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MASTER

TRANSVERSE MOMENTUM DISTRIBUTION OF DIMUONS IN THE DRELL-YAN PROCESS

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ABSTRACT

We describe recent progress in the theory of the Q_T distribution in the Drell-Yan process for $0 \leq Q_T^2 \leq Q^2$. The basic theoretical result is in transverse position (b) space. In this talk we discuss in some detail how to get from b -space to Q_T -space.

INTRODUCTION

Drell and Yan¹ proposed in 1971 that the hadronic inclusive μ -pair production process $A+B \rightarrow \mu^+ + \mu^- + X$ would proceed predominantly at high energy through the annihilation of a quark from hadron A and an antiquark from hadron B. Recent theoretical and experimental work has tended to confirm this proposal. Thus the momentum Q^u of the dimuon is the sum of the momenta of the two annihilating partons. The distribution of dimuons in Q_T measures the distribution of the partons in transverse momentum.

In the original parton model days it was thought that the transverse momentum distribution would be steeply falling for Q_T bigger than 300 MeV or so. Later it was recognized² that in a renormalizable field theory like QCD the Q_T distribution would spread out and fall off slowly at very high energy, like $1/Q_T^2$ times logarithms of Q_T for $1 \text{ GeV}^2 \ll Q_T^2 \ll Q^2$. More recently Parisi and Petronzio³ argued on the basis of a soft gluon emission picture that

$$\frac{d\sigma}{d^2Q} \propto \frac{1}{Q_T^2} \frac{\ln(Q^2/Q_T^2)}{\ln(Q_T^2/\Lambda^2)} \exp \left[-\frac{4}{3} \frac{\alpha_s(Q^2)}{2\pi} \ln^2 \left(\frac{Q_T^2}{Q^2} \right) \right]$$

in an approximation in which possible corrections of order $\alpha_s(Q^2) \ln(Q_T^2/Q^2)$ are neglected.

We are presently completing a more detailed investigation of this process in which corrections of the form $\alpha_s(Q^2)^N \ln^M(Q_T^2/Q^2)$

are not neglected. Following Parisi and Petronzio, we work in transverse coordinate space. Let $H_T(b^2)$ be the fourier transform of the cross section and define $w = \ln(Q^2/\Lambda^2)$ and $z = \ln(4/b^2\Lambda^2)/\ln(Q^2/\Lambda^2)$. Then one is concerned with large w ; $z \approx 0$ is the "confinement region" about which we cannot say anything, $z \approx 1$ is the region in which ordinary renormalization group improved perturbation theory is useful, and $0 < z < 1$ is the more difficult two scale region that we are interested in. We find that

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$H_F(b^2)$ has the form

$$H_F(b^2) = \frac{1}{4\pi} e^{-wI(z)} R(w, z) \quad (1)$$

where

$$I(z) = \frac{16}{25} \int_z^1 d\bar{z} \frac{1-\bar{z}}{\bar{z}} \quad (2)$$

and $R(w, z)$ can be computed (except for its normalization, which is proportional to the quark x -distributed) in a perturbation scheme in which the error after N terms is of order w^{-N} (all terms of order $1/Q^2$ are neglected from the start). We find that $\ln R(w, z)$ grows only like $\ln w$ for large w .

There are certain unproved assumptions made in this calculation that are related to the cancellation of infrared divergences in QCD processes with colored particles in the initial state, which is a difficult unsolved problem. (See the talk of Frenkel at this conference.) The crossed version of the Drell-Yan process, $e^+ + e^- \rightarrow A + B + X$ with nearly back to back hadrons A and B , should be free of these possible difficulties.

In this talk we assume the validity of the form (1) for the cross section in transverse position space and show how to extract the leading form for the cross section in momentum space. The essential physical point was first made by Parisi and Petronzio: as $Q^2 \rightarrow \infty$, the function $H_F(b)$ falls off so fast for large b that the small b region, in which eq. (1) for $H_F(b)$ is reliable, dominates the cross section even at small Q_T .

CALCULATION

To calculate $H(Q_T)$ we use an intermediate step. Define the mellin transform

$$H_M(s) = \int_0^\infty \frac{db^2}{b^2} \left(\frac{\Lambda^2 b^2}{4} \right)^s H_F(b^2) \quad (3)$$

then

$$H(Q_T) = \frac{4\pi}{Q^2} \frac{1}{2\pi i} \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} ds \frac{\Gamma(1-s)}{\Gamma(s)} \left(\frac{Q^2}{\Lambda^2} \right) H_M(s). \quad (4)$$

There are two steps, $H_F \rightarrow H_M$ then $H_M \rightarrow H$.

In the first step we insert the form (1) for H_F into eq. (3):

$$H_M(s) = \frac{w}{4\pi} \int_{-\infty}^{\infty} dz e^{-w(zs+I(z))} R(w, z).$$

For $w \rightarrow \infty$ with fixed s , we can use the saddle point approximation. (Here it is essential that $R(w, z)$ grows only like $\ln w$ as $w \rightarrow \infty$). The saddle point is at $z_0(s) = A/(A+s)$, $A = 16/25$ in four flavor QCD. The result of the saddle point approximation is

$$H_M(s) \sim \sqrt{\frac{Aw}{8\pi}} \frac{1}{A+s} e^{-w(z_0 s + I(z_0))} R(w, z_0) \quad (5)$$

with computable $1/w$ corrections.

In the second step, $H_M \rightarrow H$, we insert (5) into (4). Using the variable $u = \ln(Q_T^2/\Lambda^2)/\ln(Q^2/\Lambda^2)$ one has

$$H(u) \sim \frac{\sqrt{2\pi A w}}{Q_T^2} \frac{1}{2\pi i} \int_{\frac{1}{2}+i\infty}^{\frac{1}{2}+i\infty} ds \frac{\Gamma(1-s)}{\Gamma(s)} \frac{1}{A+s} R(w, z_0(s)) \quad (6)$$

$$\times \exp \left\{ -w [(z_0(s)-u)s + I(z_0(s))] \right\}$$

There are three cases to consider. First, for $w \rightarrow \infty$ with $u > 16/41$ one can again use the saddle point approximation. The saddle point is at $s_0(u) = (16/25)(1-u)/u$; there $z_0(s_0(u)) = u$. The result is

$$H(u) \sim \frac{1}{Q_T^2} \frac{\Gamma\left(1 - \frac{16}{25} \frac{1-u}{u}\right)}{\Gamma\left(\frac{16}{25} \frac{1-u}{u}\right)} e^{-wI(u)} R(w, u) \quad (7)$$

In the second case, $u < 16/41$, the saddle point $s_0(u)$ lies to the right of the pole of $\Gamma(1-s)$ at $s = 1$. When the integration contour is moved past the pole so as to run through the saddle point, $H(u)$ picks up a contribution from the integral around the pole. This pole contribution dominates the saddle point contribution for large enough w .

Neither the $u > 16/41$ approximation, nor the $u < 16/41$ approximation is accurate when $u \approx 16/41$. (For practical values of w , $u \approx 16/41$ means $0 < u < 1$.) In this interpolating region one treats the pole of $\Gamma(1-s)$ exactly and obtains an error function. There is no one "best" interpolating approximation. One fairly simple approximation is

$$H(Q_T^2) \sim \frac{1}{Q_T^2} \sqrt{\frac{\pi A w}{2}} \frac{\Gamma(2-\hat{s})}{\Gamma(\hat{s})(A+\hat{s})} R(w, \hat{u}) \quad (8)$$

$$\times \exp \left\{ -w \left[I(\hat{u}) + \hat{u} - u + \frac{u^2}{2A} (\hat{s} - s_0)^2 \right] \right\} e^{\phi^2} \text{erfc}(\phi)$$

where

$$\phi = \frac{A+1}{\sqrt{2A}} \sqrt{w} \left(u - \frac{16}{41} \right)$$

$$\hat{u} = \max \left[u, \frac{16}{41} \right], \quad \hat{s} = \min(s_0, 1), \quad s_0 = \frac{16}{25} \frac{1-u}{u}$$

$$A = 16/25; \quad w = \ln(Q^2/\Lambda^2); \quad wu = \ln(Q_T^2/\Lambda^2)$$

DISCUSSION

Several features of the result (7) and (8) as u varies from 1 to $-\infty$ deserve discussion. For $u \rightarrow 1$, the factor $\Gamma[(16/25)(1-u)/u]$ in the denominator produces a factor $(1-u) \propto \ln(Q^2/Q_T^2)$, in agreement

with the first order perturbative result. For somewhat smaller u one can see the exponential of a double log:

$\exp(-wI(u)) \sim \exp[-\frac{8}{25} w(1-u)^2]$. As Q_T decreases toward 0, the effective inverse impact parameter $4/b_{\text{eff}}^2 = \Lambda^2(Q^2/\Lambda^2)^{\hat{u}}$ does not decrease past

$$(4/b_{\text{eff}}^2)_{\text{min}} = \Lambda^2(Q^2/\Lambda^2)^{16/41}.$$

Thus the cross section is determined by short distance effects even near $Q_T^2 = 0$. Finally both the shape and normalization of the cross section for small Q_T are very sensitive to the QCD scale parameter Λ^2 . For instance, $H(Q_T^2)/H(0) \sim \frac{1}{2}$ at $Q_T^2 = (Q^2)^{16/41} (\Lambda^2)^{25/41}$.

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