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FREE-FREE GAUNT FACTORS FOR GOLD PLASMAS AT
LASER-FLASER CONDITIONS

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I. INTRODUCTION

One of the dominant energy absorption processes which convert an incident laser beam into thermal energy is the "free-free absorption" of the laser radiation. Conversely, very high energy electrons (called super-thermals) in any laser target will emit high-energy Bremsstrahlung, and it is common practice to attempt to estimate the super-thermal population by examining the Bremsstrahlung spectrum. Clearly, it is important to have a precise understanding of the accuracy of any estimates of the free-free Gaunt factor which may be embedded in the codes that follow the physics of laser interactions.

At present, it is common practice either to treat the free-free absorption in the Born approximation, or to treat the ionic scattering potential as a Coulomb or a screened Coulomb potential. The Born approximation solution to free-free scattering in the screened Coulomb potential is trivial; the exact solution to the Coulomb problem has been known for some time (Ref. 1).

In a dense plasma, however, the scattering potential is not a Coulomb potential; rather it is a self-consistent atomic potential with some appropriate boundary condition. Furthermore, for many values of photon and electron energies, the Born approximation is grossly inadequate. For these reasons, we have attempted a precise theoretical computation of the free-free Gaunt factor in gold plasmas.

Fortunately, the solution to the problem in a general central potential (for given electron and photon energies) has been worked out elsewhere (Ref. 2). A code exists which permits the calculation of the required wave functions and matrix elements for such a potential. However, previously,

calculations have been made only for the screened Coulomb potential (Ref. 3).

To address the problem at hand, we model the ionic gold scattering centers by their Thomas-Fermi temperature-dependent potential (Ref. 4). We use the Brem IV code (Ref. 3) to calculate the free-free Gaunt factors in that potential. As discussed in the concluding section, this provides us with a clear-cut and precise answer for some special cases, and a general approximation algorithm whose precision can be estimated.

II. THE FORM OF THE CALCULATIONS

The theory and the structure of the Brem IV code have been discussed elsewhere (Ref. 3). However, for convenience we have reproduced the underlying equations.

The absorption cross section per unit electron density is

$$a = \frac{\langle z^2 g(k_i, \nu) \rangle 4\pi e^6}{3\sqrt{3} \hbar^2 c^2 \nu^3} \text{ cm}^3 \quad (1)$$

where

$$\langle z^2 g \rangle = \frac{\sqrt{3}}{2k_1 k_z} \sum_{l=1}^{\infty} l \left[M^2(k_1, l; k_z, l-1) + M^2(k_1, l-1; k_z, l) \right]. \quad (2)$$

Here

$$E_i = k_i^2 = \frac{mv^2 a_0}{e^2} \quad (3)$$

is the energy of the absorbing electron measured in Rydbergs, and

$$E_f = k_f^2 = k_i^2 + \omega \quad (4)$$

is the energy the electron has after absorbing a photon whose energy is ω Rydbergs.

The sum in Equation (2) is a sum over a partial wave expansion of angular momentum l . The M 's are matrix elements of free wave function $\psi_{l,k}(r)$ of angular momentum l , and

energy k^2 moving on a central potential $V(r)$. The wave functions satisfy the following equations, where r is in units of a_0 , the Bohr radius.

$$\psi_{l,k}''(r) = \left\{ \frac{l(l+1)}{r^2} - V(r) - k^2 \right\} \psi_{l,k}(r)$$

$$\lim_{r \rightarrow \infty} \psi_{l,k} = \sin\left(kr - \frac{l\pi}{2} + \delta_l\right). \quad (5)$$

They are the "exact" free wave functions.

The potential $V(r)$ is the scattering potential; in our case, $V(r)$ is taken to be the temperature-dependent Thomas-Fermi potential. Its calculation is described by R. Latter (Ref. 4). The potential $V(r)$ must satisfy the condition

$$\lim_{r \rightarrow 0} V(r) \sim \frac{2Z}{r} \text{ Rydbergs.} \quad (6)$$

The matrix elements are the acceleration matrix elements subject to the selection rule $\Delta l = \pm 1$.

$$M(k_i, l_i; k_f, l_f) = \int_0^\infty \psi_{l_i, k_i} \left(\frac{dV}{dr} \right) \psi_{l_f, k_f} dr. \quad (7)$$

The Brem IV code integrates Equation (5), forms M according to Equation (7), and finally forms the sum in Equation (2) and makes $\langle z^2 g \rangle$ available. The phase shifts δ_l are also obtained, but are not reported here. In principle, the number of partial waves required for convergence increases rapidly with k . However, a calculational device described elsewhere (Ref. 3) makes convergence at high energies a tractable problem.

For completeness, Brem IV also produces the $\langle z^2 g \rangle_B$, the value of $\langle z^2 g \rangle$ in the Born approximation. It is well known (Ref. 6) that

$$\langle z^2 g \rangle_B = \frac{\sqrt{3}}{4\pi} \left[\int_{k_i - k_F}^{k_F + k_i} k \, dk \left(\int_0^\infty r V(r) \sin kr \, dr \right)^2 \right]. \quad (8)$$

In the case of the Thomas-Fermi potential, the results quoted in the remainder of this report show large differences between $\langle z^2 g \rangle$ and $\langle z^2 g \rangle_B$ for many regions of interest for laser experiments.

III. CALCULATIONAL RESULTS

1. THE THOMAS-FERMI POTENTIALS

If we use the Thomas-Fermi model, each separate value of temperature and density for the gold plasma will yield a different potential $V(r)$. In order to examine $\langle z^2_g \rangle$, we chose plasma temperatures of $kT = 100, 250, 500$, and 1000 eV at densities $\rho = 0.01$ and 1.0 gm/cc, and computed $V(r)$ for each of these temperature-density states (Ref. 4).

If we choose to write the potential in the form

$$V(r) = Z^*/r \quad (9)$$

Z^* can be interpreted to be the "local effective charge" at any position r . According to Equation (6)

$$Z^*(0) = Z, \text{ the nuclear charge.} \quad (10)$$

In Figures 1 and 2 we exhibit $Z^*(r)$ for two typical temperature-density points. Distances are measured in units of Bohr radii. The precise calculational scheme for computing the Thomas-Fermi potential for any particular T, ρ point is discussed in detail elsewhere (Ref. 5).

2. THE ENERGY-DEPENDENT VALUES OF $\langle z^2_g \rangle$

The effective charge times the Gaunt factor $\langle z^2_g \rangle$ depends on E_i , the energy of the absorbing electron, as well as on ω , the photon energy, and $V(r)$, the scattering potential. In Figures 3 through 10, we exhibit the values of $\langle z^2_g \rangle$ as a function of E_i/kT . Separate sets of calculations are made for the different $V(r)$'s corresponding to gold plasmas at various ρ, T points of interest. The photon energy corresponds to the laser wavelengths of $\lambda = 1.06$ μm . These energy-dependent

Figure 1. Z^* for Gold

Figure 2. Z^* for Gold

Figure 3. Effective $\langle Z^2_g \rangle$ for Free-Free Absorption from Gold Targets

Figure 4. Effective $\langle Z^2_g \rangle$ for Free-Free Absorption from Gold Targets

Figure 5. Effective $\langle Z^2_g \rangle$ for Free-Free Absorption from Gold Targets

Figure 6. Effective $\langle Z^2 g \rangle$ for Free-Free Absorption from Gold Targets

Figure 7. Effective $\langle Z^2 \rangle$ for Free-Free Absorption from Gold Targets

Figure 8. Effective $\langle Z^2_g \rangle$ for Free-Free Absorption from Gold Targets

Figure 9. Effective $\langle Z^2_g \rangle$ for Free-Free Absorption from Gold Targets

Figure 10. Effective $\langle Z^2 g \rangle$ for Free-Free Absorption from Solid Targets

values of $\langle z^2_g \rangle$ can be averaged against any existing electron distribution.

For comparison we also exhibit $\langle z^2_g \rangle_B$, the Born approximation value of $\langle z^2_g \rangle$. This quantity is computed using Equation (8) and the corresponding Thomas-Fermi potential. At low energies, the Born approximation result is in error by as much as an order of magnitude. For the Thomas-Fermi potential, the Born approximation is not reliable for electron energies such that $E_i \leq Z^2$ Rydbergs.

In the Thomas-Fermi theory we can extract a value for N_F , the number of free electrons (Ref. 4). We do this by using the value of α , the degeneracy parameter, which emerges automatically whenever $V(r)$ is calculated for a given α, T point.

If the Gaunt factor were identically equal to unity, and if the Thomas-Fermi potential behaved as if it scattered like $2N_F/r$, then $\langle z^2_g \rangle$ would equal N_F^2 .

For comparison, we display the values of N_F^2 in Figures 3 through 10. It is interesting to observe that in the region $E_i \approx kT$, $\langle z^2_g \rangle \sim N_F^2$.

3. DISPLAY OF $\overline{\langle z^2_g \rangle}$, THE BOLTZMAN AVERAGE OF $\langle z^2_g \rangle$

We compute

$$\overline{\langle z^2_g \rangle} = \int \langle z^2_g \rangle e^{-E_i/kT} d(E_i/kT) \quad (11)$$

which is the proper Boltzman average of $\langle z^2_g \rangle$. An examination of Equation (1) shows that Equation (11) gives the proper weighting provided we say

$$\bar{a} = \frac{\langle z^2_g \rangle 4\pi e^6}{3\sqrt{3}hc m^{3/2} \nu^3 (2kT)^{1/2}} \left(\frac{2}{\nu} \right). \quad (12)$$

Figure 11 displays the quantity $\langle z^2_g \rangle$ as a function of the plasma temperature, kT . Calculations have been done for two densities. The results of Figure 11 can be used directly in Lasnex calculations to provide appropriate free-free cross sections for the absorption of laser radiation in a gold plasma.

Alternatively we may seek a general algorithm which might be generally applicable. In Table 1 we compare values of $\langle z^2_g \rangle$ with the corresponding values of N_F^2 . For $\nu = 0.01$, N_F^2 is an excellent approximation to $\langle z^2_g \rangle$. At $\nu = 1$, N_F^2 does well for $kT \geq 500$. At 250 eV $\langle z^2_g \rangle$ differs from N_F^2 by a factor of two; at 100 eV the factor is three. Certainly Table 1 gives a clear picture of the precision with which $\langle z^2_g \rangle$ may be replaced by N_F^2 .

4. THE BREMSSTRAHLUNG PRODUCTION

The calculation of $\langle z^2_g \rangle$ for Bremsstrahlung production presents a different picture than that described in Sections 2 and 3. The Bremsstrahlung radiation is produced by super-thermal electrons whose energies far exceed the average electron energy in the plasma. These very high energy electrons are capable of penetrating deep into the interior of the ion and sense an electric field which is much higher than that implied by ion of average charge N_F .

Table 2 shows the results of detailed calculation of the effective $\langle z^2_g \rangle$ arising from super-thermals whose energies are as large as 110 keV. We compare $\langle z^2_g \rangle$ with N_F^2 for gold plasmas at 100 eV and 1000 eV, respectively.

Figure 11. Boltzman Average, Effective $\langle Z^2 g \rangle$ for Free-Free Absorption from Gold Targets

TABLE 1. COMPARISON OF EXACT VALUES OF $\overline{\langle z^2_g \rangle}$ WITH N_F^2 FOR
FREE-FREE ABSORPTION FROM GOLD TARGETS

$\rho = 0.01$
gm/cc

kT (eV)	$\overline{\langle z^2_g \rangle}$	N_F^2
100	6.54×10^2	7.63×10^2
250	2.03×10^3	1.97×10^3
500	3.88×10^3	3.37×10^3
1000	6.43×10^3	4.76×10^3

$\rho = 1.0$
gm/cc

kT (eV)	$\overline{\langle z^2_g \rangle}$	N_F^2
100	1.57×10^2	4.19×10^2
250	6.27×10^2	1.16×10^3
500	1.49×10^3	2.26×10^3
1000	3.06×10^3	3.76×10^3

TABLE 2. EFFECTIVE CHARGE SQUARED FOR
BREMSSTRAHLUNG EMISSION

Density ρ (gm/cc)	Plasma Temp. kT (eV)	Electron Energy E_e (keV)	Photon Energy $h\nu$ (keV)	N_f^2	Effective Charge Squared $\langle z_{eq}^2 \rangle$
0.01	1000	35	25	4.8×10^3	6.41×10^3
0.01	1000	62	52	4.8×10^3	6.53×10^3
0.01	1000	110	100	4.8×10^3	6.63×10^3
0.01	1000	1.5	1	4.8×10^3	5.36×10^3
0.01	1000	3.5	3	4.8×10^3	5.71×10^3
0.01	100	35	25	7.6×10^2	6.21×10^3
0.01	100	110	100	7.6×10^2	6.72×10^3
0.01	100	1.5	1	7.6×10^2	2.14×10^3
0.01	100	3.5	3	7.6×10^2	3.48×10^3

It is clear that for super-thermals there is little connection between $\langle z^2 g \rangle$ and $N_{\pm}^2$. Furthermore, for the high energy spectrum ($h\nu \geq 25$ keV) the super-thermals penetrate so deeply into the ionic potential that one can, in effect, approximate $\langle z^2 g \rangle$ by $Z_0^2 = (79)^2 \approx 6.24^3$, the square of the nuclear charge.

IV. CONCLUSIONS

We have made careful and detailed calculations of the free-free absorption process for electrons traveling in the region of the temperature-dependent Thomas-Fermi potential of various gold plasmas. The Born approximations folded into this potential give a poor representation of the cross section; we can nevertheless make some very simple statements about algorithms for use in the calculational codes.

1. The Boltzman averaged value $\langle z^2 g \rangle$ used to compute the absorption of laser radiation can be replaced by N_F^2 , when N_F is the number of free electrons per atom present in the plasma.
2. The effective charge $\langle z^2 g \rangle$ used to compute Bremsstrahlung emission from super-thermals can be replaced by Z_0^2 , the square of the nuclear charge. For photons whose energy, $h\nu \geq 25$ keV, the error is less than 5 percent.
3. It is incorrect to replace $\langle z^2 g \rangle$ by N_F^2 in estimating Bremsstrahlung emission by super-thermals.

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