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INTRODUCTION

The effects of thermal noise on a current-biased resistively shunted Josephson junction (RSJ) have been extensively studied. The noise source is assumed to be the Nyquist noise with spectral density $4k_BT/R$ developed in the shunt resistor R at a temperature T. This noise has two effects. First, it induces transitions from the zero voltage state to the non-zero voltage state at bias currents below the thermodynamic critical current, I_o, thereby inducing "noise rounding" of the current-voltage (I-V) characteristic [1]. Second, the Nyquist noise generates a voltage noise across the junction that, according to the theory of Likharev and Semenov [2], has a spectral density

$$S_v(0) = \frac{4k_BTR_D^2}{R}\left[1 + \frac{1}{2}\left(\frac{I_o}{I}\right)^2\right] \tag{1}$$

at frequencies much less than the Josephson frequency. In Eq. (1), $R_D \equiv \partial V/\partial I$ is the dynamic resistance of the junction.

The results of the theory have been well established experimentally [3,4]. One application of the theory is to predict the sensitivity of the dc SQUID [5] for a given set of experimental parameters. The predictions have been in quite good agreement with results obtained experimentally [6]. As the sensitivity of these devices is improved, however, the theory will eventually break down as quantum processes become important and set a limit on the performance. In this paper, we briefly outline a theory for noise in the RSJ in the quantum limit [7], and extend it to the dc SQUID [8]. Experimental results for both the single junction and the SQUID are reported.

QUANTUM NOISE THEORY FOR A RESISTIVELY SHUNTED JUNCTION

We consider a Josephson tunnel junction with critical current I_o and capacitance C shunted with a resistance R. We assume that the voltage is less than $2\Delta/e$, where Δ is the energy gap, and that T is well below the transition temperature, so that the quasiparticle tunneling conductance is small compared with the shunt conductance. Our central assumption is that the current noise in the shunt resistor has a spectral density

$$S_I(\nu) = \frac{2h\nu}{R}\coth\left(\frac{h\nu}{2k_BT}\right) \equiv \frac{4h\nu}{R}\left(\frac{1}{2} + \frac{1}{e^{h\nu/k_BT}-1}\right) \tag{2}$$

at frequency ν. The equation of motion of the junction can be written in the form

$$\frac{\hbar C}{2e}\ddot{\delta} + \frac{\hbar}{2eR}\dot{\delta} + I_o \sin\delta = I + I_N(t) \tag{3}$$

where δ is the phase difference across the junction, and $I_N(t)$ has the spectral density of Eq. (2).

We first assume that the capacitance is small ($\beta_c \equiv 2\pi I_0 R^2 C/\Phi_0 \ll 1$, where $\Phi_0 \equiv h/2e$ is the flux quantum), and that the noise is small so that noise-rounding can be neglected. The I-V characteristic is then [9] $V = R(I^2 - I_0{}^2)^{\frac{1}{2}}$, and Eq. (3) may be solved analytically using the techniques of Likharev and Semenov [2]. For frequencies much less than the Josephson frequency 2eV/h one finds

$$S_v(0) = R_D{}^2 \left[\frac{4k_BT}{R} + \frac{2eV}{R}\left(\frac{I_o}{I}\right)^2 \coth\left(\frac{eV}{k_BT}\right)\right]. \qquad (4)$$

In the limit $eV \ll k_BT$, the hyperbolic cotangent may be expanded and one recovers the thermal noise limit, Eq. (1). The first term on the right hand side of Eq. (4) represents noise generated in the shunt at the measurement frequency, while the second term arises from noise generated at frequencies near the Josephson frequency that is mixed down by the non-linearity of the junction. Equation (4) is plotted in Fig. 1 for 5 values of the parameter $\kappa \equiv eI_0R/k_BT$. When $\kappa \ll 1$, the curve becomes indistinguishable from that of the thermal limit, but as κ increases, the voltage noise is significantly enhanced above the thermal result. In the extreme quantum limit (T=0), one finds

$$S_v(0) = 2eV(I_o/I)^2\ R_D{}^2/R = 2eI_o{}^2\ R^3/V. \qquad (5)$$

The physical interpretation of the noise is as follows. Equation (3) also describes the motion of a particle on a tilted washboard. Then, C represents the mass of the particle, 1/R the viscous damping, and I_0 the amplitude of the oscillating potential. The average slope of the washboard is proportional to I. At bias currents above the critical current, the effect of $I_N(t)$ is to induce random variations in the slope, and thus produce random noise in the velocity of the mass, $\dot{\delta}$, that represents the voltage in the case of the junction. If the current is reduced to a value below I_0, in the absence of noise the ball no longer runs freely, but remains trapped in one of the potential wells. However, if I is sufficiently close to I_0, the noise may instantaneously tilt the washboard sufficiently so that the particle rolls into the next well. This effect produces noise round-of the I-V characteristic. In our model, this effect occurs even at T=0, where the noise current is due to the zero point fluctuations in the large collection of harmonic oscillators representing the resistor. Alternatively, one may view the current fluctuations as supplying pulses of energy that "activate" the particle over a stationary potential barrier. These fluctuations are virtual: The energy in the pulse, ΔE, and the length of the pulse, Δt, satisfy $\Delta E\Delta t \lesssim \hbar$. The dissipation that occurs when the phase changes by 2π is supplied by the bias current, not by the noise current. (On the other hand, in the thermal limit in which the noise currents have a spectral density $4k_BT/R$, $\Delta E\Delta t \sim k_BT\Delta t$, and the fluctuations are real).

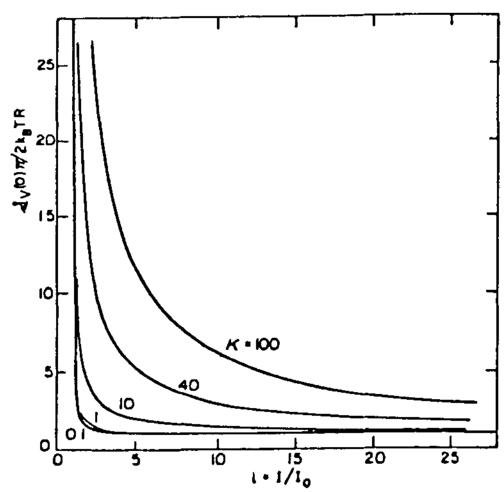

Fig. 1 Low-frequency spectral density $S_V(0)$ of the voltage noise vs current for 5 values of $\kappa \equiv eI_0R/k_BT$ with $\beta_c \ll 1$.

We emphasize that the description of the junction by a Langevin equation assumes that the wave packet representing the phase of the junction is highly localized so that a particle-like representation is appropriate. When the wave packet representing the phase is significantly broadened, a full quantum mechanical description is essential, and use of the Langevin equation may lead to unphysical results. Furthermore, we have neglected the effects of macroscopic quantum tunneling [10] which we believe are relatively unimportant for overdamped junctions.

EXPERIMENTS ON SINGLE JUNCTIONS

To observe quantum effects we require a junction with $\kappa \equiv eI_oR/k_BT = (e/k_BT)(\beta_c\Phi_oj_1/2\pi c)^{\frac{1}{2}}$ >>1, where j_1 is the critical current density and c is the capacitance per unit area. At 1K with $j_1 = 10^4$ Acm^{-2}, c = 0.04 pF μm^{-2} and $\beta_c = 0.1$ we find $\kappa \approx 3$, a value at which quantum corrections should be readily observable.

The PbIn-Ox-Pb junctions were fabricated using photolithographic techniques. The junction area was about 6 μm^2, and the critical currents ranged from 0.1 to 15 mA (0.2 to 30x10^4 Acm^{-2}). The CuAl shunt resistance was typically 0.1Ω, and had an inductance of about 0.2 pH. The low frequency voltage noise of each junction was amplified with a cooled 30- or 100-kHz LC-circuit coupled to a room-temperature low-noise preamplifier. The noise was mixed down to frequencies below 100 Hz, and its power spectrum was measured with a computer. Any 1/f noise contribution was estimated from the measurements at 30 and 100 kHz, and, when significant, was subtracted from the 100 kHz-measurement.

We first investigated junctions with κ<<1, and found excellent agreement with Eq. (1). These results demonstrate the accuracy of our measuring techniques, and give some assurance that we have taken adequate precautions to eliminate external noise. We then investigated junctions with κ>1, and discovered the resonant structure illustrated in Fig. 2. The

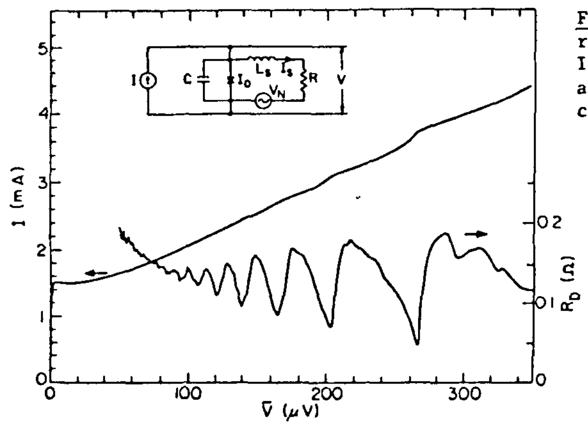

Fig. 2 I-V characteristics and dynamic resistance R_D for a junction at 1.4K with I_o = 1.53 mA, R = 0.092Ω, κ = 1.15, β_c = 0.03, and $2\pi L_sI_o/\Phi_o$ = 1.0. Inset shows equivalent circuit of junction.

resonances occur when the Josephson frequency is near a subharmonic of the resonant circuit formed by the shunt inductance L_s and the junction capacitance; the equivalent circuit is shown inset in Fig. 2. In addition to modifying the I-V characteristic, the increased nonlinearity of the phase evolution enhances the mixing down of noise from multiples of the Josephson frequency. The spectral density of the noise measured on one junction is shown in Fig. 3. The correction due to amplifier voltage and current noise is negligible. The 1/f noise was at most 20% of the total noise and was subtracted out. If we assume that the error due to an inadequate knowledge of the exact spectral density of the flicker noise is ±25%, the overall error resulting is ±5% of the total measured noise. We have subtracted the noise from the resistor at the measurement frequency, $4k_BTR_D^2/R$, from the total noise to determine the mixed down noise; the latter is also plotted in Fig. 3.

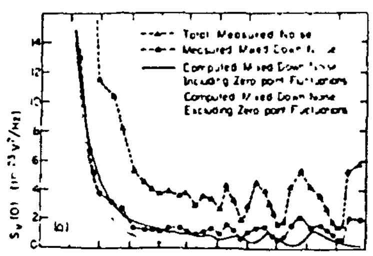

Fig. 3 Measured and computed spectral densities of the voltage noise at 1.4K for the junction in Fig. 2.

We have computed the noise expected from the circuit shown in Fig. 2 using the equations of motion

$$I = I_o \sin\delta + C\dot{V} + I_s, \tag{6}$$

and $$V = I_s R + \dot{I}_s L_s + V_N. \tag{7}$$

Here I_s is the current through the shunt inductance, L_s, and V_N has a spectral density $2h\nu R\coth(h\nu/2k_BT)$ at frequency ν. The computed mixed-down noise is plotted in Fig. 3. For comparison, the computed mixed-down noise using a noise spectral density for the resistor without the zero point term, $4h\nu R/[\exp(h\nu/k_BT)-1]$, is also shown. The latter result clearly underestimates the measured noise. The agreement between the computed and measured noise is quite good below about 120 μV, but at higher voltages the measured noise lies somewhat above the predicted value. We believe this discrepancy arises from self-heating in the junction, which increases the magnitude of the noise at the measurement frequency. Furthermore, the measured and predicted resonances are misaligned, indicating that our choices of L_s and C were slightly incorrect.

These results suggest strongly that zero point fluctuations in the shunt resistor are the source of the limiting noise in a resistively shunted Josephson junction in the quantum limit.

QUANTUM NOISE IN THE dc SQUID

The dc SQUID consists of two Josephson junctions in a superconducting loop of inductance L. The critical current of the SQUID oscillates as a function of the applied magnetic flux Φ threading the loop with a period Φ_o. Thus, when the SQUID is current-biased in the non-zero voltage regime, the voltage also oscillates as a function of the applied flux. A small change $\delta\Phi$ ($\ll\Phi_o$) produces a corresponding change in voltage $\delta V = (\partial V/\partial\Phi)\delta\Phi$. The smallest detectable change in flux that can be detected is determined by the voltage noise across the SQUID. To characterize the performance, it is convenient to define the noise energy per Hz

$$\frac{\varepsilon}{1\text{Hz}} = \frac{S_\Phi}{2L} = \frac{S_V}{2(\partial V/\partial\Phi)^2 L}, \tag{8}$$

where S_V is the spectral density of the voltage noise, and S_Φ is the spectral density of the equivalent flux noise in the SQUID.

Tesche and Clarke [5] computed the noise energy assuming that the Nyquist noises in the two resistive shunts were the only sources of noise. They found that the SQUID was optimized when $\beta_c \lesssim 1$ and $\beta = 2LI_o/\Phi_o \approx 1$, and that the optimum noise energy was given by

$$\frac{\varepsilon}{1\text{Hz}} \approx 10k_BT\,(LC)^{1/2}. \tag{9}$$

This result adequately predicts the measured performance of SQUIDs over a wide range of values of L and C. The most sensitive devices reported to date have noise energies of $\sim 6\hbar$ [11, 12, 13], in reasonable agreement with Eq. (9). When T, L, and/or C are lowered sufficiently, quantum corrections to the noise become important. The theory described earlier in this paper can be extended to the case of the SQUID, and one finds that at T=0 for an optimized device

$$\frac{\varepsilon}{1\text{Hz}} \approx \hbar. \tag{10}$$

At higher temperatures the performance is degraded: At 4K, one expects $\varepsilon/1\text{Hz} \approx 3\hbar$.

We have made SQUIDs that should approach the quantum limit in the He^4 temperature range. The fabrication procedures were similar to those used for the single junction, and the following values are typical: $L \approx 2$ pH, $C \approx 0.5$ pF, $I_o \approx 0.5$ mA, $R \approx 1\Omega$. Thus $\beta_c \approx 0.8$ and $\beta \approx 1$. The SQUID was connected across the cooled LC resonant circuits (typically 30, 100, 300 kHz) and the noise measured as for the single junction as a function of bias current and applied flux. The transfer function $\partial V/\partial \Phi$ was also measured by modulating the flux through the SQUID and lock-in detecting the resulting voltage. The best result so far was obtained at 3.4K, where the measured noise at 118 kHz was 3.8h. By measuring the noise at higher and lower frequencies, however, we estimated that approximately one-half of this noise was 1/f noise, so that the white noise contribution was approximately 2h, in rather good agreement with the predictions of the model.

CONCLUDING REMARKS

The predictions of the theory for the noise in a single junction are in good agreement with experiment when the effects of resonances are taken into account. The measured noise in dc SQUIDs also agrees well with the predictions of the theory at 3.4K, although it should be realized that the quantum corrections to the noise are small at this temperature for the parameters used. Finally, a more detailed investigation of the theory is necessary to establish the limits of validity of the Langevin approach.

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REFERENCES

[1] V. Ambegaokar and B. I. Halperin, *Phys. Rev. Lett.* 22, 1364 (1969).

[2] K. K. Likharev and V. K. Semenov, *Pis'ma Zh. Eksp. Teor. Fiz.* 15, 625 (1972) [*JETP Lett.* 15, 442 (1972)].

[3] C. M. Falco, W. H. Parker, S. E. Trullinger, and P. K. Hansma, *Phys. Rev.* B10, 1865 (1974).

[4] R. P. Giffard, P. F. Michelson, and R. J. Soulen, Jr., *IEEE Trans. Mag.* MAG-15, 276 (1979).

[5] C. D. Tesche and J. Clarke, *J. Low Temp. Phys.* 29, 301 (1977).

[6] J. Clarke, *IEEE Trans. Electron Devices* ED-27, 1896 (1980).

[7] R. H. Koch, D. J. Van Harlingen, and J. Clarke, *Phys. Rev. Lett.* 45, 2132 (1980).

[8] R. H. Koch, D. J. Van Harlingen, and J. Clarke, *Appl. Phys. Lett.* 38, 380 (1981).

[9] D. E. McCumber, *J. Appl. Phys.* 39, 3113 (1968); W. C. Stewart, *Appl. Phys. Lett.* 12, 277 (1968).

[10] A. O. Caldeira and A. J. Leggett, *Phys. Rev. Lett.* 46, 211 (1980).

[11] R. F. Voss, R. B. Laibowitz, A. N. Broers, S. I. Raider, C. M. Knoedler, and J. M. Viggiano, *IEEE Trans. Mag.* MAG-17, 395 (1981).

[12] P. Carelli and M. W. Cromar, 1980 Applied Superconductivity Conference, Santa Fe, New Mexico (unpublished).

[13] R. H. Koch, D. J. Van Harlingen, and J. Clarke, 1980 Applied Superconductivity Conference, Santa Fe, New Mexico (unpublished).