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THE COVER

A collage of some of the many people who have contributed to twenty-five years of research in magnetic fusion at LLL. Shown clockwise from top right are Richard Post, Stirling Colgate, Nicholas Christofilos, Charles Damm, Clyde Taylor, Ken Fowler, Fred Coensgen, Chet Van Atta, and Tom Simonen. In the center is the baseball-seam-shaped magnet used in such experiments as Baseball I and II. For further information on LLL's magnetic fusion program, see the article beginning on p. 1.

ABOUT THE JOURNAL

The Lawrence Livermore Laboratory is operated by the University of California for the United States Department of Energy. The Laboratory is one of two nuclear weapons design laboratories in the United States. Today nearly half of our effort is devoted to programs in magnetic and laser fusion energy, biomedical and environmental research, applied energy technology, and other research activities.

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ERRATUM

In the March 1978 *Energy and Technology Review* (UCRL-52000-78-3) an error appears in Fig. 3 on p. 14. The abscissa scale label should read "Yield—kt (1 kt = 4.18 TJ)."

BRIEF

LASER SPECTROSCOPY OF LANTHANIDES

As an extension of our laser-spectroscopy work in support of the laser isotope separation program, we have been investigating the intricate excitation spectra and ionization potentials of the lanthanides (the rare-earth elements). [Earlier laser spectroscopic work was described in the February 1976 *Energy and Technology Review* (UCRL-52000-76-2, p. 23).] Most of these spectra are too complex to be resolved by conventional absorption spectroscopy.

Laser spectroscopy operates by irradiating an atomic beam of the element being studied with light from two or more precisely tuned lasers. Atoms that become ionized (excited beyond their ionization potential) by this irradiation are separated with a quadrupole mass spectrometer and detected with a channeltron detector—a current detector with very high gain. By scanning the frequency of one of the lasers and simultaneously calibrating it with precisely known standards, we can sweep through the atom's excitation spectrum and accurately identify excited states and ionization levels.

Figure 1 presents a scan for the photoionization threshold of neodymium. The neodymium atoms start out in the ground, or totally unexcited, state. The threshold is indicated by the onset of strong autoionization peaks. The two-laser excitation scheme employed is diagrammed at the left of the illustration.

Figure 2 shows a cerium Rydberg series obtained by field ionization. As indicated in the excitation diagram at the left, in this case we started from a thermally excited state. The pulsed ionization field of 100 V/cm was delayed 4 μ s after the laser pulse to allow the obscure swarm of short-lived states to decay away. Similar results have been obtained for most of the other lanthanides.

Table 1 lists the values of ionization potentials determined from photoionization thresholds and Rydberg convergences for selected elements in the lanthanide series. Numbers in parentheses indicate

Fig. 1. Photoionization threshold spectrum for neodymium obtained by laser spectroscopy. The excitation scheme is diagrammed at the left. The threshold at 412.41 nm yields an ionization potential of 5.523 eV.

Table 1. First ionization potentials for selected elements in the lanthanide series.

Element	Photoionization threshold, eV	Rydberg convergence, eV
Cerium	5.537 (5)	5.5387 (4)
Praseodymium	5.464 ($\pm 12_2$)	[5.473 (10)]
Neodymium	5.523 (2)	5.5250 (6)
Promethium	—	[5.582 (10)]
Samarium	5.639 (2)	5.6437 (6)
Gadolinium	—	6.1502 (6)
Terbium	—	5.8639 (6)
Dysprosium	5.935 (3)	5.9390 (6)
Holmium	6.017 (3)	6.0216 (6)
Erbium	6.104 (2)	6.1077 (10)

Fig. 2. Cerium Rydberg series converging to the ground state of the ion. This series was obtained by field ionization (indicated by broad arrow on excitation scheme) of the bound Rydberg levels. The convergence limit yields the accurate ionization potential (IP) of 5.5387 (4) listed in Table I. If the ionizing field of 100 V/cm had been on during the laser pulse, the Rydberg series would have been blotted out by other peaks from short-lived excited states. Delaying the ionizing pulse 4 μ s permits these other states to die away and the Rydberg series to stand out.

the uncertainty in the last digit of the measured potentials. These values are 10 to 100 times more precise than values previously available.

This precision enables us to interpolate with confidence between measured values for the other lanthanides to estimate values thus far unmeasurable (in brackets) for praseodymium and promethium. It also greatly enhances our ability to predict the behavior of the various lanthanides in high-energy

plasmas and to check theoretical models of atomic and ionic structure. The major impetus for our work, however, comes from its impact on the laser isotope separation program, where accurate knowledge of excitation levels and ionization potentials is one key to converting a theoretical possibility into a practical accomplishment.

Contact Earl F. Worden (422-6203) for further information on this subject.

at experiments: Box Top, Table Top, and T-box

LLL Magnetic Fusion Research: The First 25 Years

Richard F. Post

From its inception, the Laboratory has supported research devoted to developing and demonstrating the scientific and fusion energy program components of the national fusion energy research effort, going back to the Laboratory's founding in 1952. This article reviews the program's beginning, progress, and present status in terms of its ultimate goal: to demonstrate a practical and economical means of generating power from controlled fusion reactions.

Keywords: Fusion energy, Magnetic fusion, Laboratory for Fusion Energy Research, LLL, Princeton University, Princeton, New Jersey, USA

From its inception, the Laboratory for Fusion Energy Research (LLL) has supported research devoted to developing and demonstrating the scientific and fusion energy program components of the national fusion energy research effort, going back to the Laboratory's founding in 1952. This article reviews the program's beginning, progress, and present status in terms of its ultimate goal: to demonstrate a practical and economical means of generating power from controlled fusion reactions. The article is written for a broad audience, including those who are not familiar with the history of the LLL and the fusion energy program in the United States. It is not intended to be a comprehensive review of the LLL's fusion energy research, but rather a summary of the program's history and current status.

The LLL was founded in 1952 as a part of the Princeton University's research program in fusion energy. It was the first of several laboratories established in the United States to study fusion energy. The LLL's research program was initially focused on the development of magnetic fusion devices, which use magnetic fields to confine and heat a plasma. The LLL's research program has since expanded to include the study of other fusion concepts, such as inertial fusion and hybrid fusion.

The LLL's research program has been supported by the National Science Foundation (NSF) and the Department of Energy (DOE). The NSF has provided the LLL with a steady stream of funding since its inception. The DOE has also provided significant funding to the LLL, particularly in the area of magnetic fusion research. The LLL's research program has been highly successful, leading to many important discoveries in the field of fusion energy. The LLL's research program has also been highly influential, leading to the development of many other fusion research programs in the United States and around the world.

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wall. In theory, a powerful magnetic field could prevent plasma particles trapped on its magnetic lines from escaping to the chamber walls long enough for fusion reactions to take place between nuclei.

Stated in this conceptually simple way, solving the fusion problem seemed like a straightforward exercise in technological development. The author was picked by York to head a small group that would tackle the problem experimentally. But it soon became apparent that the issue was a much deeper one, how deep would only be realized after many years of frustratingly difficult assaults on the confinement problem. In retrospect, it is probably just as well that in those early days we were naive in our expectations for success "just around the corner"; had we known how difficult the task would be, the fusion effort might have been prematurely abandoned.

EARLY EXPERIMENTS: FIRST STEPS AND FALSE STARTS

The earliest work on controlled fusion at LLL, which actually saw its beginnings at the Radiation Laboratory (as it was then called) in Berkeley, was purely exploratory. Three approaches were picked out for investigation. Two of these, radiofrequency confinement and the pinch effect, were studied only briefly; the third, the magnetic mirror, survived all vicissitudes and now represents the central theme of our magnetic fusion energy program. It is also the central theme of this review, which will not attempt to deal with other earlier LLL fusion programs.

The magnetic mirror effect is a simple one. In a magnetic field, charged particles move in helical orbits. These orbits can be thought of as a circular motion on which is superposed a linear motion of the instantaneous center of rotation along the field lines. The circular part is, in effect, a tiny electric current. When the gyrating particle moves along the field lines toward a region of increasing field strength (the mirror, where the field lines converge), a repulsive force will react on its circular current, tending to repel the particle. Depending on the strength of the mirror field and on the particle's amount of circular motion compared to its linear motion, the particle will be reflected, that is, its lin-

ear motion will be stopped and reversed before the particle can penetrate to the high point of the mirror field.

The intuitive leap made at the onset of the mirror program was that the mirror effect could confine a hot fusion plasma by capping the ends of a region of uniform magnetic field with regions of intensified field (the mirrors) between which the charged particles could be trapped. If the mirrors were strong enough, and if the nuclei were energetic enough, it seemed likely that the nuclei could fuse before escaping.

In its qualitative way, this intuitive leap was correct. But complexities would arise to cloud the issue and add decades to our hoped-for time to success. The mirror program story, still unfolding, is one of encountering and dealing with these complexities. In this we have paralleled all other approaches to magnetic fusion, where similar issues have impeded progress. What is heartening is that persistence and ingenuity have been rewarded, if not yet by final success, then by a firmly based conviction that success, scientifically speaking, lies well within the next decade.

The first order of business was to demonstrate, however primitively, that mirrors could contain plasma. To do this, we needed to design and build confinement apparatus, to find a technologically feasible method of generating and trapping plasma in our apparatus, and to devise noninterfering diagnostics to tell us if we were successful or, if not, what had gone wrong.

The first experiment was indeed primitive, yet it showed clearly that mirror confinement occurred. Carried out in Berkeley by the author, assisted by J. Steller (also now at LLL), it consisted of creating a transient plasma in a glass-tube confinement chamber, using a pulsed rf source to ionize low-density hydrogen gas within. Surrounding the tube were magnet coils that could produce a uniform central field with or without mirrors at the ends. The tube's metal-coated inner wall became a waveguide propagating microwaves through the confinement region. With plasma present at high density, the propagated signal was attenuated; as the plasma density decayed, the attenuation diminished. Com-

paring the microwave data with and without mirrors showed that the plasma lingered on in the tube for a longer time with mirrors on. Crude though it was, this experiment encouraged us to press on.

During these early years, while the U.S. fusion program was still classified, several important developments occurred in the LLL mirror program. These were reported at the 1958 Geneva "Atoms for Peace" conference, when all work on controlled fusion was declassified. Some of the LLL milestones were:

- Creation of hot, mirror-confined plasma by plasma injection and magnetic compression (Toy: F. Coensgen and co-workers).

- Millisecond-long stable confinement of a hot-electron plasma between mirrors (Table Top: F. Ford, R. Post, and co-workers). In a hot-electron plasma, the electrons are preferentially heated. Though not of direct fusion interest, this plasma could be used to investigate certain aspects of the confinement problem.

- Development of the microwave interferometer as a tool for measuring plasma density (C. Wharton, A. Gardner, and co-workers).

- Development of ultrahigh vacuum techniques and their application to a mirror experiment using energetic ion injection (Felix: F. Eby and C. Damm).

- Development of fiber-composite-reinforced pulsed magnets and high-power condenser banks (F. Coensgen, E. Hartwig, and others).

Important theoretical steps were also taken, among them:

- Theoretical calculation of the collisionally induced rate of escape of hot plasma from a mirror machine (M. Rosenbluth, D. Judd, and co-workers).

- Formulation of the idea of the *flute instability*, that is, hydromagnetic (MHD) instability of plasma confined in a simple mirror machine. (Proposed conceptually by E. Teller for mirror systems, this instability is of central importance in magnetic confinement.)

- Development of the theory of adiabatic confinement of charged particles in mirror systems. This refers to the ability of mirror systems to con-

fine individual charged particles for long periods of time (millions of reflections) provided their circular orbits are small in diameter compared to the dimensions of the confining field (E. Teller and T. Northrop).¹

- Evaluation of impurity radiation losses from plasmas as a critical problem to be overcome in achieving fusion temperatures (R. Post).

- Identification of the problem of charge exchange as an important loss mechanism that must be controlled in magnetic fusion systems (J. Carothers).

- Proposal of the idea of creating high-temperature fusion plasma through the injection and capture of energetic ions, followed by magnetic compression (R. Post).

- Proposal of *neutral beam injection*, a method of injecting energetic particles into fusion plasma by shooting them through the confining field as neutral atoms, to be ionized upon impact with the confined plasma (S. Colgate, then at LLL, and independently W. Brobeck, then at LBL).

The above experimental and theoretical developments provided the LLL mirror program with a base on which progress in recent years has been built. These early efforts did not, however, uncover some of the recondite problems that have had to be faced and overcome.

DOG DAYS AND THE DEPRESSION YEARS

The decade following the Geneva conference was a period of frustration and disappointment throughout magnetic fusion research, both in the U.S. and abroad. The widespread and pernicious nature of plasma instabilities emerged as the main barrier to achieving fusion.

Attempting to confine a hot plasma, free from contact with surrounding material walls, defies natural tendencies. Being a gas, plasma attempts to fill all available volume and to reach temperature equilibrium with its surroundings. If restrained magnetically, it tries to escape across the confining field. Plasma instability—that is, spontaneously excited turbulence and oscillations in the plasma—provides a short cut, allowing escape to occur on time scales

thousands or even millions of times shorter than those required for useful fusion.

There was one bright light on an otherwise bleak landscape of negative results. Building on earlier theoretical work, Ioffe and co-workers in the U.S.S.R. showed that upon reshaping a mirror field so that it became a magnetic well (a field possessing a minimum in intensity for motion in any direction), all tendencies toward gross plasma instability were suppressed.

Theory had already predicted, and experiments confirmed, that plasma in a simple mirror (one with circular coils) is grossly unstable against sideways motion. The field produced weakens in directions perpendicular to the coil axis, so that the plasma (so to speak, located on a saddle-shaped region) will move sideways at the slightest provocation. In a magnetic well, this is impossible: the most damaging of all plasma instabilities, gross MHD instability, is eradicated.

Within months, Ioffe's success was duplicated at LLL (in Table Top, and then in Toy Top II and ALICE). In fact, LLL went on to pioneer new and improved magnetic well configurations, starting with Baseball I, which used a coil shaped like a baseball seam (see the cover). (Such a coil was first suggested by British physicists, who called it a tennis ball coil.) More recently, we invented the so-called "yin-yang" coil, which has still greater flexibility and efficiency.

Thus, while other approaches to magnetic fusion were still wrestling with the problem of MHD instabilities, the LLL mirror program moved into two problem areas that lay beyond those concerns. The technical problem was how to create and then sustain mirror-confined plasma at fusion temperatures. The scientific problem was how to overcome a different class of plasma instabilities—high-frequency oscillations—predicted by theory. Though much less violent in their effect on confinement than the MHD instability, these microinstabilities could speed the diffusion of particles through the mirrors and thus prevent the achievement of net power. Unraveling this problem has taken many years, and the task is not yet completed.

THE PUSH TOWARD FUSION TEMPERATURES

We first launched a two-pronged attack on the problem of creating a fusion plasma. The simplest solution found, and the one that first achieved success, was to follow injection by magnetic compression. In fact, in the Toy Top experiment, where extremely high, pulsed magnetic fields in excess of 20 T (200 kG) were used, fusion neutrons were produced. This feat was among the earliest such achievements in the controlled fusion program.

The plasma injection/magnetic compression technique is relatively simple. An array of plasma guns (pulsed devices producing microsecond bursts of streaming deuterium plasma with translational kinetic energies of about 1 keV) is fired into a guiding magnetic field. A plasma cloud, composed of electrons and the most energetic deuterons channeled by this field, arrives at the confinement zone a few microseconds later. Before slower deuterons and still slower impurity ions can arrive, the confining field is rapidly pulsed up from its initial low value (one-tenth of a tesla or so), reaching peak value in a few hundred microseconds. This action has three consequences:

- A plasma at kilovolt energies is trapped between rising mirrors.
- This trapped plasma is further heated by being compressed on the inward-moving flux surfaces of the rising magnetic field.
- Impurity ions from the plasma sources are kept from entering the confinement chamber by encountering the pulsed-on mirror field.

The above technique resulted in the creation of high-temperature (1 to 5 keV, or 10 to 50 million °C), mirror-confined deuterium plasmas. With such plasmas, we could begin a systematic study of the processes that lead to plasma losses.

Early on we found that, unless controlled, residual gas and associated charge-exchange effects caused rapid plasma loss. Loss of hot plasma by charge exchange occurs when a fast deuterium ion of the trapped plasma captures an electron from a slow-moving gas atom and becomes a fast neutral atom. Unaffected by the confining field, this fast neutral escapes, leaving behind a useless slow ion

(the atom from which it captured the electron). In the 2X experiment this problem was overcome by evaporatively coating the vacuum chamber walls with a thin, clean, titanium metal surface that removed impurity atoms by surface adsorption. Pioneered at LLL, this technique has found its way into fusion experiments throughout the world.

With charge-exchange-induced losses under control, we could now determine the rates of plasma-related loss processes. In brief, three important physics results emerged from 2XII, a follow-on experiment to 2X:

- In the 2XII magnetic well, MHD stability was preserved even at beta² values as high as could be achieved at that time (40%).

- Up to plasma ion temperatures of about 1 keV, plasma instabilities did not dominate, and plasma losses agreed well with theoretical values for collision-induced losses.

- As the plasma temperature was increased beyond about 1 keV, so that the rate of collision-induced losses (varying inversely with the three-halves power of the ion temperature) decreased, plasma confinement time did not improve, losses now being dominated by unidentified high-frequency instabilities.

ALICE, BASEBALL, AND NEUTRAL BEAMS

In the long run, a critical problem for any approach to magnetic fusion energy is how to heat and replenish the fuel in the firebox. A great practical advantage of the mirror is that it can operate steady-state, that is, it can use dc magnetic fields. Neutral beams seemed to us an almost ideal solution to the problem of creating, heating, and sustaining plasmas in dc confining fields at the billion-degree kinetic temperatures believed required for an operating mirror fusion power plant.

The ALICE experiment (for adiabatic low-energy injection and capture experiment) and its programmatic descendants, Baseball I and II, were begun to find out if neutral beams could build a hot plasma from scratch and maintain it in steady state (see Fig. 1 and the photographs on p. 6). Although

they were successful in demonstrating the idea at low plasma densities, it took 2XII:3 (2XII fitted with high-power neutral beam injectors) to make the push to fusion densities. What the earlier experiments did was to launch us into neutral beam technology and to expose the microinstability problem in mirror systems.

Neutral atoms, though unaffected by a magnetic field, may be ionized by passage through a plasma. Thus an energetic neutral atom directed from outside at a magnetically confined plasma can cross the confining field, become ionized in transit through the plasma, and deposit therein a new electron and a new energetic ion. If intense enough, neutral beams can create a high-density hot plasma, starting from a seed plasma. If maintained at a current level balancing plasma losses, neutral beams can sustain a plasma at temperatures and densities adequate for fusion power.

Success in using neutral beams to create a hot plasma depends sensitively on quantitative issues. In ALICE we faced two technological problems: available sources produced only a few milliamperes equivalent of neutral beam current, and theory indicated that extremely high vacuums—less than 0.1 μ Pa (10^{-10} Torr)—would be required for

Fig. 1. Fabricating the Baseball II superconducting coil. This was the largest high-field superconducting coil used in fusion research. It provided 2 T (20 kG) in the center and up to 8 T (80 kG) at the surface of the coil.

A.T.C.T. and its programmatic descendants, Baseball I and II. Baseball II used a magnet coil wound with superconductor—the largest high-field superconducting magnet built at that time.

plasma buildup, even with neutral beams 100 times more powerful than those then available. Both obstacles were overcome: a tightly focused neutral beam of hundreds of milliamperes equivalent was developed (H. Gordon), and vacuums good enough to permit this beam to initiate plasma buildup, essentially from scratch, were achieved.

At first all went well: a hot plasma was created with an ion temperature of 20 keV and mean particle lifetime of about 1 second. But attempts to push plasma density above 10^{19} particles cm^{-3} (about 1/100,000 of that needed for fusion power) ran into the onset of high-frequency instabilities. Weak though these instabilities were at low plasma density, they were strong enough to prevent the also weak trapping rate from driving the density higher.

In trying to learn how to overcome these instabilities, we studied the stabilizing effects of ion-ion collisions in randomizing the particle velocities in the plasma. Baseball II showed that with low beam energies (about 1 keV), a stable, well-randomized, low-density plasma could be achieved, one whose loss rate agreed almost exactly with that calculated theoretically. But our lack of knowledge at that time kept us from pushing the plasma through the unstable intermediate-density regime to the higher densities where theory predicted the microinstability problem would be ameliorated.

Baseball II had another important technological result. It used a magnet coil wound with superconductor—at the time, the largest high-field superconducting magnet constructed. Through this design, construction, and operation of this magnet, we gained invaluable experience in the practical application of superconductors, knowledge now being put to use in the design of the new mirror fusion test facility (MFTF).

Though our early attempts to achieve high plasma density through neutral beam injection failed, we remained convinced of its potential. It seemed clear that beam current limits were the problem and that given intense enough beams, buildup to high density could be achieved. The truth of this hunch has been amply demonstrated by the successes with 2XIIIB.

BEAMS, STREAMS, AND BETA: THE 2XIIIB STORY

Although 2XII achieved plasma densities believed adequate for a mirror fusion power plant, we were stymied in our attempts to use it to study the confinement of plasmas hotter than about 10 million degrees (1 keV). This barrier was imposed both by technical limitations inherent to the plasma injection magnetic compression approach, and (as understood in retrospect) by the emergence as temperature increased of losses from microinstabilities. We decided that it was essential to take a bold step in order to push 2XII into the high-temperature, high-plasma-density regime required for a critical test of mirror physics. The step: to upgrade neutral beam current a thousandfold as compared with that used in Baseball II.

The gamble paid off. Sources delivering 50-A-equivalent, tightly focused neutral beams were developed in a joint LLNL effort; 12 such sources were mounted on 2XII, now dubbed 2XIIIB (see photographs on p. 8). Furthermore, contrary to the fears of some, the beams did not self-destruct by destroying the seed plasma used to initiate plasma buildup. In fact, plasma density and temperature built up rapidly to higher values than those ever achieved in 2XII.

Despite this initial encouragement, however, all was not well. The product of particle containment time and density ($n\tau$) should have increased more than tenfold with the increased ion temperature. It did not. Instead, strong high-frequency turbulence appeared, arising from a long-expected instability mode whose apparent absence had puzzled us for years.

Ten years earlier, together with M. Rosenbluth at Princeton, we had predicted a mode called the drift cyclotron loss cone, or DCLC for short, a microinstability feeding on the presence of steep radial density gradients in mirror-confined plasmas. In the small-diameter plasma in 2XIIIB (only 3 or 4 ion-orbit diameters across), we would have expected this instability. But not until we could maintain the plasma temperature at the hundred million degree level did the DCLC mode clearly appear.

Presented with evidence that we were dealing with

2X, 2XII, and 2XIII. The twelve neutral beam sources of 2XIII (six each in the two circular tanks on either side) enabled us to reach the high-temperature, high-plasma-density regime needed for a critical test of plasma physics.

a known instability, the problem could now be tackled head-on. F. Coenegen, in charge of 2XHB, made a critical decision: go all out to try a stabilization technique—warm plasma streaming—predicted theoretically years earlier. The idea was to suppress the DCLC mode by injecting a low-density stream of relatively low-temperature plasma through one of the mirrors. This idea was quickly tested. By modifying circuitry driving the plasma guns used to create the seed plasma, we produced a long-duration, low-density plasma stream. Upon penetrating the hot plasma, this stream dramatically reduced the level of the DCLC mode. Confinement times increased tenfold to 5 ms. Best of all, the confinement factor, $n\tau$, now increased with increasing ion temperature as theory said it should.

With the experimental picture clarified, FFI theorists (H. Berk, D. Baldwin, and others) formulated a comprehensive picture of the DCLC mode. Their theory explained additional details of warm plasma stabilization and, gratifyingly, also explained why the mode had lain apparently dormant in earlier years: through the particle diffusion it caused, the DCLC mode would tend to generate its own warm plasma component. This in turn kept the instability level low—too low to be observed until the ion temperature was high enough (and thus the classical confinement time long enough) to expose these relatively weak additional losses.

These developments marked the successful end to our long quest in mirror research: how to achieve a stable plasma state at high plasma temperature and density. The whole thrust of the mirror program then changed from a search for encouragement that mirror confinement was a *qualitatively* feasible concept, to a forward-looking commitment to finding the best ways to achieve the *quantitative* confinement requirements needed for economic fusion power.

The stabilized 2XHB produced one additional result. We mentioned previously the important plasma parameter beta: the ratio of plasma energy density to energy density of the confining magnetic field (as evaluated without plasma)—that is, the energy density of the so-called vacuum magnetic field. Starting with a vacuum field, as plasma density

builds up and beta increases, the plasma depresses the magnetic field within its interior; the plasma acts as a diamagnetic medium tending to exclude magnetic field. Mechanically, diamagnetism results from elementary electrical currents associated with the circular motion of the plasma's charged particles. These elementary currents, summed over the totality of confined particles, result in the diamagnetic current, whose interaction with the confining field produces inward-acting forces balancing the plasma pressure.

Significantly, in 2XHB the plasma beta value built to such high levels (up to 2.0) that the magnetic field deep inside the plasma was markedly depressed relative to its vacuum value. That such high beta values can be obtained without gross instability, though at first surprising, is consistent with the capabilities of the deep magnetic-well field in 2XHB.

The 2XHB beta values are far higher than those attained in the other major magnetic confinement approach, the doughnut-shaped tokamak. They have three implications. The first one is economic: fusion power varies as the square of beta at a given confining field strength. Thus, high beta greatly enhances the efficient use of the confining field, reducing the cost per kilowatt of fusion of providing that field. Second, confinement of plasma in a mirror magnetic well improves with the well's depth. Since plasma diamagnetism acts to deepen the well, a high-beta plasma tends toward becoming better confined owing to its own presence. This concept leads to the third, and most tantalizing, implication: the possibility of using plasma diamagnetic currents to achieve field reversal.

TWO WAYS TO HIGHER Q

As mentioned earlier, with the proof in 2XHB that mirrors represent a viable approach to the confinement of dense plasmas at fusion temperatures, our concern has shifted from the qualitative to the quantitative. Our goal now is to find the most direct route to applying mirror-confinement physics in a fusion power plant. To be sure, there still are knotty scientific problems to unravel, but the climate is different.

We visualize a mirror fusion power plant as a

Fig. 2. Power flow diagram for a fusion reactor. Mirror fusion reactors are visualized as driven systems, that is, ones in which plasma temperature and particle density are maintained in steady state by injecting intense beams of energetic neutral atoms. The electrical power needed to generate these beams will be tapped from the reactor's power conversion system; most if not all of this recirculated power will be obtained from the direct converter. This converter will extract energy both from the charged fusion-reaction products and from unreacted fuel ions.

"driven" steady-state system, that is, one in which a fusing plasma is maintained by neutral beam injection. Electricity needed to power the injectors is tapped off from fusion power derived from the plasma. As shown schematically in Fig. 2, the fusion energy is converted both through conventional thermal means (steam or helium gas turbines, for example) and through direct conversion to electricity of the kinetic energy of plasma particles escaping through the mirrors. Direct conversion is highly efficient: in the laboratory, we have demonstrated efficiencies up to 86%; in a practical device, 60 to 75% would be feasible.

The cost per kilowatt of fusion power from a driven system depends sensitively on the efficiency and capital cost of its elements, including the confinement system. The parameter Q quantifies the sensitivity to confinement; Q is defined as the ratio of fusion-power release to the input of plasma heating power. But now an important point: a deuterium-tritium-fueled mirror machine with economically feasible values of the mirror ratio (ratio of mirror field to vacuum field in the center) can have a theoretical maximum Q of about 1.5. To make net power at this low a Q requires high efficiencies in the neutral beam injectors and in the thermal and direct converters and implies high re-

circulated power, all severe economic penalties. On the other hand, even modest improvements in Q result in major reductions in costs, a trend illustrated in Fig. 3. In fact, reaching a Q of 5.0 would reduce recirculated power to a comfortably low level.

Our answer to this essentially economic requirement for higher Q has been to embark on two modifications of the mirror idea, the tandem mirror (proposed by K. Fowler and G. Logan)³ and the field-reversed mirror.

Fig. 3. Impact of Q enhancement on overall reactor costs. A Q of 5.0 would reduce recirculated power to a comfortably low level.

Fig. 4. Tandem mirror concept. The magnetic fields of the baseball coils at the two ends serve as plugs to reflect ions escaping from the ends of the central confinement chamber—or solenoid. The end cells, fed by high-energy neutral beams, stabilize the central fusion plasma and provide longitudinal confinement.

The tandem mirror, shown schematically in Fig. 4, places mirror cells at the ends of a central confinement chamber. These end cells, fed by neutral beams of very high energy (500 keV or higher in a power plant), are electrostatic dams confining a lower density, lower temperature D-T plasma that produces the fusion power release. The idea takes advantage of an inherent characteristic of mirror confinement: the naturally arising ambipolar electrical potential holding in the electrons, which, being much more mobile, would otherwise tend to be lost more rapidly than the magnetically confined, more massive ions. As Fowler explains it, just as a narrow dam can hold back a large lake, two small-volume mirror plugs can, through their ambipolar potential, electrostatically hold in the much larger volume of reacting plasma in the chamber between the plugs.

In a tandem mirror, where Q is determined by the ratio of the volume of the central plasma to the volume of the plasma in the end mirror cells, Q values of 5.0 seem readily accessible. Apparatus to test this idea, the TMX facility, is due for completion in late 1978.

Although relatively new to the LLL mirror program, the field-reversed mirror has much earlier origins, both at LLL and elsewhere in the fusion community. Its earliest roots are from the ASTRON program at LLL, the brainchild of the late Nicholas Christofilos. In ASTRON it was proposed to produce the confining field by trapping (between magnetic mirrors) a ring-shaped entity made up of multi-MeV electrons. The diamagnetic current created by these circulating electrons represents a nonmaterial magnet winding, one that would create in the vicinity of the ring a closed pattern of magnetic lines within which a fusing plasma could be confined. Though ASTRON had not reached its objectives, for largely technical reasons, at Christofilos' untimely death in 1972, his idea helped to pave the way to the field-reversed mirror.

In this concept, shown schematically in Fig. 5, powerful neutral beams would be used to create field reversal, not with electrons as in ASTRON, but with the fusion plasma ions themselves. Given powerful enough neutral beams, aimed tangentially as shown in the figure, the ionic diamagnetic currents should be high enough to create field reversal,

that is, to close field lines deep within the mirror cell back on themselves. Relative to the standard mirror, confinement should improve since deeply trapped particles must diffuse across the closed field lines before they can escape through a mirror. A plasma cell the size of a basketball might produce tens of megawatts of fusion power at a predicted Q of 5.0 or more.

The field-reversed mirror concept is indeed attractive, embodying desirable features of both the mirror and the toroidal magnetic confinement approaches. But being an extrapolation beyond present experience, it raises unanswered questions concerning stability and confinement. We are making first attempts, therefore, to explore field reversal in 2XIIB. These attempts, though encouraging in that a plasma state close to field reversal was achieved, have not yet succeeded, suggesting that more powerful beams may be needed. Pulsed neutral beam sources to provide an additional "push" are under development.

There may also be fundamental obstacles barring the achievement of steady-state field reversal through the particular avenues we are now pursuing. Nevertheless, we are encouraged by the transient field-reversed states already seen at other lab-

oratories, using other approaches.

To pursue both the physics of the tandem mirror plugs and the possibility of a field-reversed mirror, we are now building a scaled-up mirror experiment, MFTF, that, with its operating temperature of 500 million degrees and its plasma volume nearly 100 times that of 2XIIB, could come close to proving the scientific feasibility of the mirror approach.

IS IT WORTH THE CANDLE?

Throughout this article I have tried to emphasize the search through unknown territory that the magnetic fusion energy program has represented and still represents. Although fusion power research, taken as a whole, has now advanced so far that it is virtually assured that some approach will succeed, we cannot as yet be sanguine about any one specific approach.

We are still in the exciting time of an evolving field, and involvement with the discovery of the unknown implies that there will be disappointments as well as victories. But those of us who are in the mirror program feel no need to apologize for the fact that we have not yet produced a working power plant. We are pursuing with growing scientific success what is at once one of the most challenging and most rewarding scientific and technological quests that man has ever undertaken—to achieve, once and for all, a safe source of energy that will be available for as long as man will need it on earth, and that may one day take him to the stars.

Key Words: ALICE; ASTRON; Baseball; CTR project; controlled thermonuclear reactions; magnetic fusion; magnetic fusion energy; neutral particle beams—*injection*.

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1. An important experimental test of this concept was the serendipitous result of the famous ARGUS experiment in 1958, proposed by Nicholas Christofilos, in which a small nuclear bomb was detonated in the earth's magnetosphere. Energetic electrons created by this explosion were trapped in the earth's dipole mirror field, and some have remained there, reflecting back and forth between the north and south mag-

netic poles, for over a decade—that is, having traveled light years.

2. Beta refers to the parameter ratio of plasma energy density to the energy density of the confining field. Since fusion power varies as the square of beta, if beta becomes too small (a few percent), it would not be possible to satisfy economic constraints in a fusion power system as imposed by the capital cost of the confining field.
3. Also proposed independently by G. Dimov in Novosibirsk, U.S.S.R.

Propagation of High-Energy Laser Beams Through the Atmosphere

The atmosphere can degrade the propagation of high-energy laser beams in two ways. First, minute temperature fluctuations in the atmosphere cause density fluctuations and, hence, refractive index fluctuations that cause beam defocusing and wandering. Second, absorption of minute amounts of energy from a laser beam creates density gradients that turn the transmitting air into an extended divergent lens, causing a spreading—or thermal blooming—of the beam. Funded by several Department of Defense agencies, we have mounted an extensive calculational effort to analyze these atmospheric propagation effects.

Although there has been considerable progress in developing high-energy lasers, any military application of these devices will depend on many developments, one of the most important of which is the successful propagation of their energy through the atmosphere to a target. Studying the atmospheric propagation of laser beams is thus quite important to the military high-energy-laser (HEL) program.

The two main effects that can limit the intensity and fluence of laser beams on a target are beam degradation and air breakdown. (Air breakdown is the condition in which the air becomes heated, turns into a slightly ionized plasma, and prevents, by reflection or absorption, the propagation of the laser

beam.) In this article, we will discuss beam degradation.

There are four principal sources of beam degradation. The first of these is the laser itself. Because of inhomogeneities in the laser medium and of imperfections in the laser's optical train, it is never possible to produce a diffraction-limited beam, that is, a beam that can be focused down to the minimum spot size permitted by wave optics. Thus, practically attainable laser systems and their focusing optics produce beams with aberrations that create larger focal spot areas and weaker peak intensities than would be obtained with diffraction-limited beams of the same power.

A second source of beam degradation is beam jitter, caused by the unavoidable random instabilities in the laser's pointing and tracking system.

The two remaining sources of beam degradation, optical turbulence and thermal blooming, are due to the atmosphere itself. These two sources are the subjects of this article. Optical turbulence is a purely linear phenomenon that arises from minute temperature fluctuations in the atmosphere that in turn create density and refractive index fluctuations. These random fluctuations lead to both a defocusing of the beam and an erratic wandering of its direction.

Thermal blooming is a nonlinear defocusing of the laser beam caused by the atmosphere absorbing minute amounts of laser energy along the propagation path. The result of this absorption—heating—creates density and refractive index gradients that

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convert the propagation path into a kind of extended divergent lens. Since the air density is most reduced near the center of the laser beam, light will tend to be diverted out of the beam toward regions of higher density and higher refractive index; as a result of this effect, the beam spreads. Although the attenuation of laser beams by the atmosphere is exceedingly weak in the so-called atmospheric windows¹ (the absorption coefficient is about 0.2/km for carbon dioxide laser beams operating at a 10.6- μ m wavelength), thermal blooming can, under some conditions, lead to severe beam degradation for 500-kW to 1-MW beams propagating over kilometre distances.

FOUR: COMPUTER MODEL FOR STUDYING BEAM-DEGRADATION SOURCES

A general theoretical modeling of atmospheric laser-beam propagation to study these beam-degradation sources requires a computer model of considerable complexity and versatility. It must take into account the particular conditions under which the laser beam will be used. These conditions can vary greatly and, consequently, any one or any combination of the degradation sources may become of overriding importance, or all of them may assume comparable importance.

The theoretical model must also be prepared to treat beams from lasers that differ fundamentally in their mode of operation. Lasers may put out their energy in a steady flow (continuous-wave lasers), or they may emit a train of pulses of microsecond duration (multipulse or repetitively pulsed lasers).

We have developed such a comprehensive computer model under contract to various Department of Defense agencies. (It is now funded by the Army's Missile Research and Development Command; previously, it was funded by the U.S. Navy and DARPA, the Department of Defense Advanced Research Projects Agency.) It is embodied in a code called FOUR²⁻⁴ that calculates in four dimensions, three spatial directions plus time. This code is currently being used to assess the capability of HEL weapon systems by the Army, Navy, and Air Force.

Mathematical Basis of FOUR. Since both refraction and diffraction are important in long-range atmospheric propagation of laser beams, we describe the laser field by means of a three-space-dimensional wave equation. We describe density changes due to atmospheric heating by a two-transverse-dimensional linearized system of hydrodynamic equations, which includes the effect of forced convection due to wind. The presence of the wind is essential for the effective performance of the laser, since the wind clears the laser-heated air from the beam and thus limits the magnitude of the density changes.

Both the natural background wind and the kinematics of the military engagement scenario contribute to the resultant effective wind. The latter contribution is by far the most important. In the typical engagement, the laser platform is moving, and the laser is tracking a moving target. As a result of this tracking, the laser beam executes a rotating motion, referred to as slewing. In the most general case the laser beam and the velocity vectors describing the motion of the platform and the target do not lie in a single plane, and in this noncoplanar scenario the laser slewing motion will be three-dimensional. The component of the resulting wind velocity transverse to the direction of propagation will rotate, and the magnitude of this transverse component will be a nonlinear function of the propagation distance. The important parameters for a typical noncoplanar engagement scenario that can be handled by FOUR are shown in Fig. 1.

FOUR solves both the optical wave equation and the accompanying linearized hydrodynamic equations by highly accurate, discrete Fourier transform methods. The transverse spatial dependence of the electric field amplitude and the hydrodynamic variables are expressed as two-dimensional Fourier series with a finite number of terms. The problem variables are thus assumed to be bandwidth-limited in a spatial frequency sense, and the coefficients of the Fourier series can be evaluated by means of the highly efficient fast Fourier transform algorithm. For strictly bandwidth-limited solutions that vanish on the boundaries of the configuration-space computational mesh (as established from the sampling

Fig. 1. Diagram of a typical engagement scenario that can be handled by the FOURD code. The laser platform (transmitter), perhaps a ship, moves with velocity V_T in x direction. The missile target (receiver) is launched with velocity V_R and is aimed to impact at point P on the platform, a time t_c later. Background wind velocity is V_W . The laser is located at position L, which is distance l in front of the target-sighted point P. For a coplanar scenario, the instantaneous laser beam trajectory R , missile path $V_R t_c$, and platform path are in the same plane. However, it is likely that the laser is situated at a height h above the platform, at position L' . In such a case, the laser beam trajectory R' and missile and platform paths do not lie in the same plane. Laser slewing is thus three-dimensional, and complete stagnant wind conditions are avoided. FOURD can treat the fully three-dimensional, noncoplanar engagement scenario.

theorem of information theory), the above method provides an exact numerical solution at the mesh points.⁵ Of course, the above two conditions are never strictly fulfilled; but as long as the solution is reasonably bounded, both spatially and spectrally, we can expect a much higher degree of accuracy for the numerical solution than can be achieved by conventional finite-difference methods.

Because of focusing by the laser optics and defocusing by the atmospheric thermal lens, the laser beam undergoes gross changes in size as it propagates toward the target. Consequently, to make the calculation more efficient, we invoke a coordinate

transformation that changes the size of the computational grid to accommodate the changing size of the laser beam. This coordinate transformation is based on the beam's local phase front and is adaptively updated at each propagation step. The effect of turbulence is modeled by interposing at each propagation step a special phase screen constructed by a Monte Carlo algorithm from the statistical properties of the assumed atmospheric turbulence model.

Beam jitter can be considered as either low frequency or high frequency, depending on whether the beam direction shifts in a time that is long or

s. It is in comparison with a wind transit time across the beam. In the former case we account for the effect by convoluting the target irradiance pattern with a jitter distribution function. In the latter case we assume that the spatial pattern for the laser heating distribution along the propagation path can be obtained from a convolution of a jitter distribution with the local intensity distribution.

Important Applications of FOURD. FOURD was designed to treat a number of special situations that could not be handled on existing propagation codes

at other research laboratories, including:

- Transonic slewing.
- Stagnation (zero-velocity wind conditions).
- A complete treatment of multipulse-laser thermal blooming.
- Versatile predictive lens corrections for thermal blooming.

When a laser tracks a supersonic target, it is possible for the wind velocity to go through Mach 1 conditions at some point along the propagation path. Presumably, the resulting density changes can be large, and this might pose a propagation problem. To account for this, FOURD employs a solution of the linearized hydrodynamic equations—good for all Mach numbers. Examples of isodensity contours are shown in Fig. 2 for wind speeds slightly below Mach 1 (Mach 0.875) and somewhat above Mach 1 (Mach 1.5). Calculations with FOURD for realistic transonic slewing scenarios, however, have shown that, although density changes can be sizable near Mach 1, the total integrated phase changes along the propagation path tend to be small. Hence, transonic slewing is no longer regarded as a deterrent to the use of HEL beams.

Stagnation can result at some point along the propagation path if the laser platform and the target move in opposite directions. In this case, the effective wind due to the motion of the laser platform can just cancel the wind due to the laser slewing motion. Large density changes can result over significant distances on either side of the actual stagnation position.

In contrast to the transonic slew situation, stagnation conditions can lead to large phase changes and very serious thermal blooming. Figure 3 shows the time evolution of the thermal blooming for a multipulse, 50-kW carbon dioxide laser beam (operating at a $10.6\text{-}\mu\text{m}$ wavelength) that passes through a coplanar stagnation region. The repetition rate is 100 pulses/second. The figure shows contours of constant intensity in the focal or target plane; the initial pulse is, of course, unbloomed. The magnitude of the thermal blooming after 0.25 s is very significant and is typical of coplanar stagnation-zone situations.

Most actual scenarios, however, are to some

Fig. 2. Isodensity contours of the heating by a Gaussian-shaped laser beam with (a) a Mach 0.875 wind velocity and (b) a Mach 1.5 wind velocity. Wind is from left to right along the x axis. Contours represent changes of 10%.

degree noncoplanar. Thus, a nominal vertical wind component will generally exist even when the horizontal wind component in the plane transverse to the beam direction vanishes. This vertical wind component will have an ameliorating effect on the thermal blooming. It will also allow the establishment of a steady state, which does not require a time-dependent calculation to describe it. FOURD has full capability for treating all noncoplanar

effects.

Most treatments of the thermal blooming of multipulse beams include only the effects of earlier pulses in the train on the current pulse. This greatly simplifies the hydrodynamic calculation, since we can determine density changes based on equalized pressures. Under certain conditions, however, the effect of the current pulse on itself, or the so-called self-blooming effect, must also be taken into ac-

Fig. 4. (a) Change in the isointensity contour at the target as a function of time for a typical pulse in a train from a multipulse laser. The steady-state situation is assumed in which a given pulse is periodically repeated. In a real situation, a pulse is affected by the heating from all of the previous pulses and, in addition, by its own heating, as assumed here. This latter self-blooming effect is difficult to calculate because it requires a two-dimensional time-dependent hydrodynamic calculation for each propagation step. Also shown are (b) the isointensity contour pattern for the case where self-blooming is neglected and (c) the time-averaged isointensity contour pattern obtained by integrating over the complete pulse.

count. This requires a fully time-dependent treatment of the complete set of linearized hydrodynamic equations.

Such a calculation has been performed for the first time on FOURD, and results are given in Fig. 4. Shown are the target isointensity contours as a function of time for a typical pulse in a steady-state train. (After an initial transient stage, a train of pulses will reach a steady-state condition in which all pulses are alike.) These target contours are compared with the isointensity contours for a typical pulse under the assumption that only previous pulses affect the density distribution. Clearly, the effect of self-blooming can be all-important.

Modern laser mirror technology has produced adaptive focusing optics—"rubber" mirrors—that can be adjusted on a millisecond time scale. These adaptive optic systems can compensate at least partially for thermal blooming if most of the atmospheric heating occurs near the laser aperture, a situation that prevails when slow rates are high. Under these conditions, the heated atmosphere can be approximated by a thin lens, which can in turn be compensated for by the laser's focusing optics. Typical beam corrections have resulted in factors of two to three improvement in the peak intensity in the target plane, and factors of five have been obtained

for some scenarios.

Adaptive optical systems may operate in either an open-loop or a closed-loop manner. In the closed-loop mode, the adjustable mirror is driven by a return signal from a bright spot, or "glint," on the target. In the open-loop mode, the mirror must be adjusted to compensate for thermal blooming, based on whatever information is available. FOURD is equipped with an efficient algorithm for providing this predictive-phase correction information for the open-loop mode.⁶

The predictive-phase algorithm, in effect, propagates the beam through a normal atmosphere to a position intermediate between the laser and the target and then propagates the beam backward through a vacuum over the same path to the laser. The phase of the resulting beam at the aperture is the conjugate of the desired phase correction. The intermediate position is determined by an iteration involving two or three separate calculations. Figure 5 shows examples of the improvements in the isointensity contours in the target plane obtainable from this type of correction.

PROPAGATION CHARACTERISTICS OF REAL LASER BEAMS

In the initial studies of HEL propagation, idealis-

tic assumptions were made regarding the amplitude and phase distributions at the laser aperture. The amplitude distribution was usually assumed to be a truncated Gaussian curve, and the phase distribution was assumed to correspond to an ideal-focusing spherical wave front. However, we are currently interested in determining the propagation characteristics of beams coming from real laser hardware.

Figure 6 shows the intensity and phase distributions for the Abel laser, a repetitively pulsed carbon dioxide laser under construction for the Army by Hughes Aircraft and the Pratt and Whitney Corporation.

Figure 7 shows the isointensity profiles at a 1-km range for a typical Army engagement scenario. Note that thermal blooming always tends to broaden the laser beam in the direction perpendicular to that of the wind. Since the high-density air tends to bunch in the upwind direction, the beam also undergoes a focusing in the upwind direction.

Fig. 7. Isointensity contours in the target plane for the Army's repetitively pulsed Abel laser corresponding to the incoming flying-target scenario. The wind blows from left to right. The range is 1 km; the pulse repetition rate is 125 Hz.

For beams that are circularly symmetric at first, a thermally bloomed beam will consequently take on a crescent shape. The shapes shown here are considerably more complicated, due to the asymmetry characteristics of the aperture beam. However, despite the appearance of the beam, the intensity was degraded by only a factor of two below what it would be if an ideal spherical phase front were placed on the beam at the aperture.

FUTURE PLANS

We have made a number of theoretical contributions to the Department of Defense's HEL propagation program, including:

- Development of a general, versatile, fully time-dependent, three-space-dimensional laser propagation code.
- An accurate treatment and assessment of the effect of transonic slewing on thermal blooming.
- A quantitative treatment of stagnant wind conditions and their effect on laser propagation in practical situations.
- The first treatment of wind conditions intro-

duced by noncoplanar scenarios.

- The first treatment of self-blooming for repetitively pulsed laser beams.
- Development of a simple, versatile algorithm for making phase corrections to compensate for thermal blooming.

Our HEL propagation program will continue. In the future we expect to complete the assessment of single-pulse thermal blooming effects on the performance of multipulse lasers. This will involve providing theoretical support to a research effort at United Technology Research Corporation intended to measure these effects. We will also investigate the impact of real beam characteristics on the performance of proposed laser devices, including the calculation of far-field irradiance properties for high-energy lasers with real amplitude and phase as well as multiline spectral characteristics. Finally, we will assess the accuracy of existing corrective optics methods and schemes for compensating atmospheric lens effects; to do this, we will use the numerical techniques and code subroutines developed during the last year for predicting the optical corrections required to correct for thermal blooming.

Key Words: laser beams—applications; laser beams—distortion; laser beams—effects of atmosphere; laser beams—focusing; laser pulses—nonlinear effects; laser pulses—self-focusing effects; laser weapons—computations; lasers—military applications.

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PARET: A Parameter Estimation Technique for Physical Applications

We have developed a technique for processing time-domain data to obtain the parameters required to represent the data by a sum of complex exponentials (or damped sinusoids). This is a significant capability since the exponential parameters can usually be related to fundamental physical quantities or attributes of the observed phenomena. We have used the method for data compression, curve fitting, linear prediction, radar target identification, radar and acoustic array processing, and structural dynamics and characterization. The parameter estimation technique (PARET) is based on Prony's method, anti-aliasing filtering, and statistical averaging and is incorporated in a user-oriented computer program.

Tall buildings continue to shake for many seconds after an earthquake stops. Radar signals return distorted by their interaction with a target. Shock waves from an explosion, traveling through the ground, are modified by reflections and refractions. These processes are only three of many that can be characterized by complex exponential behavior—processes whose time history can be represented by

$$f(t) = \sum_{n=1}^N A_n e^{s_n t} \quad (1)$$

A_n is the initial complex amplitude of the n th vibrational term (at $t = 0$), and s_n is the complex natural frequency of vibration.

An important feature of this formulation is that the complex natural frequencies of a system are invariant: they depend only on the physical dimensions and properties of the system and remain the same regardless of how the system is being excited. Hence, determining the s_n of a system becomes a prime objective of such diverse disciplines as structural analysis and radar target recognition.

We have developed a straightforward technique for determining the complex natural frequencies (s_n) and amplitudes (A_n) of a system by analyzing its output in response to an arbitrary input. Although Prony's method,¹⁻⁵ the mathematical formulation on which our work is based, was first described almost 200 years ago, it is seldom used because of serious difficulties in applying it. Recent advances in digital computers and an interest in system identification⁶ and digital signal processing⁷ have refocused attention on the process, however. Our contribution has been to devise new ways of organizing and processing the data to circumvent the difficulties and make the method practical.

Equation (1) can be simplified for computation by decomposing s_n into a damping constant, α_n , and a circular oscillation frequency, ω_n , by the relation $s_n = \alpha_n + j\omega_n$, where j is the square root of -1 . Under certain conditions, Eq. (1) can be written in terms of damped cosinusoids

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$$f(t) = \sum_{n=1}^N B_n e^{\alpha_n t} \cos \omega_n t \quad (2)$$

Figure 1, a plot of a single term from Eq. (2), explicitly shows the definitions of B_n , α_n , and ω_n for $n = 1$.

The vibration of a cantilevered beam loaded with masses at various discrete points is an example of a response that can be described by such a complex exponential series. The motion of any given point on the beam, though quite complicated, is the superposition of damped sinusoids. This behavior is very common; in fact, any process that can be described by a linear n th-order differential equation with constant coefficients will have a response composed of damped sinusoids.

The widespread occurrence of these processes with such behaviors makes their time-series analysis very important. However, when more than one term

Fig. 1. A single term in the complex exponential series

$$f(t) = \sum_{n=1}^N B_n e^{\alpha_n t} \cos \omega_n t$$

illustrating the decay induced by the damping constant α_1 and the period of the sinusoidal oscillation $2\pi/\omega_1$. B_1 is the initial amplitude of the envelope at $t = 0$.

is involved (that is, when several waveforms like that in Fig. 1 are superimposed), the determination of the natural frequencies and their amplitudes and damping constants from observations of system response becomes a complex task. For example, simple Fourier analysis

$$f(t) = \sum_{n=1}^N (a_n \sin \omega_n t + b_n \cos \omega_n t)$$

is inapplicable because damping precludes modeling by simple sines and cosines.

The invariance of the complex natural frequencies simplifies the work of predicting the response of the system to any arbitrary driving function. It also presents new opportunities in areas such as radar target identification. At the same time, access to the damping constants (α_n) in various oscillation modes is of great importance in such applications as earthquake safety analysis, where the rapid decay of oscillations would help minimize structural damage.

WAVEFORM PARAMETERIZATION BY PRONY'S METHOD

Since both α_n and A_n appear in each term of Eq. (1), solving for both directly would be a very difficult process involving coupled nonlinear equations. Prony's method is to convert Eq. (1) to more tractable form, based on the recognition that Eq. (1) is the solution of a linear difference equation whose coefficients are also those of a polynomial equation with roots related to the same complex natural frequencies. We set up a data record of values of $f(t)$ sampled uniformly, manipulate the data into appropriate form and solve for the coefficients of the linear difference equation, use the coefficients in the polynomial equation and solve for the roots, convert the roots into the complex natural frequencies, and combine these frequencies and the sampled data back into Eq. (1) to evaluate the complex amplitudes. This method condenses the nonlinear portion of the process into the solution of a polynomial equation, a significantly simpler task than that of solving coupled simultaneous nonlinear equations.

Prony's method, however, replaces a large tech-

nical difficulty with several practical difficulties that, together, had severely limited its usefulness. These are the problems of "aliasing," "rank deficiency," and noise.

Aliasing is the tendency of high-frequency components of the data to masquerade as low-frequency components when the data-sampling rate is inadequate. Communication theory states that each component in a signal being analyzed must be sampled at least twice per cycle. Hence the sampling frequency in Prony's method must be at least twice that of the component of highest frequency. Otherwise the highest frequencies will be "folded" downward and distort the computed spectrum.

The aliasing problem has two components: there is a practical limit to the sampling frequency that can be used, and most signals will contain a large number of very high frequencies. The solution comes from recognizing that the higher frequency components are usually weak and can be removed without seriously affecting the outcome. An "anti-aliasing filter" assures that the signal contains no component approaching the sampling frequency.

The rank deficiency problem concerns the matter of having enough terms in the exponential sum to adequately describe the waveform. Since each term in Eq. (1) contains two unknowns, the number of sampled data points must be at least twice the number of terms in the series. If the recorded waveform is "clean," that is, if there is a large signal-to-noise ratio, it is often simple to decide how many frequency terms (and hence how many data points) are needed to describe it. If the signal is noisy, the number of frequency terms may well be infinite.

Noise also enters into the aliasing problem. Noise, being atonal, contains a wide range of frequencies, especially high frequencies. The aliasing filter helps to remove some of this background, but it can do nothing about spurious noise within the frequency band occupied by the signal. Hence, we need a method that permits reliable analysis of the signal in spite of the noise.

SEMPEX,⁸ our original computer program based on Prony's method and an anti-aliasing filter, has been in use for several years and performs well on relatively clean signals. Noise in the data record

can make it fail catastrophically, however. Unlike the exponentials we have been considering, noise does not die away, thereby violating a basic premise of this type of data processing. Hence the attempt to represent it with exponentials leads to infinite numbers of frequency terms and arbitrarily high frequencies, with the attendant rank deficiency and aliasing problems.

We have developed a statistical averaging technique that suppresses noise effects. This technique, embodied in our new code PARET, performs a SEMPEX analysis on a slice of the data, stores the result, and then repeats the process on additional overlapping slices of the data and averages all the various results together. Noise displaces the computed natural resonances by random amounts in each of the intermediate results, but averaging tends to cancel out these random displacements and focuses on the true values. This ability to extract meaningful parameters from noisy data has opened up a broad range of applications.

APPLICATIONS

Our "moving window" statistical averaging technique is so new that its potential applications are still largely unexplored. Some idea of its scope may be obtained from its accomplishments to date, which include such diverse tasks as data compression, curve fitting, linear prediction, radar target identification, evaluation of the impulse response of electrical networks, study of the physical mechanisms of electromagnetic radiation from conducting wires, processing of signals from acoustic and radio arrays to determine angle of arrival and multipath reflections, synthesis of antenna arrays, determination of decay constants in nuclear processes, and structural dynamics analysis of the way buildings respond to earthquake shocks. We will confine our discussion to three of these applications: data compression and related topics, radar target identification, and structural dynamics analysis.

One way to define a transient waveform is by specifying its displacement from a baseline at each of a long series of times. This is the method an oscilloscope uses in displaying a waveform, and it results in a point-by-point graph. If the waveform con-

Fig. 3. An s -plane plot (upper left quadrant only) displaying the various component frequencies in the radar reflections from a 60-m straight wire loaded in the center (closed dots) and a similar wire bent in the center (open dots). Although the complex waveform of the radar response undergoes tremendous changes as the target moves or turns, the complex natural frequencies are essentially constant. Hence this technique shows great promise in radar target identification.

of automatic radar target identification.

Previously proposed automatic radar target-identification schemes involved comparing the waveform from an unknown target with a library of waveforms containing responses for all possible incidence angles and polarizations—a virtually impossible task. PARET describes the waveform with a small set of numbers that are essentially independent of orientation or polarization, greatly simplifying the identification. The radar return from an aircraft or a rocket is much more complicated than that from the straight wire illustrated in Fig. 3, but the principle is the same.

There are a number of very useful applications of PARET in structural analysis. One in particular is in analyzing the earthquake response of tall structures and indicating structural modifications that will limit or prevent damage. Figure 4 is an accelerometer recording of north-south motion on the

21st floor of a large hotel in Los Angeles during the 1971 San Fernando earthquake. Although the earthquake itself lasted less than 14 s (the driven response at the start of the record), the building continued to shake measurably for almost a minute. A similar record was obtained from an accelerometer in the basement of the same building.

The portion of the response selected for PARET analysis extended from 16.2 to 28.8 s. The "moving window" consisted of ten frames, the first from 16.2 to 27 s, the second from 16.4 to 27.2 s, and so on up to the tenth at 18 to 28.8 s. The accelerometer recorder had a low-frequency cutoff at 0.125 Hz, and we set the anti-aliasing filter at 1.3 Hz.

Figure 5 displays the frequencies and damping constants of the major oscillation modes encountered on the 21st floor and in the basement (color)

Fig. 4. Accelerometer measurement of motion recorded on the 21st floor of a large hotel in Los Angeles during the 1971 San Fernando earthquake. The earthquake lasted less than 14 s, but the building continued to shake for almost a minute. The colored band indicates the portion of the data selected for analysis with the PARET code. The "moving window" viewed this data in ten overlapping frames, beginning at 16.2 to 27 s and ending at 18 to 28.8 s.

Fig. 5. An s-plane plot (upper half only) of the complex natural frequency components of the earthquake-response waveform pictured in Fig. 4, together with similar information for an accelerogram recorded at the same time in the basement of the same building (color). The length of the vector at each point indicates the complex amplitude of that vibration mode on a modified logarithmic scale (numbers beside each vector indicate the amplitude in arbitrary units), and the direction indicates the relative phase. The upper and lower pairs of points are artifacts introduced by the necessary filtering. For the points between these extremes, the natural frequencies for each vibration mode agree well (within about 10%) between the top and bottom of the building. The damping constants show wide variation. The positive damping constants (to the right of the zero line) indicate modes that, instead of dying away, were increasing in amplitude during the interval sampled. We are investigating the mechanism of this effect.

together with the low-frequency cutoff and the anti-aliasing filter frequency. The vector from each point indicates the complex amplitude of each vibration and its relative phase. (Because of the wide differences in strengths of vibration, the vectors are plotted on a modified logarithmic scale; the number beside each vector indicates the amplitudes in arbitrary units.) Only the strongest vibration mode at 0.4551 Hz had been previously predicted in the literature.^{9,10}

A great deal of information can be extracted from this plot. Some of the damping constants are positive, for example, indicating that the oscillations of these modes were *increasing* during the time interval examined. Although this phenomenon is still under investigation, preliminary analysis suggests that other modes were apparently transferring energy into these modes. Precession, for example, would permit a direct transfer from an east-west mode into a north-south mode.

Comparison of corresponding vectors at roughly the same frequency might someday aid us in determining which modes are elastic motion and which are rigid-body motion.

This application of PARET should have great impact, since the valuable information on natural frequencies and damping coefficients it provides has never before been so readily available. These data enable structural engineers to design building modifications that will increase the damping of certain modes to help prevent earthquake damage. They will also undoubtedly have applications in the design of new buildings, dams, and bridges to withstand unusual stresses.

Although our analysis technique using Prony's method is relatively new, a wide variety of applications have already been demonstrated. The basic advantages of this method over previous techniques of analyzing complex waveforms are that the calculation stays in the time domain (in which the data

was originally recorded), and that the frequencies and damping constants it provides can often be related directly to physical properties of the system. The breadth and scope of the application: since the development of the "moving-window" scheme suggest a broad range of further applications as the technique is perfected and its capabilities become better known.

Key Words: curve fitting; data compression; earthquake-resistant construction; PARET; Prony's method; radar signals; system identification; structural dynamics.

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PAST TITLES

Articles in the *Energy and Technology Review* have been organized into subject areas approximately corresponding to the Assistant Secretaries' areas of responsibility in the Department of Energy and to other funding agencies. These subject areas are listed below with references to some recent articles in each category. (A semiannual index appears in the June and December issues.)

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