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Transient Analysis of Vibrations in Nonideal Multilayered Piezoelectric Devices

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Marion L. Hodgdon

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TRANSIENT ANALYSIS OF VIBRATIONS IN
NONIDEAL MULTILAYERED PIEZOELECTRIC DEVICES

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ABSTRACT

A numerical method of solving the equations involved in the transient vibration analysis of nonideal multilayered piezoelectric is presented. The use of the computer code WONDY IV in obtaining the solution, a numerical example, and experimental data are described. In addition, a method is included for approximating the values of creep or relaxation functions from steady-state attenuation data.

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TRANSIENT ANALYSIS OF VIBRATIONS IN NONIDEAL MULTILAYERED PIEZOELECTRIC DEVICES

Introduction

The importance of considering nonideal material behavior in the steady-state vibration analysis of piezoelectric devices has been demonstrated.¹ In this report, the study of nonideal piezoelectric behavior is extended from the steady-state to the transient case. In transient analysis, the actual wave motion and information concerning the mechanical and electrical fields within materials need to be calculated. In the steady-state solution, the time dependence is assumed to be of the form $e^{i\omega t}$ and no attempt is made to derive actual particle motion or the temporal form of other time-dependent variables. While steady-state analysis is convenient for calculating resonant frequencies and is appropriate in many situations, several important problems, such as charging a capacitor with a piezoelectric device like the one shown in Figure 1, require transient analysis.

Figure 1. Cross Section of a Multilayered Piezoelectric Device. Layers 1 and 5 are piezoelectric transducers. Layer 3 is a nonpiezoelectric barrier. Layers 2 and 4 are conductive epoxy bonds. For the example presented in this paper, V_{in} is a 300-V pulse, R is $1\text{ M}\Omega$, and C is 20 pF . The transducers are PZT. The barrier is aluminum. The electroded surfaces are considered ideal.

Numerical solutions to transient problems involving ideal piezoelectric materials were presented by Lawrence and Davison² for the axial mode case and by Montgomery³ for normal mode propagation. Axial mode propagation has application to the power transmitting devices studied in Reference 1 and is the mode of propagation considered here. As in References 2 and 3, a numerical solution is obtained through the use of the finite difference wave propagation code WONDY.⁴

The type of devices to be analyzed consists of layers of piezoelectric and nonpiezoelectric materials. All materials are considered to be nonideal; that is, signals propagating through the various layers suffer attenuation because of the time-dependent responses of the materials themselves. This time dependence is included in the constitutive relations of each material through the use of time-dependent material parameters.

Motion is assumed to be one-dimensional and perpendicular to the electroded surfaces shown in Figure 1, i.e., axial mode. The information to be obtained includes stress, electric field, and particle motion as functions of time and position within the layers. If a voltage is applied to the layer, input current is part of the solution. If the layer is connected to an external circuit, the voltage and current in that circuit are calculated.

Constitutive Relations

Part of the solution is obtained by evaluating the constitutive relations that define stress and electric field for nonideal piezoelectric materials;⁵

$$\begin{aligned} T(x, t) = & c(o)S(x, t) + \int_0^t \frac{d}{dt} c(t - \tau) S(x, \tau) d\tau \\ & - h_D(o)D(t) - \int_0^t \frac{d}{dt} h_D(t - \tau) D(\tau) d\tau \end{aligned} \quad (1)$$

$$\begin{aligned} E(x, t) = & h_S(o)S(x, t) - \int_0^t \frac{d}{dt} h_S(t - \tau) S(x, \tau) d\tau \\ & + \beta(o)D(t) + \int_0^t \frac{d}{dt} \beta(t - \tau) D(\tau) d\tau . \end{aligned} \quad (2)$$

where

$S(x, t)$ = strain

$D(t)$ = electric displacement

$c(t)$ = elastic stiffness

$h_D(t), h_S(t)$ = piezoelectric coupling

$\beta(t)$ = inverse dielectric permittivity .

While other forms of the constitutive relations could be used, this choice with T and E as dependent variables is convenient for axial mode problems.

Assuming exponential time dependence and single relaxation times, the time-dependent material parameters used in Equations (1) and (2) can be expressed as⁵

$$\begin{aligned} c(t) &= c_e - (c_e - c_i) e^{-t/\tau_S} \\ h_D(t) &= h_e - (h_e - h_i) e^{-t/\tau_D} \\ h_S(t) &= h_e - (h_e - h_i) e^{-t/\tau_S} \\ \beta(t) &= \beta_e - (\beta_e - \beta_i) e^{-t/\tau_D} \end{aligned} \quad (3)$$

The subscripts e and i refer to equilibrium and instantaneous values, respectively.

To simplify notation and for ease of solution, Equations (1) and (2) can be written in terms of two state variables q_1 and q_2 :

$$T(x, t) = c_i S(x, t) + \frac{c_e - c_i}{\tau_S} q_1(x, t) - h_i D(t) - \frac{h_e - h_i}{\tau_D} q_2(t) \quad (4)$$

$$E(x, t) = -h_i S(x, t) - \frac{h_e - h_i}{\tau_S} q_1(x, t) + \beta_i D(t) + \frac{\beta_e - \beta_i}{\tau_D} q_2(t) \quad (5)$$

where

$$q_1(x, t) = \int_0^t e^{-(t-\tau)/\tau_S} S(x, \tau) d\tau \quad (6)$$

$$q_2(x, t) = \int_0^t e^{-(t-\tau)/\tau_D} D(\tau) d\tau \quad (7)$$

Values for q_1 and q_2 can be obtained by performing the integration specified in the above expressions. Alternately, by using Laplace transform techniques, differential equations can be developed for these state variables. Using the Laplace transform for convolutions,⁶

$$\mathcal{L} \left[\int_0^t f_1(\lambda) f_2(t - \lambda) d\lambda \right] = F_1(p) F_2(p) \quad ,$$

the Laplace transform of q_1 is

$$Q_1(x, p) = \frac{1}{p + \frac{1}{\tau_S}} S(x, p) \quad .$$

Rearranging and taking the inverse transform gives the following differential equation for q_1 :

$$\frac{\partial q_1(x, t)}{\partial t} + \frac{1}{\tau_S} q_1(x, t) = S(x, t) . \quad (8)$$

In the same way, a differential equation can be developed for q_2 :

$$\frac{dq_2(t)}{dt} + \frac{1}{\tau_D} q_2(t) = D(t) . \quad (8)$$

In order to solve for q_1 and q_2 , as well as to solve Equations (1) and (2) and calculate input or output current and voltage, values for strain and electric displacement must be obtained. The strain $S(x, t)$ can be calculated from changes in the material density $\rho(x, t)$:⁴

$$S(x, t) = \frac{\rho(x, 0)}{\rho(x, t)} - 1.0 . \quad (9)$$

The density depends on the value of the stress in the equation of motion and on the conservation of mass:

$$\rho = \frac{M}{\delta x} = \frac{M}{x_1 - x_2} \quad (\text{conservation of mass}) \quad (10)$$

$$\frac{d^2 x}{dt^2} = \frac{1}{\rho} \frac{\partial T(x, t)}{\partial x} \quad (\text{equation of motion}) \quad (11)$$

where

M = mass

x_i = location of the i th particle .

The electric displacement $D(t)$ is evaluated by using the definition of voltage and Equation (2).

$$\begin{aligned} V(t) &= - \int_{\Delta x} E(x, t) dx \\ &= h_i \int_{\Delta x} S(x, t) dx + \frac{(h_e - h_i)}{\tau_S} \int_{\Delta x} q_2(x, t) dx - \Delta x \beta_i D(t) \\ &\quad - \Delta x \frac{(\beta_e - \beta_i)}{\tau_D} q_2(t) \end{aligned} \quad (12)$$

where

Δx = distance between electrodes.

This equation may be rearranged to obtain

$$D(t) = \frac{h_i \int_{\Delta x} S(x, t) dx + \frac{(h_e - h_i)}{\tau_S} \int_{\Delta x} q_1(x, t) dx - \Delta x \frac{(\beta_e - \beta_i)}{\tau_D} q_2(t) - V(t)}{\Delta x \beta_i} \quad (13)$$

For input layers in which $V(t)$ is specified, this equation defines the electric displacement.

For an output layer connected to a resistive load, a differential equation for the electric displacement is derived from the relationship between voltage and current:

$$\begin{aligned} V(t) &= RI(t) \\ &= RA \frac{dD(t)}{dt} \end{aligned} \quad (14)$$

where

R = resistance

A = area of the electrode .

After substituting for $V(t)$ and rearranging, Equation (13) becomes

$$\begin{aligned} \frac{dD(t)}{dt} + \frac{\Delta x \beta_i}{RA} D(t) &= \frac{h_i}{RA} \int_{\Delta x} S(x, t) dx + \frac{(h_e - h_i)}{RA \tau_S} \int_{\Delta x} q_1(x, t) dx \\ &\quad - \frac{\Delta x (\beta_e - \beta_i)}{RA \tau_D} q_2(t) \end{aligned} \quad (15)$$

If the output circuit consists of a parallel combination of a resistance R and a capacitance C , the current into the load is

$$I(t) = \frac{V(t)}{R} + C \frac{dV}{dt} \quad (16)$$

The time derivative of the voltage is calculated from Equation (10) and substituted along with Equation (12) into the above expression for output current. The differential equation for electric displacement is written by using

$$I(t) = A \frac{dD(t)}{dt} \quad (17)$$

$$\begin{aligned}
\frac{dD(t)}{dt} + \frac{\Delta x \beta_1 K}{RC} D(t) = & \frac{-\Delta x K(\beta_e - \beta_1)}{\tau_D} \left(\frac{dq_2(t)}{dt} + \frac{q_2(t)}{RC} \right) \\
& + \frac{K(h_e - h_i)}{\tau_S} \left(\int_{\Delta x} \frac{\partial q_1}{\partial t}(x, t) dx + \frac{1}{RC} \int_{\Delta x} q_1(x, t) dx \right) \\
& + h_i K \left(\int_{\Delta x} \frac{\partial S(x, t)}{\partial t} dx + \frac{1}{RC} \int_{\Delta x} S(x, t) dx \right)
\end{aligned} \tag{18}$$

where

$$K = \frac{C}{\Delta x \beta_1 C + A}$$

Similarly, additional output circuits could be included by using Equation (12) and the appropriate expression for the relationship between voltage and current.

Numerical Solution

A finite difference approach to the set of equations outlined above provides the solution. To implement this approach, the material layers are subdivided into zones. A numerical solution is obtained at each zone at discrete time intervals. The sizes of the zones and the time intervals are determined by stability criteria. Each layer of material retains its own material parameters and perhaps its own constitutive relations. Finite difference analogs are substituted for many of the equations. In particular, finite difference substitutions are made for the equations involved in calculating the strain. Considerable storage must be arranged since values for all variables that are functions of x , such as stress, strain, electric field, and density, are obtained for each zone, of which there may be several hundred per layer. The size of the time step must be monitored and adjusted after each cycle to insure that stability criteria are not violated.

An existing wave propagation computer code that uses this finite difference approach and provides the necessary storage and stability monitoring is WONDY IV.⁴ In this code, the different constitutive relations used in stress calculations are contained in equation of state subroutines. The value of stress calculated by one of these routines is sent to the WONDY subroutine MOTION that uses a finite difference form of the equation of motion to solve for zone thickness, density, and strain. One of these equation-of-state routines can be replaced by one that solves the above constitutive relations so that WONDY can be used for nonideal piezoelectric problems. The necessary modifications to WONDY are presented in Appendix A.

Basically, material parameters, material zoning, and the size of the time step are transferred from WONDY to the replacement equation of state STAT4. Because of the need for the integral of strain for each zone calculation of electric displacement (Equations (13), (15), and (18)), the strain calculation is repeated in the new STAT4.

Values for the output layer state variable q_2 and the output electric displacement are calculated by using Equation (8), Equations (15) or (18), and the external differential equation solver RKF.⁷ For the state variable q_1 and the input state variable q_2 , the following integration scheme for Equations (6) and (7) is used. From Equation (6),

$$q_1(x, t) = e^{-t/\tau_S} \int_0^t e^{\tau/\tau_S} S(x, \tau) d\tau$$

Since the strain calculated by WONDY is constant over each time step, this expression can be written as

$$\begin{aligned} q_1(x, t_n) &= e^{-t_n/\tau_S} \sum_{i=1}^n \int_{t_{i-1}}^{t_i} e^{\tau/\tau_S} S(x, \tau) d\tau \\ &= e^{-t_n/\tau_S} \sum_{i=1}^n S(x, t_i) \int_{t_{i-1}}^{t_i} e^{\tau/\tau_S} d\tau \\ &= \tau_S e^{-t_n/\tau_S} \sum_{i=1}^n S(x, t_i) \left(e^{t_i/\tau_S} - e^{t_{i-1}/\tau_S} \right) \end{aligned} \quad (19)$$

where n is the number of elapsed time steps from $t = 0$ current time t_n . The corresponding equation for q_2 for the input layer is

$$\begin{aligned} q_2(t) &= \left[1.0 + \left(\frac{\beta_e}{\beta_i} - 1.0 \right) \left(1.0 - e^{(t_{n-1} - t_n)/\tau_D} \right) \right]^{-1} \\ &\quad \cdot \tau_D e^{-t_n/\tau_D} \left[\sum_{i=1}^n IS(t_i) \left(e^{t_i/\tau_D} - e^{t_{i-1}/\tau_D} \right) \right. \\ &\quad \left. - \frac{(\beta_e - \beta_i)}{\beta_i \tau_D} \sum_{i=1}^n q_2(t_i) \left(e^{t_i/\tau_D} - e^{t_{i-1}/\tau_D} \right) \right] \end{aligned} \quad (20)$$

where

$$IS(t_i) = \frac{h_i}{\Delta x \beta_i} \int_{\Delta x} S(x, t_i) dx + \frac{h_e - h_i}{\Delta x \beta_i \tau_S} \int_{\Delta x} q_1(x, t_i) dx - \frac{V(t_i)}{\Delta x \beta_i}$$

This expression was obtained by substituting Equation (13) into Equation (7) and using the finite difference approximation that strain, q_1 and q_2 are constant over each time step.

Information from STAT4 that is transferred back to WONDY for use in stability calculations or in the output formats include these state variables. strain, electric displacement, voltage, current, sound speed, stress, and electric field.

Use of this equation of state is not restricted to nonideal piezoelectric materials. Conductive and ideal nonpiezoelectric materials are also acceptable as long as the material parameters listed on the input cards are specified according to the instructions given in Appendix B.

Calculations Using Attenuation Data

While most of the materials which make up the layers of piezoelectric devices of the type shown in Figure 1 exhibit nonideal responses characteristic of the creep or relaxation functions given in Equation (3), values for the time constants or the equilibrium and instantaneous parameters for many of these materials are not given in handbooks or other references. Instead, a single material parameter, such as a dielectric constant or an elastic stiffness, is specified along with a value for attenuation α in units of decibels/meter or nepers/meter and the frequency in Hertz at which α was measured. The attenuation is related to sinusoidal waves and is assumed to be proportional to the frequency squared.⁸ Values for the material parameters for the creep or relaxation function could be calculated from these data. For several reasons, this calculation is seldom feasible.⁹ However, if two of the three material parameters are estimated, a value can be computed for the third. One choice is to use the given material parameter as the equilibrium value. This choice is reasonable since the material response is closer to equilibrium than instantaneous for the range of frequencies at which α is measured. If a value for the time constant τ is estimated, the instantaneous material parameter can be calculated by solving a quadratic equation. A less attractive choice involves solving a fourth order equation for the time constant when the instantaneous value is supplied. Either equation may be derived from the definition of the complex propagation constant used in steady state sinusoidal analysis. For the case of the elastic parameters,¹

$$k^* = \omega \sqrt{\rho/c^*} \quad (21)$$

where

ρ = density

$k^* = k - i\alpha$

k = propagation constant

α = attenuation

$$c^* = c_i + \frac{(c_e - c_i)}{1 + \omega^2 \tau^2} - i \frac{(c_e - c_i)}{1 + \omega^2 \tau^2} .$$

With c , α , and $\omega = 2\pi f$ given by a handbook or materials' manufacturer and

$c_e = c$

$c_i = Kc$

where

K = constant,

the imaginary parts of Equation (18) yield

$$\alpha = \omega \sqrt{\frac{\rho}{c}} \left[\frac{(\omega\tau - K\omega\tau)(1 + \omega^2 \tau^2)}{(K\omega^2 \tau^2 + 1)^2 + (\omega\tau - K\omega\tau)^2} \right]^{1/2} . \quad (22)$$

Rearranging Equation (22) results in a quadratic equation for K :

$$\begin{aligned} & \left(\alpha^2 \omega^4 \tau^4 + \alpha^2 \omega^2 \tau^2 \right) K^2 + \left(\frac{\omega^3}{c} \rho \tau + \frac{\omega^5}{c} \rho \tau^3 \right) K \\ & + \alpha^2 + \omega^2 \tau^2 \alpha^2 - \frac{\omega^3}{c} \rho \tau - \frac{\omega^5}{c} \rho \tau^3 = 0 . \end{aligned} \quad (23)$$

In Appendix A, the solution for K , and hence for $c_i = Kc$, is included as part of the equation of state.

Time-dependent dielectric or piezoelectric values could be estimated from related attenuation data, if available, by substituting β or h for c in Equation (23). These calculations are not shown in the examples in the appendices.

Example

Using the above method of solution and the modification to WONDY listed in Appendix A, the output voltage displayed in Figure 2 was calculated for the multilayered device of Figure 1. The following material parameters were used for the PZT transducers:^{1, 10, 11}

$$c_e = 1.52 \times 10^{11}$$

$$c_i = 1.48 \times 10^{11}$$

$$h_e = 2.26 \times 10^9$$

$$h_i = 2.15 \times 10^9$$

$$\beta_e = 1.36 \times 10^8$$

$$\beta_i = 2.84 \times 10^8$$

For the conductive epoxy bonds, the material parameters used in this calculation are¹²

$$c_e = 1.11 \times 10^{11}$$

$$\alpha = 7500 \text{ nepers/meter at } 1 \text{ GHz}$$

The parameters for the aluminum barrier are⁸

$$c_e = 1.03 \times 10^{10}$$

$$\alpha = 505 \text{ nepers/meter at } 1 \text{ MHz}$$

Figure 2. Calculated Output Waveform for the Device Shown in Figure 1. The 300-V input pulse occurred at $t = 0$.

Experimental data were obtained using a device of 1/8-in.-thick square plate transducers with 6-mil-thick bonds and a 0.1-in.-thick aluminum barrier. These dimensions were used in the numerical solution. The experimental data are shown in Figure 3.

Figure 3. Output Voltage From a Device Similar to That Shown in Figure 1. The input pulse for this device consisted of a pulse from 300 to 0 V, which occurred approximately $3.5 \mu\text{s}$ after $t = 0$, as shown here. This accounts for the 180° phase difference between the computed voltage shown in Figure 2 and the above waveform. In addition it should be noted that an actual device consists of irregularities in materials and surfaces, especially in the bonds which are difficult to measure and cannot be modelled by a one-dimensional code. The effects of these irregularities occur in addition to the multidimensional effects of edges and boundaries which become more important as time progresses.

Conclusion

A numerical solution has been presented for the fully coupled equations that describe nonideal piezoelectric materials. The results of calculations provided by this solution are shown to be in good agreement with experimental data.

APPENDIX A
Modifications to WONDY

```

*IDENT,PZEM
*DELETE,WONDY2.30,WONDY2.374
  SUBROUTINE STAT4
*CALL,SHORTEN
  COMMON/DOT/F2,F3,F4,F5,RC,IQ1D,IQ1,ISTD,IST,TAUD
  LOGICAL PFRACT,QFRACT,NONE
  REAL IST,IQ1,MI,KM,KT1,M,NOMESH,MP,IQ1D,ISTD,ISTOL,IQ1OL
  INTEGER PLATE,U4020,RHBT
  DIMENSION YY(2),W3(163),IW3(6)
  EXTERNAL DBYDT,DERIN,DEROUT
  CALL ERXSET(10,0)

```

```

C
C   EXTRA VARIABLES ARE STORED AS FOLLOWS:
C   13-CURRENT
C   14-VOLTAGE
C   15-STRAIN
C   16-ELECTRIC FIELD
C   17-STATE VARIABLE Q1
C   18-SUMQ1
C   19-D
C

```

```

  ACQUIRE NECESSARY INPUT VARIABLES

```

```

  RH00=CES(1,PLATE)
  AREA=CES(3,PLATE)
  U0=CES(4,PLATE)
  OMEGA=CES(5,PLATE)
  RLOAD=CES(6,PLATE)
  CAP=CES(7,PLATE)
  CE=CES(8,PLATE)
  CI=CES(9,PLATE)
  HE=CES(10,PLATE)
  HI=CES(11,PLATE)
  BE=CES(12,PLATE)
  BI=CES(13,PLATE)

```

```

    TAUS=CES(15,PLATE)
    TAUD=CES(16,PLATE)
    RERR=1.0E-4
C    WONDY SUPPLIES THESE
    TEND=T
    JSAVE=J
    DXI=DELXJ
    IF(NEWPLAT)101,100
C    COMPUTE STATE VARIABLE Q1 AND THE INTEGRALS OF STRAIN
C    AND Q1 ONLY ONCE FOR EACH PLATE
101 IST=0.0
    IQ1=0.0
    THIK=0.0
C    START WITH COMPUTATION AT THE BEGINNING OF MOTION
    JANI=JAN
    JI=J
    PAI=PA
    PI=P
    QAI=QA
    QI=Q
    RAI=RA
    RI=R
    SAI=SA
    SI=S
    UAI=UA
    UI=U
    UBI=UB
    UBNI=UBN
    XAI=XA
    XI=X
    XBI=XB
    XBNI=XBN
    MI=M

```



```

DO 10 LL=LOL,LOR
AI=2.0*(SI-SAI+QI-QAI)/(RI*(XI-XBI)+RAI*(XAI-XI))
UNI=UI+DELT(5)*AI
XNI=XI+UNI*DELT(1)
DXII=XNI-XBNI
RNI=RI*MI/(MI+RI*DELT(1)*(UNI-UBNI))
C   COMPUTE STRAIN AND Q1
THIK=THIK+DXII
ST=RHOO/RNI-1.0
SUMQ1=STORE(JI+18)
EX1=EXP(-TEND/TAUS)
EX2=EXP(TEND/TAUS)
EX3=EXP(TENDB/TAUS)
SUMQ1=SUMQ1+ST*(EX2-EX3)
Q1=EX1*TAUS*SUMQ1
C   WORK ON INTEGRALS
IST=IST+ST*DXII
IQ1=IQ1+Q1*DXII
C   STORE Q1 AND ST IN STORE ARRAY.
STORE(JI+15)=ST
STORE(JI+17)=Q1
STORE(JI+18)=SUMQ1
C   INCREMENT USING THE SAME PROCEDURE AS IN JLOOP.
JNI=JANI
JI=(JNI-1)*NVAR
JANI=NEXT(JNI)
JAI=(JANI-1)*NVAR
UBNI=UNI
XBNI=XNI
MI=STORE(JI+3)
PI=PAI
PAI=STORE(JAI+4)
QI=QAI

```

```

QAI=STORE(JAI+5)
RI=RAI
RAI=STORE(JAI+6)
SI=SAI
SAI=STORE(JAI+7)
UBI=UI
UI=UAI
UAI=STORE(JAI+8)
XBI=XI
XI=XAI
XAI=STORE(JAI+9)
10 CONTINUE
C   SKIP ELECTRICAL CALCULATIONS FOR PLATES THAT ARE CONDUCTIVE
   IF(CES(12,PLATE).NE.0.0)GO TO 14
   VOLT=0.0
   CUR=0.0
   D=0.0
   Q2=0.0
   GO TO 13
C   APPROXIMATE THE TIME DERIVATIVE OF THE STRAIN INTEGRAL AND IQ1
14  ISTOL=CES(34,PLATE)
   ISTD=(IST-ISTOL)/DELT(1)
   IQ10L=CES(35,PLATE)
   IQ1D=(IQ1-IQ10L)/DELT(1)
C   IDENTIFY PLATE FOR OUTPUT BY SIGN OF RLOAD
   IOUT=0
   IF(RLOAD.GE.0.0)IOUT=1
   IF(IOUT.EQ.1)GO TO 12
C   COMPUTE STATE VARIABLES AND ELECTRIC DISPLACEMENT FOR INPUT PLATES
   VOLT=U0
   IF(OMEGA.NE.0.0)VOLT=U0*SIN(OMEGA*TEND)
   SUMQ2=CES(38,PLATE)
   EX1=EXP(-TEND/TAUD)

```

```

EX2=EXP(TEND/TAUD)
EX3=EXP(TENDB/TAUD)
EX4=EX3*EX1
DENOM=1.0+(BE/BI-1.0)*(1.0-EX4)
AIS=HI*IST+(HE-HI)*IQ1/TAUS-VOLT
AIS=AIS/(THIK*BI)
SUMQ2=SUMQ2+AIS*(EX2-EX3)
Q2=TAUD*EX1*SUMQ2/DENOM
SUMQ2=SUMQ2-(BE-BI)*Q2*(EX2-EX3)/(TAUD*BI)
D=HI*IST+(HE-HI)*IQ1/TAUS-VOLT-THIK*(BE-BI)*Q2D/TAUD
D=D/(THIK*BI)
C   CALCULATE INPUT CURRENT
VOLT D=V0*OMEGA*COS(OMEGA*TEND)
DDOT=HI*ISTD+(HE-HI)*IQ1D/TAUS-VOLT D-THIK*(BE-BI)*Q2D/TAUD
DDOT=DDOT/(THIK*BI)
CUR=AREA*DDOT
GO TO 13
C   COMPUTE STATE VARIABLES, ELECTRIC DISPLACEMENT, CURRENT
C   AND VOLTAGE FOR OUTPUT PLATE
12 YY(1)=0.0
   YY(2)=0.0
   IF(IFLAG.NE.1)IFLAG=2
   TBEG=0.0
   RC=RLOAD*CAP
   F1=CAP/(THIK*BI*CAP+AREA)
   F2=THIK*F1*(BI/RC+BE/TAUD-BI/TAUD)
   F3=THIK*F1*(BE/TAUD-BI/TAUD)
   F3=F3*(1.0/RC-1.0/TAUD)
   F4=F1*(HE-HI)/TAUS
   F5=F1/HI
   CALL RKF(DEROUT,2,YY,TBEG,TEND,RERR,RERR,IFLAG,W3,IW3)
   Q2=YY(1)
   D=YY(2)

```

```

DDOT=-F2*D-F3*Q2+F4*IQ1D+F4*IQ1/RC+F5*ISTD+F5*IST/RC
CUR=AREA*DDOT
VOLT=HI*IST+(HE-HI)*IQ1/TAUS-THIK*BI*D-THIK*(BE-BI)*Q2/TAUD
C   STORE VOLT, CUR, Q2, D AND THE INTEGRALS FOR STRAIN AND
C   Q1 IN CES
13  CES(32,PLATE)=CUR
    CES(33,PLATE)=VOLT
    CES(34,PLATE)=IST
    CES(35,PLATE)=IQ1
    CES(36,PLATE)=D
    CES(37,PLATE)=Q2
    CES(38,PLATE)=SUMQ2
C   COMPUTE STRESS, ELECTRIC FIELD AND SOUND SPEED
C   GET PREVIOUSLY COMPUTED AND STORED STATE VARIABLES AND
C   STRAIN FOR CURRENT ZONE
100 Q1=STORE(JSAVE+17)
    ST=STORE(JSAVE+15)
    CUR=CES(32,PLATE)
    VOLT=CES(33,PLATE)
    IST=CES(34,PLATE)
    IQ1=CES(35,PLATE)
    D=CES(36,PLATE)
    Q2=CES(37,PLATE)
    SN=CI*ST+(CE-CI)*Q1/TAUS-HI*D-(HE-HI)*Q2/TAUD
    ELEC=-HI*ST-(HE-HI)*Q1/TAUS+BI*D+(BE-BI)*Q2/TAUD
C   FOR WONDY ONLY
    SN=-SN
    DATB(3)=CUR
    DATB(4)=VOLT
    DATB(5)=STORE(JSAVE+15)
    DATB(6)=ELEC
    DATB(7)=STORE(JSAVE+17)
    DATB(8)=STORE(JSAVE+18)

```

```

DATB(9)=D
DATB(10)=Q2
CN=SQRT(CE/RH00)
EN=E
ZN=Z
PN=P
RETURN
ENTRY STIN4
C   DEFAULT CAPACITANCE IS .0001PF
    IF(CES(7,PLATE).EQ.0.0)CES(7,PLATE)=0.0001E-12
C   DEFAULTS FOR TIME CONSTANTS
    IF(CES(15,PLATE).EQ.0.0)CES(15,PLATE)=0.1E-6
    IF(CES(16,PLATE).EQ.0.0)CES(16,PLATE)=0.001E-6
    CES(32,PLATE)=0.0
    CES(33,PLATE)=0.0
    CES(34,PLATE)=0.0
    CES(35,PLATE)=0.0
    CES(36,PLATE)=0.0
    CES(37,PLATE)=0.0
    CES(38,PLATE)=0.0
C   COMPUTE CI FROM ALFA IN DB IF FLAG IS SET
    IF(CES(19,PLATE).NE.1.0)GO TO 605
C   CHANGE ALFA FROM DB TO NEPERS AND SQUARE
    ALFA2=(CES(17,PLATE)/8.686)*(CES(17,PLATE)/8.686)
    OM=CES(18,PLATE)*2.0*3.14159
    TAUS=CES(15,PLATE)
    CE=CES(8,PLATE)
    RH00=CES(1,PLATE)
    W2=OM*OM
    W3=W2*OM
    W4=W3*OM
    W5=W4*OM
    T2=TAUS*TAUS

```

```

T3=T2*TAUS
T4=T3*TAUS
E1=ALFA2*W4*T4+ALFA2*W2*T2
E2=(W3*RH00*TAUS+W5*RH00*T3)/CE
E3=ALFA2+W2*T2*ALFA2-E2
E4=E2*E2-4.0*E1*E3
IF(E4.LT.0.0)GO TO 603
F1=(-E2+SQRT(E4))/(2.0*E1)
F2=(-E2-SQRT(E4))/(2.0*E1)
IF(F1.LT.0.0.AND.F2.LT.0.0)GO TO 603
G1=ABS(1.0-F1)
G2=ABS(1.0-F2)
ANS=F1
IF(G2.LT.G1)ANS=F2
IF(ANS.LT.0.8.OR.ANS.GT.1.2)GO TO 603
GO TO 600
C   IGNORE ATTENUATION WHEN COMPUTATIONAL DIFFICULTIES ARISE
603 ANS=1.0
C   STORE CI
600 CES(9,PLATE)=ANS*CE
605 CONTINUE
RETURN
END
SUBROUTINE DEROUT(TIME,YY,YYD)
COMMON/DOT/F2,F3,F4,F5,RC,IQ1D,IQ1,ISTD,IST,TAUD
REAL IST,IQ1,ISTD,IQ1D
DIMENSION YY(2),YYD(2)
YYD(1)=YY(2)-YY(1)/TAUD
YYD(2)=-F2*YY(2)-F3*YY(1)+F4*IQ1D+F4*IQ1/RC+F5*ISTD+F5*IST/RC
RETURN
END

```

APPENDIX B
Input Instructions for WONDY

APPENDIX B

Input Instructions for WONDY

General input instructions for WONDY are given in Reference 4. To use the equation-of-state routine presented here, the following changes to those instructions should be made.

On card 2, LPHA must equal 3 and RHBT must equal 3. Also, on card 2, NVAR must be set to 20. On card 10, STAT should be 4 only for the layers that require this equation of state; other layers may use one of the other WONDY equation-of-state routines. For STATE equal to 4, cards 15, 16, and 17 become:

NAME	DESCRIPTION
CARDNO	15
PLATE	Plate number for data on this card
CES (1, PLATE)	ρ Reference density of uncompressed material
CES (2, PLATE)	c_o Bulk sound speed of uncompressed material
CES (3, PLATE)	A Area of plate perpendicular to the direction of wave propagation.
CES (4, PLATE)	V_o Magnitude of the input voltage. This is needed only for input plates. Note that CES (6, PLATE) must be less than 0 for input plates.
CES (5, PLATE)	ω Frequency in radians for sinusoidal input voltages. If $\omega = 0$, the input voltage is a pulse of magnitude V_o .
CES (6, PLATE)	R Load resistance. Set R less than zero for an input plate.
CES (7, PLATE)	C Load capacitance. Defaults to 1.0E-16 farads.
CARDNO	16
PLATE	Plate number for data on this card
CES (8, PLATE)	c_e Equilibrium value of the elastic stiffness
CES (9, PLATE)	c_i Instantaneous value of the elastic stiffness. For ideal materials $c_e = c_i$. Note also CES (19, PLATE).
CES (10, PLATE)	h_e Equilibrium value of the piezoelectric coupling. For nonpiezoelectric materials $h_e = 0$.
CES (11, PLATE)	h_i Instantaneous value of the piezoelectric coupling. For ideal materials, $h_e = h_i$.
CES (12, PLATE)	β_e Equilibrium value of the inverse dielectric permittivity. For conductive materials, $\beta_e = 0$.
CES (13, PLATE)	β_i Instantaneous value of the inverse dielectric permittivity. For ideal materials, $\beta_e = \beta_i$.

NAME	DESCRIPTION
CARDNO	17
PLATE	Plate number for data on this card
CES (15, PLATE)	τ_s Time constant for nonideal elastic materials. Defaults to 0.1E-6 sec.
CES (16, PLATE)	τ_D Time constant for nonideal electric materials. Defaults to 0.1E-8 sec.
CES (17, PLATE)	α Attenuation in dB/unit of length, e.g., dB/meter for MKS units, from which c_i is computed if CES (19, PLATE) = 1.0.
CES (18, PLATE)	f Frequency in Hertz at which α was measured.
CES (19, PLATE)	Flag for attenuation calculations to be set to 1.0 if c_i is to be calculated from CES (18, PLATE).

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