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CONFINEMENT OF NONNEUTRAL PLASMA IN UNCONVENTIONAL GEOMETRIES

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Our interest in efficient storage of cold, nonneutral plasma has been motivated by the elegant studies on cryogenic nonneutral electron plasmas at UCSD and by the remarkable results obtained from the laser-cooled ion plasmas at the NIST, Boulder, Colorado. Also motivating our study is the perceived need to develop the most expedient means of storing antimatter, whether it be antiprotons for gravitational studies or positrons for a variety of physics experiments and diagnostic purposes.

One of the most explored technologies of confining nonneutral plasmas is the Penning trap. The maximum number density of cold nonneutral plasma that can be stored in such a trap is $B^2/2\mu_0 mc^2$, in which $B^2/2\mu_0$ is the (homogeneous) magnetic energy density and mc^2 is the rest energy of the stored charges.

Below, we shall present a synopsis of the results of our theoretical exploration of the effect on this hydrostatic limit, the so-called "Brillouin" limit, of altering the geometry of the confining vacuum magnetic field while maintaining the field's azimuthal symmetry. In particular, we shall analyze equilibrium confinement by, first, a poloidal magnetic field, $B_r(r,z)\hat{r} + B_z(r,z)\hat{z}$, and, second, a toroidal magnetic field, along with the concomitant electrostatic fields.

In either case, the following set of equilibrium equations must be satisfied:

$$m\mathbf{v}(\mathbf{r}) \cdot \nabla\mathbf{v}(\mathbf{r}) = q[\mathbf{E}(\mathbf{r}) + \mathbf{v}(\mathbf{r}) \times \mathbf{B}(\mathbf{r})] , \text{ (force balance);}$$

$$\nabla \times \mathbf{E}(\mathbf{r}) , \text{ (Paraday's Law); } \quad \frac{qn(\mathbf{r})}{\epsilon_0} = \nabla \cdot \mathbf{E}(\mathbf{r}), \text{ (Poisson's Equation);}$$

(1)

$$\nabla \times \mathbf{B}(\mathbf{r}) = 0, \text{ (Ampere's Law); } \quad \nabla \cdot \mathbf{B}(\mathbf{r}) = 0, \text{ (solenoidal constraint);}$$

$$\nabla \cdot [n(\mathbf{r})\mathbf{v}(\mathbf{r})] = 0, \text{ (equation of number density conservation) .}$$

If one lets $q\mathbf{E}(\mathbf{r})/m = -\nabla\mathbf{V}(\mathbf{r})$, one immediately notes not only that Paraday's Law is satisfied, but also that the force balance equation becomes

$$\underline{v}(\underline{r}) \times \underline{g}(\underline{r}) = \nabla \phi(\underline{r}), \quad (2)$$

in which

$$\underline{g}(\underline{r}) = \nabla \times \left[\underline{v}(\underline{r}) + \frac{qA_{\theta}(\underline{r})}{m} \right] \quad \text{and} \quad \phi(\underline{r}) = \Psi(\underline{r}) + \frac{v^2(\underline{r})}{2}.$$

In the case of the poloidal field, if we assume that the fluid velocity is purely azimuthal, we may set

$$\underline{g}(\underline{r}) = \nabla \times \chi(r, z) \hat{\theta}$$

where

$$\chi(r, z) = v_{\theta}(r, z) + \frac{qA_{\theta}(r, z)}{m}. \quad (3)$$

Equation (2) then implies $\phi(\underline{r}) = \nabla^2 [r\chi(r, z)]$ and hence that

$$v_{\theta}(r, z) = r \nabla^2 [r\chi(r, z)], \quad (4)$$

where the prime denotes differentiation with respect to the argument of ϕ . Now restricting the flow, v_{θ} , to be a rigid rotation with an angular velocity ω , we find that Eqs. (3) and (4) yield

$$\nabla^2 [r\chi(r, z)] = \omega r \chi(r, z) = v_{\theta}^2(r, z) + \frac{qv_{\theta}(r, z)A_{\theta}(r, z)}{m}.$$

If we define the plasma frequency by

$$\omega_p^2(r,z) = \frac{q^2 n(r,z)}{\epsilon_0 m} ,$$

we find from Poisson's equation that

$$\omega_p^2(r,z) = \nabla^2 \left[\frac{v_\theta^2(r,z)}{2} - \phi(r,z) \right] = -2\omega^2 - \frac{q\omega}{m} \nabla^2 [rA_\theta(r,z)] . \quad (5)$$

For vacuum poloidal magnetic fields, one easily may verify that

$$\nabla^2 [rA_\theta(r,z)] = 2B_z(r,z) .$$

For the case of poloidal magnetic field geometry, using Eq. (5) we thus arrive at the result that

$$\omega^2 + \omega_{cz}(r,z) + \frac{\omega_p^2(r,z)}{2} = 0, \quad (6)$$

in which $\omega_{cz}(r,z)$ ($= qB_z(r,z)/m$) represents the local cyclotron frequency. For the solution for ω to be physical, the discriminant of Eq. (6) must be real; i.e.,

$$\omega_p^2(r,z) \leq \frac{\omega_{cz}^2(r,z)}{2} . \quad (7)$$

One observes that Eq. (7) implies

$$n(r,z) \leq \frac{B_z^2(r,z)}{2\mu_0 mc^2} \leq \frac{B^2(r,z)}{2\mu_0 mc^2} .$$

We shall choose the ratio of the rest energy density of the stored charges $[n(r,z)mc^2]$ to the magnetic energy density to be the measure of storage efficiency. Then we note that the storage efficiency of the purely poloidal magnetic field is maximized by eliminating its radial component, leaving a necessarily homogeneous field in the z-direction, such as is found in the Penning trap.

We shall now redirect our attention toward the solution of Eqs. (1) for a toroidal nonneutral plasma whose major radius is R_0 and whose minor radius is r_0 confined by a purely toroidal magnetic field.

Utilizing (R, ϕ, Z) as the standard set of cylindrical coordinates, we shall let the Z-axis be the axis of azimuthal symmetry of the toroidal plasma and let the $Z = 0$ plane bisect the torus into equal annular volumes. The circular poloidal cross section of the toroidal plasma is assumed to have a radius r_0 . Its center is located a distance R_0 from the Z-axis. We shall let this center serve as an origin for a polar coordinate system (r, θ) for the particular poloidal plane. We shall denote distance along the circular toroidal axis at $r = 0$ by s . The transformation between the two coordinate systems for the domain within the toroidal plasma then can be defined by

$$R = R_0 + r \cos(\theta) ,$$

$$\phi = - \frac{s}{R_0} ,$$

$$Z = r \sin(\theta) .$$

The minus sign guarantees that (r, θ, s) is a right-handed coordinate system. We shall denote the triad of unit basis vectors by $\hat{r}$, $\hat{\theta}$, and $\hat{s}$ and perform our analysis using this (r, θ, s) coordinate system.

Our approach to the solution entails expanding the differential operators and fields in terms of a power series in the inverse aspect ratio parameter r_0/R_0 . We assume that

$$\underline{B} = \frac{R_0 B_0}{R} \hat{s} = \frac{B_0}{1 + \frac{r}{R_0} \cos(\theta)} \hat{s} .$$

As the first three terms of an inverse aspect ratio expansion of $\underline{B}$, we therefore obtain

$$B^{(0)}(r, \theta) = B_0 \hat{s}, \quad B^{(1)}(r, \theta) = - \frac{r B_0 \cos(\theta)}{R_0}, \quad \text{and} \quad B^{(2)}(r, \theta) = \frac{r^2 \cos^2(\theta) B_0}{R_0^2} ,$$

where the superscript, i , for example, labels the contribution of order $(r/R_0)^i$. We set $v_{\theta}^{(0)}(r, \theta) = r\omega$ and impose the boundary condition $v_r(r_0, \theta) = 0$. This condition guarantees a constant poloidal radius r_0 of the toroidal plasma. We then solve the coupled set of equations, Eqs. (1), order-by-order in the inverse aspect ratio and obtain the following zeroth and first-order terms of the expansion of the fluid velocity, $\underline{v}(r, \theta)$, the electrostatic potential, $\Psi(r, \theta)$, [normalized as stated following Eq. (1)] and the square of the plasma frequency, $\omega_p^2(r, \theta)$, or number density, $n(r, \theta)$:

$$v_r^{(0)} = 0, \quad v_{\theta}^{(0)} = r\omega ;$$

$$\Psi^{(0)} = \omega(\omega + \omega_c) \frac{r^2}{2} ;$$

$$\omega_p^2(0) = \frac{q^2 n(0)}{\epsilon_0 m} = -2\omega(\omega + \omega_c) ;$$

$$v_r^{(1)} = \left[\frac{15\omega^2 + 12\omega\omega_c + 5\omega_c^2}{8(\omega + \omega_c)^2} \right] \left(\frac{\omega}{R_0} \right) (r_0^2 - r^2) \sin(\theta) ;$$

$$v_{\theta}^{(1)} = \left[\frac{(15\omega^2 + 12\omega\omega_c + 5\omega_c^2)r_0^2 - (21\omega^2 + 20\omega\omega_c + 7\omega_c^2)r^2}{8(\omega + \omega_c)^2} \right] \left(\frac{\omega}{R_0} \right) \cos(\theta) ;$$

$$v_r^{(1)} = \left[\frac{(15\omega^2 + 12\omega\omega_c + 5\omega_c^2)r_0^2 - (9\omega^2 + 10\omega\omega_c + 5\omega_c^2)r^2}{8(\omega + \omega_c)} \right] \left(\frac{\omega r}{R_0} \right) \cos(\theta) ;$$

$$\omega_p^{2(1)} = \frac{q^2 n^{(1)}}{\epsilon_0 m} = \left(\frac{2\omega^2 + 2\omega\omega_c + \omega_c^2}{\omega + \omega_c} \right) \left(\frac{4\omega r}{R_0} \right) \cos(\theta) .$$

Our solution describes a toroidal plasma without toroidal flow; i.e., $v_{\theta}(r, \theta) = 0$.

We have also solved for the second-order terms and will display them elsewhere.¹ Of particular physical interest is the finite aspect ratio modification of the storage efficiency. Defining $v \equiv (\text{average number density} \times mc^2) / (\text{average magnetic energy density})$ and $\eta \equiv \omega/\omega_c$, we have found that

$$v = -4\eta(1 + \eta) \left\{ 1 - \frac{r_0^2}{32R_0^2} \left[\frac{8 + 31\eta + 50\eta^2 + 23\eta^3}{(1 + \eta)^3} \right] + 0 \left(\frac{r_0^4}{R_0^4} \right) \right\} . \quad (8)$$

For $R_0/r_0 \rightarrow \infty$, this yields the standard result for a homogeneous field: $\omega^2 + \omega\omega_c + \omega_c^2/2 = 0$, whose solutions satisfy $-1 \leq \eta \leq 0$. On this range of η , the square bracket of Eq. (8) is always positive. Hence at a given value of η , the second-order inverse aspect ratio correction reduces the storage efficiency.

We have extended the toroidal geometry analysis summarized above to include viscous effects.¹

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REFERENCES

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