2
11-26-80
UCRL 15294

PO#8011009

RDA-TR-114000-004

MASTER

THE USE OF RADIATIVE LINE BROADENING BY AN
INTENSE LASER FIELD AS A PLASMA DIAGNOSTIC

AUGUST 1980

By:
L. SCHLESSINGER
J. WRIGHT

Sponsored By:
LAWRENCE LIVERMORE LABORATORY
Post Office Box 808
Livermore, California 94550

R & D ASSOCIATES
Post Office Box 9695
Marina del Rey,
California 90291

4640 ADMIRALTY WAY • MARINA DEL REY • TELEPHONE: (213) 822-1715

RDA-TR-114000-004

THE USE OF RADIATIVE LINE BROADENING BY AN
INTENSE LASER FIELD AS A PLASMA DIAGNOSTIC

AUGUST 1980

By:
L. SCHLESSINGER
J. WRIGHT

Sponsored By:
LAWRENCE LIVERMORE LABORATORY
Post Office Box 808
Livermore, California 94550

R & D ASSOCIATES
Post Office Box 9695
Marina del Rey,
California 90291

4640 ADMIRALTY WAY • MARINA DEL REY • TELEPHONE: (213) 822-1715

EB
DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

THE USE OF RADIATIVE LINE BROADENING BY AN INTENSE LASER FIELD AS A PLASMA DIAGNOSTIC

In many laser fusion applications it would be of significant interest to have an independent measurement of the strength of the laser electric field at various positions in the plasma. In this report we show that in many cases of interest the line broadening caused by an intense laser field can provide such a measurement. For low- Z elements at moderate temperatures and for typical laser intensities and densities, the line broadening caused by the intense field is the dominant source of line width. In this situation the line width depends on the strength of the laser field at that position and the line profile is quite unique, having a characteristic and calculable dependence on the angle between the laser electric field and the direction of the radiated photon. These special properties should allow the lines of low- Z elements seeded at various positions in the plasma to be identified, their widths determined, and the value of the laser electric field at that position derived.

We consider the case where the plasma is seeded with a low- Z element that is almost totally stripped at the temperatures of interest. In this case the dominant transitions would be those from the $2P$ to the $1S$ state of a hydrogenic atom of charge Z . The calculation could be performed for any states and while the angular distribution and the magnitude of the effect would be different, the principle would be the same. However, when the atom is not nearly stripped and representable by hydrogenic wave functions, the calculation of the transition energies and widths becomes more difficult and other sources of line broadening must be included. Thus, we limit this discussion to transitions from the $2P$ to the $1S$ state of a hydrogenic atom.

In Reference 1 we have shown that for the transition described in the field of an intense laser with maximum electric field strength E , the line width Δ caused by the intense laser field is the Stark width in that field:

$$\Delta = \frac{3eEa_0}{Z} \quad (1)$$

where a_0 is the Bohr radius. The electric field of the laser only broadens the lines coming from the $\ell=1, m=0$ to $\ell=0$ transitions, but does not broaden the lines coming from the $\ell=1, m=\pm 1$ to $\ell=0$ transitions. Since the photons coming from these transitions have different angular dependence with respect to the direction of the laser field, the line shape is a function of the angle between the emitted photon direction and the laser electric field. For a traveling wave with group velocity v_g , the laser electric field is related to the laser intensity, I , by

$$E(V/\text{\AA}) \approx 15 \left(\frac{I(\text{w/cm}^2)}{3 \times 10^{15}} \frac{c}{v_g} \right)^{1/2}. \quad (2)$$

Thus, the line width caused by the intense field is

$$\Delta \approx \frac{24}{Z} \left(\frac{I}{3 \times 10^{15}} \frac{c}{v_g} \right)^{1/2}. \quad (3)$$

The Doppler width for a line of energy

$$\hbar\omega_0 = E_{1S} - E_{2P} \approx 10Z^2(\text{eV}) \quad (4)$$

in a plasma of temperature, T , for ions of mass, M , is

$$\Delta(\hbar\omega) \approx \hbar\omega_0 \left(\frac{2kT}{Mc^2} \right)^{1/2} \approx 0.01Z^{3/2} \sqrt{T(\text{keV})}. \quad (5)$$

The pressure width for the same situation is (Ref. 2)

$$\Delta(\hbar\omega) \approx 3 \times 10^{-22} \frac{N_e}{Z^2 \sqrt{T(\text{keV})}} \quad (6)$$

where N_e is the electron density. Thus, for temperatures, densities and laser intensities of interest, the width caused by the laser field is about 5 to 10 times larger than the width caused by Doppler broadening and the pressure width is negligible.

The expression derived below for the line profile for the 2P to 1S transition in a hydrogenic atom including the effects of the laser field and the Doppler broadening is

$$\frac{d^2 I}{d\omega d\Omega}(\theta, \xi, \varepsilon, \omega) = G \left[(1 + \cos^2 \theta) F_1(\varepsilon/\xi) + \sin^2 \theta F_2(\xi, \varepsilon) \right]. \quad (7)$$

Here $\frac{d^2 I}{d\omega d\Omega}$ is the rate of photon emission per frequency interval in the direction $\hat{\Omega}$ in the interval $d\Omega$; $GF_1(\varepsilon/\xi)$ (given below) is the rate when the laser field is zero, and θ is the angle between the laser electric field and the direction of propagation of the radiated photons. The function F_1 is the Doppler broadening function

$$F_1(\varepsilon/\xi) = \exp\left(-\varepsilon^2/\xi^2\right). \quad (8)$$

F_2 is the function that describes the combination of Doppler broadening and the broadening caused by the laser field

$$F_2(\xi, \varepsilon) = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\phi \exp \left[-(\varepsilon + \cos \phi)^2 / \xi^2 \right] \quad (9)$$

where

$$\varepsilon = \frac{\hbar\omega - \hbar\omega_0}{\Delta}$$

$$\xi^2 = \frac{2kT}{Mc^2} \left(\frac{\hbar\omega}{\Delta} \right)^2. \quad (10)$$

The profiles F_1 and F_2 for aluminum at $T=1$ keV ($\xi=0.25$) are plotted in Figure 1, and the profiles F_1 and F_2 for argon at $T=1$ keV ($\xi=0.58$) are plotted in Figure 2. The same quantities are plotted in Figure 3 for neon at $T=0.5$ keV, all for an intensity $I=3 \times 10^{15}$ W/cm² and $v_g=c$. Some properties of F_2 can be obtained by using the method of steepest descent. The maximum value of F_2 occurs at $\varepsilon \approx 1$ and is

$$(F_2)_{\max} \sim 0.4\sqrt{\xi}. \quad (11)$$

The minimum value of F_2 in the line center occurs at $\varepsilon=0$ and is

$$(F_2)_{\min} \sim \xi/\sqrt{\pi}. \quad (12)$$

For the case when $\varepsilon > 1$, we have

$$F_2(\xi, \varepsilon) \sim \frac{\xi}{2\sqrt{\pi}} \frac{\exp(-(|\varepsilon| - 1)^2 / \xi^2)}{(|\varepsilon| - 1)^{1/2}}. \quad (13)$$

For the case when $|\varepsilon| < 1$, we have

Figure 1. Profile functions F_1 and F_2 vs ϵ for aluminum at $T=1$ keV, $I=3 \times 10^{15}$ W/cm², $\xi=0.25$.

Figure 2. Profile functions F_1 and F_2 vs ϵ for argon
at $T = 1$ keV, $I = 3 \times 10^{15}$ W/cm², $\xi = 0.58$.

Figure 3. Profile functions F_1 and F_2 vs ϵ for neon
at $T = 0.5$ keV, $I = 3 \times 10^{15}$ W/cm², $\xi = 0.09$.

$$F_2(\xi, \epsilon) \sim \frac{\xi}{\sqrt{\pi} (1 - \epsilon^2)^{1/2}}. \quad (14)$$

For $\epsilon \approx 1$, we have

$$F_2(\xi, \epsilon) \sim 0.4\sqrt{\xi}. \quad (15)$$

It would seem that the distinctive shape and strong angular dependence of the line profile would allow identification of the line and determination of the laser electric field.

The derivation of these results makes use of the techniques developed in Reference 1 in which we found the expression for the transition rate for bound-bound processes in an intense laser field. There we showed that first-order perturbation theory could be used to calculate the bound state wave functions in the intense laser field and that a static approximation for the laser field is adequate for the states and intensities of interest. The derivation of the result in Eq. (7) proceeds by a generalization of the results from Reference 1 to include the ion motion, the summing of the radiation rate over the photon polarization and over the states available for the transition, and the folding of the transition rate per ion into a Maxwellian distribution of ion velocities.

Following Reference 1, the rate of bound-bound emission per unit volume per frequency interval in the direction $\vec{\Omega}$ in the interval $d\Omega$ for the transition from the state 1 to the state 2 if the ions have a momentum distribution $f(p)$ is given by

$$\frac{d^2 I}{d\omega d\Omega} = \frac{n_1 e^2}{2\pi} \left(\frac{\omega}{c}\right)^3 \int_{\phi=-\pi}^{\phi=\pi} \frac{d\phi}{d\pi} \int d^3 p_1 d^3 p_2 |T_{12}|^2 f(p_1) \cdot \delta(\vec{p}_1 - \vec{p}_2 - \vec{k}) \delta\left(\bar{\epsilon}_1 + \hbar\omega - \bar{\epsilon}_2 + \frac{p_1^2 - p_2^2}{2M}\right) \quad (16)$$

where $\vec{p}_1, \vec{p}_2$ are the initial and final momenta of the ions, $\vec{k}$ is the photon momentum, $\hbar\omega$ is the photon energy, and $\bar{\epsilon}_1$ and $\bar{\epsilon}_2$ are the energies of the states 1 and 2 in the laser field and include the Stark shift. That is,

$$\bar{\epsilon} = \epsilon + \Delta \cos \phi$$

where ϵ is the unperturbed energy of the bound state. Δ is the "first-order Stark shift in the laser field and is given by

$$\Delta = \int d^3 r \psi^*(r) \hat{H}' \psi(r) \quad (17)$$

where the perturbation Hamiltonian in the laser field is given by

$$H' = \hat{H}' \cos(\omega_L t + \phi) \quad (18)$$

where

$$\hat{H}' = eE\vec{c} \cdot \vec{r} \quad (19)$$

and E is the magnitude of the laser electric field, $\vec{\epsilon}$ is the direction of the laser electric field, and ω_0 is the laser frequency. Because we use the static approximation for the laser field ($\omega_0 \rightarrow 0$), the result in Eq. (16) is averaged over the unobservable phase, ϕ , of the laser field. In Eq. (16), T_{12} is given by

$$T_{12} = \vec{V} \cdot \vec{R} \quad (20)$$

where $\vec{V}$ is the polarization of the emitted photon and

$$\vec{R} = \int d^3r \psi_1^*(r) \vec{r} \psi_2(r). \quad (21)$$

Summing $|T_{12}|^2$ over photon polarizations yields

$$|\bar{T}_{12}|^2 = \vec{R} \cdot \vec{R} - \left(\frac{\vec{R} \cdot \vec{R}}{K} \right)^2. \quad (22)$$

To proceed further we must specify the wave functions in detail. In the example we chose ψ_1 to be the 1S hydrogenic state and considered transitions from the 2P states. The 1S state is unaffected by the perturbation of the laser field. Initially, the four 2P states are degenerate with energy ϵ_2 . The $n=2$, $\ell=1$, $m=\pm 1$ states are also unaffected by the intense laser field while the $n=2$, $\ell=1$, $m=0$ state is mixed with the $n=2$, $\ell=0$, $m=0$ states. The states that diagonalize the perturbation and their energies are

$$\psi_{l=1, m=+1}, \mathcal{E} = \mathcal{E}_2$$

$$\frac{1}{\sqrt{2}} \left(\psi_{l=1, m=0} \pm \psi_{l=0, m=0} \right), \mathcal{E} = \mathcal{E}_2 \pm \Delta. \quad (23)$$

Only the $l=1$ component of the states contributes to the matrix elements in Eq. (21).

Averaging Eq. (16) over the possible initial states (i.e., over all the 2P states) yields

$$\begin{aligned} \frac{d^2 I}{d\omega d\Omega} = & \frac{n_i e^2}{18\pi} \left(\frac{\omega}{c} \right)^3 |t_{12}|^2 \int_{-\pi}^{\pi} \frac{d\phi}{2\pi} \int d^3 p_1 d^3 p_2 \delta(\vec{p}_1 - \vec{p}_2 - \vec{k}) f(p_1) \\ & \cdot \left\{ \delta(\rho) (1 + \cos^2 \theta) + \delta(\rho - \Delta \cos \phi) \sin^2 \theta \right\} \end{aligned} \quad (24)$$

where

$$\rho = \mathcal{E}_1 - \mathcal{E}_2 + \hbar\omega + \frac{p_1^2 - p_2^2}{2M} \quad (25)$$

and

$$t_{12} = \int r^3 dr L_{n=1, l=0}^{(r)} L_{n=2, l=1}^{(r)} \quad (26)$$

and $L_{n, l}$ is the radial portion of the hydrogenic wave function.

Integrating Eq. (24) over P_1 and P_2 yields

$$\frac{d^2 I}{d\omega d\Omega} = \frac{n_i e^2}{18\pi} \left(\frac{\omega}{c}\right)^3 \frac{|t_{12}|^2}{\hbar\omega} \left(\frac{Mc^2}{2\pi kT}\right)^{1/2} \left\{ \frac{(1 + \cos^2 \theta)}{2\pi} \int_{-\pi}^{\pi} d\phi \exp\left(-\frac{P_+^2}{2MkT}\right) + \frac{\sin^2 \theta}{2\pi} \int_{-\pi}^{\pi} d\phi \exp\left(\frac{-P_0^2}{2MkT}\right) \right\} \quad (27)$$

where

$$P_+ = (\mathcal{E}_1 - \mathcal{E}_2 + \hbar\omega) \frac{M}{K} \quad (28)$$

and

$$P_0 = (\mathcal{E}_1 - \mathcal{E}_2 + \hbar\omega - \Delta \cos \phi) \frac{M}{K}.$$

Eq. (6) follows from Eq. (25) with the identification

$$G = \frac{n_i e^2}{18\pi} \left(\frac{\omega}{c}\right)^3 \frac{|t_{12}|^2}{\hbar\omega} \left(\frac{Mc^2}{2kT}\right)^{1/2} \quad (29)$$

$$F_2(\xi, \epsilon) = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\phi \exp\left[-(\epsilon + \cos \phi)^2 / \xi^2\right]. \quad (30)$$

The forms obtained for $F_2(\xi, \epsilon)$ given in Eq. (14) are obtained from Eq. (30) by steepest descent integration.

For the purpose of evaluating the rate it is convenient to write the equation (28) in a different form. We can write

$$\frac{d^2 I}{d\omega d\Omega} = n_i \Gamma_0 \left[\left(\frac{Mc^2}{2\pi kT} \right)^{\frac{1}{2}} \frac{1}{\omega} \cdot \left\{ (1 + \cos^2 \theta) F_1(\xi, \epsilon) + \sin^2 \theta F_2(\xi, \epsilon) \right\} \right] \quad (31)$$

where

$$\Gamma_0 = \frac{e^2}{8\pi\hbar} |t_{12}|^2 \left(\frac{\omega}{c} \right)^3 = 2.5 \times 10^7 \text{ Z}^4 \text{ sec}^{-1} \quad (32)$$

the quantity in brackets, when integrated over ω has value approximately equal to 1.

REFERENCES

1. Schlessinger, L., and Wright, J., *Free-Bound and Bound-Bound Radiative Transitions in an Intense Laser Field*, R & D Associates, RDA-TR-114000-002, July 1980; also submitted to *Physical Review A*.
2. Stewart, J. C., and Pyatt, K. D., Jr., *Theoretical Study of Optical Properties: Photon Absorption Coefficients, Opacities, and Equations of State of Light Elements, Including the Effect of Lines, Vol. I*, General Dynamics Corporation, GA-2528, September 1961.