Beam Tilt Due to e^+e^- Crossing at an Angle

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The problem of beam tilt caused by beam-beam crossing at an angle has been treated by B. Richter and D. Ritson.¹ An electron at a longitudinal position z relative to the center of the bunch experiences on the average a transverse kick proportional to z from the on-coming bunch. This kick produces a closed orbit for the electron being considered. The closed orbit is different for electrons with different values of z . Consequently, the beam will become tilted relative to its direction of motion.

On the other hand, there is a longitudinal kick proportional to z of the particle due to both the rf focussing and the beams crossing at an angle.² This longitudinal kick will cause the particle's energy to change and the energy change will, in turn, change the particle's z in the next revolution. It is, therefore, clear that a complete treatment of this problem should take the synchrotron oscillation, as well as the transverse betatron oscillation, into account. We will assume in a later calculation that the longitudinal defocussing effect due to beam-beam crossing at an angle is dominated by the focussing effect provided by the rf cavities.

The present note is to re-do Ref. 1 with the synchrotron dimension included. The result shows the structure of a synchrobetatron resonance.³ The difference between this calculation and that of Ref. 1 is expected to be small away from the synchrobetatron coupling resonances since the synchrotron oscillation is indeed slow compared with the revolution frequency. Numerical estimate shows that this beam tilt is negligible away from synchrobetatron resonances. Only linear terms are kept in this calculation. Non-linear effects are ignored.

Consider a vertical crossing of two beams with crossing angle 2α . Let the beams be tilted by θ_1 and θ_2 as shown below

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For two equal beams, $\theta_1 = \theta_2$. For unequal beams, the strong beam will not tilt; only the weak beam tilts.

Consider a single particle in beam 1. Let the coordinates of this particle be (y, z) relative to the center of beam 1, with $\hat{z}$ along the direction of beam motion:

Let the particle density distribution of beam 2 be that of a ribbon beam:

$$\lambda(u, v) = \frac{N}{2\pi \sigma_u \sigma_v L_x} \exp \left(-\frac{u^2}{2\sigma_u^2} - \frac{v^2}{2\sigma_v^2} \right) \quad (1)$$

where L_x is the bunch width and u, v are coordinates relative to the center of bunch 2, but with $\hat{v}$ along the direction of bunch orientation:

It is straightforward to obtain the impulse that a particle with (y, z) in bunch 1 receives from one passage through the bunch 2. In the relativistic limit, it is given by:

$$\Delta \vec{p} = - \frac{\sqrt{8\pi} N e^2}{c L_x \sum \cos \alpha} \hat{n} \int_0^y \cos \alpha - z \sin \alpha \, dx \exp \left(- \frac{x^2}{2 \Sigma^2} \right) \quad (2)$$

where

$$\Sigma^2 = \sigma_v^2 \sin^2 (\theta_2 + \alpha) + \sigma_u^2 \cos^2 (\theta_2 + \alpha)$$

and $\hat{n}$ is the unit vector shown below:

In the linear approximation, let

$$|y \cos \alpha - z \sin \alpha| \ll \Sigma,$$

we obtain

$$\Delta \vec{p} = - \frac{\sqrt{8\pi} N e^2}{c L_x \Sigma} \hat{n} (y - z \tan \alpha). \quad (3)$$

This impulse can be decomposed into a longitudinal component and a transverse component.^{2,3} The longitudinal component causes the energy of the particle to change by an amount

$$\Delta \delta = \frac{\Delta p_z}{p_0} = G \sin \alpha (y - z \tan \alpha), \quad (4)$$

where

$$G = \frac{\sqrt{8\pi} N r_e}{\gamma L_x \Sigma}. \quad (5)$$

The transverse component of the impulse gives a vertical kick to the particle

$$\Delta y' = \frac{\Delta p_y}{p_0} = - G \cos \alpha (y - z \tan \alpha), \quad (6)$$

In the absence of a crossing angle, G is related to the beam-beam tune shift according to

$$\Delta \nu = \frac{\beta_0 G}{4 \pi}. \quad (7)$$

If we define the state vector by

$$\begin{bmatrix} y \\ y' \\ z \\ \delta \end{bmatrix}, \quad \delta = \frac{\Delta E}{E},$$

the transformation matrix for the beam-beam crossing can be written:

$$T_{\alpha} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -G \cos \alpha & 1 & G \sin \alpha & 0 \\ 0 & 0 & 1 & 0 \\ G \sin \alpha & 0 & -G \tan \alpha \sin \alpha & 1 \end{bmatrix}. \quad (8)$$

To proceed, we assume that there are two crossing points in the storage ring. Let $T_{1/2}$ be the transformation from one crossing point to another. In the absence of longitudinal-transverse coupling aside from the beam-beam effect, $T_{1/2}$ can be written as

$$T_{1/2} = \begin{bmatrix} \cos \pi \nu_y & B_0 \sin \pi \nu_y & 0 & 0 \\ -\frac{1}{B_0} \sin \pi \nu_y & \cos \pi \nu_y & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & c & d \end{bmatrix}, \quad (9)$$

where $ad-bc = 1$ and $a+d = 2 \cos \pi \nu_s$. The total transformation around one revolution is then given by

$$T_0 = T_{1/2} T_{\pm \alpha} T_{1/2} T_{\alpha} \quad (10)$$

If the two crossings have the same polarity, i.e., if the two crossing angles have the same sign, the upper (plus) sign is used in Eq.(10). If the two crossings have opposite polarities, the lower (minus) sign is used in Eq.(10).

The transformation T_0 , which describes the motion of a single particle in beam 1, has two eigenmodes. The eigenmode axes are tilted relative to the y-z axes. The orientation of beam 1 must be such that the principle axes of the beam distribution coincide with these eigenmode axes. Knowing T_0 , we can thus obtain θ_1 by looking for the eigenmode axes of the transformation T_0 .

Assuming for simplicity in algebra that the beam-beam interaction is weak, i.e.,

$$|\beta_0 G| \ll 1, \quad (11)$$

and keeping terms only up to first order in G, it can be shown that

$$\begin{bmatrix} -\frac{1}{2} \beta_0 G \sin \alpha \sin \pi \nu_y / (\cos \pi \nu_y \mp \cos \pi \nu_s) \\ -\frac{1}{2} G \sin \alpha (\cos \pi \nu_y \mp e^{-i\pi \nu_s}) / (\cos \pi \nu_y \mp \cos \pi \nu_s) \\ 1 \\ e^{i\pi \nu_s} - \frac{a}{b} + \frac{i}{2} G \tan \alpha \sin \alpha e^{-i\pi \nu_s} / \sin \pi \nu_s \end{bmatrix} \quad (12)$$

is an eigenvector of T_0 . It follows that the tilting angle of beam 1 is given by

$$\theta_1 \approx \tan \theta_1 = \frac{G \beta_0 \sin \alpha \sin \pi \nu_y}{2(\cos \pi \nu_y \mp \cos \pi \nu_s)}. \quad (13)$$

The upper (minus) sign is used in Eq.(13) if the two crossings have the same polarity, while the lower (plus) sign is used if the two crossings have opposite polarities.

In practice, this effect of beam tilt is rather weak away from synchro-betatron resonances. To see this, we note that in order to avoid any appreciable loss of luminosity due to this beam tilt, one must have

$$\theta_1 \lesssim \frac{\sigma_u}{\sigma_v} . \quad (14)$$

This can only be fulfilled provided that the tunes satisfy the condition

$$|\nu_y \mp \nu_s - k| \geq \delta\nu , \quad (15)$$

where k is some integer and $\delta\nu$ is the resonance width⁴ given by

$$\delta\nu = \frac{\sigma_v}{\sigma_u} \frac{\beta_0 G \sin \alpha}{2\pi} \quad (16)$$

For example, if

$$\sigma_v / \sigma_u = 600$$

$$\beta_0 G = 0.6$$

$$\alpha = 0.1 \text{ mr},$$

we obtain a resonance width of $\delta\nu = 6 \times 10^{-3}$.

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2. J.E. Augustin, Note Interne 35-69 (1969).
3. A. Piwinski, DESY 77/18 (1977).
4. This resonance width is not for the beam stability as calculated in Ref.3.

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