**A Two-Dimensional Transient Far-Field
Analysis for the Excess Temperature
from an Arbitrary Source**

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MASTER

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A TWO-DIMENSIONAL TRANSIENT FAR-FIELD ANALYSIS FOR THE
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Alan J. Witten* Ed C. Long†

ABSTRACT

An analytic solution is presented for the two-dimensional time-dependent advective diffusion equation governing the distribution of excess temperature in a river of uniform width, depth, and downstream flow. The solution is also applicable to a straight coastline with uniform longshore flow. Exact solutions are obtained for a point heat source and a particular line heat source, while an approximate representation is given for an arbitrary time-varying heat source. These solutions are incorporated into a computer program which calculates excess temperature and time rate-of-change of excess temperature in a river or coast as a result of waste heat discharged from various transient sources.

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INTRODUCTION

The most efficient system for dissipating excess heat from a nuclear reactor, as well as from other thermal sources, is once-through cooling. However, this system can frequently be the least desirable, because of its adverse environmental impacts, notably that the excess temperature produced in the receiving body of water can destroy or inhibit the growth of aquatic organisms. In rivers, it is particularly important to avoid full channel blockage so that aquatic life can have passage without exposure to significant temperature differences. Therefore, a predictive model in at least two dimensions is required to determine if a given discharge will satisfy an imposed restriction on the spatial distribution of excess temperature. Because aquatic life is susceptible to cold shock, it is also desirable to have the capability to predict the transient thermal behavior of a plume from a time-varying source.

In the past, the problems of cold shock and excess temperature have been considered separately in analytic models. A number of simple two-dimensional steady-state thermal models are reviewed in a state-of-the-art report by Policastro and Tokar (1972). In addition, a number of more sophisticated analytic solutions have been given by Sayre and Chang (1968), Cleary and Adrian (1973), and Yeh and Tsai (1976). Yeh and Tsai address a quite general problem considering both transient flow and three-dimensional sources; however, for this problem numerical integration is required. The work of Cleary and Adrian is restricted to a point-source instantaneous discharge. Little analytical work has been done on the cold shock problem. There are one-dimensional transient models such as that by Pilati (1973), which solves the initial-value problem for sudden plant shutdown.

The two-dimensional transient model presented here is for the far-field and is valid only in the region where the excess momentum from the discharge is insignificant. Consideration is restricted in this paper to straight coastlines and rivers of uniform depth and width with a steady uniform downstream or longshore flow. The restriction to uniform width is a result of the time-dependence of the model. Coordinate

transforms such as that used by Dresner (1973) are invariant only for the steady-state form of the equation and, therefore, cannot be used here.

Subject to these assumptions, the program developed here enables exact solutions to be obtained for a point source and for a Gaussian line source normal to the shore. This paper also discusses an approximate technique for dealing with other types of source distributions.

ANALYSIS FOR A RIVER

The present analysis establishes a coordinate system for the hypothetical river under study, with the x-axis pointing downstream, the y-axis pointing cross-stream, and the origin on one shore, as shown in Fig. 1. The width of the river is b , and the uniform downstream velocity is u_0 . The equation governing the excess temperature $\Delta T(x, y, t)$ is

$$\frac{\partial(\Delta T)}{\partial t} + u_0 \frac{\partial(\Delta T)}{\partial x} - \alpha \frac{\partial^2(\Delta T)}{\partial y^2} = q(t)f(x, y) - \frac{K}{\rho c_p d} \Delta T, \quad (1)$$

where α is the eddy diffusivity, $q(t)f(x, y)$ is the two-dimensional transient source strength, K is the surface heat exchange coefficient, ρ is the density, c_p is the specific heat, and d is the depth. Downstream diffusion is omitted in Eq. (1). This is acceptable in most cases because the dominant downstream transport mechanism is advection. Analytic solutions can still be obtained if the term for downstream diffusion is retained [see Cleary and Adrian (1973) and Yeh and Tsai (1976)]; however, numerical integration is required to consider transient sources. The appropriate boundary conditions are insulated river banks and zero excess temperature far upstream. These conditions can be written as

$$\left. \frac{\partial(\Delta T)}{\partial y} \right|_{y=0, b} = \left. \Delta T \right|_{x=-\infty} = 0. \quad (2)$$

Fig. 1. Basic coordinate system.

For mathematical convenience, the source strength $q(t)$ and the excess temperature ΔT are set equal to zero initially ($t = 0$). It is also assumed that $q(t)$ is, or can be approximated as, a piecewise continuous function, and $|q(t)|$ does not increase too rapidly as t goes to infinity. With these assumptions, Eq. (1) can be Laplace transformed to give

$$s\hat{T} + u_0 \frac{\partial \hat{T}}{\partial x} - \alpha \frac{\partial^2 \hat{T}}{\partial y^2} = \hat{q}(s)f(x, y) - k\hat{T}, \quad (3)$$

where

$$\hat{T}(x, y, s) = \mathcal{L}[\Delta T(x, y, t)] = \int_0^\infty \Delta T(x, y, t) e^{-st} dt, \quad (4)$$

and $k = K/\rho c_p d$. Equation (3) can be simplified by multiplying by $\exp[(s + k)x/u_0]$ and defining

$$\hat{\chi}(x, y, s) = \exp[(s + k)x/u_0] \hat{T}(x, y, s). \quad (5)$$

This results in the differential equation

$$\frac{\partial^2 \hat{\chi}}{\partial y^2} - \frac{u_0}{\alpha} \frac{\partial \hat{\chi}}{\partial x} = - \frac{\hat{q}(s)}{\alpha} f(x, y) \exp[(s + k)x/u_0], \quad (6)$$

subject to

$$\left. \frac{\partial \hat{\chi}}{\partial y} \right|_{y=0, b} = \left. \hat{\chi} \right|_{x=-\infty} = 0. \quad (7)$$

Formally, the solution to Eq. (6), with the boundary conditions prescribed by Eq. (7), can be written

$$\hat{\chi}(x, y, s) = \int_{-\infty}^x dx' \int_0^b dy' F(x', y', s) G(x, y, x', y'), \quad (8)$$

where

$$F(x, y, s) = \frac{\hat{q}(s)}{4\pi\alpha} f(x, y) \exp[(s + k)x/u_0] . \quad (9)$$

The Green's function $G(x, y, x', y')$ satisfies

$$\frac{\partial^2 G}{\partial y^2} - \frac{u_0}{\alpha} \frac{\partial G}{\partial x} = -4\pi\delta(x - x')\delta(y - y') , \quad (10)$$

where $\delta(x)$ is the Dirac delta function and subject to the boundary conditions

$$\left. \frac{\partial G}{\partial y} \right|_{y=0, b} = \left. G \right|_{x=-\infty} = 0 , \quad (11)$$

The free-space Green's function [Morse and Feshbach (1953)] is

$$G_\infty(x, y, x', y') = \frac{2\sqrt{\pi}}{\alpha\sqrt{x - x'}} U(x - x') \exp[-\alpha^2(y - y')^2/4(x - x')] , \quad (12)$$

where $\alpha = \sqrt{u_0/\alpha}$ and $U(x)$ is the unit step function. To satisfy the boundary conditions at $y = 0$ and $y = b$, the point source at (x', y') must be imaged in these boundaries. This results in sources located at $(x', y' \pm 2nb)$ and $(x', -y' \pm 2nb)$ for $n = 0, 1, 2, \dots$. The desired Green's function is obtained by summing over these sources to give

$$G(x, y, x', y') = \frac{2\sqrt{\pi}}{\alpha\sqrt{x - x'}} U(x - x') \cdot \sum_{n=-\infty}^{\infty} \left\{ \exp\left[-\frac{\alpha^2(y - y' - 2nb)^2}{4(x - x')}\right] + \exp\left[-\frac{\alpha(y + y' - 2nb)^2}{4(x - x')}\right] \right\} . \quad (13)$$

Substituting Eqs. (9) and (13) into Eq. (8) gives

$$\begin{aligned} \hat{\chi}(x, y, s) = & \frac{\hat{q}(s)}{2a\alpha\sqrt{\pi}} \int_{-\infty}^x dx' \int_0^b dy' \frac{f(x', y')}{\sqrt{x-x'}} \exp[(s+k)x'/u_0] U(x-x') \\ & \cdot \sum_{n=-\infty}^{\infty} \left\{ \exp\left[-\frac{a^2(y-y'-2nb)^2}{4(x-x')}\right] \right. \\ & \left. + \exp\left[-\frac{a^2(y+y'-2nb)^2}{4(x-x')}\right] \right\}. \end{aligned} \quad (14)$$

If the spatial distribution of the source $f(x, y)$ has a simple form, Eq. (14) can be easily integrated. A few such cases will be considered here.

Point Source

For a point source located at (x_0, y_0) , the function $f(x, y)$ becomes

$$f(x, y) = \delta(x - x_0)\delta(y - y_0). \quad (15)$$

The integration of Eq. (14) now reduces to the evaluation of the integrand at $x' = x_0$ and $y' = y_0$ to give

$$\begin{aligned} \hat{\chi}(x, y, s) = & \frac{\hat{q}(s)}{2a\alpha\sqrt{\pi}(x-x_0)} \exp[(s+k)x_0/u_0] U(x-x_0) \\ & \cdot \sum_{n=-\infty}^{\infty} \left\{ \exp\left[-\frac{a^2(y-y_0-2nb)^2}{4(x-x_0)}\right] \right. \\ & \left. + \exp\left[-\frac{a^2(y+y_0-2nb)^2}{4(x-x_0)}\right] \right\}. \end{aligned} \quad (16)$$

To obtain $\hat{T}(x, y, s)$, Eq. (16) is multiplied by $\exp[-(s+k)x/u_0]$. The time dependence can be recovered by performing the inverse transformation

$$\begin{aligned}
\mathcal{L}^{-1} \left\{ \exp \left[- \frac{s+k}{u_0} (x - x_0) \right] \hat{q}(s) \right\} \\
= \frac{1}{2\pi i} \exp \left[- \frac{k}{u_0} (x - x_0) \right] \int_{c-i\infty}^{c+i\infty} \hat{q}(s) \exp \left[- \frac{s}{u_0} (x - x_0) \right] \exp(st) ds .
\end{aligned} \tag{17}$$

By translation, this inverse transform is simply

$$\exp \left[- \frac{k}{u_0} (x - x_0) \right] q \left(t - \frac{x - x_0}{u_0} \right) U \left(t - \frac{x - x_0}{u_0} \right) .$$

Finally, the functional form for the excess temperature is

$$\begin{aligned}
\Delta T(x, y, t) = \frac{q \left(t - \frac{x - x_0}{u_0} \right)}{2\sqrt{\pi u_0 \alpha} (x - x_0)} \exp \left[- \frac{k}{u_0} (x - x_0) \right] U \left(t - \frac{x - x_0}{u_0} \right) U(x - x_0) \\
\cdot \sum_{n=-\infty}^{\infty} \left\{ \exp \left[- \frac{u_0 (y - y_0 - 2nb)^2}{4\alpha (x - x_0)} \right] \right. \\
\left. + \exp \left[- \frac{u_0 (y + y_0 - 2nb)^2}{4\alpha (x - x_0)} \right] \right\} .
\end{aligned} \tag{18}$$

Some insight can be gained by inspection of this equation. First, there is an excess temperature front being advected downstream. This feature is apparent in the step function $U \left(t - \frac{x - x_0}{u_0} \right)$. At time t , the excess temperature only extends to $x = x_0 + u_0 t$, and there is no excess temperature beyond this point. Another feature is the linear dependence of the excess temperature on the magnitude of the source strength. In other words, if ΔT is known for a source strength q , then the excess temperature from a source of strength γq is merely $\gamma \Delta T$.

Gaussian Line Source Normal to Shore

Most near-field models are Gaussian. Therefore, it is desirable to integrate Equation (14) for a source strength having such a distribution. This integration should produce, in most situations, a more realistic

plume than a point source. For simplicity, consider a Gaussian line source normal to the shore with the mean at a point (x_0, y_0) . In this case, the function $f(x, y)$ has the form

$$f(x, y) = \delta(x - x_0) A \exp\left[-\frac{(y - y_0)^2}{\sigma^2}\right], \quad (19)$$

where σ is defined as the standard deviation and A is a normalization constant. Now, Equation (14) becomes

$$\begin{aligned} \hat{\chi}(x, y, s) = & \frac{\hat{q}(s)}{2\alpha\sqrt{\pi}(x - x_0)} A \exp\left[\left(\frac{s + k}{u_0}\right)x_0\right] U(x - x_0) \int_0^b dy' \exp\left[-\frac{(y' - y_0)^2}{\sigma^2}\right] \\ & \cdot \sum_{n=-\infty}^{\infty} \left\{ \exp\left[-\frac{\alpha^2(y - y' - 2nb)^2}{4(x - x_0)}\right] \right. \\ & \left. + \exp\left[-\frac{\alpha^2(y + y' - 2nb)^2}{4(x - x_0)}\right] \right\}. \end{aligned} \quad (20)$$

To perform this integration, consider the integral

$$I = \int_0^b dy' \exp\left[-\frac{(y' - y_0)^2}{\sigma^2}\right] \exp\left[-\frac{\alpha^2(y \pm y' - 2nb)^2}{4(x - x_0)}\right]. \quad (21)$$

This can be written in the form

$$I = \int_0^b dy' \exp[-(c_1 y'^2 + 2c_2 y' + c_3)], \quad (22)$$

where

$$c_1 = \frac{1 + \beta^2}{\sigma^2}, \quad c_2 = -\frac{[y_0 \pm \beta^2(y - 2nb)]}{\sigma^2}, \quad c_3 = \frac{y_0^2 + \beta^2(y - 2nb)^2}{\sigma^2},$$

and $\beta^2 = \frac{\alpha^2 \sigma^2}{4(x - x_0)}$. In this form, Eq. (22) can be integrated to give

$$I = \frac{1}{2} \sqrt{\frac{\pi}{c_1}} \exp\left[\frac{c_2^2 - c_1 c_3}{c_1}\right] \left\{ \operatorname{erf}\left(\sqrt{c_1} b + \frac{c_2}{\sqrt{c_1}}\right) - \left(\operatorname{erf} \frac{c_2}{\sqrt{c_1}}\right) \right\}, \quad (23)$$

or

$$I = \frac{\sigma}{2} \sqrt{\frac{\pi}{1 + \beta^2}} \exp\left[-\frac{\beta^2}{\sigma^2(1 + \beta^2)} (y \pm y_0 - 2nb)^2\right] \cdot \left\{ \operatorname{erf}\left[\sqrt{1 + \beta^2} \frac{b}{\sigma} \mp \frac{(\beta^2 y \pm y_0 - 2n\beta^2 b)}{\sigma\sqrt{1 + \beta^2}}\right] \pm \operatorname{erf}\left[\frac{\beta^2 y \pm y_0 - 2n\beta^2 b}{\sigma\sqrt{1 + \beta^2}}\right] \right\}. \quad (24)$$

Substituting Eq. (24) into Eq. (20), multiplying this result by $\exp\left[-\frac{(s+k)}{u_0} x\right]$ and taking the inverse of the Laplace transform in the manner shown by Eq. (17) results in

$$\begin{aligned} \Delta T(x, y, t) = & -\frac{\sigma g[t - (x - x_0)/u_0]}{2\sqrt{u_0}\sqrt{4\alpha(x - x_0) + u_0\sigma^2}} A \exp\left[-\frac{k}{u_0}(x - x_0)\right] U\left(t - \frac{x - x_0}{u_0}\right) \\ & \cdot U(x - x_0) \sum_{n=-\infty}^{\infty} \left\{ \exp\left[-\frac{u_0(y \pm y_0 - 2nb)^2}{4\alpha(x - x_0) + u_0\sigma^2}\right] \right. \\ & \cdot \left[\operatorname{erf}\left(\sqrt{1 + \beta^2} \frac{b}{\sigma} - \frac{\beta^2 y + y_0 - 2n\beta^2 b}{\sigma\sqrt{1 + \beta^2}}\right) \right. \\ & \quad \left. + \operatorname{erf} \frac{\beta^2 y + y_0 - 2n\beta^2 b}{\sigma\sqrt{1 + \beta^2}} \right] \quad (25) \\ & + \exp\left[-\frac{u_0(y - y_0 - 2nb)^2}{4\alpha(x - x_0) + u_0\sigma^2}\right] \\ & \cdot \left[\operatorname{erf}\left(\sqrt{1 + \beta^2} \frac{b}{\sigma} + \frac{\beta^2 y - y_0 - 2n\beta^2 b}{\sigma\sqrt{1 + \beta^2}}\right) \right. \\ & \quad \left. - \operatorname{erf} \frac{\beta^2 y - y_0 - 2n\beta^2 b}{\sigma\sqrt{1 + \beta^2}} \right] \left. \right\}. \end{aligned}$$

The difficult form of Equation (25) is deceiving. It is computationally straightforward and, in most instances, converges rapidly for downstream distances that are not too excessive.

Arbitrary Source Distribution

An arbitrary source distribution, or multiple sources, can be approximated by a number of point sources. To do this, assume that the spatial source function has the form

$$f(x, y) = \sum_{m=1}^M f_m \delta(x - x_m) \delta(y - y_m) . \quad (26)$$

This is the sum of M point sources of various strengths f_m located at points (x_m, y_m) , where $m = 1, 2, \dots, M$. From substituting Eq. (26) in Eq. (6), it is evident that the excess temperature resulting from multiple point sources is simply the sum of the excess temperatures resulting from the individual point sources. Therefore, the excess temperature can be represented by a superposition of the point source solutions given by Eq. (18) and can be written as

$$\begin{aligned} \Delta T(x, y, t) = & \sum_{m=1}^M \left(\frac{q \left(t - \frac{x - x_m}{u_0} \right)}{2\sqrt{\pi u_0 \alpha} (x - x_m)} f_m \exp \left[-\frac{k}{u_0} (x - x_m) \right] U \left(t - \frac{x - x_m}{u_0} \right) \right. \\ & \cdot U(x - x_m) \sum_{n=-\infty}^{\infty} \left\{ \exp \left[-\frac{u_0 (y - y_m - 2nb)^2}{4\alpha (x - x_m)} \right] \right. \\ & \left. \left. + \exp \left[-\frac{u_0 (y + y_m - 2nb)^2}{4\alpha (x - x_m)} \right] \right\} \right) \quad (27) \end{aligned}$$

With careful selection of point source strengths and locations, many realistic sources can be represented. This technique will be used in a later section to consider the excess temperature from a line source at some angle to the shore.

COLD SHOCK

The time dependence is the same in all the cases considered in the previous section. The time rate of change of the excess temperature at any point is proportional to the time rate of change of the function $q\left(t - \frac{x - x_0}{u_0}\right)$ and, therefore,

$$\begin{aligned} \frac{\partial(\Delta T)}{\partial t} &\propto U\left(t - \frac{x - x_0}{u_0}\right) \frac{dq}{dt}\left(t - \frac{x - x_0}{u_0}\right) \\ &+ q\left(t - \frac{x - x_0}{u_0}\right) \delta\left(t - \frac{x - x_0}{u_0}\right). \end{aligned} \quad (28)$$

The delta function in this equation is a result of the excess temperature front and has no real significance when calculating the time derivative. It will be disregarded so that

$$\frac{\partial(\Delta T)}{\partial t} \propto U\left(t - \frac{x - x_0}{u_0}\right) \frac{dq}{dt}\left(t - \frac{x - x_0}{u_0}\right). \quad (29)$$

This form of the time rate of change is appropriate for any of the results in the previous section. For example, the time rate of change of excess temperature at any point from multiple point sources is

$$\begin{aligned} \frac{\partial(\Delta T)}{\partial t} &= \sum_{m=1}^M \left(\frac{U\left(t - \frac{x - x_m}{u_0}\right)}{2\sqrt{\pi}u_0\alpha(x - x_m)} \frac{\partial q}{\partial t}\left(t - \frac{x - x_m}{u_0}\right) f_m \exp\left[-\frac{k}{u_0}(x - x_m)\right] \right. \\ &\quad \cdot U(x - x_m) \sum_{n=-\infty}^{\infty} \left\{ \exp\left[-\frac{u_0(y - y_m - 2nb)^2}{4\alpha(x - x_m)}\right] \right. \\ &\quad \left. \left. + \exp\left[-\frac{u_0(y + y_m - 2nb)^2}{4\alpha(x - x_m)}\right] \right\} \right) \quad (30) \end{aligned}$$

LAKE OR OCEAN WITH A STRAIGHT COASTLINE

Results for a straight coastline of an ocean or lake are easily obtained from the river results if the flow is uniform and longshore. In this case, the far boundary can be moved to infinity and the boundary conditions are:

$$\left. \frac{\partial(\Delta T)}{\partial y} \right|_{y=0} = \left. \Delta T \right|_{x=-\infty} = \left. \Delta T \right|_{y=\infty} = 0 . \quad (31)$$

For the Green's function to satisfy these conditions, only a single image source located at $y = -y'$ is required, and the Green's function becomes

$$G(x, y, x', y') = \frac{2\sqrt{\pi}}{a\sqrt{x - x'}} U(x - x') \cdot \left\{ \exp\left[-\frac{a^2(y - y')^2}{4(x - x')}\right] + \exp\left[-\frac{a^2(y + y')^2}{4(x - x')}\right] \right\} . \quad (32)$$

This is simply the $n = 0$ term in Equation (13). Therefore, the solution for a coastal situation can be obtained for any of the three cases in Section II by merely considering the $n = 0$ term in the respective solutions. For example, the solution for the excess temperature from a point source is

$$T(x, y, t) = \frac{q\left(t - \frac{x - x_0}{u_0}\right)}{2\sqrt{\pi u_0 \alpha}(x - x_0)} \exp\left[-\frac{k}{u_0}(x - x_0)\right] U\left(t - \frac{x - x_0}{u_0}\right) U(x - x_0) \cdot \left\{ \exp\left[-\frac{u_0(y - y_0)^2}{4\alpha(x - x_0)}\right] + \exp\left[-\frac{u_0(y + y_0)^2}{4\alpha(x - x_0)}\right] \right\} . \quad (33)$$

COMPUTATION OF EXCESS TEMPERATURE BY COMPUTER

A computer program TWOD has been written in FORTRAN IV to calculate excess temperature and time rate-of-change of excess temperature

using the results obtained in previous sections. A listing of TWOD is given in Appendix A. Because some of the results derived above are too general for efficient programming, the program is limited to specific useful situations, as described below.

The source strength as a function of time is taken to be

$$q(t) = q_0[1 - \exp(-t/\tau_1)] \left(1 - U(t - t_2) \left\{ 1 - \exp[-(t - t_2)/\tau_2] \right\} \right). \quad (34)$$

This form for the source strength is fairly versatile. If one is only concerned with the steady-state result, t_2 can be taken very large, and τ_1 can be taken very small. This will give a step function for the transient source strength at time equal zero, as shown in Fig. 2a. To simulate plant shutdown, τ_1 is small, t_2 is taken sufficiently large to insure steady-state, and τ_2 is the shutdown e-folding time. This form for the source function is shown in Fig. 2b.

The method used for an arbitrary source distribution is programmed to approximate a Gaussian line source at an arbitrary angle to the shore. Since this form of the function has the capability to approximate a line source normal to the shore, the exact solution given in Section II is not programmed.

For a prescribed mean at (x_0, y_0) , standard deviation σ , and angle to shore θ (see Fig. 3), the line distribution is approximated by five point sources. To determine the strengths and locations of these point sources, the Gaussian is truncated at two standard deviations to either side of the mean, provided that the source has not yet intersected either shore. The line will intersect the near shore at a distance $-y_0/\sin \theta$ from the mean and will intersect the far shore at a distance $(b - y_0)/\sin \theta$ from the mean. To avoid creating point sources beyond the boundaries of the river, the line source will be truncated at the near shore at $\xi_1 = -y_0/\sin \theta$, if $y_0/\sin \theta \leq 2\sigma$. Similarly, the line source will be truncated at the far shore at $\xi_2 = (b - y_0)/\sin \theta$, if $(b - y_0)/\sin \theta \leq 2\sigma$. In all other instances, $\xi_1 = -2\sigma$ and $\xi_2 = 2\sigma$.

The line segment from ξ_1 to ξ_2 is divided into five equal intervals of width Δ , so that

(a) STEP-FUNCTION APPROXIMATION FOR STEADY-STATE RESULTS

(b) SIMULATION OF SOURCE SHUT-DOWN FROM STEADY-STATE

Fig. 2. Examples of the transient source function $q(t)$ used in the program TWOD.

Fig. 3. Definition sketch for the approximation of a Gaussian line source. The angle of the line source with respect to the horizontal is θ , and ξ is the cross plume coordinate.

$$\Delta = (\xi_2 - \xi_1)/5 . \quad (35)$$

A point source is located at the center of each of these five intervals,

$$x_m = x_0 - [\xi_1 + (2m - 1)\frac{\Delta}{2}] \cos \theta , \quad (36a)$$

$$y_m = y_0 + [\xi_1 + (2m - 1)\frac{\Delta}{2}] \sin \theta , \quad (36b)$$

where m goes from 1 to 5. The strength of each point source, f_m , will be proportional to the total strength of the Gaussian source over a particular interval Δ , so that

$$f_m \propto \int_{\xi_1 + (m-1)\Delta}^{\xi_1 + m\Delta} \exp[-\xi'^2 \sigma^2] d\xi' . \quad (37)$$

Since the total source strength is specified by the constant q_0 , all the point source strengths must be normalized by the total strength of the Gaussian over the interval ξ_1 to ξ_2 . Performing the necessary integrations, the point source strength can be written

$$f_m = \frac{\operatorname{erf}\left(\frac{\xi_1 + m\Delta}{\sigma}\right) - \operatorname{erf}\left(\frac{\xi_1 + (m-1)\Delta}{\sigma}\right)}{\operatorname{erf}\left(\frac{\xi_2}{\sigma}\right) - \operatorname{erf}\left(\frac{\xi_1}{\sigma}\right)} . \quad (38)$$

The capability for representing an arbitrary source by a number of point sources is also incorporated in the TWOD program. The user simply specifies the strength and location of each point source, as input, and the program automatically calculates the excess temperature from all of these sources, using Eq. (27). (The program is dimensioned for a maximum of 10 point sources.) This feature is particularly useful for calculating the effects from multiple discharges.

INPUT AND OUTPUT INFORMATION FOR COMPUTER ANALYSIS

The input parameters are defined so that the computer program TWOD contains no dimensional constants. Therefore, the program is not limited to any specific system of units. However, it is important to use consistent units for all input data. The output will be in the same units as the input. Input specifications are shown in Table 1.

If the multiple point source option is used, there should be one card for each point source. For example, if five point sources are used, there should be a total of ten input cards.

The input parameters σ and θ are required only for the Gaussian line source. If the point source is selected, whatever values of σ and θ that are entered will be ignored. The input parameter x_{max} is optional. It is used to specify a downstream value at which calculations are terminated. If the fifth card is left blank, calculations will continue to the edge of the plume, which is at $x = x_0 + u_0 t$ (for the specified values of x_0 , u_0 , and t). Usually, the unit of time used for the input variables will be seconds. If this is the case and the time rate-of-change of excess temperature is requested, the values of the output could be quite small (particularly if τ_2 is large). Therefore, it might be desirable to multiply the input value for q_0 by 60 (3600) and thereby produce an output in degrees per minute (degrees per hour), rather than degrees per second.

To run multiple cases, successive sets of input cards may be placed one after the other.

Printed output will consist of an array of values for either the excess temperature or the time rate-of-change of the excess temperature. Values are listed in the cross-stream (y) direction at intervals of one-tenth of the river width and in the downstream direction at multiples of the river width (b , $2b$, $3b$, etc.). If the plotting option is requested, a finer grid is used for calculations, but the printout will be the same. The contour plots produced will be 10 in. high and 24 in. long and will show five contours. The values of these contours are generated internally and will be listed at the bottom of the printout. No contour values will be given on the plots.

Table 1. Input for TWOD

Columns	Data (right justified in assigned columns)
<u>First card</u>	
[Format (20A4)]	
1-80	Any title or identification.
<u>Second card</u>	
[Format (3I5)]	
1-5	Source function type: = 0 for Gaussian line source; = 1 for point source; = n ($n = 2-10$) for n arbitrary point sources.
6-10	Plot: = 1, if contour plots are desired.
11-15	Time-derivative: = 1, if the time rate-of-change of excess temperature is desired rather than excess temperature.
<u>Third card</u>	
[Format (6F10.0)]	
1-10	t - time at which excess temperature is to be calculated.
11-20	q_0 - total source strength, equal to the product of the excess temperature of the discharge and the discharge flow rate divided by the river depth (length squared·degrees/time).
21-30	t_2 - plant shutdown time.
31-40	τ_1 - start-up e-folding time.
41-50	τ_2 - shutdown e-folding time.
51-60	x_0 - downstream coordinate of point source or mean of Gaussian line source.
<u>Fourth card</u>	
[Format (7F10.0)]	
1-10	y_0 - cross-stream coordinate of point source or mean of Gaussian line source.
11-20	α - eddy diffusivity.
21-30	u_0 - ambient river flow.
31-40	k - the surface heat exchange coefficient divided by the product of the density, the heat capacity, and the depth. This should have dimensions of time^{-1} .
41-50	b - width of the river.
51-60	σ - standard deviation (temperature e-folding width) of the line source.
61-70	θ - the angle of the line source with respect to shore (= 0° : discharge centerline is normal to shore; = 90° : discharge centerline is parallel to shore).
<u>Fifth card</u>	
[Format (7F10.0)]	
1-10	x_{max} - maximum value of downstream coordinate (optional).
<u>Sixth card-fifteenth card</u>	
[Format (3F10.0)]	
(used only for multiple sources)	
1-10	x_m - downstream coordinate of m th point source.
11-20	y_m - cross-stream coordinate of m th point source.
21-30	f_m - source strength of m th point source, which should have dimensions of length squared·degrees/time.

APPLICATION OF THE MODEL

To demonstrate the use of the program TWOD, an example has been run; the results are reproduced here. The example consists of three cases for the same physical conditions, but at three different times during a plant shutdown. The values for the input parameters are listed in Table 2. The ambient conditions are a river velocity of 3.0 ft/sec, a river width of 600 ft, and a river depth of 20 ft.

Table 2. Input parameters for three executions of the program TWOD

Variable	Data supplied		
	Case 1	Case 2	Case 3
Source function ^a	0	0	0
Plot ^b	1	1	0
Time-derivative ^b	0	0	1
t (sec)	7,200	10,800	12,600
q_0	3,000 (F°·ft ² /sec)	3,000 (F°·ft ² /sec)	180,000 (F°·ft ² /min)
t_2 (sec)	7,200	7,200	7,200
τ_1 (sec)	10	10	10
τ_2 (sec)	1,800	1,800	1,800
x_0 (ft)	0	0	0
y_0 (ft)	100	100	100
α (ft ² /sec)	2	2	2
u_0 (ft/sec)	3	3	3
k (sec ⁻¹)	0	0	0
b (ft)	600	600	600
σ (ft)	20	20	20
θ (deg)	70	70	70
x_{max} (ft)	20,000	22,000	Blank

^a0 = Gaussian.

^b1 = yes; 0 = no.

At some point along a shore of the river, a power plant is discharging cooling water normal to the shore with an excess temperature of 20°F and at a rate of 3000 cfs. Away from the point of discharge, the jet spreads and loses some momentum.

At 100 ft (y_0) from shore, the jet has lost most of its initial momentum, and, at the same time, has been turned through an angle of 70° (θ) by the ambient flow. The temperature distribution across the jet can be represented by a Gaussian with a standard deviation of 20 ft (σ). This is the beginning of the far field, and this physical description of the jet (plume) at this point is the boundary conditions for the TWOD far-field model.

The first case is at a time of 7200 sec (2 hr); steady-state conditions exist and plant shutdown is imminent. The printed output for this case is given in Fig. 4, and the plotted result is shown in Fig. 5. As expected, the maximum excess temperatures are found near the source. The second case is at a time of 10,800 sec, 1 hr after plant shutdown has begun. The discharge temperature has decreased, and the warm water near the source has been advected downstream. This can be seen in the printed output (Fig. 6) or the plotted output (Fig. 7). The warmest water is in a "pool" 6000 ft downstream. This pool is cooling, through diffusion, as it is being swept downstream at a rate of 3.0 ft/sec (u_0). Surface heat exchange was not included in this calculation since its effect would be negligible in this case. For instance, if the surface heat exchange coefficient was assumed to be 3 Btu/hr·ft²·°F, a parcel of heated water would have to travel six miles downstream before surface heat transfer could produce a 2% temperature change. The final case gives the time rate-of-change of excess temperature at 1.5 hr after the start of plant shutdown. The results are shown as printed output in Fig. 8. All values are negative, since the temperature is decreasing. The greatest rate-of-change is approximately $-0.13^{\circ}\text{F}/\text{min}$ ($-7.8^{\circ}\text{F}/\text{hr}$), 15,600 ft downstream along the near shore. The rate-of-change is zero beyond 16,200 ft downstream, since insufficient time has elapsed for changes at the source to be advected to these distances.

Fig. 5. Excess temperature isotherms for Case 1.

TWO DIMENSIONAL TRANSIENT FAR-FIELD ANALYSIS
 EXAMPLE: 1 HOUR AFTER INITIATION OF PLANT SHUT-DOWN
 FUNCTION NUMBER (FN)--0=GAUSSIAN LINE SOURCE AT ARBITRARY ANGLE TO SHORE, 1= POINT SOURCE,
 , AND >1= MULTIPLE POINT SOURCE: FUNCTION NUMBER= 0.
 CONTOUR PLOT IF PLOT=1: PLOT=1
 USE DERIVATIVE OF Q FUNCTION IF QK=1: QK=0
 T=10800.0 Q0= 3000.0 T2= 7200.0 TAU1= 10.0 TAU2= 1800.0 X0= 0.0 Y0= 100.0
 ALPHA= 2.0 UC= 3.000 K= 0.0 R= 600.0 XMAX=2.200000E 04 SIGMA= 20.0 THETA= 70.00

I	XN(I)	YN(I)	PN(I)
1	-1.0945E 01	6.9930E 01	4.2704E-02
2	5.4723E 00	8.4965E 01	2.4209E-01
3	0.0	1.0000E 02	4.3341E-01
4	-5.4723E 00	1.1504E 02	2.4209E-01
5	-1.0945E 01	1.3007E 02	4.2704E-02

	Y	0	60	120	180	240	300	360	420	480	540	600
X												
600		0.0229	0.8581	1.5663	0.0760	0.0001	0.0000	0.0000	0.0000	0.0300	0.0000	0.0000
1200		0.1936	1.0230	1.4267	0.2669	0.0067	0.0000	0.0000	0.0000	0.0300	0.0000	0.0300
1800		0.4290	1.0996	1.3725	0.4307	0.0337	0.0007	0.0000	0.0000	0.0300	0.0000	0.0000
2400		0.6618	1.1771	1.3664	0.5632	0.0804	0.0040	0.0001	0.0000	0.0300	0.0000	0.0300
3000		0.8821	1.2713	1.3924	0.6781	0.1403	0.0123	0.0005	0.0000	0.0300	0.0000	0.0000
3600		1.0941	1.3844	1.4438	0.7854	0.2091	0.0271	0.0017	0.0001	0.0300	0.0000	0.0000
4200		1.3037	1.5163	1.5176	0.8915	0.2846	0.0489	0.0045	0.0002	0.0300	0.0000	0.0300
4800		1.5167	1.6668	1.6126	1.0009	0.3663	0.0779	0.0096	0.0007	0.0300	0.0000	0.0300
5400		1.7376	1.8363	1.7284	1.1170	0.4544	0.1141	0.0176	0.0017	0.0301	0.0000	0.0000
6000		1.9706	2.0254	1.8653	1.2427	0.5499	0.1579	0.0293	0.0035	0.0303	0.0000	0.0303
6600		2.2194	2.2366	2.0240	1.3805	0.6537	0.2094	0.0451	0.0065	0.0306	0.0000	0.0300
7200		2.4872	2.4706	2.2057	1.5325	0.7673	0.2693	0.0657	0.0111	0.0313	0.0001	0.0000
7800		2.7776	2.7294	2.4119	1.7011	0.8921	0.3380	0.0916	0.0177	0.0324	0.0002	0.0300
8400		3.0940	3.0169	2.6446	1.8897	1.0300	0.4166	0.1236	0.0268	0.0342	0.0005	0.0301
9000		3.4399	3.3346	2.9061	2.0977	1.1827	0.5061	0.1624	0.0389	0.0369	0.0009	0.0302
9600		3.8192	3.6861	3.1989	2.3307	1.3524	0.6077	0.2088	0.0546	0.0408	0.0016	0.0304
10200		4.2359	4.0751	3.5262	2.5907	1.5414	0.7229	0.2639	0.0744	0.0462	0.0028	0.0307
10800		4.6932	4.5042	3.8901	2.8797	1.7514	0.8529	0.3282	0.0992	0.0535	0.0044	0.0313
11400		4.6514	4.4584	3.8464	2.8672	1.7783	0.8955	0.3613	0.1161	0.0596	0.0061	0.0319
12000		4.6104	4.4118	3.8048	2.8548	1.8018	0.9349	0.3936	0.1335	0.0664	0.0081	0.0328
12600		4.5665	4.3662	3.7660	2.8432	1.8231	0.9717	0.4251	0.1515	0.0739	0.0106	0.0339
13200		4.5221	4.3215	3.7296	2.8322	1.8423	1.0062	0.4556	0.1699	0.0820	0.0134	0.0353
13800		4.4775	4.2778	3.6952	2.8218	1.8598	1.0384	0.4853	0.1885	0.0907	0.0167	0.0370
14400		4.4331	4.2350	3.6624	2.8118	1.8758	1.0687	0.5140	0.2072	0.0999	0.0204	0.0391
15000		4.3890	4.1932	3.6312	2.8022	1.8904	1.0972	0.5418	0.2260	0.0795	0.0246	0.0415
15600		4.3453	4.1523	3.6012	2.7930	1.9039	1.1240	0.5686	0.2448	0.0896	0.0291	0.0443
16200		4.3021	4.1123	3.5724	2.7839	1.9163	1.1493	0.5946	0.2636	0.1000	0.0341	0.0475
16800		4.2599	4.0732	3.5445	2.7751	1.9277	1.1733	0.6197	0.2822	0.1108	0.0395	0.0511
17400		4.2182	4.0349	3.5175	2.7664	1.9383	1.1959	0.6439	0.3007	0.1219	0.0453	0.0551
18000		4.1773	3.9976	3.4914	2.7578	1.9480	1.2173	0.6673	0.3190	0.1332	0.0514	0.0595
18600		4.1372	3.9611	3.4659	2.7493	1.9570	1.2377	0.6899	0.3371	0.1447	0.0580	0.0642
19200		4.0978	3.9254	3.4412	2.7409	1.9653	1.2570	0.7119	0.3550	0.1564	0.0649	0.0694
19800		4.0593	3.8906	3.4170	2.7325	1.9729	1.2753	0.7329	0.3726	0.1682	0.0721	0.0750
20400		4.0216	3.8565	3.3934	2.7242	1.9799	1.2928	0.7533	0.3900	0.1802	0.0797	0.0809
21000		3.9847	3.8232	3.3704	2.7158	1.9864	1.3094	0.7731	0.4071	0.1924	0.0876	0.0872
21600		3.9486	3.7907	3.3479	2.7075	1.9924	1.3252	0.7921	0.4240	0.2046	0.0958	0.0939

CONTOUR LEVELS: 0.2500 0.5000 1.0000 2.0000 4.0000
 PLOT CALL OUTSIDE LIMITS X=0.0 Y=-.1100E 02 IPEN= 3

Fig. 6. Print-out of excess temperature for Case 2, plant shutdown (the input parameters are given in Table 1).

Fig. 7. Excess temperature isotherms for Case 2.

TWO DIMENSIONAL TRANSIENT PAR-FIELD ANALYSIS
 EXAMPLE: TIME RATE OF CHANGE OF TEMPERATURE IN DEGREES F PER MINUTE
 FUNCTION NUMBER (FN)--0=GAUSSIAN LINE SOURCE AT ARBITRARY ANGLE TO SHORE, 1= POINT SOURCE,
 , AND >1=MULTIPLE POINT SOURCE: FUNCTION NUMBER= 0
 CONTOUR PLOT IF PLOT=1: PLOT=0
 USE DERIVATIVE OF Q FUNCTION IF QK=1: QK=1
 T=12600.0 Q0=180000.0 T2= 7200.0 TAU1= 10.0 TAU2= 1800.0 X0= 0.0 Y0= 100.0
 ALPHA= 2.0 B0= 3.000 K= 0.0 B= 600.0 XMAX=7.237005E 75 SIGMA= 20.0 THETA= 70.00

I	XH(I)	YH(I)	PH(I)
1	1.0945E 01	6.9930E 01	4.2704E-02
2	5.4723E 00	8.4965E 01	2.4209E-01
3	0.0	1.0000E 02	4.3041E-01
4	-5.4723E 00	1.1504E 02	2.4209E-01
5	-1.0945E 01	1.3007E 02	4.2704E-02

Y	0	60	120	180	240	300	360	420	480	540	600
X											
600	-0.0003	-0.0105	-0.0192	-0.0009	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000
1200	-0.0024	-0.0125	-0.0175	-0.0033	-0.0001	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000
1800	-0.0053	-0.0135	-0.0168	-0.0053	-0.0004	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000
2400	-0.0081	-0.0144	-0.0168	-0.0069	-0.0010	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000
3000	-0.0108	-0.0155	-0.0171	-0.0083	-0.0017	-0.0002	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000
3600	-0.0134	-0.0170	-0.0177	-0.0096	-0.0026	-0.0003	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000
4200	-0.0160	-0.0186	-0.0186	-0.0109	-0.0035	-0.0006	-0.0001	-0.0000	-0.0000	-0.0000	-0.0000
4800	-0.0186	-0.0204	-0.0198	-0.0123	-0.0045	-0.0013	-0.0001	-0.0000	-0.0000	-0.0000	-0.0000
5400	-0.0213	-0.0225	-0.0212	-0.0137	-0.0056	-0.0014	-0.0002	-0.0000	-0.0000	-0.0000	-0.0000
6000	-0.0242	-0.0248	-0.0229	-0.0152	-0.0067	-0.0019	-0.0004	-0.0000	-0.0000	-0.0000	-0.0000
6600	-0.0272	-0.0274	-0.0248	-0.0169	-0.0080	-0.0026	-0.0006	-0.0001	-0.0000	-0.0000	-0.0000
7200	-0.0305	-0.0303	-0.0270	-0.0188	-0.0094	-0.0033	-0.0008	-0.0001	-0.0000	-0.0000	-0.0000
7800	-0.0341	-0.0335	-0.0296	-0.0209	-0.0109	-0.0041	-0.0011	-0.0002	-0.0000	-0.0000	-0.0000
8400	-0.0379	-0.0370	-0.0324	-0.0232	-0.0126	-0.0051	-0.0015	-0.0003	-0.0001	-0.0000	-0.0000
9000	-0.0422	-0.0409	-0.0356	-0.0257	-0.0145	-0.0062	-0.0020	-0.0005	-0.0001	-0.0000	-0.0000
9600	-0.0468	-0.0452	-0.0392	-0.0286	-0.0166	-0.0075	-0.0026	-0.0007	-0.0001	-0.0000	-0.0000
10200	-0.0519	-0.0500	-0.0432	-0.0318	-0.0189	-0.0089	-0.0032	-0.0009	-0.0002	-0.0000	-0.0000
10800	-0.0576	-0.0553	-0.0477	-0.0353	-0.0215	-0.0105	-0.0040	-0.0012	-0.0003	-0.0001	-0.0000
11400	-0.0638	-0.0611	-0.0527	-0.0393	-0.0244	-0.0123	-0.0050	-0.0016	-0.0004	-0.0001	-0.0000
12000	-0.0706	-0.0676	-0.0583	-0.0437	-0.0276	-0.0143	-0.0063	-0.0020	-0.0006	-0.0001	-0.0000
12600	-0.0781	-0.0747	-0.0644	-0.0487	-0.0312	-0.0166	-0.0073	-0.0026	-0.0008	-0.0002	-0.0001
13200	-0.0865	-0.0826	-0.0713	-0.0542	-0.0352	-0.0192	-0.0087	-0.0032	-0.0010	-0.0003	-0.0001
13800	-0.0957	-0.0914	-0.0790	-0.0603	-0.0399	-0.0222	-0.0104	-0.0049	-0.0013	-0.0004	-0.0002
14400	-0.1059	-0.1011	-0.0875	-0.0672	-0.0448	-0.0255	-0.0123	-0.0050	-0.0017	-0.0005	-0.0002
15000	-0.1171	-0.1119	-0.0969	-0.0748	-0.0505	-0.0293	-0.0145	-0.0060	-0.0021	-0.0007	-0.0003
15600	-0.1296	-0.1238	-0.1074	-0.0833	-0.0568	-0.0335	-0.0173	-0.0073	-0.0027	-0.0009	-0.0004
16200	-0.0749	-0.0711	-0.0606	-0.0459	-0.0306	-0.0177	-0.0089	-0.0038	-0.0014	-0.0005	-0.0002
16800	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

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Fig. 8. Print-out for Case 3, time rate-of-change of excess temperature in Fahrenheit degrees per minute (the input parameters are given in Table 1).

ADDITIONAL REMARKS

An analytic solution has been presented for the advective diffusion equation in two dimensions. This solution treats an arbitrary transient source exactly and approximates an arbitrary spatial source distribution. With the associated computer code TWOD, this solution provides an efficient method for dealing with transient power plant thermal output, particularly the cold shock problem.

This work does not, however, represent a universally applicable model. The simplifying assumptions required to obtain an analytic solution preclude its use for a number of cases. In particular, problems involving complex channel geometries or variable natural flows cannot adequately be analyzed with this model. However, this model could be useful in such cases for both a scoping study as well as providing initial conditions to a more sophisticated numerical model.

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APPENDIX A
FORTRAN-IV PROGRAM "TWOD"

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C      PROGRAM TWOD                                TWOD  1
C      TWO-D MAIN PROGRAM                          TWOD  2
C      TWO-DIMENSIONAL TRANSIENT FAR-FIELD THERMAL ANALYSIS "TWOD" TWOD  3
C      WITH OPTIONAL CONTOUR PLOT                  TWOD  4
C                                                    TWOD  5
C      PROGRAM AUTHOR:  A J WITTEN, ENERGY DIVISION, OAK RIDGE NATIONAL TWOD  6
C      LABORATORY (ORNL) OAK RIDGE, TENNESSEE 37830 TWOD  7
C      PROGRAMMED BY E C LONG, COMPUTER SCIENCES DIVISION, ORNL TWOD  8
C                                                    TWOD  9
C      *****TWOD 10
C      *                                           TWOD 11
C      *                                           TWOD 12
C      *           *****TWOD 12
C      *           *INPUT INFORMATION FOR TWCD* TWOD 13
C      *           *****TWOD 14
C      *                                           TWOD 15
C      * IT IS SUGGESTED THAT SETS OF DATA CARDS BE NUMBERED TWOD 16
C      * COLS. 73-80 ARE AVAILABLE ON ALL DATA CARDS TWOD 17
C      *                                           TWOD 18
C      * CARD 1  TITLE                                20A4 TWOD 19
C      * ANY SORT OF OUTPUT AND PLOTTING IDENTIFICATION TWOD 20
C      *                                           TWOD 21
C      * CARD 2  FN|PLOT|QK                            3I5 TWOD 22
C      * FN IS FUNCTION TYPE TWOD 23
C      * FN=0:  GAUSSIAN LINE SOURCE AT AN ARBITRARY ANGLE TWOD 24
C      * FN=1:  POINT SOURCE TWOD 25
C      * FN>1:  MULTIPLE POINT SOURCES TWOD 26
C      * FOR PLOT=1, PRODUCE CONTOUR PLOT TWOD 27
C      * FOR QK=1, USE DERIVATIVE OF Q FUNCTION TWOD 28
C      *                                           TWOD 29
C      * CARD 3  T1|Q0|T2|TAU1|TAU2|X0                6F10.0 TWOD 30
C      * T--TIME AT WHICH EXCESS TEMPERATURE IS CALCULATED TWOD 31
C      * Q0--SOURCE STRENGTH, SHOULD HAVE DIMENSIONS OF TWOD 32
C      * DEGREES-LENGTH**2/TIME TWOD 33
C      * T2--TIME AT WHICH PLANT SHUTS DOWN TWOD 34
C      * START UP TIME IS ASSUMED TO BE ZERO TWOD 35
C      * TAU1--E-FOLDING TIME FOR PLANT START UP TWOD 36
C      * TAU2--E-FOLDING TIME FOR PLANT SHUT DOWN TWOD 37
C      * X0--DOWNSTREAM COORDINATE OF POINT SOURCE OR TWOD 38
C      * MEAN OF GAUSSIAN LINE SOURCE TWOD 39
C      *                                           TWOD 40
C      * CARD 4  Y0|ALPHA|U0|K|B|SIGMA|THETA            7F10.0 TWOD 41
C      * Y0--CROSS STREAM COORDINATE OF POINT SOURCE OR TWOD 42
C      * MEAN OF GAUSSIAN LINE SOURCE TWOD 43
C      * ALPHA--EDDY DIFFUSIVITY TWOD 44
C      * U0--AMBIENT RIVER FLOW (ALWAYS POSITIVE) TWOD 45
C      * K--SURFACE HEAT TRANSFER COEFFICIENT/(DENSITY*HEAT CAPACITY TWOD 46
C      * *DEPTH), SHOULD HAVE DIMENSIONS OF (1/TIME) TWOD 47
C      * B--WIDTH OF RIVER TWOD 48
C      * SIGMA--TEMPERATURE STANDARD DEVIATION OF LINE SOURCE TWOD 49
C      * (EQUAL TO 1.44*HALF-VALUE WIDTH) TWOD 50
C      * THETA--ANGLE OF LINE SOURCE WITH RESPECT TO THE SHORE TWOD 51
C      * THETA=0 DEGREES DISCHARGE CENTERLINE NORMAL TO SHORE, TWOD 52
C      * THETA=90 DEGREES DISCHARGE CENTERLINE PARALLEL TO SHORE TWOD 53
C      *                                           TWOD 54
C      * CARD 5  XMAX                                7F10.0 TWOD 55
C      * XMAX IS DOWNSTREAM DISTANCE AT WHICH CALCULATIONS CEASE TWOD 56
C      * IF XMAX=0.0, XMAX IS SET TO LARGEST MACHINE NUMBER TWOD 57
C      *                                           TWOD 58
C      * CARDS 6-A,B--  XM(I)|YM(I)|PM(I),                3F10.0 TWOD 59
C      * I=1,PN (FOR PN>1) TWOD 60
C      * XM(I), YM(I), & PM(I) ARE COORDINATES AND STRENGTH OF TWOD 61
C      * OF POINT SOURCE I TWOD 62
C      *                                           TWOD 63
C      *                                           TWOD 64
C      *           *****TWOD 64
C      *           *END OF INPUT INFORMATION FOR TWOD* TWOD 65
C      *           *****TWOD 66
C      *                                           TWOD 67
C      *****TWOD 68
C      *                                           TWOD 69
C      THE CONTOUR PLOTTING ROUTINE USES THE "ORGRAPH" PLOTTING PACKAGE TWOD 70
C      ORNL-4596 TWOD 71
C      EACH CALL TO "RCONTR" MUST BE PRECEDED BY A CALL TO "PIXPLT" TWOD 72
C      SEE "ORGRAPH" MANUAL FOR USE OF "PIXPLT" TWOD 73

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C   ARRAYS:
C   X(NX) & Y(NY) DEFINE THE GRID POINTS FROM WHICH
C   XY(NX,NY) DATA POINTS WILL HAVE A CONTOUR PLOT GENERATED
C   C(NC) CONTAINS THE CONTOUR LINE VALUES WHICH WILL BE PLOTTED
C   IN THE CALL TO "RCONTR":
C   CALL RCONTR(NC,C,XY,NX,X,NXF,NX,NDLTX,Y,NYP,NY,NDLTY)
C   VALUES OF X(IX), Y(IY) & XY(IX,IY) WHICH WILL BE USED ARE
C   DETERMINED BY DO LOOPS
C   DO --- IX=NXF,NX,NDLTX &
C   DO --- IY=NYP,NY,NDLTY
C   "PLID" IDENTIFIES PLOT WITH DATE, HASP LOG NUMBER, & THE
C   TIME AT WHICH JOB WAS EXECUTED
C   "PLID" USES "GTJN" (AN ASSEMBLY LANGUAGE ROUTINE)
C   USUALLY SUPPLIED AS A HEX DECK
C   IF NO IDENTIFICATION OF PLOTS IS DESIRED, REMOVE THE CALL
C   TO "PLID"
C
COMMON/LGND/C(20),DESCR(40),SYMBL(20),NC,NSYMBL
COMMON/XYTN/Q0,T2,TAU1,TAU2,X0,Y0,ALPHA,U0,K,B,SIGMA,THETA,
1   QK,PN
COMMON/XYT2C/FN(10),XM(10),YM(10),U4A,KU0,PU0A2,B2
REAL K,KU0
DIMENSION X(500),Y(500),XY(500,50),BUFFER(3000),TITLEP(20)
INTEGER FY(50),FN,PLOT,DY10,NPLOT/0/,PB,QK
DATA BIG/Z7FFFFF/,SPACE/' '/
NSYMBL=-1
NC=5
CALL OONEWS(0)
100 READ (5,8000,END=165) TITLEP,PN,PLOT,QK,T,Q0,T2,TAU1,TAU2,
1   X0,Y0,ALPHA,U0,K,B,SIGMA,THETA,XMAX
8000 FORMAT (20A4/3I5/6F10.0/(7F10.0))
TITLEP(19)=SPACE
TITLEP(20)=SPACE
IF (XMAX.EQ.0.0) XMAX=BIG
DX=B
YI=0.0
DY=B/10.0
DY10=1
PB=B
C   PLOT=0, NO CONTOUR PLOTTING
IF (PLOT.EQ.0) GO TO 105
DX=DX/4.0
DY=DY/4.0
DY10=4
105 XI=DX
DO 110 I=1,500
X(I)=XI
XI=XI+DX
110 CONTINUE
DO 115 IY=1,50
Y(IY)=YI
FY(IY)=YI
IF (YI.GE.B) GO TO 120
YI=YI+DY
115 CONTINUE
120 PRINT 8005,TITLEP,PN,PLOT,QK,T,Q0,T2,TAU1,TAU2,X0,Y0,ALPHA,U0,
1   K,B,XMAX
8005 FORMAT ('1',10X,'TWO DIMENSIONAL TRANSIENT FAR-FIELD ANALYSIS'/
1   10X,20A4/10X,'FUNCTION NUMBER (FN)--0=GAUSSIAN LINE ',
2   'SOURCE AT ARBITRARY ANGLE TO SHORE, 1= POINT SOURCE,'/
3   14X,', AND >1=MULTIPLE POINT SOURCE: FUNCTION NUMBER=',I2/
4   10X,'CONTOUR PLOT IF PLOT=1: PLOT=',I1/
5   10X,'USE DERIVATIVE OF Q FUNCTION IF QK=1: QK=',I1/
6   10X,'T=',F7.1,' Q0=',F8.1,' T2=',F7.1,
7   ' TAU1=',F7.1,' TAU2=',F7.1,' X0=',F6.1,' Y0=',F6.1/
8   10X,'ALPHA=',F4.1,' U0=',F6.3,' K=',1PE10.3,' B=',0PF7.1,
9   ' XMAX=',1PE12.6)
IF (FN.EQ.0) PRINT 8010,SIGMA,THETA
8010 FORMAT ('+',71X,'SIGMA=',F6.1,' THETA=',F6.2)
C   INITIALIZE ARRAYS FOR LINE SOURCE IF FN=0
C   OR MULTIPLE POINT SOURCE IF FN>1

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IF (FN.GT.1) READ 8015, (XM(I),YM(I),PM(I),I=1,PN)	TWOD 146
8015 FORMAT (3F10.0)	TWOD 147
IF (FN.GE.1) CALL XYP1	TWOD 148
IF (FN.EQ.0) CALL XYP2	TWOD 149
PRINT 8020, (I,YM(I),YM(I),PM(I),I=1,PN)	TWOD 150
8020 FORMAT (1X/15X,'I',4X,'XM(I)',7X,'YM(I)',7X,'PM(I)'/	TWOD 151
1 (14X,I2,1P3B12.4))	TWOD 152
PRINT 8025, (PY(I),I=1,IY,DY10)	TWOD 153
8025 FORMAT (1X/9X,'Y',15,10I9/8X,100(' ')/5X,'X ',100X,' ')	TWOD 154
XMYPN=0.0	TWOD 155
DO 125 I=1,PN	TWOD 156
XMYPN=AMAX1(XM(I),XMYPN)	TWOD 157
125 CONTINUE	TWOD 158
TT2U0=(T-T2)/U0	TWOD 159
XYMAX=0.0	TWOD 160
DO 140 IX=1,500	TWOD 161
XI=X(IX)	TWOD 162
XY0=0.0	TWOD 163
DO 130 I=1,IY	TWOD 164
XY(IX,I)=0.0	TWOD 165
130 CONTINUE	TWOD 166
IF (XI.GT.XMAX.AND.XI.GT.XMYPN) GO TO 145	TWOD 167
DO 135 IYI=1,IY	TWOD 168
YI=Y(IYI)	TWOD 169
XY(IX,IYI)=XYT2(XI,YI,T)	TWOD 170
XY0=XY0+XY(IX,IYI)	TWOD 171
IF (XI.GE.B) XYMAX=AMAX1(XY(IX,IYI),XYMAX)	TWOD 172
135 CONTINUE	TWOD 173
IXI=XI	TWOD 174
IF (MOD(IXI,PN).EQ.0) PRINT 8030,IXI,(XY(IX,J),J=1,IY,DY10)	TWOD 175
8030 FORMAT (1X,I5,' ',11F9.4,' ')	TWOD 176
IF (XI.LE.XMYPN.OR.XI.LE.TT2U0) GO TO 140	TWOD 177
IF (XY0.EQ.0.0) GO TO 145	TWOD 178
140 CONTINUE	TWOD 179
145 PRINT 8035	TWOD 180
8035 FORMAT (7X,' ',100(' '), ' ')	TWOD 181
WRITE (40) IX,IY,XY	TWOD 182
IF (PLOT.EQ.0) GO TO 100	TWOD 183
NPLOT=1	TWOD 184
C SET LENGTH OF PLOT	TWOD 185
CALL QQQCPA(5,24.0)	TWOD 186
C(NC)=AINT(XYMAX)	TWOD 187
NC1=NC-1	TWOD 188
J=NC1	TWOD 189
DO 150 I=1,NC1	TWOD 190
C(J)=C(J+1)/2.0	TWOD 191
J=J-1	TWOD 192
150 CONTINUE	TWOD 193
PRINT 8040, (C(I),I=1,NC)	TWOD 194
8040 FORMAT ('O CONTOUR LEVELS:',10F10.4)	TWOD 195
XMIN=X(1)	TWOD 196
PMIN=BIG	TWOD 197
DO 155 I=1,PN	TWOD 198
XMIN=AMIN1(XM(I),XMIN)	TWOD 199
PMIN=AMIN1(PM(I),PMIN)	TWOD 200
155 CONTINUE	TWOD 201
PMIN=PMIN*50.0	TWOD 202
CALL FIXPLT(XMIN,X(IX),Y(1),Y(IY),'LINEAR','MECH',-1.0,-1.0,	TWOD 203
1 BUFFER,3000,'LONG\$')	TWOD 204
CALL RCONTR(NC,C,XY,500,X,1,IX,1,Y,1,IY,1)	TWOD 205
CALL LEFT(TITLEP)	TWOD 206
CALL TITLE(0.4,TITLEP,'XS','YS')	TWOD 207
DO 160 I=1,PN	TWOD 208
X(1)=XM(I)	TWOD 209
X(2)=XM(I)	TWOD 210
Y(1)=YM(I)	TWOD 211
Y(2)=YM(I)	TWOD 212
CALL QQQCPA(9,AMIN1(PM(I)/PMIN,0.1))	TWOD 213
CALL LINPLT(X,Y,2,11,-1)	TWOD 214
160 CONTINUE	TWOD 215
CALL QQQCPA(9,0.1)	TWOD 216

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CALL PLID  
GO TO 100  
165 IF (NPLOT.EQ.1) CALL ADVANS  
STOP 9999  
END
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TWOD 217  
TWOD 218  
TWOD 219  
TWOD 220  
TWOD 221
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	SUBROUTINE XYTI	XYTI	1
	COMMON/XYTN/Q0,T2,TAU1,TAU2,X0,Y0,ALPHA,U0,K,B,SIGMA,THETA,	XYTI	2
1	OK,FN	XYTI	3
	INTEGER FN,OK	XYTI	4
	COMMON/XYT2C/FM(10),XM(10),YM(10),U4A,KU0,PU0A2,B2	XYTI	5
	REAL K,KU0	XYTI	6
	ENTRY XYP1	XYTI	7
C	INITIALIZE TERMS FOR POINT SOURCE	XYTI	8
C	OR MULTIPLE POINT SOURCE	XYTI	9
	IF (FN.GT.1) GO TO 100	XYTI	10
	XM(1)=X0	XYTI	11
	YM(1)=Y0	XYTI	12
	FM(1)=1.0	XYTI	13
100	U4A=U0/(4.0*ALPHA)	XYTI	14
	KU0=K/U0	XYTI	15
C	PU0A2=2*SQRT(PI*U0*ALPHA)	XYTI	16
	PU0A2=SQRT(12.53664*U0*ALPHA)	XYTI	17
	B2=2.0*B	XYTI	18
	RETURN	XYTI	19
	ENTRY XYP2	XYTI	20
C	INITIALIZE VALUES FOR FUNCTION XYT2 (GAUSSIAN SOURCE AT	XYTI	21
C	ARBITRARY ANGLE)	XYTI	22
C	CONVERT THETA TO RADIANS	XYTI	23
	THETA=THETA*1.745329E-2	XYTI	24
C	CALCULATE THE P-FUNCTIONS	XYTI	25
	SINT=SIN(THETA)	XYTI	26
	COST=COS(THETA)	XYTI	27
C	CALCULATE THE XI FUNCTIONS	XYTI	28
	S2=2.0*SIGMA	XYTI	29
	FST=S2*SINT	XYTI	30
	XI1=-S2	XYTI	31
	IF (Y0.LE.FST) XI1=-Y0/SINT	XYTI	32
	BY0=B-Y0	XYTI	33
	XI2=S2	XYTI	34
	IF (BY0.LE.FST) XI2=BY0/SINT	XYTI	35
C	CALCULATE THE XM & YM VALUES FROM THE XI FUNCTIONS	XYTI	36
	DELTA=(XI2-XI1)/5.0	XYTI	37
	DENOM=ERP(XI2/SIGMA)-ERP(XI1/SIGMA)	XYTI	38
	DEL2=DELTA/2.0	XYTI	39
	XMR=1.0	XYTI	40
	DO 105 M=1,5	XYTI	41
	FM(M)=(ERP((XI1+XMR*DELTA)/SIGMA)-ERP((XI1+(XMR-1.0)*DELTA)/	XYTI	42
1	SIGMA))/DENOM	XYTI	43
	TWOM1=(2.0*XMR-1.0)*DEL2	XYTI	44
	YM(M)=Y0+(XI1+TWOM1)*SINT	XYTI	45
	XM(M)=X0-(XI1+TWOM1)*COST	XYTI	46
	XMR=XMR+1.0	XYTI	47
105	CONTINUE	XYTI	48
	FN=5	XYTI	49
	GO TO 100	XYTI	50
	END	XYTI	51

C	FUNCTION XYT2(X,Y,T)	XYT2	1
	SUM FUNCTION	XYT2	2
	COMMON/XYT2C/PN(10),XM(10),YM(10),U4A,KU0,PU0A2,B2	XYT2	3
	COMMON/XYTN/Q0,T2,TAU1,TAU2,X0,Y0,ALPHA,U0,K,B,SIGMA,THETA,	XYT2	4
1	QK,PN	XYT2	5
	REAL N,NB2,K,KU0	XYT2	6
	INTEGER PN,QK	XYT2	7
	XYT2=0.0	XYT2	8
	DO 110 M0=1,PN	XYT2	9
	XM0=XM(M0)	XYT2	10
	IF (X.LE.XM0) GO TO 110	XYT2	11
	XXM=X-XM0	XYT2	12
	XXMU0=XXM/U0	XYT2	13
	TXXMU0=T-XXMU0	XYT2	14
	IF (TXXMU0.LE.0.0) GO TO 110	XYT2	15
	QT=Q(TXXMU0)	XYT2	16
	IF (QT.EQ.0.0) GO TO 110	XYT2	17
	YM0=YM(M0)	XYT2	18
	YMYM=Y-YM0	XYT2	19
	YPYM=Y+YM0	XYT2	20
	U4AXXM=U4A/XXM	XYT2	21
	N=1.0	XYT2	22
	SUM2=0.0	XYT2	23
	SUM2S=0.0	XYT2	24
100	NB2=N*N	XYT2	25
	SUM2=SUM2+EXP(-U4AXXM*(YMYM-NB2)**2)+EXP(-U4AXXM*(YPYM-NB2)**2)	XYT2	26
1	+EXP(-U4AXXM*(YMYM+NB2)**2)+EXP(-U4AXXM*(YPYM+NB2)**2)	XYT2	27
	IF (SUM2.EQ.0.0) GO TO 105	XYT2	28
	IF (ABS((SUM2-SUM2S)/SUM2).LE.1.0E-4) GO TO 105	XYT2	29
	SUM2S=SUM2	XYT2	30
	N=N+1.0	XYT2	31
	GO TO 100	XYT2	32
105	XYT2=XYT2+(QT/(PU0A2*SQRT(XXM))*PN(M0)*EXP(-KU0*XXM)	XYT2	33
1	* (EXP(-U4AXXM*YMYM**2)+EXP(-U4AXXM*YPYM**2)+SUM2)	XYT2	34
110	CONTINUE	XYT2	35
	RETURN	XYT2	36
	END	XYT2	37

FUNCTION Q(T)	Q	1
COMMON/XYTN/Q0,T2,TAU1,TAU2,X0,Y0,ALPHA,U0,K,E,SIGMA,THETA,	Q	2
1 OK,FN	Q	3
REAL K	Q	4
INTEGER FN,OK	Q	5
Q=0.0	Q	6
S4=1.0	Q	7
TT2=T-T2	Q	8
UTT2=0.5	Q	9
IF (TT2.EQ.0.0) GO TO 100	Q	10
UTT2=UTT2+SIGN(0.5,TT2)	Q	11
100 S3=0.0	Q	12
IF (TT2.GE.0.0) S3=EXP(-TT2/TAU2)	Q	13
IF (TT2.GT.0.0) S4=1.0-UTT2*(1.0-S3)	Q	14
S1=EXP(-T/TAU1)	Q	15
S2=1.0-S1	Q	16
IF (OK.EQ.1) GO TO 105	Q	17
C USE NORMAL Q FUNCTION	Q	18
Q=Q0*S2*S4	Q	19
RETURN	Q	20
C USE DERIVATIVE OF Q FUNCTION	Q	21
105 Q=Q0*(S1/TAU1*S4-S3/TAU2*UTT2*S2)	Q	22
RETURN	Q	23
ENTRY U(X)	Q	24
C FOR X<0, U(X)=0.0	Q	25
C FOR X=0, U(X)=0.5	Q	26
C FOR X>0, U(X)=1.0	Q	27
U=0.5	Q	28
IF (X.EQ.0.0) RETURN	Q	29
U=U+SIGN(0.5,X)	Q	30
RETURN	Q	31
END	Q	32

	SUBROUTINE LEFT(X)	LEFT	1
C	LEFT ADJUST X AND PUT \$ IN FIRST NON-SPACE ON RIGHT (FOR ORGRAPH)	LEFT	2
	LOGICAL*1 X(80),Y(80),SPACE,DOLLAR	LEFT	3
	DATA SPACE,DOLLAR/' ','\$'/	LEFT	4
	DO 100 I=1,80	LEFT	5
100	Y(I)=SPACE	LEFT	6
	DO 105 I=1,80	LEFT	7
	IF (X(I).NE.SPACE) GO TO 110	LEFT	8
105	CONTINUE	LEFT	9
C	I=80, X ALL SPACES	LEFT	10
	X(1)=DOLLAR	LEFT	11
	RETURN	LEFT	12
C	X(I) IS FIRST NON-SPACE ON LEFT	LEFT	13
110	K=1	LEFT	14
	DO 115 J=I,80	LEFT	15
	Y(K)=X(J)	LEFT	16
	K=K+1	LEFT	17
115	CONTINUE	LEFT	18
	J=80	LEFT	19
	DO 120 I=1,80	LEFT	20
	IF (Y(J).NE.SPACE) GO TO 125	LEFT	21
120	J=J-1	LEFT	22
C	Y(J) FIRST NON-SPACE FROM RIGHT	LEFT	23
125	J=MIN0(J+1,80)	LEFT	24
	IF (Y(J-1).NE.DOLLAR) Y(J)=DOLLAR	LEFT	25
	DO 130 I=1,80	LEFT	26
	X(I)=Y(I)	LEFT	27
130	CONTINUE	LEFT	28
	RETURN	LEFT	29
	END	LEFT	30

	SUBROUTINE RCONTR(K1,C,A,MAX,X,IMIN,IMAX,ISTEP,Y,JMIN,JMAX,JSTEP)	RCON	1
C	ADAPTED FROM LAWRENCE RADIATION LAB REPORT CIC-RC.3-003	RCON	2
C	ASSUMES THAT A(I,J)=PCN(X(I),Y(J))	RCON	3
	DIMENSION A(MAX,1),C(1),X(1),Y(1)	RCON	4
	COMMON/Q7QUAD/XC(2),YC(2),ZC(4),ZMIN,ZMAX,KK1	RCON	5
	KK1=K1	RCON	6
	CALL QQQCPA(48,0.1)	RCON	7
C	BEGIN LOGIC FOR SWEEPING THROUGH GRID	RCON	8
	IMA=IMAX-ISTEP	RCON	9
	JMA=JMAX-JSTEP	RCON	10
	DO 105 I=IMIN,IMA,ISTEP	RCON	11
	XC(1)=X(I)	RCON	12
	XC(2)=X(I+ISTEP)	RCON	13
	DO 100 J=JMIN,JMA,JSTEP	RCON	14
	YC(1)=Y(J)	RCON	15
	YC(2)=Y(J+JSTEP)	RCON	16
C	ELEMENT(I,JMIN)	RCON	17
	ZC(1)=A(I,J)	RCON	18
C	ELEMENT(I+ISTEP,JMIN)	RCON	19
	ZC(2)=A(I+ISTEP,J)	RCON	20
C	ELEMENT(I,J+JSTEP)	RCON	21
	ZC(3)=A(I,J+JSTEP)	RCON	22
C	ELEMENT(I+ISTEP,J+JSTEP)	RCON	23
	ZC(4)=A(I+ISTEP,J+JSTEP)	RCON	24
	ZMAX=AMAX1(ZC(1),ZC(2),ZC(3),ZC(4))	RCON	25
	ZMIN=AMIN1(ZC(1),ZC(2),ZC(3),ZC(4))	RCON	26
	CALL Q7PLOT(C)	RCON	27
100	CONTINUE	RCON	28
105	CONTINUE	RCON	29
	RETURN	RCON	30
	END	RCON	31

SUBROUTINE Q7PLOT(CN)	Q7PL 1
DIMENSION CN(1),C(50)	Q7PL 2
COMMON/Q7QUAD/X(2),Y(2),Z(2,2),ZMIN,ZMAX,K1	Q7PL 3
IF (ZMAX.EQ.ZMIN) RETURN	Q7PL 4
IF (CN(1).GT.ZMAX) RETURN	Q7PL 5
IF (CN(K1).LT.ZMIN) RETURN	Q7PL 6
DO 100 I=1,K1	Q7PL 7
IF (CN(I).GE.ZMIN) GO TO 105	Q7PL 8
100 CONTINUE	Q7PL 9
105 DO 110 J=I,K1	Q7PL 10
IF (CN(J).GT.ZMAX) GO TO 115	Q7PL 11
110 CONTINUE	Q7PL 12
J=K1+1	Q7PL 13
115 J=J-1	Q7PL 14
CALL CONTUR(Z,CN(I),X(1),X(2),Y(1),Y(2),2,2,J-(I-1),1,2,1,2)	Q7PL 15
RETURN	Q7PL 16
END	Q7PL 17

SUBROUTINE CONTUR(Z,C,P1MIN,P1MAX,P2MIN,P2MAX,M,N,NCL,NMIN,	CONT	1
1 MMAX,NMIN,NMAX)	CONT	2
COMMON/CONT/II,ISAVE,JSAVE,KSAVE,X(2,50),Y(2,50),CL	CONT	3
COMMON/SUB/XINC,YINC,PMIN1,PMIN2	CONT	4
DIMENSION Z(M,N),C(NCL)	CONT	5
KM=M	CONT	6
KN=N	CONT	7
KNCL=NCL	CONT	8
XINC=(P1MAX-P1MIN)/FLOAT(NMAX-NMIN)	CONT	9
YINC=(P2MAX-P2MIN)/FLOAT(NMAX-NMIN)	CONT	10
PMIN1=P1MIN	CONT	11
PMIN2=P2MIN	CONT	12
NN=MMAX-1	CONT	13
NN=MMAX-1	CONT	14
DO 190 J=NMIN,NN	CONT	15
DO 185 I=MMIN,MM	CONT	16
K=1	CONT	17
II=1	CONT	18
100 IF (Z(I,J).GE.Z(I+1,J)) GO TO 110	CONT	19
AMAX=Z(I+1,J)	CONT	20
105 IF (AMAX.LT.Z(I,J+1)) AMAX=Z(I,J+1)	CONT	21
GO TO 115	CONT	22
110 AMAX=Z(I,J)	CONT	23
GO TO 105	CONT	24
115 IF (Z(I,J).LE.Z(I+1,J)) GO TO 125	CONT	25
AMIN=Z(I+1,J)	CONT	26
120 IF (AMIN.GT.Z(I,J+1)) AMIN=Z(I,J+1)	CONT	27
GO TO 130	CONT	28
125 AMIN=Z(I,J)	CONT	29
GO TO 120	CONT	30
130 IF (C(K).GT.AMAX) GO TO 135	CONT	31
IF (C(K).LT.AMIN) GO TO 135	CONT	32
JSAVE=J	CONT	33
ISAVE=I	CONT	34
KSAVE=K	CONT	35
GO TO 175	CONT	36
135 IF (Z(I,J+1).GE.Z(I+1,J+1)) GO TO 145	CONT	37
AMAX=Z(I+1,J+1)	CONT	38
140 IF (AMAX.LT.Z(I+1,J)) AMAX=Z(I+1,J)	CONT	39
GO TO 150	CONT	40
145 AMAX=Z(I,J+1)	CONT	41
GO TO 140	CONT	42
150 IF (Z(I,J+1).LE.Z(I+1,J+1)) GO TO 160	CONT	43
AMIN=Z(I+1,J+1)	CONT	44
155 IF (AMIN.GT.Z(I+1,J)) AMIN=Z(I+1,J)	CONT	45
GO TO 165	CONT	46
160 AMIN=Z(I,J+1)	CONT	47
GO TO 155	CONT	48
165 IF (C(K).LT.AMIN) GO TO 170	CONT	49
JSAVE=J	CONT	50
ISAVE=I	CONT	51
KSAVE=K	CONT	52
GO TO 180	CONT	53
170 K=K+1	CONT	54
IF (K.LE.NCL) GO TO 100	CONT	55
GO TO 185	CONT	56
175 CALL LTRIS(Z,C,XINC,YINC,KM,KN,KNCL)	CONT	57
II=1	CONT	58
GO TO 135	CONT	59
180 CALL UTRIS(Z,C,XINC,YINC,KM,KN,KNCL)	CONT	60
II=1	CONT	61
GO TO 170	CONT	62
185 CONTINUE	CONT	63
190 CONTINUE	CONT	64
RETURN	CONT	65
END	CONT	66

SUBROUTINE LTRIS (Z,C,XINC,YINC,KM,KN,KNCL)	LTRI	1
COMMON/CONT/II,ISAVE,JSAVE,KSAVE,X(2,50),Y(2,50),CL	LTRI	2
DIMENSION Z(KM,KN),C(KNCL)	LTRI	3
K=KSAVE	LTRI	4
I=ISAVE	LTRI	5
J=JSAVE	LTRI	6
IF (C(K).LE.Z(I,J+1)) GO TO 100	LTRI	7
IF (C(K).LE.Z(I,J)) GO TO 105	LTRI	8
GO TO 115	LTRI	9
100 IF (C(K).LT.Z(I,J)) GO TO 115	LTRI	10
ZSAVE=Z(I,J+1)-Z(I,J)	LTRI	11
IF (ZSAVE.EQ.0.0) ZSAVE=1.0	LTRI	12
Y(II,K)=(C(K)-Z(I,J))/ZSAVE+FLOAT(J)	LTRI	13
X(II,K)=I	LTRI	14
GO TO 110	LTRI	15
105 IF (C(K).LT.Z(I,J+1)) GO TO 115	LTRI	16
ZSAVE=Z(I,J)-Z(I,J+1)	LTRI	17
IF (ZSAVE.EQ.0.0) ZSAVE=1.0	LTRI	18
Y(II,K)=(Z(I,J)-C(K))/ZSAVE+FLOAT(J)	LTRI	19
X(II,K)=I	LTRI	20
110 II=II+1	LTRI	21
115 IF (C(K).LE.Z(I,J)) GO TO 120	LTRI	22
IF (C(K).LE.Z(I+1,J)) GO TO 125	LTRI	23
GO TO 135	LTRI	24
120 IF (C(K).LT.Z(I+1,J)) GO TO 135	LTRI	25
ZSAVE=Z(I,J)-Z(I+1,J)	LTRI	26
IF (ZSAVE.EQ.0.0) ZSAVE=1.0	LTRI	27
Y(II,K)=J	LTRI	28
X(II,K)=(Z(I,J)-C(K))/ZSAVE+FLOAT(I)	LTRI	29
GO TO 130	LTRI	30
125 IF (C(K).LT.Z(I,J)) GO TO 135	LTRI	31
ZSAVE=Z(I+1,J)-Z(I,J)	LTRI	32
IF (ZSAVE.EQ.0.0) ZSAVE=1.0	LTRI	33
Y(II,K)=J	LTRI	34
X(II,K)=(C(K)-Z(I,J))/ZSAVE+FLOAT(I)	LTRI	35
130 II=II+1	LTRI	36
IF (II.EQ.3) GO TO 155	LTRI	37
135 IF (C(K).LE.Z(I+1,J)) GO TO 140	LTRI	38
IF (C(K).LE.Z(I,J+1)) GO TO 145	LTRI	39
RETURN	LTRI	40
140 IF (C(K).LT.Z(I,J+1)) RETURN	LTRI	41
ZSAVE=Z(I+1,J)-Z(I,J+1)	LTRI	42
IF (ZSAVE.EQ.0.0) ZSAVE=1.0	LTRI	43
DX=(C(K)-Z(I,J+1))/ZSAVE+FLOAT(I)	LTRI	44
DY=(Z(I+1,J)-C(K))/ZSAVE+FLOAT(J)	LTRI	45
II=II+1	LTRI	46
IF (II.EQ.3) GO TO 150	LTRI	47
RETURN	LTRI	48
145 IF (C(K).LT.Z(I+1,J)) RETURN	LTRI	49
ZSAVE=Z(I,J+1)-Z(I+1,J)	LTRI	50
IF (ZSAVE.EQ.0.0) ZSAVE=1.0	LTRI	51
DX=(Z(I,J+1)-C(K))/ZSAVE+FLOAT(I)	LTRI	52
DY=(C(K)-Z(I+1,J))/ZSAVE+FLOAT(J)	LTRI	53
II=II+1	LTRI	54
IF (II.EQ.3) RETURN	LTRI	55
150 X(2,K)=DX	LTRI	56
Y(2,K)=DY	LTRI	57
155 CALL SUBPLT(C(K))	LTRI	58
CL=C(K)	LTRI	59
RETURN	LTRI	60
END	LTRI	61

SUBROUTINE UTRIS (Z,C,XINC,YINC,KM,KN,KNCL)	UTRI	1
COMMON/CONT/II,ISAVE,JSAVE,KSAVE,X(2,50),Y(2,50),CL	UTRI	2
DIMENSION Z(KM,KN),C(KNCL)	UTRI	3
I=ISAVE	UTRI	4
J=JSAVE	UTRI	5
K=KSAVE	UTRI	6
IF (C(K).LE.Z(I,J+1)) GO TO 100	UTRI	7
IF (C(K).LE.Z(I+1,J+1)) GO TO 105	UTRI	8
GO TO 110	UTRI	9
100 IF (C(K).LT.Z(I+1,J+1)) GO TO 110	UTRI	10
ZSAVE=Z(I,J+1)-Z(I+1,J+1)	UTRI	11
IF (ZSAVE.EQ.0.0) ZSAVE=1.0	UTRI	12
X(II,K)=(Z(I,J+1)-C(K))/ZSAVE+FLOAT(I)	UTRI	13
Y(II,K)=J+1	UTRI	14
II=II+1	UTRI	15
GO TO 110	UTRI	16
105 IF (C(K).LT.Z(I,J+1)) GO TO 110	UTRI	17
ZSAVE=Z(I+1,J+1)-Z(I,J+1)	UTRI	18
IF (ZSAVE.EQ.0.0) ZSAVE=1.0	UTRI	19
Y(II,K)=J+1	UTRI	20
X(II,K)=(C(K)-Z(I,J+1))/ZSAVE+FLOAT(I)	UTRI	21
II=II+1	UTRI	22
110 IF (C(K).LE.Z(I+1,J+1)) GO TO 115	UTRI	23
IF (C(K).LE.Z(I+1,J)) GO TO 120	UTRI	24
GO TO 125	UTRI	25
115 IF (C(K).LT.Z(I+1,J)) GO TO 125	UTRI	26
ZSAVE=Z(I+1,J+1)-Z(I+1,J)	UTRI	27
IF (ZSAVE.EQ.0.0) ZSAVE=1.0	UTRI	28
X(II,K)=I+1	UTRI	29
Y(II,K)=FLOAT(J+1)-(Z(I+1,J+1)-C(K))/ZSAVE	UTRI	30
II=II+1	UTRI	31
IF (II.EQ.3) GO TO 130	UTRI	32
GO TO 125	UTRI	33
120 IF (C(K).LT.Z(I+1,J+1)) GO TO 125	UTRI	34
ZSAVE=Z(I+1,J)-Z(I+1,J+1)	UTRI	35
IF (ZSAVE.EQ.0.0) ZSAVE=1.0	UTRI	36
X(II,K)=I+1	UTRI	37
Y(II,K)=(Z(I+1,J)-C(K))/ZSAVE+FLOAT(J)	UTRI	38
II=II+1	UTRI	39
IF (II.EQ.3) GO TO 130	UTRI	40
125 IF (II.NE.2) RETURN	UTRI	41
IF (CL.NE.C(K)) RETURN	UTRI	42
CALL SUBPLT(C(K))	UTRI	43
RETURN	UTRI	44
130 CALL SUBPLT(C(K))	UTRI	45
RETURN	UTRI	46
END	UTRI	47

SUBROUTINE SUBPLT(CC)	SUBP 1
DIMENSION V(2),W(2),VP(2),WP(2)	SUBP 2
COMMON/SUB/XINC,YINC,PMIN1,PMIN2	SUBP 3
COMMON/CONT/II,ISAVE,JSAVE,KSAVE,X(2,50),Y(2,50),CL	SUBP 4
COMMON/LGND/C(13),NC	SUBP 5
DATA SYMB/-1.0/	SUBP 6
K=KSAVE	SUBP 7
JJ=2	SUBP 8
IF(II.EQ.2) JJ=1	SUBP 9
DO 100 I=1,JJ	SUBP 10
X(I,K)=(X(I,K)-1.0)*XINC+PMIN1	SUBP 11
Y(I,K)=(Y(I,K)-1.0)*YINC+PMIN2	SUBP 12
V(I)=X(I,K)	SUBP 13
W(I)=Y(I,K)	SUBP 14
100 CONTINUE	SUBP 15
IF (V(1).EQ.V1.AND.V(2).EQ.V2.AND.W(1).EQ.W1.AND.W(2).EQ.W2)	SUBP 16
1 RETURN	SUBP 17
N=2	SUBP 18
IF (V(1).EQ.V(2).AND.W(1).EQ.W(2)) N=1	SUBP 19
V1=V(1)	SUBP 20
V2=V(2)	SUBP 21
W1=W(1)	SUBP 22
W2=W(2)	SUBP 23
IF (V(2).GE.V(1)) GO TO 105	SUBP 24
VP(1)=V(2)	SUBP 25
VP(2)=V(1)	SUBP 26
WP(1)=W(2)	SUBP 27
WP(2)=W(1)	SUBP 28
CALL LINPLT(VP,WP,N,SYMB,0)	SUBP 29
C PRINT 8000,VP,WP,SYMB	SUBP 30
8000 FORMAT (' V(1)=' ,1PE10.3,' V(2)=' ,E10.3,' W(1)=' ,E10.3,	SUBP 31
1 ' W(2)=' ,E10.3,' SYMB=' ,I5)	SUBP 32
RETURN	SUBP 33
105 CALL LINPLT(V,W,N,SYMB,0)	SUBP 34
C PRINT 8000,V,W,SYMB	SUBP 35
RETURN	SUBP 36
END	SUBP 37

	SUBROUTINE PLID	PLID 1
C	IDENTIFY PLOT WITH DATE, HASP JOB NUMBER, AND TIME.	PLID 2
C	PLOT ON RIGHT SIDE OF GRAPH	PLID 3
C	FUNCTION GTJN MUST BE FURNISHED	PLID 4
	DIMENSION DAY(3), HASP(4), TIM(3)	PLID 5
	DATA IDTHS, DAY, HASP, TIM/0,0,0,'\$\$\$\$','HASP',' JOB',0,'\$\$\$\$',	PLID 6
1	0,0,'\$\$\$\$' /	PLID 7
	IF (IDTHS.EQ.1) GO TO 100	PLID 8
	CALL IDAY(DAY)	PLID 9
	HASP(3)=GTJN(0)	PLID 10
	IDTHS=1	PLID 11
100	CALL TIME(TIM)	PLID 12
	CALL LABEL(-4,0.15,DAY,-1)	PLID 13
	CALL LABEL(-2,0.15,HASP,-1)	PLID 14
	CALL LABEL(-6,0.15,TIM,-1)	PLID 15
	RETURN	PLID 16
	END	PLID 17

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