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RESIDUAL STRESS CHARACTERIZATION BY USE OF ELASTIC WAVE SCATTERING MEASUREMENTS

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INTRODUCTION

The presence of a state of residual stress in a material can impair its structural quality by adversely affecting its elastic limit, yield point, etc.¹ Most common nondestructive measurements of residual stress use x-ray techniques.² However, these techniques determine only the surface residual stresses, while in many practical cases knowledge of the bulk residual stresses is desired. Ultrasonic methods^{3,4} appear most natural for measuring bulk residual stress but are used infrequently, in part because of difficulty in adequately measuring small effects and in part because of the absence of theoretical results treating the inhomogeneous nature of residual stress fields.

In this paper we derive the appropriate equations for the use of elastic waves to probe an inhomogeneous state of residual stress. As in other treatments of ultrasonic residual stress measurement, we start with nonlinear effects and require knowledge of third order elastic constants. Unlike other treatments, which relate these nonlinear effects to small relative changes in propagation speed of an incident wave,^{4,5} we identify these effects as a source of scattering of the incident wave. Like other treatments, one difficulty with ultrasonic residual stress measurements is separating small residual stress effects from other effects. However, we will give an example of at least one class of problems where this separation appears possible using our approach.

Our analysis is guided by the following picture: we start with an elastic material that initially is in an undeformed state and

subsequently placed (by cold working, heat treatment, etc.) in a state of residual stress. Since after the deformation the material still behaves elastically, and since the residual stress state is one of mechanical equilibrium, the probing linear elastic wave can couple (i.e., interact anharmonically) with the residual stress state only through nonlinear effects. These interaction terms are viewed as local variations in the elastic stiffness tensor (magnitude and symmetry changes), produced by residual deformation gradients, that cause scattering of the probing elastic wave propagating through the residual stress field. By examining this scattering, we wish to characterize the residual stress state.

BASIC EQUATIONS⁵

If x_i is a point in the undeformed material and z_i is this point in the deformed material, then the Lagrangian density associated with the deformation is⁶

$$\mathcal{L} = \frac{1}{2}\rho_0 \dot{z}_i \dot{z}_i - W(z_{i,j})$$

where ρ_0 is the density and W is the elastic strain energy per unit of undeformed volume. If $z_i(\vec{x}, t) = y_i(\vec{x}) + u_i(\vec{x}, t)$ where $y_i(\vec{x})$ describes the displacements in the residual stress state and $u_i(\vec{x}, t)$ is to be the propagating displacement, then

$$\mathcal{L} = \frac{1}{2}\rho_0 \dot{u}_i \dot{u}_i - (W^{(0)} + W_{j,k}^{(1)} u_{j,k} + \frac{1}{2} W_{i,j,k,l}^{(2)} u_{i,j} u_{k,l} + \dots)$$

where

$$u_{i,j} = \frac{\partial u_i(\vec{x}, t)}{\partial x_j}$$

$$y_{i,j} = \frac{\partial y_i(\vec{x})}{\partial x_j}$$

$$W^{(0)} = W(y_{i,j})$$

$$W_{i,j}^{(1)} = \left. \frac{\partial W}{\partial z_{i,j}} \right|_{z_{i,j} = y_{i,j}}$$

$$w_{ijkl}^{(2)} = \frac{\partial^2 W}{\partial z_{i,j} \partial z_{j,k}} \bigg|_{z_{i,j} = y_{i,j}}$$

Physically, $w_{ij}^{(1)}$ is the residual stress, which satisfies $w_{ij,j}^{(1)} = 0$, and $w_{ijkl}^{(2)}$ is the elastic stiffness of the deformed material. These quantities in general are inhomogeneous fields.

In terms of the Lagrangian, the equations of motion are

$$\frac{\partial}{\partial t} \frac{\partial \mathcal{L}}{\partial \left(\frac{\partial u_i}{\partial t} \right)} + \frac{\partial}{\partial x_j} \frac{\partial \mathcal{L}}{\partial \left(\frac{\partial u_i}{\partial x_j} \right)} - \frac{\partial \mathcal{L}}{\partial x_i} = 0$$

but to terms linear in the u_i , we have

$$\rho_0 \ddot{u}_i = \left(w_{ijkl}^{(2)} u_{k,l} \right)_{,j} \quad (1)$$

where to find this equation we used the static equilibrium condition on the residual stress. Thus, the equation of motion is simply that for a material with density ρ_0 and "stiffness" $w_{ijkl}^{(2)}$.

To find $w_{ijkl}^{(2)}$, we take

$$W = \frac{1}{2} C_{ijkl} z_{ij} z_{kl} + \frac{1}{3!} C_{ijklmn} z_{ij} z_{kl} z_{mn} + \dots$$

where z_{ij} is the finite strain,

$$z_{ij} = \frac{1}{2} (z_{i,j} + z_{j,i} + z_{p,i} z_{p,j})$$

From its definition, we have

$$w_{ijkl}^{(2)} = \frac{\partial^2 W}{\partial z_{i,j} \partial z_{k,l}} \bigg|_{z_{i,j} = y_{i,j}}$$

but from the chain rule we also have

$$\frac{\partial^2 W}{\partial z_{i,j} \partial z_{k,l}} = \frac{\partial^2 z_{pq}}{\partial z_{i,j} \partial z_{k,l}} \frac{\partial W}{\partial z_{pq}} + \frac{\partial z_{pq}}{\partial z_{i,j}} \frac{\partial z_{rs}}{\partial z_{k,l}} \frac{\partial^2 W}{\partial z_{pq} \partial z_{rs}}$$

So

$$\frac{\partial^2 w_{ij}}{\partial z_{k,\ell}} = I_{ijkl} + \frac{1}{2} (\delta_{i\ell} z_{k,j} + \delta_{j\ell} z_{k,i})$$

and

$$\frac{\partial^2 w_{ij}}{\partial z_{k,\ell} \partial z_{m,n}} = I_{ijkln} \delta_{km}$$

where $I_{ijkl} = \frac{1}{2}(\delta_{i\ell}\delta_{jk} + \delta_{ik}\delta_{j\ell})$ is the identity fourth rank tensor, we find that

$$w_{ijkl}^{(2)} = C_{ijkl} + D_{ijklmn} y_{m,n} + \dots \quad (2)$$

with

$$D_{ijklmn} = C_{jklmn} + C_{ij\ell n} \delta_{km} + C_{jinkl} \delta_{in} + C_{mnjl} \delta_{ik}$$

In what follows we will drop from $w_{ijkl}^{(2)}$ terms quadratic and higher in $y_{m,n}$. At this level of approximation consistency requires keeping the third order stiffness constants C_{ijklmn} . Keeping terms higher order in $y_{i,j}$ would require higher order elastic constants.

In applying the equation of motion to the scattering from residual stress fields, we will assume that the undeformed material is elastically isotropic. In this case, we found it easiest to determine $w_{ijkl}^{(2)}$ by directly differentiating the elastic strain energy. To do this, we start with the expression for W given by Landau and Lifshitz⁶,

$$W = \mu z_{ik}^2 + \frac{1}{2} \lambda z_{\ell\ell}^2 + \frac{1}{3} \lambda z_{ik} z_{i\ell} z_{k\ell} + \mu z_{ik}^2 z_{\ell\ell} + \frac{1}{3} \lambda z_{\ell\ell}^3 \quad (4)$$

in which C_{ijklmn} is implicitly and compactly defined by the three constants A, B, and C and by the accompanying scalar invariants. We then substitute

$$z_{ij} = \frac{1}{2}(z_{i,j} + z_{j,i} + z_{\ell,i} z_{\ell,j})$$

into this expression and expand W as a power series in $z_{i,j}$. Through third order

$$\begin{aligned} W = & \frac{1}{2}\mu(z_{i,k} + z_{k,i})^2 + \frac{1}{2}\lambda z_{\ell,\ell}^2 + (\mu + \frac{1}{4}A)z_{i,k}z_{\ell,i}z_{\ell,k} \\ & + \frac{1}{2}(\lambda+B)z_{\ell,\ell}(z_{i,k})^2 + \frac{1}{12}A z_{i,k}z_{k,\ell}z_{\ell,i} \\ & + \frac{1}{2}B z_{i,k}z_{k,i}z_{\ell,\ell} + \frac{1}{3}C z_{\ell,\ell}^3 \end{aligned} \quad (5)$$

Then using the definition,

$$w_{pqrs}^{(2)} = \left. \frac{\partial^2 W}{\partial z_{p,q} \partial z_{r,s}} \right|_{z_{i,j} = y_{i,j}}$$

we find that

$$\begin{aligned} w_{pqrs}^{(2)} = & \lambda \delta_{pq} \delta_{rs} + \mu (\delta_{pr} \delta_{qs} + \delta_{ps} \delta_{qr}) \\ & + \left(\mu + \frac{1}{4}A \right) (\delta_{ps} y_{r,q} + \delta_{qs} y_{r,p} + \delta_{pr} y_{q,s} + \delta_{qr} y_{p,s} + \delta_{qs} y_{p,r} + \delta_{pr} y_{s,q}) \\ & + (\lambda+B) (\delta_{pq} y_{r,s} + \delta_{rs} y_{p,q} + \delta_{pr} \delta_{qs} y_{\ell,\ell}) \\ & + \frac{1}{4}A (\delta_{pr} y_{s,p} + \delta_{ps} y_{q,r}) \\ & + B (\delta_{qr} \delta_{ps} y_{\ell,\ell} + \delta_{pq} y_{s,r} + \delta_{rs} y_{q,p}) \\ & + 2C \delta_{pq} \delta_{rs} y_{\ell,\ell} \end{aligned} \quad (6)$$

What is buried in these expressions, but what seems obvious physically, is that the symmetry of $w_{ijkl}^{(2)}$, and hence the wave motion, is determined by the symmetry of $y_{i,j}$. This property is most easily demonstrated by returning to (2) and observing that if all the quantities defining $w_{ijkl}^{(2)}$ (δ_{ij} , C_{ijkl} , C_{ijklmn} , and $y_{i,j}$) are invariant under the symmetry operations of some point group then $w_{ijkl}^{(2)}$ must also be invariant.

We remark that the symmetry is governed by a second rank tensor instead of a fourth rank tensor, and since the forms of second rank tensors for cubic and isotropic symmetries are identical, the elastic wave motion will be determined (for a cubic distortion) by two and not three independent parameters. In fact, the next simplest wave motion beyond isotropic is transverse isotropic (hexagonal symmetry) with five independent parameters. We will now consider several examples for the wave motion in regions where the displacement gradients are homogeneous. As such we will be discussing what is often called the acoustoelastic effect.

ACOUSTOELASTICITY⁵

We start by writing the general form for the equation of motion

$$\begin{aligned} \rho_0 \ddot{u}_p = & (\lambda + \mu) u_{q,pq} + \mu u_{p,qq} \\ & + (\mu + \frac{1}{4}A) \left(u_{s,pq} y_{s,q} + u_{s,qq} y_{s,p} + u_{p,qs} y_{q,s} \right. \\ & \quad \left. + u_{q,qs} y_{p,s} + u_{s,qq} y_{p,s} + u_{p,qs} y_{s,q} \right) \\ & + (\lambda + B) \left(u_{s,qp} y_{s,q} + u_{s,sq} y_{p,q} + u_{p,qq} y_{s,s} \right) \\ & + \frac{1}{4}A \left(u_{q,qs} y_{s,p} + u_{s,pq} y_{q,s} \right) \\ & + B \left(u_{q,pq} y_{s,s} + u_{q,ps} y_{s,q} + u_{s,sq} y_{q,p} \right) \\ & + 2C u_{s,sp} y_{q,q} \end{aligned} \quad (7)$$

If the deformation is hydrostatic, i.e., $y_{i,j} = \frac{1}{3} \Delta \delta_{ij}$, then the equation of motion reduces to that of an isotropic material,

$$\rho_o \ddot{u}_p = (\lambda' + \nu') u_{q,pq} + \mu' u_{p,qq} \quad (8)$$

with λ' , μ' , and ν' defined by

$$\begin{aligned} \lambda' &= \lambda + \frac{1}{3}\Delta(2\lambda + 4B + 6C) \\ \mu' &= \mu + \frac{1}{3}\Delta \left[4\left(\mu + \frac{1}{4}A\right) + 3(\lambda + B) \right] \\ \nu' &= \mu + \frac{1}{3}\Delta [2\mu + A + 3B] \end{aligned} \quad (9)$$

If the deformation is uniaxial, $y_{i,j} = \Delta \delta_{i3} \delta_{j3}$, as the displacement gradients have transverse isotropic symmetry, the equations of motion become

$$\begin{aligned} \rho_o \frac{\partial^2 u_1}{\partial t^2} &= c_{11} \frac{\partial^2 u_1}{\partial x_1^2} + c_{12} \frac{\partial^2 u_2}{\partial x_1 \partial x_2} + c_{13} \frac{\partial^2 u_3}{\partial x_1 \partial x_3} \\ &\quad + c_{66} \left(\frac{\partial^2 u_1}{\partial x_2^2} + \frac{\partial^2 u_2}{\partial x_1 \partial x_2} \right) + c_{44} \left(\frac{\partial^2 u_1}{\partial x_3^2} + \frac{\partial^2 u_3}{\partial x_1 \partial x_3} \right) \\ \rho_o \frac{\partial^2 u_2}{\partial t^2} &= c_{11} \frac{\partial^2 u_2}{\partial x_2^2} + c_{12} \frac{\partial^2 u_1}{\partial x_1 \partial x_2} + c_{13} \frac{\partial^2 u_3}{\partial x_2 \partial x_3} \\ &\quad + c_{66} \left(\frac{\partial^2 u_2}{\partial x_1^2} + \frac{\partial^2 u_1}{\partial x_1 \partial x_2} \right) + c_{44} \left(\frac{\partial^2 u_2}{\partial x_3^2} + \frac{\partial^2 u_3}{\partial x_2 \partial x_3} \right) \\ \rho_o \frac{\partial^2 u_3}{\partial t^2} &= c_{33} \frac{\partial^2 u_3}{\partial x_3^2} + c_{13} \left(\frac{\partial^2 u_1}{\partial x_1 \partial x_3} + \frac{\partial^2 u_2}{\partial x_2 \partial x_3} \right) \\ &\quad + c_{44} \left(\frac{\partial^2 u_3}{\partial x_1^2} + \frac{\partial^2 u_1}{\partial x_1 \partial x_3} + \frac{\partial^2 u_3}{\partial x_2^2} + \frac{\partial^2 u_3}{\partial x_2 \partial x_3} \right) \end{aligned} \quad (10)$$

with

$$\begin{aligned}
 C_{11} &= \lambda + 2\mu + \Delta(\lambda + 2B + 2C) \\
 C_{12} &= \lambda - \Delta(\lambda - 2C) \\
 C_{13} &= \lambda - \Delta(\mu - 2B - 2C) \\
 C_{33} &= \lambda + 2\mu + \Delta[3(\lambda + 2\mu) + 2(A + 3B + C)] \\
 C_{44} &= \mu + \Delta(\lambda + 2\mu + \frac{1}{2}A + B) \\
 C_{66} &= \frac{1}{2}(C_{11} - C_{12})
 \end{aligned}
 \tag{11}$$

Equations like these form the basis of the theoretical interpretation of most experiments on acoustoelasticity. In the experiments a sample is subjected to a homogeneous deformation, and it is found that the fractional change in wave speed is proportional to the deformation and in certain directions the constants of proportionality are directly related to principal stresses. Since the third-order elastic constants (A, B, and C) are often an order of magnitude larger than are the second-order elastic constants (λ and μ), the small magnitude of the relative change in speed is because Δ is very small.

THE MISFIT PROBLEM

While in practical situations the residual stresses (deformation gradients) are likely to be nearly homogeneous over regions larger than the grain size, the residual stress field is essentially an inhomogeneous one. Besides experiencing an apparent change in speed, a wave propagating through an inhomogeneous medium is also attenuated. Although attenuation and speed measurements might prove to be useful in characterizing the average residual stress state of the entire material, for most nondestructive evaluation purposes regions where the residual stress is concentrated are of particular concern. As these may be well separated, it is of considerable interest to consider the scattering of an elastic wave from a single center of an inhomogeneous state of residual stress. To model such a center, we use the work of Eshelby⁷ who described how the uniformity of an elastic medium is disturbed by a region which has changed its form by twinning, thermal expansion, martensitic transformation, etc. (or which has elastic constants differing from the host material). Such problems we call "misfit" problems. A particularly simple one is a

hydrostatically-deformed spherical region. Since the residual stress only couples weakly to the incident field, we will calculate the scattering from this spherical region by the Born approximation.⁸

When $\delta\rho = 0$, the Born approximation result for the f-vector⁸ for the scattering is

$$f_i(\vec{k}) = \frac{ik^3 \hat{r}_j}{4\pi\rho\omega^2} \int d\vec{r}' \delta C_{ijkl}(\vec{r}') u_{k,l}^o(\vec{r}') e^{-i\vec{k} \cdot \vec{r}'}$$

where u_i^o is the incident wave. For an incident direction along the positive z-axis and longitudinal scattering from an incident longitudinal plane wave, the scattered amplitude $A = \hat{r}_i f_i(\alpha\hat{r})$ reduces to

$$A = - \frac{\alpha^4}{4\pi\rho\omega^2} \hat{r}_i \hat{r}_j \int d\vec{r}' \delta C_{ij33} e^{i\alpha(z' - \hat{r} \cdot \vec{r}')} \quad (12)$$

where α is the longitudinal wave number. To use the Born approximation, we thus need the perturbation in elastic stiffness induced by residual stress.

To find the induced stiffness perturbations, we start by writing $\delta C_{ijkl} = W_{ijkl}^{(2)} - C_{ijkl}$. Then, since the induced displacement field (Eshelby's $u_i^c(\vec{r})$) by symmetry is of the form

$$y_i(\vec{r}) = \hat{r}_1 y_1(r)$$

we know that the deformation gradients are given by

$$y_{i,j}(\vec{r}) = g(r) \delta_{ij} + f(r) \hat{r}_i \hat{r}_j \quad (13)$$

where

$$g = y_r/r \quad (14a)$$

and

$$f = \partial y_r / \partial r - g \quad (14b)$$

Next, we use these gradients in (6) to find that

$$\begin{aligned}
\delta C_{ijkl} = & \left[g(2\lambda+4B+6C) + 2fC \right] \delta_{ij} \delta_{kl} \\
& + \left[g(3\lambda+4\mu+A+3B) + f(\lambda+B) \right] \delta_{ik} \delta_{jl} \\
& + \left[g(2\mu+A+3B) + fB \right] \delta_{il} \delta_{jk} \\
& + f \left[(\lambda+2B)(\delta_{ij} \hat{r}_k \hat{r}_l + \delta_{kl} \hat{r}_i \hat{r}_j) \right. \\
& + (\mu+\frac{1}{2}A)(\delta_{jk} \hat{r}_i \hat{r}_l + \delta_{il} \hat{r}_j \hat{r}_k) \\
& \left. + (2\mu+\frac{1}{2}A)(\delta_{jl} \hat{r}_i \hat{r}_k + \delta_{ik} \hat{r}_j \hat{r}_l) \right] \quad (15)
\end{aligned}$$

The final parts needed are the functions y_r , f , and g . These are straightforwardly obtained from Eshelby: Since our deformation is hydrostatic, Eshelby's $P_{ij}^T = \frac{1}{3} \Delta K \delta_{ij}$ where $K = \lambda + \frac{2}{3}\mu$ is the bulk modulus; furthermore, from Eshelby's equation (2.15) it follows that

$$y_i = \begin{cases} \frac{\Delta V K}{4\pi(\lambda+2\mu)} \frac{\hat{r}_i}{r^2} & , r \geq a \quad (16a) \\ \frac{\Delta V K}{4\pi(\lambda+2\mu)} \frac{r}{a^3} \hat{r}_i & , r < a \quad (16.b) \end{cases}$$

where V is the volume of a sphere of radius a . With these equations and (14), we find that outside the sphere

$$\begin{aligned}
f &= -3g \\
g &= \frac{\Delta V K}{4\pi(\lambda+2\mu)r^3} \quad (17a)
\end{aligned}$$

and inside the sphere

$$\begin{aligned}
f &= 0 \\
g &= \frac{\Delta V K}{4\pi(\lambda+2\mu)a^3} \quad (17b)
\end{aligned}$$

Assembling all the parts and performing the necessary integrations, we finally are able to write the longitudinal scattered amplitude as

$$A = \frac{-\alpha^4}{4\pi\rho\omega^2} \left[\frac{VK}{4\pi(\lambda+2\mu)} \right] (a^{(0)} + a^{(i)}) \quad (18)$$

The term $a^{(i)}$ is the contribution from the region inside the sphere which gives the scattered amplitude a part which resembles the Born approximation for the scattering from an elastic impurity

$$a^{(i)} = \left[\delta\lambda + (\delta\mu + \delta\nu) \cos^2\theta \right] S(\vec{q})/a^3 \quad (19)$$

where $\vec{q} = \alpha(\hat{z} - \hat{r})$ and $S(\vec{q})$ is the shape factor for a sphere of radius a

$$S(\vec{q}) = 4\pi a^3 \frac{\sin qa - qa \cos qa}{(qa)^3} \quad (20)$$

and

$$\begin{aligned} \delta\lambda &= \lambda' - \lambda = \frac{1}{3}\Delta(2\lambda + 4B + 6C) \\ \delta\mu &= \mu' - \mu = \frac{1}{3}\Delta(3\lambda + 4\mu + A + 3B) \end{aligned} \quad (21)$$

$$\delta\nu = \nu' - \mu = \frac{1}{3}\Delta(2\mu + A + 3B)$$

with λ' , μ' , and ν' being defined by (9). The term $a^{(0)}$ is the contribution from the distortion outside the sphere; its value is slightly more involved than that of $a^{(i)}$:

$$\begin{aligned} a^{(0)} &= \Delta \left[2\lambda + 4B + (6\mu + 2A) \cos^2\theta \right] I^{(3)} \\ &\quad - 3\Delta \left[(\lambda + 2B) (I_{33} + \hat{r}_i \hat{r}_j I_{ij}) + (6\mu + 2A) \hat{r}_i \hat{r}_3 I_{i3} \right] \end{aligned} \quad (22)$$

where

$$I^{(3)} = \int_{r>a} d\vec{r} \frac{e^{i\vec{q}\cdot\vec{r}}}{r^3} 4\pi \int_{aq}^{\infty} dx \frac{\sin x}{x^2} \quad (23)$$

and

$$I_{ij} = \int d\vec{r} \frac{\hat{r}_i \hat{r}_j e^{i\vec{q} \cdot \vec{r}}}{r^3} = - \frac{\partial^2}{\partial q_i \partial q_j} \int_{r>a} d\vec{r} \frac{e^{i\vec{q} \cdot \vec{r}}}{r^5} = - \frac{\partial^2}{\partial q_i \partial q_j} I^{(5)} \quad (24)$$

After some algebra we find

$$I^{(5)} = 4\pi q^2 (2\sin aq + aq \cos aq) / 6(aq)^3 - q^2 I^{(3)} / 6 \quad (25)$$

and also

$$I_{nm} = \tilde{U} \delta_{nm} - \tilde{V} \hat{q}_n \hat{q}_m \quad (26)$$

with

$$\begin{aligned} \tilde{U} &= (S(q)/a^3 + I^{(3)})/3, \\ \tilde{V} &= S(q)/a^3 \end{aligned} \quad (27)$$

Substituting (26-27) into (22) we note that all terms involving $I^{(3)}$ exactly cancel, and we obtain

$$\begin{aligned} a^{(0)} &= -2\Delta \frac{S(q)}{a^3} \left[(\lambda + 2B)(1 - 3\sin^2 \theta/2) \right. \\ &\quad \left. + (3\mu + A)\cos \theta (\cos \theta + \sin^2 \theta/2) \right] \end{aligned} \quad (28)$$

The exact cancellation of $I^{(3)}$ in $a^{(0)}$ is quite surprising. It means that as far as the frequency dependence is concerned, the long-range tail of the residual stress field does not result in any broadening of the scattered signal in the time domain; it will be similar to that obtained for a spherical defect. This result may well be an artifact of the Born Approximation, since on physical grounds one would expect that an extended volume inhomogeneity will scatter an impulse well before the actual misfit region is reached.

We do note, however, that the angular distribution of the power scattered by the region outside the sphere differs from that obtained by spherical impurity scattering. We performed cal-

culations of the scattering cross section associated with the misfit region only ($a^{(i)}$) and with the combined ($a^{(i)} + a^{(0)}$) scatterer. The results of this calculation are presented in Figs. 1-3. From there one can observe that for some materials (such as Armco iron and Pyrex) the angular distribution of scattered power changes drastically when the contribution of the stress field outside the sphere is included, while for polystyrene only small changes are seen. The elastic constants of the materials mentioned above are summarized in Table I.

One should keep in mind, though, that these results are based on the first Born Approximation. In particular, we consider the lack of signal broadening a clear indication that a different approximation scheme, such as the eikonal approximation, may be better suited to treat this scattering problem.

SUMMARY AND SUGGESTIONS OF FUTURE WORK

We have demonstrated that elastic wave propagation in the presence of non-uniform residual stress can be viewed as a scattering problem. One should note that in various limits, such as that of short wavelength, this scattering problem (as well as any other) can be treated by optical methods (ray bendings, diffraction, etc.). The special features of a scattering situation are expected to be important for smaller wavelengths, and therefore their experimental observability is questionable, and can be resolved only by careful and thorough measurements.

Table 1. Elastic constants (in 10^5 bars) of materials studied.^a
The anharmonic constants A, B, C are related to the Murnaghan constants via $A = n$, $B + C = l$, $C = l - m + \frac{n}{2}$.

Material	λ	μ	A	B	C
Polystyrene	.2889	.1381	-1.00	-.83	-1.06
ARMCO Iron	11.00	8.2	110.0	-153.0	123.2
Pyrex	1.353	2.75	42.7	-4.17	13.2

^aD. S. Hughes and A. J. Kelly, Second-Order Elastic Deformation of Solids, Phys. Rev. 92:1145 (1953).

Fig. 1 Longitudinal-longitudinal scattering cross section from the residual stress field associated with a spherical misfit region in Pyrex; (a) Contribution from inside the spherical region (b) outside and inside; as a function of scattering angle θ and ka .

Fig. 2 Longitudinal-longitudinal scattering cross section from the residual stress field associated with a spherical misfit region in Armco Iron; (a) Contribution from inside the spherical region (b) outside and inside; as a function of scattering angle θ and ka .

Fig. 3 Longitudinal-longitudinal scattering cross section from the residual stress field associated with a spherical mistit region in Polystyrene; (a) Contribution from inside the spherical region (b) outside and inside; as a function of scattering angle θ and ka .

We do hope that our interpretation can be used in various contexts, such as surface residual stress characterization, attenuation by randomly distributed regions of residual stress, etc. We hope to pursue these directions of research in future work. Also, we plan to study the scattering caused by the residual stress state associated with spherical and cylindrical misfit regions, using the Born as well as other approximation methods.

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