.0	2.4	4.8	.72	96	240
<hr/>					
$\epsilon_\eta = 2 \times 10^{-6}$ m rad					
<hr/>					
5.9	6.2	6.1	.61	97	170
6.1	5.9	5.7	.63	97	180
6.3	5.6	5.4	.65	97	200
6.5	5.3	5.1	.67	96	210
6.7	5.1	4.8	.69	96	220

The magnetic field might have to be slightly higher to account for electron transverse temperature. The second emittance value is from the present experiment. The first value will probably be closer to reality since many of the grids in the present experiment will be removed. Observe that a factor of 5 decrease in the radius is indicated.

Transport Experiment

When $K = \epsilon^2/r_0^4$, eqn. 9 implies that the beam size remains constant as the beam translates longitudinally. In other words the collective focusing mechanism provides a means to transport the beam. Robertson's fluid model does not include this possibility. One can try to propagate the beam in this manner. The magnetic field needed for transport is

$$B^2 = \frac{\epsilon^2}{r_b^4} \frac{8W}{r_{ce}} = -\frac{4m_e c^2}{r_{ce}} \frac{\epsilon_n^2}{r_b^4} \quad (20)$$

For the single beam transport experiment this value is $B = 15$ Gauss and V can be computed to be about 6 Volts. It would be nice if the emittance could be varied so the details of eqn. 20 could be checked. The large source can be used in a transport experiment only if the beam is compressed first. Otherwise, it turns out that the magnetic field would be too small, i.e. smaller than the earth's ambient field.

In all these cases the question of how to neutralize the beam has been carefully avoided. The naive plan would be to have the beam neutralized with some sort of grid source. Whether this is sufficient is not clear.

More sophisticated electron sources have already been used to neutralize intense light ion beams with good success. It should be noted that if some of the ions are lost during neutralization, the focused percentages will become somewhat lower.

These results indicate that experiments could be done using the Cs^{+1} sources at LBL. They could extend Robertson's findings if the effects of transverse emittance were studied. It will be important to have such information since in large-scale applications the emittance effects are important. Finally, the suggested experiments would be unique in that heavy ions will be focused.

Conclusion

The principal purpose of this note is to show when RCF is suitable for final focusing applications. Calculations are done using an ion envelope equation and an expression is presented which indicates when RCF is useful. Applications to the heating experiment appear possible. The small magnetic fields required and the opportunity to neutralize the beam space charge forces during longitudinal compression and final focusing make the use of this idea very attractive.

It is important to do several experiments to demonstrate this collective focusing idea. In particular, experiments using the intense cesium beams at LBL are suggested. There are many important checks to be done before such a system is taken seriously in larger-scale applications. For example, determining the current limits more precisely would be useful.

Finally it should be emphasized that much needs to be done. Potential matching and neutralization appear to be the major problems. Also because large space charge potentials are present before neutralization, the electrons will be "born" hot. This might not be a problem for the transverse degrees of freedom as the magnetic field may be increased to bind them. It is not so clear what happens longitudinally, and this must be checked.

The author thanks Dr. Lloyd Smith for providing the initial encouragement to study collective focusing, and for considerably clarifying these results.

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Fig. 1 Lens Geometry

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Fig. 2 r_o/r_t vs. $\sqrt{KL}$ for various α

Fig. 3 ψ vs. $\sqrt{\kappa L}$, Note that corrections for various α are small.

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Fig. 4 Percentage of the beam focused vs. ϵ_n for various r_t . 100MeV sodium is the ion.

Fig. 5 Percentage of beam focused vs. B for Cesium transport experiment parameters.