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SEABED WASTE DISPOSAL PROGRAM
ONE-DIMENSIONAL HOLE CLOSURE SIMULATIONS

P. R. Dawson and P. F. Chavez

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ABSTRACT

One-dimensional hole closure simulations have been performed to determine the effect of different constitutive models and assumed boundary conditions on the predicted rebound behavior of ocean sediments following penetrometer emplacement. It has been found that the type of yield criterion and the post yield behavior strongly influence the ability of sediment to reform a barrier behind a penetrometer. Elastic moduli, density, and overburden affect the rebound, but to a lesser degree than the yield criterion and post yielding response. Further, varying the imposed displacement constraints at assumed sediment boundaries can change the qualitative behavior of the cavity response.

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I. Introduction

The feasibility of seabed disposal of nuclear wastes depends on having sediments surrounding the wastes form a barrier to their possible release. To be assured that the sediment barrier will exist, it is necessary to verify that the hole opened during emplacement of the wastes closes to establish a complete barrier. As part of the feasibility study of seabed waste disposal, analytical and experimental hole closure studies are being conducted to assess the closure characteristics of holes resulting from waste emplacement.

A preliminary study of the effects of different sediment material models and boundary conditions on hole closure rates following emplacement has been performed. In this study the sediment has been modeled as a homogeneous, isotropic material with elastic or elastoplastic behavior. Only one-dimensional (radial) deformations have been considered. First, an analytical solution of the closure behavior is presented that predicts the rates of closure that can be expected following quasi-static penetrometer emplacement if elastic rebound is the only restoring mechanism present. Closure rate predictions for two different initial displacement functions are presented. Numerical studies for dynamic penetrometer emplacement and hole closure are then presented in which the effect of different elastoplastic models with von Mises or Mohr-Coulomb failure surfaces has been considered. The influence of assumed boundary constraints is also addressed.

The calculations are for a system geometry that is similar to the hole closure simulation tests being performed at the University of Rhode Island (URI). The projectile geometry and penetration velocity used in the experiments are also used in this study.

II. Summary

Analytical and numerical one-dimensional hole closure simulations have been performed to assess the influence of material modeling assumptions on predicted hole closure response and to evaluate the effect of experimental boundary constraints on observed closure behavior.

The analytical model provides an estimate of the rebound rates for elastic spring-back of a cavity following quasi-static penetrometer emplacement. For varying amounts of assumed plastic flow during cavity formation, it can be seen that rebound to an equilibrium (but not zero) cavity radius occurs in approximately 0.05 ms.

The numerical simulations of the cavity opening and rebound process indicated a strong dependence of assumed plastic behavior (yield surface and hardening parameters). Increased work hardening behavior resulted in the cavity wall rebounding more quickly. The numerical simulations also indicated that the effect of assumed boundary conditions can change the qualitative nature of cavity response. With zero axial strain imposed and the outside radius fixed, closure is predicted quickly after release of cavity wall velocity constraint. However, if either of these two constraints is replaced with a stress free condition, closure is not predicted.

III. Analytical Hole Closure Model

An analytical solution has been obtained for the closure of a hole opened during emplacement of a waste canister. The one-dimensional equations of motion have been solved for radial deformations of a cylindrical cavity. Linear elastic, homogeneous, isotropic behavior of the sediment has been assumed. This restricts the validity of the results to the initial closing of the hole. Further, the magnitude of tensile stress is not limited in this solution. A deformed state corresponding to the opened configuration has been imposed as an initial condition for the rebound analysis. Details of the solution are provided here for completeness.

The governing equations for the cylindrical one-dimensional hole closure are

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = \rho \ddot{u} \quad (1)$$

$$\epsilon_r = \frac{\partial u}{\partial r} \quad (2)$$

$$\epsilon_\theta = \frac{u}{r} \quad (3)$$

$$\epsilon_{rz} = 0 \quad (4)$$

$$\epsilon_r = \frac{1}{E} (\sigma_r - \mu(\sigma_\theta + \sigma_z)) \quad (5)$$

$$\epsilon_\theta = \frac{1}{E} (\sigma_\theta - \mu(\sigma_r + \sigma_z)) \quad (6)$$

$$\epsilon_z = \frac{1}{E} (\sigma_z - \mu(\sigma_r + \sigma_\theta)) \quad (7)$$

where σ_r = r-direction normal stress
 σ_θ = θ -direction normal stress
 σ_z = z-direction normal stress
 u = r-direction displacement
 ϵ_r = r-direction longitudinal strain
 ϵ_θ = θ -direction longitudinal strain
 $\epsilon_{r\theta}$ = shear strain
 E = Young's modulus
 μ = Poisson's ratio

We first assume that $\epsilon_z = 0$, giving

$$\sigma_z = \mu(\sigma_r + \sigma_\theta) \quad \text{and} \quad (8)$$

$$\epsilon_r = \frac{1}{E} (\sigma_r(1 - \mu^2) - \sigma_\theta(1 + \mu)\mu) \quad (9)$$

$$\epsilon_\theta = \frac{1}{E} (\sigma_\theta(1 - \mu^2) - \sigma_r(1 + \mu)\mu) \quad (10)$$

Using Equations (2), (3), (9) and (10), the displacements are related to stresses

$$\frac{\partial u}{\partial r} = \frac{1}{E} (\sigma_r(1 - \mu^2) - \sigma_\theta(1 + \mu)\mu) \quad (11)$$

$$\frac{u}{r} = \frac{1}{E} (\sigma_\theta(1 - \mu^2) - \sigma_r(1 + \mu)\mu) \quad (12)$$

Equations (11) and (12) can be inverted to yield the stresses as functions of displacement. Thus,

$$\sigma_{\theta} = \frac{E\mu}{(1+\mu)(1-2\mu)} \left[\frac{\partial u}{\partial r} + \frac{(1-\mu)}{\mu} \frac{u}{r} \right] \quad (13)$$

and

$$\sigma_r = \frac{E\mu}{(1+\mu)(1-2\mu)} \left[\frac{u}{r} + \frac{(1-\mu)}{\mu} \frac{\partial u}{\partial r} \right] \quad (14)$$

These relationships (Equations (13) and (14)) are substituted into Equation (1) to yield a single differential equation in displacement alone:

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} = K \frac{\partial^2 u}{\partial \tau^2} \quad (15)$$

where

$$K = \frac{\rho(1+\mu)(1-2\mu)}{E(1-\mu)} \quad (16)$$

We proceed by assuming a separation of variables solution of the form

$$u(r, \tau) = R(r) \cdot T(\tau) \quad (17)$$

Substituting into Equation (15) yields

$$\frac{R''}{R} + \frac{1}{r} \frac{R'}{R} - \frac{1}{r^2} = \frac{KT}{T} \quad (18)$$

Since the left hand side depends only on r and the right hand side depends only on τ , each side must equal some constant, κ . This yields two ordinary differential equations

$$R'' + \frac{1}{r} R' - \left(\kappa + \frac{1}{r^2} \right) R = 0 \quad (19)$$

$$\ddot{T} - \frac{\kappa}{K} T = 0 \quad (20)$$

These equations are subject to the boundary and initial conditions

$$u(r_0, \tau) = 0 \quad (21)$$

$$\sigma_r(r_i, \tau) = 0 \quad (22)$$

where r_i and r_0 are the inside and outside radii and

$$u(r, 0) = f(r) \quad (23)$$

$$\dot{u}(r, 0) = g(r) = 0. \quad (24)$$

Equation (19) is a Bessel's equation with the general solution

$$R(r) = C_1 J_1(\alpha r) + C_2 Y_1(\alpha r) \quad (25)$$

in which $\alpha = \sqrt{k}$. Applying boundary conditions (21) and (22) we obtain

$$\left[\begin{array}{cc} J_1(\alpha r_0) & Y_1(\alpha r_0) \\ \frac{J_1(\alpha r_i)}{r_i} + \frac{(1-\mu)}{\mu} \frac{dJ_1(\alpha r_i)}{dr} & \frac{Y_1(\alpha r_i)}{r_i} + \frac{(1-\mu)}{\mu} \frac{dY_1(\alpha r_i)}{dr} \end{array} \right] \left\{ \begin{array}{c} C_1 \\ C_2 \end{array} \right\} = \left\{ \begin{array}{c} 0 \\ 0 \end{array} \right\} \quad (26)$$

In order to obtain values of C_1 and C_2 different from the trivial solution ($C_1 = C_2 = 0$), the determinant of the coefficient matrix in Equation (26) must be equal to zero. Thus,

$$\begin{aligned} J_1(\alpha_m r_0) \left[\frac{Y_1(\alpha_m r_i)}{r_i} + \frac{(1-\mu)}{\mu} \frac{d}{dr} \left(Y_1(\alpha_m r) \right) \Big|_{r=r_i} \right] \\ - Y_1(\alpha_m r_0) \left[\frac{J_1(\alpha_m r_i)}{r_i} + \frac{(1-\mu)}{\mu} \frac{d}{dr} \left(J_1(\alpha_m r) \right) \Big|_{r=r_i} \right] = 0 \end{aligned} \quad (27)$$

Use of the identities

$$\frac{d}{dr} \left(J_n(r) \right) = \frac{1}{2} \left(J_{n-1}(r) - J_{n+1}(r) \right)$$

and

$$\frac{d}{dr} (Y_n(r)) = \frac{1}{2} (Y_{n-1}(r) - Y_{n+1}(r))$$

allows Equation (27) to be written as

$$J_1(\alpha_m r_0) \left[\frac{Y_1(\alpha_m r_1)}{r_1} + \frac{(1-\mu)}{\mu} \left\{ \frac{\alpha_m}{2} (Y_0(\alpha_m r_1) - Y_2(\alpha_m r_1)) \right\} \right] - Y_1(\alpha_m r_0) \left[\frac{J_1(\alpha_m r_1)}{r_1} + \frac{(1-\mu)}{\mu} \left\{ \frac{\alpha_m}{2} (J_0(\alpha_m r_1) - J_2(\alpha_m r_1)) \right\} \right] = 0 \quad (28)$$

An infinite number of values of α_m provide roots to Equation (28). This allows us to express the solution of Equation (19) as an infinite series:

$$R(r) = \sum_{m=1}^{\infty} C_{2m} \left(Y_1(\alpha_m r) - \frac{Y_1(\alpha_m r_0)}{J_1(\alpha_m r_0)} J_1(\alpha_m r) \right) \quad (29)$$

Turning now to Equation (20) the characteristic equation can be written as

$$n^2 = \frac{\kappa}{K} \quad n = \sqrt{\frac{\kappa}{K}} \quad (30)$$

giving

$$T(\tau) = A_1 \cos n\tau + A_2 \sin n\tau \quad \frac{\kappa}{K} < 0$$

$$T(\tau) = A_1 + A_2 \tau \quad \frac{\kappa}{K} = 0$$

$$T(\tau) = A_1 e^{n\tau} + A_2 e^{-n\tau} \quad \frac{\kappa}{K} > 0$$

The second and third solutions are ruled out for the initial condition $\dot{u}(r,0) = 0$ (since for the second solution $\dot{u}(r,0) = 0$ would yield no time dependence and for the third solution $\dot{u}(r,0) = 0$ would yield either the trivial solution or an unbounded solution), leaving the solution

$$T(-) = A_1 \cos n\tau + A_2 \sin n\tau$$

Application of $u'(r, 0) = 0$ results in

$$T'(-) = A_1 \cos n\tau \quad (31)$$

The total solution for the displacement can then be written as

$$u(r, \tau) = R(r) \cdot T(\tau) = \sum_{m=1}^{\infty} D_m \left(Y_1(\alpha_m r) - \frac{Y_1(\alpha_m r_0)}{J_1(\alpha_m r_0)} J_1(\alpha_m r) \right) \cos \left(\frac{\alpha_m}{\sqrt{K}} \tau \right) \quad (32)$$

The coefficients D_m are evaluated from the remaining initial condition

$u(r, 0) = f(r)$. Thus,

$$f(r) = \sum_{m=1}^{\infty} D_m \left(Y_1(\alpha_m r) - \frac{Y_1(\alpha_m r_0)}{J_1(\alpha_m r_0)} J_1(\alpha_m r) \right) \quad (33)$$

Multiplying by

$$r \left(Y_1(\alpha_n r) - \frac{Y_1(\alpha_n r_0)}{J_1(\alpha_n r_0)} J_1(\alpha_n r) \right)$$

and integrating over the region gives

$$\int_{r_i}^{r_0} r f(r) \left(Y_1(\alpha_n r) - \frac{Y_1(\alpha_n r_0)}{J_1(\alpha_n r_0)} J_1(\alpha_n r) \right) dr = \int_{r_i}^{r_0} \sum_{m=1}^{\infty} \left[\left(Y_1(\alpha_m r) - \frac{Y_1(\alpha_m r_0)}{J_1(\alpha_m r_0)} J_1(\alpha_m r) \right) \cdot D_m \right] \cdot r \left(Y_1(\alpha_n r) - \frac{Y_1(\alpha_n r_0)}{J_1(\alpha_n r_0)} J_1(\alpha_n r) \right) dr$$

Orthogonality gives

$$D_m = \frac{\int_{r_i}^{r_o} f(r) \cdot r \cdot \left(Y_1(\alpha_n r) - \frac{Y_1(\alpha_n r_o)}{J_1(\alpha_n r_o)} J_1(\alpha_n r) \right) dr}{\int_{r_i}^{r_o} r \left(Y_1(\alpha_n r) - \frac{Y_1(\alpha_n r_o)}{J_1(\alpha_n r_o)} J_1(\alpha_n r) \right)^2 dr} \quad (34)$$

Two forms of $f(r)$ have been assumed for the initial displacement. These are $f_1(r) = K_1 r + K_2/r$ and $f_2(r) = K_1 + K_2/r$. The first represents the elastic displacements while the second represents a uniform decay of displacement as a function of the inverse of the radius. For $f_1(r) = K_1 r + K_2/r$ values of D_m are of the form

$$D_{mn} = \frac{u_m r_i}{\alpha_m} \left[Y_0(\alpha_m r_i) - \frac{Y_1(\alpha_m r_o)}{J_1(\alpha_m r_o)} J_0(\alpha_m r_i) \right] - \frac{2K_1 r_i}{\alpha_m^2} \left[Y_1(\alpha_m r_i) - \frac{Y_1(\alpha_m r_o)}{J_1(\alpha_m r_o)} J_1(\alpha_m r_i) \right] \quad (35)$$

$$D_{md} = \int_{r_i}^{r_o} r Y_1^2(\alpha_m r) dr - \int_{r_i}^{r_o} 2r \frac{Y_1(\alpha_m r_o)}{J_1(\alpha_m r_o)} Y_1(\alpha_m r) J_1(\alpha_m r) dr + \int_{r_i}^{r_o} \left(\frac{Y_1(\alpha_m r_o)}{J_1(\alpha_m r_o)} \right)^2 r J_1^2(\alpha_m r) dr \quad (36)$$

$$D_m = \frac{D_{mn}}{D_{md}} \quad (37)$$

where $K_1 = \frac{-u_a r_i}{(r_o^2 - r_i^2)}$, $K_2 = -K_1 r_o^2$, and u_a are the inside radius initial displacements. For $f_2(r) = K_1 + K_2/r$, the value of the D_{mn} becomes:

$$D_{mn} = \frac{-r_i u_a}{r_o - r_i} \int_{r_i}^{r_o} r \left(Y(\gamma_n r) - \frac{Y_1(\gamma_n r_o)}{J_1(\gamma_n r_o)} J_1(\gamma_n r) \right) dr$$

$$+ \frac{r_i r_o u_a}{r_o - r_i} \left(Y_o(\gamma_n r_i) - Y_o(\gamma_n r_o) + \frac{Y_1(\gamma_n r_o)}{J_1(\gamma_n r_o)} (J_o(\gamma_n r_o) - J_o(\gamma_n r_i)) \right)$$

while D_{md} remains the same.

Program ROOTS was written to determine the roots, γ_n , to Eq. (26) and to evaluate the coefficients D_n . ROOTS performs necessary summations to determine the displacement as a function of time and radial location. The infinite summation for $u(r, \tau)$ has been truncated after 500 terms. The use of 300, 400, or 500 terms in the series showed little difference in the predicted displacements. MATHLIB [7] Bessel function routines have been used to compute the values of J_o , J_1 , J_2 , Y_o , Y_1 , and Y_2 .

Cavity closure rates have been computed for rebound of the sediment following quasi-static emplacement of a penetrometer into a sediment region 5 m in radius. The elastic behavior of the sediment was characterized by the elastic constants of Model 2 in Table I. A total of six cases have been examined. In each case the assumed inside cavity radius plus the inside radius displacement (initial) equaled 2.5 cm. Three different values of r_i have been used (1.0, 1.5, and 2.0 cm). The initial displacement was determined by the $f(r)$ functions described earlier. Thus, for each one of the two $f(r)$ functions, three combinations of r_i and u_a were analyzed (each combination giving $r_i + u_a = 2.5$ cm). Different combinations

of r_i and u_a were used to approximate the effect of different amounts of plastic flow occurring as the cavity opened.

The closure response for the six cases are plotted in Figures 1-16. Figures 1-9 show the sediment response at three radii for an initial displacement distribution of $f_1(r) = K_1 r + K_2/r$ and for the three different combinations of r_i and u_a . With $r_i = 2$ cm and $u_a = .5$ cm rebound of the cavity wall begins immediately. The displacement overshoots its equilibrium position within 0.05 ms followed by a period in which the displacement is nearly zero. However, after approximately 11 ms the cavity wall experiences the return of the reflected relief pulse. The pulse can be observed from Figure 1 to return every 11 ms, corresponding to two transit times of a compressional wave in the sediment. The responses of sediment at $r = .85$ m and $r = 2.51$ m are shown in Figures 2 and 3 for $r_i = 2.0$ cm and $u_a = .5$ cm. Initial rebound at these radii does not occur until the relief wave has had time to propagate into the sediment. Values of displacement are more than an order of magnitude smaller than at the cavity wall.

Since terms of the solution series expansion are incommensurable, the displacement histories do not repeat in time. It is important to note that the system is conservative and thus does not experience decay in total energy with time. Dissipative mechanisms in the sediment would result in decay of the pulses with the motion eventually ceasing.

The responses for other combinations of r_i and u_a with $f_1(r) = K_1 r + K_2/r$ are shown in Figures 4-9. The initial cavity rebound again occurs within .05 ms. The displacement histories are similar to the case with $r_i = 2$ cm and $u_a = .5$ cm with the exception of larger magnitudes for the displacements.

Assuming an initial displacement corresponding to $f_2(r) = K_1 + K_2/r$ did not alter the initial rebound times of the sediment ($\sim .05$ ms). Some differences are apparent in the stress histories at the radii shown (Figures 10-18), but these are minor.

The rates of rebound for the cavity wall determined from the analytical solution represent an upper bound for closure rates. The stress distribution for elastic deformations include tensile hoop stresses near the inside radius. As the elastic deformation becomes large these tensile stresses probably could not be supported by the material. As will be seen in the next section, the analytically determined rebound rates correspond closely to the rates predicted using an elastoplastic model with a von Mises failure criterion and with large amounts of strain hardening. In these cases large tensile hoop stresses are also predicted.

IV. Numerical Hole Closure Models

One-dimensional analyses of the sediment response during penetrometer emplacement have been conducted using the finite element computer code, HONDO [1]. HONDO is capable of two-dimensional, large strain, elastoplastic analysis of either plane strain or axisymmetric geometries. Dynamic analyses are performed using an explicit time integration method. HONDO was modified to allow a prescribed velocity to be applied along a given boundary for a specified time followed by application of a stress-free condition.

The simulations were performed on an axisymmetric region in which only radial deformations were considered. A cylindrical cavity was opened at a fixed rate to correspond to the projectile pushing the sediment outward as it passed through the sediment. Once the cavity had been opened to a radius equal to that of the projectile, the velocity constraint was removed from the cavity wall. From that time on the cavity wall was required to be stress free. The sediment was then free to continue to move radially outward or to rebound inward. Various values of overburden stress or axial deformation were simulated. Some of the overburden stress values used corresponded to the experiments on hole closure being conducted at URI. The outside radius was constrained to have zero displacement. Outside radii of 5 m and 0.5 m were examined in order to determine the magnitude of boundary effects.

Two material models available in HONDO were used in the hole closure simulations. Elastoplastic behavior can be specified with either a von Mises or Mohr-Coulomb yield criteria. Tensile stresses are not explicitly limited in these models. Strain hardening can be included

when using the elastoplastic model with a von Mises failure surface, but only perfectly plastic behavior can be simulated when using the Mohr-Coulomb yield surface. Schematic drawings of these models are shown in Figures 19 and 20.

The hole closure simulations were first used to study the effect of variations in material properties on hole closure rates. For these simulations the outside radius extended to 5 m in order to minimize boundary effects. The elastoplastic model with a von Mises failure surface requires values of density, Young's modulus, Poisson's ratio, yield stress, and hardening modulus. Typical values of these parameters were taken from the sources indicated in Table I. The values were then varied to evaluate their effects on hole closure response. The study was not intended to determine the closure response for all possible combinations of parameters, but rather was intended to obtain a qualitative understanding of which parameters are the most significant in the closure process.

A summary of the simulation results has been tabulated in Table II. The effect of variation in Young's modulus is indicated in simulations 11 and 12. The value of Young's modulus of 1.35×10^9 Pa is an equivalent modulus for saturated sediment and pore water. The sediment and water were assumed to act in parallel with the proportion of total load carried by each constituent being proportional to the porosity. The value of 6.9×10^6 Pa is the value of Young's modulus for the saturated clay alone. The simulation indicated that for either value of Young's modulus the hole continued to open following removal of velocity constraint. However, after approximately 25 ms the inside surface has begun to rebound and is

traveling slowly inward. The hole dimension at the end of the analyses is approximately the same for both values of moduli.

Closure characteristics using values of Poisson's ratio equal to 0.25 and 0.45 were determined with other parameters held constant (cases 7 and 13). This parameter had relatively small influence on the inside surface position for perfectly plastic models. However, at the end of the analyses the less compressible case ($\mu = 0.45$), because it performed more work along the upper and lower boundaries, had a slower outward velocity.

The shear strength of $.2 \times 10^6$ Pa used in the analysis is a typical value obtained from shear vane tests on ocean sediments (UMI). Sediment having a shear strength one order of magnitude larger ($.2 \times 10^7$ Pa) was also simulated. The cavity responded in a very similar manner for both cases. At the end of the analyses, the cavity is rebounding at a high velocity and has closed significantly. The simulation with the larger shear strength rebounded more, but the difference in cavity diameter was less than 10% after 2 ms.

The significant difference in closure behavior resulted from variations in the hardening modulus. With nearly perfect plastic behavior the cavity continues to open at a decreasing rate after the radius exceeds the projectile diameter. After 4 ms the cavity is still opening. However, with a significant amount of strain hardening, the cavity ceases to open quickly after reaching the projectile diameter (at 1.9 ms). Rebound of the cavity to less than half of the projectile diameter occurs in the next 0.1 ms (similar rate to analytical solution). Since the work hardening material requires significantly higher stresses to continue deformations after the velocity constraint is released, the kinetic energy in the sediment is dissipated quickly by the work performed in

continuing the outward deformations. Thus the outward motion stops and elastic rebound occurs. With the nearly perfect plastic model, however, little work is required to continue opening the cavity. A long time is required to dissipate the inertial energy of the sediment and rebounding is not observed until after a much larger radius is reached. Typical inside radius displacement histories for strain hardening (case 4) and nearly perfect plastic (case 5) models are shown in Figure 21.

The stress distribution at the time the velocity constraint at the inside radius is released is quite different for the nearly perfect plastic model and the strain hardening model. Using the nearly perfect plastic model stresses near the cavity wall all remain compressive. The stress state is nearly hydrostatic with the maximum shear being approximately 50% larger than the initial yield stress ($.2 \times 10^5$ Pa). However, with the strain hardening model, stresses near the cavity wall become 2 to 5 orders of magnitude greater than the initial yield stress. Further tensile zones (hoop stress) occur near the sediment inside radius. This stress distribution for the strain hardening model is similar to the initial stress distribution assumed in the analytical solution.

Although variations in density affected the results of the closure, the differences were small in comparison to the differences observed for the hardening modulus variations. Increased density results in increased kinetic energy that requires more time to dissipate. Thus, as density increases, the time for rebound also increased.

The effect of the failure surface shape was studied using a material model with separate volumetric and shear behavior. Either von Mises or Mohr-Coulomb failure criteria may be specified to define the initiation of plastic shear behavior. The Mohr-Coulomb failure criterion allows the yield

limit to be specified as a function of pressure; whereas, the von Mises failure criteria is independent of the pressure value. In both cases perfectly plastic behavior with the strain increment normal to the yield surface occurs after yielding. Again, a 5 m outside radius was specified.

URI quasi-static triaxial compression data were used to evaluate the needed volumetric and shear behavior relationships [4]. The values of the parameters are indicated in Table III. The model using the von Mises failure criterion provides the same material response as the elastoplastic model without strain hardening except that the yield strength increases with mean pressure, giving an apparent hardening as pressure is increased. Figure 20 shows the volumetric and shear response used in the model in which the yield point stress can depend on the mean pressure (Mohr-Coulomb).

The hole closure behavior determined using the two different failure surfaces was found to be quite different (Table IV). Using the von Mises failure criterion, the sediment cavity continues to open after the velocity constraints are removed. (This behavior is the same as predicted using the perfectly plastic elastoplastic model.) However, using the Mohr-Coulomb failure surface the inside surface begins to rebound soon after removal of the velocity constraints. As material near the inside radius unloads, the change in hoop stress (becoming tensile) results in the material failing since the strength is pressure dependent. The inside surface then tends to flow plastically inward. Variations in bulk modulus and overburden pressure modified the rates of these phenomena, but did not alter the basic qualitative difference. Thus the yield criterion used to define the transition from elastic to plastic deformations also has a significant impact on overall closure behavior. Figure 22 shows the difference in rebound predicted by von Mises and Mohr-Coulomb models.

In order to evaluate the effect of boundary constraints on the hole rebound, penetration into a region extending radially only to 0.5 m was analyzed. The cavity was opened to a radius of 0.019 m at a rate of 10 m/s. The velocity constraint placed on the cavity wall was removed when the cavity reached the penetrometer diameter and a stress free condition was prescribed in its place. Both constant pressure and zero axial strain conditions were imposed through the region thickness (axial direction).

An elastoplastic model with a von Mises yield criterion was used to represent the sediment behavior. The computations were performed using a very low hardening modulus (see Table V for model parameters).

The presence of rigid constraints near the cavity had a pronounced effect on hole closure. With a nearly perfect plastic model, the hole formed in a region 5 m in radius either for constant pressure or zero strain in the axial direction exhibit closure for times exceeding 2 ms after being released. When a cavity was formed by the penetrometer in a region 0.5 m in radius with constant pressure applied in the axial direction, the hole also did not close. However, when the firing of a penetrometer into a region 0.5 m in radius with zero axial strain imposed, the cavity began to close quickly after being released (Figure 23). The cavity had closed to only 20% of penetrometer size within 0.2 ms after being released.

Thus, the presence of stiff boundaries near the cavity (such as occurs in the URI tests) can have a significant influence on cavity rebound. The boundaries reflect the stress pulse created by the cavity opening (Figure 24). After the stress pulse is reflected from the wall it can travel back into the sediment and cause sediment deformation that result in hole closure.

However, in an infinite medium there are no reflective surfaces sufficiently near the penetrometer to return the stress pulse to the cavity. Thus, walls associated with the test apparatus can themselves aid in causing apparent hole closure.

The pressures predicted using the elastoplastic model were used to evaluate the validity of the assumption that pore pressure dissipation is small during the deformation process. The pressures at the end of the imposed velocity period were used as an initial excess pore pressure distribution in a porous flow computation performed using the finite element code COUPLEFLO [6], [7]. The dissipation of the pore pressure is indicated in Figure 25. For time periods far exceeding the closure analysis period, the dissipation was not significant. This indicates that dissipation of pore pressure does not result in load redistribution to the sediment skeleton in the time frames of the closure analysis as a consequence of pore water migration.

V. Comparison to Laboratory Tests

The experimental hole closure simulations [3] indicated a qualitative difference between dynamic emplacement (projectile initial velocity of approximately 20 m/s) and quasi-static emplacement (projectile pushed slowly into the sediment at 0.1 m/s). Following dynamic emplacement immediate cavity closure was visually observed when a confining plate used to simulate overburden pressure on the upper surface of the sediments was present; whereas, closure following quasi-static emplacement took hours. Further, in trial tests performed for evaluating instrumentation, the hole did not close during dynamic penetration without the upper surface confining plate present. This difference in behavior indicates that either a restoring mechanism (such as the release of elastic strain energy) is active in the dynamic case but not in the quasi-static case or that boundary conditions more seriously affect the dynamic case.

The numerical hole closure simulations of the dynamic emplacement have examined the cavity rebound relative to different types of material behavior and distance to the tank wall. Simulating nearly perfect plastic behavior without a pressure dependent failure surface yields a continued opening of the cavity unless severe constraints are placed on the deformations. On the other hand specifying either a significant degree of strain hardening or pressure dependence of the failure surface yields rebounding of the cavity wall soon after the hole has opened to the projectile diameter. Although none of the material models can yet be ruled out based on comparison of the experimental and numerical simulations the need to quantify the hardening behavior and yield criteria of the sediment is evident. Further, close examination of the boundary constraints imposed in testing is needed.

VI. Limitations of the One-Dimensional Analysis

There are several aspects of the numerical analysis that limit their usefulness. The first is that the closure problem is at best a two-dimensional problem. A one-dimensional model neglects shear stresses resulting from drag along the projectile and from the opening history being dependent on depth. Two-dimensional analyses now underway should help to evaluate the importance of the shear stresses present. Curvature of the projectile path or impact at angles different than 90° to the sediment surface add complexity that would require three-dimensional analyses for accurate modeling.

A second aspect of the analysis that limits the results of the study is the uncertainty in the material properties. In general, the values for material parameters are derived from quasi-static tests. Further, some values are typical of saturated clays but may not be accurate for deep ocean clay sediments. Dynamic tests being planned for sediments typical of deep sea clays should provide data in the future for more accurate computations.

The third aspect is the use of one constituent material model for the two constituent sediment-pore water system. Although the dissipation of excess pore pressures has been shown to be very small during the cavity forming and rebounding process, the stress distribution between the sediment and water may not be appropriately modeled with only a one constituent model. This aspect is currently being studied in more detail.

VII. Conclusions

The one-dimensional closure study has indicated that the sediment failure criterion and post yielding behavior significantly affect the hole closure response. The degree to which material strain hardens determines its ability to dissipate the sediment's inertia and allow rebound to occur. The definition of the transition from elastic to plastic behavior (failure criterion) has a large impact on the closure response. Because the mean stress is positive near the inside surface, some material near the surface fails and flows inward, readily following release of the velocity constraint when modeled using a Mohr-Coulomb criterion. This behavior is not observed for a von Mises model. Thus, the need to define the sediment's plastic behavior is important to numerical modeling of deformation processes.

The imposed boundary constraints in penetration simulations also have strong effects on cavity closure behavior. The presence of a nearby wall results in reloading of sediments near the cavity wall that aids in closing the cavity.

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TABLE I

Sediment Constitutive Models

Elastoplastic Models with Von Mises Failure Criteria and Strain Hardening

Model	E^* (Pa)	μ^{**}	Y_0^* (Pa)	$E_+^{\dagger}$ (Pa)	$c^{\ddagger}$ (kg/m ³)	Porosity
1	$.135 \times 10^{10}$	0.25	$.2 \times 10^6$	$.1 \times 10^9$	1438.	50%
2	$.135 \times 10^{10}$	0.25	$.2 \times 10^5$	$.2 \times 10^5$	1875.	50%
3	$.135 \times 10^{10}$	0.25	$.2 \times 10^6$	$.2 \times 10^6$	1438.	50%
4	$.135 \times 10^{10}$	0.25	$.2 \times 10^5$	$.1 \times 10^9$	1875.	50%
5	$.135 \times 10^{10}$	0.25	$.2 \times 10^5$	$.2 \times 10^5$	1438.	50%
6	$.135 \times 10^{10}$	0.25	$.2 \times 10^7$	$.1 \times 10^9$	1438.	50%
7	$.69 \times 10^9$	0.25	$.2 \times 10^5$	$.2 \times 10^5$	1875.	50%
8	$.135 \times 10^{10}$	0.25	$.2 \times 10^5$	$.2 \times 10^5$	1875.	50%
9	$.135 \times 10^{10}$	0.45	$.2 \times 10^6$	$.2 \times 10^6$	1438.	50%

*Young's Modulus (E) was taken from Reference [2] and [5].

**Poisson's ratio (μ) was assumed.

† Yield stress (Y_0) was taken from Reference [3].

‡ Hardening modulus (E_+) was assumed.

$^{\#}$ Density (ρ) was taken from Reference [2].

TABLE II

Sediment Response for Models Having Strain Hardening[‡]

Case	Elastoplastic Model ^{**}	Overburden Pressure	Inward [*] Velocity	Final Time	Final KE	Final IE	Final Velocity	Final Opening
1	1	$.1 \times 10^5$ Pa	10. m/s	$.201 \times 10^{-2}$ s	$.756 \times 10^4$ Nm	$.412 \times 10^4$ Nm	$-.233 \times 10^3$ m/s	$.73 \times 10^{-2}$ m
2	1	$.434 \times 10^6$	10.	$.201 \times 10^{-2}$	$.775 \times 10^4$	$.755 \times 10^4$	$-.272 \times 10^3$	$.60 \times 10^{-2}$
3	1	$.855 \times 10^5$	10.	$.201 \times 10^{-2}$	$.760 \times 10^4$	$.415 \times 10^4$	$-.310 \times 10^3$	$.46 \times 10^{-2}$
4	1	$.855 \times 10^6$	10.	$.200 \times 10^{-2}$	$.786 \times 10^4$	$.197 \times 10^4$	$-.233 \times 10^3$	$.741 \times 10^{-2}$
5	1	$.434 \times 10^5$	10.	$.200 \times 10^{-2}$	$.757 \times 10^4$	$.413 \times 10^4$	$-.231 \times 10^3$	$.738 \times 10^{-2}$
6	2	$.855 \times 10^5$	10.	$.444 \times 10^{-2}$	$.801 \times 10^2$	$.166 \times 10^3$	$-.36 \times 10^1$	$.327 \times 10^{-1}$
7	3	$.855 \times 10^5$	10.	$.411 \times 10^{-2}$	$.168 \times 10^2$	$.370 \times 10^3$	$.73 \times 10^{-1}$	$.22 \times 10^{-1}$
8	4	$.855 \times 10^5$	10.	$.203 \times 10^{-2}$	$.761 \times 10^4$	$.300 \times 10^3$	$-.260 \times 10^3$	$.50 \times 10^{-2}$
9	5	$.855 \times 10^5$	10.	$.412 \times 10^{-2}$	$.422 \times 10^3$	$.170 \times 10^3$	$.412 \times 10^1$	$.324 \times 10^{-1}$
10	6	$.434 \times 10^5$	10.	$.701 \times 10^{-2}$	$.713 \times 10^2$	$.541 \times 10^4$	$-.246 \times 10^3$	$.673 \times 10^{-2}$
11	7	fixed upper surface	1.	$.270 \times 10^{-1}$	.2731	$.262 \times 10^2$	$-.128 \times 10^{-2}$	$.193 \times 10^{-1}$
12	8	fixed upper surface	1.	$.296 \times 10^{-1}$	$.732 \times 10^{-1}$	$.289 \times 10^2$	$-.147 \times 10^{-2}$	$.193 \times 10^{-1}$
13	9	$.855 \times 10^5$	10.	$.415 \times 10^{-2}$	$.77 \times 10^4$	$.311 \times 10^3$	$.518 \times 10^{-1}$	$.229 \times 10^{-1}$

^{*}Velocity at the inside radius during cavity opening.[†]Kinetic Energy[‡]Internal Energy[§]Outside radius = 5 m^{**}See Table I.

TABLE II

Sediment Constitutive Models

Elastoplastic Models with either Mohr-Coulomb or Von Mises Failure Criteria and No Strain Hardening

Model	G	K ₀	a ₀ [†]	a ₁ [†]	a ₂ [†]	p-v	σ	Porosity
1	.4 x 10 ⁹	.5 x 10 ¹⁰	.883 x 10 ³	.9826 x 10 ⁴	.989 x 10 ⁴	*	1438.	50%
2	.23 x 10 ⁹	.189 x 10 ¹⁰	.133 x 10 ⁹	0.	0.	constant	1438.	50%
3	.5 x 10 ⁹	.5 x 10 ¹⁰	.1E + 09	0.	0.	constant	1438.	50%

$$* \quad \frac{p}{\sigma_0} \quad \frac{\ln \epsilon_{ii}}{\sigma_0}$$

$$.14366 \times 10^5 \quad -.606 \times 10^{-2}$$

$$.2442 \times 10^5 \quad -.00763$$

$$.3643 \times 10^5 \quad -.00853$$

$$.7937 \times 10^7 \quad -.01005$$

$$^{\dagger} \text{Yield Criterion: } \frac{1}{2} s'_{ij} s'_{ij} = a_0 + a_1 p + a_2 p^2 \quad [1]$$

G = Shear Modulus

TABLE IV

Sediment Response[†] for Models Having No Strain Hardening

Case	Model [‡]	Overburden Pressure (Pa)	Inward* Velocity (m/s)	Final Time (s)	Final KE (Nm)	Final IE (Nm)	Final Velocity (m/s)	Final Opening (m)
1	1	Fixed boundary $u_z = 0$	10.	$.412 \times 10^{-2}$	$.278 \times 10^1$	$.281 \times 10^3$	$-.365 \times 10^1$	$.11 \times 10^{-1}$
2	2	$.434 \times 10^5$	10.	$.417 \times 10^{-2}$	$.533 \times 10^2$	$.698 \times 10^2$	$.429 \times 10$	$.338 \times 10^{-1}$
3	2	$.100 \times 10^6$	10.	$.416 \times 10^{-2}$	$.439 \times 10^2$	$.147 \times 10^2$	$.361 \times 10^1$	$.332 \times 10^{-1}$
4	1	$.100 \times 10^6$	10.	$.403 \times 10^{-2}$	$.23 \times 10^2$	$.428 \times 10^3$	$-.381 \times 10^0$	$.206 \times 10^{-1}$
5	3	$.100 \times 10^6$	10.	$.405 \times 10^{-2}$	$.34 \times 10^2$	$.835 \times 10^2$	$.373 \times 10^1$	$.321 \times 10^{-1}$

*Velocity at inside radius during cavity opening.

[†]Outside radius = 5 m[‡]See Table 3.

TABLE V

Cavity Response with a 0.5 m Radius Region

<u>Case</u>	<u>Overburden Pressure</u> (Pa)	<u>Inward Velocity</u> (m/s)	<u>Final Time</u> (s)	<u>Final Kinetic Energy</u> (Nm)	<u>Final Internal Energy</u> (Nm)	<u>Final Velocity</u> (m/s)	<u>Cavity Size</u> (m)
1	$.855 \times 10^5$	10.	$.735 \times 10^{-2}$	$.413 \times 10^2$	$.874 \times 10^2$	$.401 \times 10^1$ m/s	3.95×10^{-1} m
	$U_z = 0$	10.	$.385 \times 10^{-2}$	$.173 \times 10^3$	$.529 \times 10^2$	$-.54 \times 10^2$	$.281 \times 10^{-2}$

Elastoplastic Model Parameters:

 $E = .135 \times 10^9$ Pa (Young's Modulus) $\mu = .25$ (Poisson's Ratio) $Y_0 = .2 \times 10^5$ Pa (Yield Stress) $E_t = .2 \times 10^3$ Pa (Hardening Modulus)

Von Mises Yield Criterion

Figure 1. Displacement History for Initial Displacement Function f_1 .

Figure 2. Displacement History for Initial Displacement Function f_1 .

Figure 3. Displacement History for Initial Displacement Function f_1 :

Figure 4. Displacement History for Initial Displacement Function f_1 .

Figure 5. Displacement History for Initial Displacement Function f_1 .

Figure 6. Displacement History for Initial Displacement Function f_1 .

Figure 7. Displacement History for Initial Displacement Function f_1 .

Figure 8. Displacement History for Initial Displacement Function f_1 .

Figure 9. Displacement History for Initial Displacement Function f_1 .

Figure 10. Displacement History for Initial Displacement Function f_2 .

Figure 11. Displacement History for Initial Displacement Function f_2 .

Figure 12. Displacement History for Initial Displacement Function f_2 .

Figure 13. Displacement History for Initial Displacement Function f_2 .

Figure 14. Displacement History for Initial Displacement Function f_2 .

Figure 15. Displacement History for Initial Displacement Function f_2 .

Figure 16. Displacement History for Initial Displacement Function f_2 .

Figure 17. Displacement History for Initial Displacement Function f_2 .

Figure 18. Displacement History for Initial Displacement Function f_{10} .

Figure 19. Elastoplastic Models with Strain Hardening.

Figure 20. Elastic-Perfectly Plastic Models with von Mises or Mohr-Coulomb Yield Criterion.

Figure 21. Sediment Response for Strain Hardening Models.

Figure 22. Sediment Response for Elastic-Perfectly Plastic Models.

Figure 23. Sediment Response for Various Boundary Conditions.

Figure 24. Effect of Rigid Boundary Constraints on Stress History.

Figure 25. Excess Pore Pressure Dissipation.