Weak Turbulence Theory of Collisionless Trapped Electron Driven Drift Instability in Tokamaks

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Abstract

The toroidal collisionless trapped electron modes are analyzed in the weak turbulence regime treating both ions and trapped electrons nonlinearly in the presence of ion and electron temperature gradients. The spectral intensity of the density fluctuations in the nonlinearly saturated state is analytically obtained from the steady state solution of the wave-kinetic equation. Distant nonlinear interactions between low- k_θ and high- k_θ modes of similar frequencies via trapped electron scattering (the resonance between the beat wave and the trapped electron precession drift frequencies) suppress the low- k_θ [$k_\theta \rho_s \ll (L_n/R)^{1/2}$] modes while close interactions via ion Compton scattering (nonlinear ion Landau damping) produce a monotonically decreasing spectrum from $k_\theta \rho_s \equiv (L_n/R)^{1/2}$ to $k_\theta \rho_s \equiv 1$ according to an approximate power law k_θ^{-3} . Various fluctuation amplitudes at saturation and the fluctuation-induced anomalous particle and heat fluxes are found to be smaller than the mixing length estimates. The plasma confinement is predicted to improve with higher T_i/T_e , more peaked density profile, larger aspect ratio, and higher plasma current. Also, a significant dependence of transport on the electron temperature gradient is found which could contribute to the rigidity of the electron temperature profile often experimentally observed.

I. Introduction

It is generally believed that low frequency microinstability-driven electrostatic turbulence plays an important role in determining the plasma transport in the interior region of tokamaks.^{1,2} Indeed, transport models³⁻⁵ based on rather simple trapped electron drift wave turbulence pictures have performed relatively well in reproducing a number of tokamak confinement trends. For most of the present day large tokamak auxiliary heated plasmas, the collisionless trapped electron mode (CTEM) with $\omega_{be} > \omega$, $\omega_{*e} > \omega_{de} > v_{eff}$ rather than the dissipative trapped electron mode (DTEM) with $\omega_{be} > v_{eff}$, ω , $\omega_{*e} > \omega_{de}$ appears to be more relevant.^{6,7} Here, ω is the mode frequency, $\omega_{*e} = C_s \rho_s k_\theta / L_n$ is the electron diamagnetic drift frequency. $\omega_{te} = v_{Te} / qR$ and $\omega_{be} = \epsilon^{1/2} v_{Te} / qR$ are the electron transit and bounce frequencies, and ω_{de} is the orbit-averaged trapped electron precession drift frequency.

The CTEM, which is destabilized by the trapped electron precession drift resonance has been analyzed extensively in the linear regime.⁸⁻¹⁰ The spatial structure of the most unstable eigenmode has been found to be localized in the bad curvature side of the tokamak with the parallel wavelength characterized by the connection length.¹¹ This is true for a wide range of the poloidal wavelengths.¹² Since the mode is dispersive in the linearly unstable range, a number of weak turbulence approaches have been attempted based upon the wave-particle-wave nonlinear interactions. Most of these studies have been carried out within the context of local theory¹³ or in a slab geometry.¹⁴⁻¹⁵

Chen et al.,¹³ have considered both the ion nonlinearity and the trapped electron nonlinearity in a slightly different collisionality regime; i.e., weakly dissipative trapped electron (WDTE) regime where $\omega_{be} > \omega$, $\omega_{*e} > v_{eff} > \omega_{de}$. The dominant nonlinearity in this case has been found to be the trapped electron nonlinearity which transfers the fluctuation energy to shorter wavelengths. Recently, Gang et al.,¹⁵ have developed a weak turbulence theory of CTEM in a sheared slab geometry in the absence of temperature gradients assuming the Pearlstein-Berk-type eigenmode structure.¹⁶ They found that the trapped electron nonlinearity

becomes weaker and that the ion Compton scattering, which transfers fluctuation energy to longer wavelengths, is the dominant nonlinear mechanism. The main reason for the different conclusions of these two previous studies is the difference in the eigenmode structure incorporated in the analyses; i.e., a local theory¹³ vs. Pearlstein-Berk-type eigenmode structure¹⁵ in a sheared slab geometry. The ion Compton scattering has also been found to be the dominant nonlinear saturation mechanism for strongly dissipative trapped electron mode (SDTEM) with $\omega_{be} > v_{eff} > \omega$, $\omega_{*e} > \omega_{de}$ for sheared slab¹⁴ and toroidal geometry¹⁷ where the trapped electron dynamics can be treated linearly.

In this work, we develop a weak turbulence theory of CTEM in toroidal geometry in the presence of both ion and electron temperature gradients. We consider a simple model representation of the toroidal eigenmode structure which is found to be the most unstable form over a wide range of poloidal wavelengths. Ion Compton scattering and the trapped electron scattering are treated on an equal footing. The spectral intensity of the density fluctuation in the nonlinearly saturated state is analytically obtained from the steady state solution of the wave-kinetic equation. It is found that both nonlinearities are important in determining the spectral shape of the saturated fluctuations. Distant nonlinear interactions between low- k_θ and high- k_θ modes of similar frequencies via trapped electron scattering (the resonance between the beat wave and the trapped electron precession drift frequencies) suppress the low- k_θ [$k_\theta \rho_s \ll (L_n/R)^{1/2}$] modes while close interactions via ion Compton scattering (nonlinear ion Landau damping) produce a monotonically decreasing spectrum from $k_\theta \rho_s \equiv (L_n/R)^{1/2}$ to $k_\theta \rho_s \equiv 1$ according to an approximate power law k_θ^{-3} . Fluctuation amplitude at saturation and the fluctuation-induced anomalous particle and heat fluxes are then obtained. They are found to be smaller than the mixing length estimates. The plasma confinement is predicted to improve with higher T_i/T_e , more peaked density profile, larger aspect ratio, and higher plasma current. These trends are in agreement with recent experimental observations from TFTR transport studies.⁶ Also, a significant dependence of transport on the electron temperature gradient is found. This

might be relevant to the rigidity of the electron temperature profiles¹⁸ often experimentally observed.¹⁹

The remainder of this paper is organized as follows. In Sec. II, the nonlinear wave-kinetic equation, which describes the evolution of the fluctuation spectral intensity, is derived from the nonlinear ion gyrokinetic equation and the bounce-averaged nonlinear trapped-electron drift kinetic equation. Here, the respective roles of ion Compton scattering and trapped-electron scattering are also discussed. The spectral intensity at saturation is obtained as a steady-state solution of the wave-kinetic equation in Sec. III, and the fluctuation-induced fluxes as well as the anomalous electron-ion heat exchange rate are calculated in Sec. IV. Finally, the implications of the principal results from the present studies for the tokamak confinement are discussed in Sec. V.

II. Derivation of Nonlinear Wave-Kinetic Equation

In this section, we derive a wave-kinetic equation which describes the nonlinear evolution of the fluctuation spectral intensity. Before presenting the derivation, we briefly discuss the linear properties of collisionless trapped electron modes (CTEM) in toroidal geometry. Since the focus of this paper is on nonlinear theory, we adopt a simple model which captures the essential features of these instabilities. Our approach is to incorporate the relevant results from previous linear theories in a simple context in order to allow a systematic and tractable development of a weak turbulence nonlinear analysis for the CTEM.

The theoretical model considered in this paper consists of the Boltzmann response for the untrapped electrons and the collisionless bounce-averaged drift-kinetic equation for trapped electrons. Ions are treated by the collisionless gyrokinetic equation. The relevant frequencies in the linear analysis are ordered as $\omega_{te} > \omega_{be} > \omega$, $\omega_{*e} > \omega_{de} > \nu_{eff}$, and ω , $\omega_{*e} > \omega_{di}, \omega_{ti}$.

Here, $\omega_{ti} = v_{Ti}/qR$ is the ion transit frequency, the ion magnetic drift frequency is ω_{di} , and the orbit-averaged trapped electron precession drift frequency is $\omega_{de} = \omega_{*e} (L_n E / R T_e) G(S, \kappa)$, where $S = r d \ln q / dr$ is the magnetic shear parameter, and κ is the pitch angle variable related to the azimuthal angle θ_0 of the turning point of a trapped particle by $\kappa = \sin \theta_0$. For a concentric high aspect ratio model equilibrium, $G(S, \kappa) = 2E/K - 1 + 4S(E/K + \kappa^2 - 1)$,²⁰ where $E = E(\kappa)$ and $K = K(\kappa)$ are the Elliptic functions. Note that for higher q (which leads naturally to higher shear) G increases. This trend is also true for more realistic toroidal equilibria.²¹ Since $\omega > \omega_{bi}$ (the trapped ion bounce frequency), trapped ion dynamics are ignored. By taking appropriate perturbed density moments for each species in the linear regime, the quasi-neutrality condition yields the following eigenmode equation,

$$(1 + \tau) \Phi_{\mathbf{k}} + \tau \int d^3 v (i L_{i\mathbf{k}}) [\omega - \omega_{*i} (1 + (u^2 - 3)\eta_i/2)] F_{mJ_0}^2 \Phi_{\mathbf{k}} + \int d^3 v (i L_{e\mathbf{k}}) [\omega - \omega_{*e} (1 + (E/T_e - 3/2)\eta_e)] F_{mJ_0} \Phi_{\mathbf{k}} = 0, \quad (1)$$

where $\Phi_{\mathbf{k}} \equiv e \phi_{\mathbf{k}} / T_e$, and $L_{e\mathbf{k}} = i(\omega - \omega_{de} + i0^+)^{-1}$ and $L_{i\mathbf{k}} = i(\omega - \omega_{di} - k_{\parallel} v_{ti} + i0^+)^{-1}$ are the linear test particle propagators in a toroidal geometry with magnetic shear. Although we have adopted for simplicity notations which are familiar from the local theory, it should be remembered that some of these quantities are really operators. In general, the solution to Eq. (1) requires numerical analysis and there have been numerous papers on this subject.⁸⁻¹⁰ The following results are of particular importance and interest for our purpose of developing a weak turbulence theory of collisionless trapped electron modes in a toroidal geometry.

i) A local dispersion relation has been derived for CTEM driven unstable by wave-trapped-electron-precession resonances using a fluid approximation ($\omega > \omega_{ti}$) for ions.⁸ This driving mechanism remains robust even when progressively complicated and realistic configurations are considered.⁸⁻¹⁰

ii) A consequence of toroidal geometry is that each Fourier eigenmode radial width for the most unstable mode is approximately given by the distance between the nearest neighboring rational surfaces, $W_n = 1/k_{\theta} S$, due to linear interactions between different poloidal harmonics.²²

Consequently, magnetic shear-induced ion Landau damping (which could be significant in a sheared slab geometry¹⁶) is rendered ineffective for most of the k_θ range of our interest where $\omega > \omega_{ti}$ is satisfied.^{9,10,22} Therefore, the local dispersion relation of Ref.8 (where ion Landau damping effects are ignored) is a reasonable approximation.

iii) Describing the nonlinear evolution of toroidal drift waves using the ballooning mode formalism is extremely complicated.¹⁷ In this paper, we develop a nonlinear theory in a configuration space assuming a simple gaussian-shaped radial structure²³ with a mode-width characteristic of results obtained from numerical eigenmode studies.¹¹

In order to simply model the main k_θ spectral features in the present problem, we adopt the following approximate form of the linear dispersion relation generated by Eq. (1),

$$\omega(k_\theta) = v^* k_\theta / (1 + k_\theta^2 / k_M^2), \quad (2)$$

where $k_M^{-2} \equiv \rho_s^2 / \{1 + \tau^{-1}(1 + \eta_i)\}$, and $v^* = \omega_{*e} / k_\theta$. Note that for $k_\theta^2 / k_M^2 \ll 1$, this reduces to the usual small Larmor radius expansion procedure result, $\omega(k_\theta) = v^* k_\theta (1 - k_\theta^2 / k_M^2)$ which Eq. (2) successfully approximates for $k_\theta^2 / k_M^2 \ll 1$. The advantage of adopting the form of the dispersion relation given in Eq. (2) is that it makes the analysis of the nonlinear distant interaction of low k and high k modes much simpler. Our major conclusions are not affected by the approximate nature of this dispersion relation since most of the fluctuation energy exists in the long wave length range, $k_\theta^2 / k_M^2 \ll 1$. This will be demonstrated in the next section. With regard to the destabilizing mechanism, the linear growth rate is assumed to be solely due to trapped electron precession resonances. From Ref. 8,

$$\gamma / \omega_{*e} = 2(2\pi\epsilon)^{1/2} (R/GL_n)^{3/2} \exp(-R\Omega/GL_n) \Omega^{1/2} \{1 - \Omega + \eta_e (R\Omega/GL_n - 3/2)\}, \quad (3)$$

where $\Omega = 1/(1 + k_\theta^2 / k_M^2)$.

Other nonresonant type driving mechanisms²⁴⁻²⁶ are not considered here since their nonlinear behaviors are not readily amenable to the weak turbulence approach²⁷ pursued in the present study. Finally, the radial mode structure is taken to be of the form,

$$|\Phi_k(x)|^2 = |\Phi_k(0)|^2 \exp(-x^2 / W_n^2), \quad (4)$$

with $W_n = 1/k_\theta S$ being the approximate distance between the nearest neighboring rational surfaces.

Equations (2)-(4) summarize the linear properties of CTEM which are used in the following nonlinear calculations. Both ion and trapped electron nonlinearities are treated on an equal footing in the analysis. The trapped electron dynamics are given by the bounce-averaged nonlinear drift-kinetic equation,

$$(\partial_t + i\omega_{de})h - i[\omega - \omega_{*e}(1 + (E/T_e - 3/2)\eta_e)]\langle\Phi\rangle F_m = \nabla\langle\Phi\rangle \times \mathbf{b} \cdot \nabla h, \quad (5)$$

where h is the nonadiabatic part of the perturbed distribution function of trapped electrons; i.e., $f_{e1} = h + F_m e\phi/T_e$, $F_m = n_0(m_e/2\pi T_e)^{3/2} \exp(-E/T_e)$, $\mathbf{b} = \mathbf{B}/B$, $\langle\Phi\rangle$ is the bounce-averaged value of $\Phi = e\phi/T_e$, and ϕ is the electrostatic potential. The untrapped electron dynamics can be approximated by the Boltzmann response as stated before. The collisionless nonlinear gyrokinetic equation²⁸ for ions is

$$(\partial_t + i\omega_{di} + v_{||}\mathbf{b} \cdot \nabla)g + i\tau[\omega - \omega_{*i}(1 + (u^2 - 3)\eta_i/2)]\Phi J_0 F_m = \nabla\Phi J_0 \times \mathbf{b} \cdot \nabla g, \quad (6)$$

where g is the nonadiabatic part of the perturbed distribution function; i.e., $g = f_{i1} + F_m e\phi/T_i$, $F_m = n_0(M_i/2\pi T_i)^{3/2} \exp(-u^2/2)$, $u^2 = M_i v^2/T_i$, and $J_0(k_\perp \rho_i)$ is a Bessel function.

We begin our analysis of Eq. (6) by noting that the right side represents the $\mathbf{E} \times \mathbf{B}$ convective nonlinearity. This term is assumed to be smaller than the linear terms in weak turbulence theory, i.e., Φ^2 is on the order of $\gamma/\delta\omega < 1$, where $\delta\omega = \omega_{*e} - \omega$. To the dominant order, the test mode at $\mathbf{k}$ obeys the linear relation.

$$\begin{aligned} L_{ek}^{-1} h_k^{(1)} &= i[\omega - \omega_{*e}(1 + (E - 3/2)\eta_e)]\langle\Phi\rangle_k F_m, \\ L_{ik}^{-1} g_k^{(1)} &= -i\tau[\omega - \omega_{*i}(1 + (u^2 - 3)\eta_i/2)]J_0\Phi_k F_m, \end{aligned} \quad (7)$$

where the linear ion propagator has been simplified to $L_{ik} = i(\omega - k_{||}v_{||} + i0^+)^{-1}$ ignoring ω_{di} compared to $k_{||}v_{||}$. The primary role of ω_{di} is to help set up a toroidal drift mode structure with $W_n \equiv 1/k_\theta S$ by introducing linear coupling between neighboring poloidal harmonics.²² Trapped electron dynamics can also contribute to this coupling.^{29,30} The influence of ω_{di} on the growth rate is not appreciable and its influence on the downward shift of the real frequency is smaller

than the finite gyroradius effects for $k_0^2/k_M^2 > L_n/R$. Since the radial mode width is on the order of W_n for toroidal drift-wave, $\langle \Phi \rangle_k \equiv \Phi_k$. It is worthwhile to note that the bounce-average reduction ($\langle \Phi \rangle_k < \Phi_k$) tends to be more significant for the Pearlstein-Berk-type modes which have characteristically broader radial widths.¹⁵

To the next order, the mode at k'' is driven by the nonlinear interaction of the test mode at k and the background modes at k' ; i.e.,

$$L_{ek}''^{-1} h_{k''}^{(2)} - \nabla \Phi_k \times b \cdot \nabla h_{k'}^{(1)*} - \nabla \Phi_{k'}^* \times b \cdot \nabla h_k^{(1)} = 0, \quad (8)$$

$$L_{ik}''^{-1} g_{k''}^{(2)} - \nabla \Phi_k J_0 \times b \cdot \nabla g_{k'}^{(1)*} - \nabla \Phi_{k'}^* J_0' \times b \cdot \nabla g_k^{(1)} = 0, \quad (9)$$

where $k'' = k - k'$, and $\langle \Phi \rangle_k$ has been approximated by Φ_k . The "induced potential" $\Phi_{k''}^{(2)}$ is obtained by imposing the quasi-neutrality condition at the second order.

$$\begin{aligned} \epsilon^{(1)}(\omega'', k'') \Phi_{k''}^{(2)} = & -i \{ (\omega_* e / \omega_k) - (\omega_* e' / \omega_{k'}) \} \\ & \left[\int d^3 v (\omega'' - k_{||} v_{||} + i0^+)^{-1} [J_0 J_0' J_0'' \{1 + (u^2 - 3)\eta_i/2\}] F_m \right. \\ & \left. - \int dE E^{1/2} (\omega'' - \omega_{de}'' + i0^+)^{-1} \{1 + (E - 3/2)\eta_e\} F_m \right] (C_s \rho_s k \times k' \cdot b) \Phi_k \Phi_{k'}, \end{aligned} \quad (10)$$

where $J_0' = J_0(k'_{\perp} \rho_i)$, $J_0'' = J_0(k''_{\perp} \rho_i)$, E has been normalized to T_e , and

$$\begin{aligned} \epsilon^{(1)}(\omega'', k'') \Phi_{k''} & \equiv (1 + \tau) \Phi_{k''} \\ & + \tau \int d^3 v (i L_{ik}'') J_0^2 [\omega'' - \omega_{*i}'' \{1 + (u^2 - 3)\eta_i/2\}] F_m \Phi_{k''} \\ & + \int d^3 v (i L_{ek}'') [\omega'' - \omega_{*e}'' \{1 + (E/T_e - 3/2)\eta_e\}] F_m \Phi_{k''}, \end{aligned} \quad (11)$$

is the expression for the linear dielectric function. It is important to remember here that (ω'', k'') does not satisfy a linear dispersion relation $\epsilon^{(1)}(\omega'', k'') = 0$, and that $\Phi_{k''}$ is not a linear eigenmode. Furthermore, the assumed frequency ordering for the linear eigenmode is no longer valid; i.e., $\omega'' < \omega_{de}'', k_{||} v_{||}$. Consequently, both trapped electron and ion responses are weighted heavily by the contribution from the kinetic regime, and $\epsilon^{(1)}(\omega'', k'')$ deviates significantly from the local expression. The induced potential $\Phi_{k''}^{(2)}$ is accordingly quite small, and the nonlinear shielding contribution is subdominant to the "bare" scattering terms. Of course, in the local approximation^{13,31} or in the situation where the ion Landau damping is ignorable,³² significant cancellation occurs between the shielding contribution and the "bare" scattering

contributions at long wavelength.

To the third order, the back reaction of the driven modes on the test mode gives the nonlinear correction to the distribution function at $\mathbf{k}$;

$$L_{e\mathbf{k}}^{-1} h_{\mathbf{k}}^{(3)} = -\sum_{\mathbf{k}'} (C_s \rho_s \mathbf{k} \times \mathbf{k}' \cdot \mathbf{b})^2 \{ \Phi_{\mathbf{k}'} (h_{\mathbf{k}}^{(2)} - i L_{e\mathbf{k}} [\omega - \omega_{*e} \{ 1 + (E/T_e - 3/2) \eta_e \}] \Phi_{\mathbf{k}}) - \Phi_{\mathbf{k}}^{(2)} h_{\mathbf{k}'}^{(1)} \}, \quad (12)$$

and

$$L_{i\mathbf{k}}^{-1} g_{\mathbf{k}}^{(3)} = -\sum_{\mathbf{k}'} (C_s \rho_s \mathbf{k} \times \mathbf{k}' \cdot \mathbf{b})^2 \{ J_0' \Phi_{\mathbf{k}'} (g_{\mathbf{k}}^{(2)} - i \tau L_{i\mathbf{k}} [\omega - \omega_{*i} \{ 1 + (u^2 - 3) \eta_i / 2 \}] \Phi_{\mathbf{k}}) - J_0'' \Phi_{\mathbf{k}}^{(2)} g_{\mathbf{k}'}^{(1)} \}. \quad (13)$$

Here, we have retained only the "coherent" response $g_{\mathbf{k}}^{(3)}$ which is proportional to $\Phi_{\mathbf{k}}$ by construction. Other "incoherent" contributions, which are not proportional to $\Phi_{\mathbf{k}}$, are ignored. Although within this "coherent approximation" the fluctuation energy is no longer strictly conserved, we can nevertheless obtain useful information about the shape of the fluctuation $\mathbf{k}$ -spectrum. In Eqs. (12) and (13), the first term on the right side will hereafter be referred to as the "bare" contribution because this distribution function response is directly driven through the nonlinear interaction of $\Phi_{\mathbf{k}}$ with $\Phi_{\mathbf{k}'}$ as described by Eqs. (8) and (9). The remaining two terms will be referred to as the "shielding" contribution since these are due to the $\Phi_{\mathbf{k}}^{(2)}$ fluctuation self-consistently induced by $g_{\mathbf{k}}^{(2)}$ and $h_{\mathbf{k}}^{(2)}$. Finally, by requiring quasi-neutrality up to third order, we obtain the formal nonlinear eigenmode equation

$$\epsilon^{(1)}(\omega, \mathbf{k}) \Phi_{\mathbf{k}} = - \int d^3v h_{\mathbf{k}}^{(3)} + \int d^3v J_0 g_{\mathbf{k}}^{(3)}. \quad (14)$$

Here, the left hand side is the linear response; i.e.,

$$\begin{aligned} \epsilon^{(1)}(\omega, \mathbf{k}) \Phi_{\mathbf{k}} &= (1 + \tau) \Phi_{\mathbf{k}} \\ &+ \tau \int d^3v (i L_{i\mathbf{k}}) [\omega - \omega_{*i} \{ 1 + (u^2 - 3) \eta_i / 2 \}] F_m J_0^2 \Phi_{\mathbf{k}} \\ &+ \int d^3v (i L_{e\mathbf{k}}) [\omega - \omega_{*e} \{ 1 + (E/T_e - 3/2) \eta_e \}] F_m \Phi_{\mathbf{k}}, \end{aligned} \quad (15)$$

which is approximately equivalent to Eqs. (1)-(3) based on the frequency ordering for the linearly unstable range, $L_n/R < k_0/k_M < 1$. At very short and very long wavelengths outside of this k_0 range, modes are linearly damped by ion Landau damping since the frequency becomes

as low as the ion transit frequency. The first two terms on the right hand side of Eq. (15) are from the adiabatic responses of electrons and ions, respectively. The third term is the nonadiabatic ion linear response, and the last term represents the trapped electron response.

The right hand side of Eq. (14) contains the trapped electron nonlinear response from the trapped electron "bare" scattering and the nonlinear ion response from the ("bare") ion Compton scattering.

$$-\int d^3v h_{\mathbf{k}}^{(3)} = \sum_{\mathbf{k}'} (C_S \rho_S \mathbf{k} \times \mathbf{k}' \cdot \mathbf{b})^2 \langle L_{e\mathbf{k}} L_{e\mathbf{k}'} \rangle \{ (\omega_{*e}/\omega_{\mathbf{k}}) - (\omega_{*e}'/\omega_{\mathbf{k}'}) \} \{ 1 + (E/T_e - 3/2) \eta_e \} |\Phi_{\mathbf{k}'}|^2 \Phi_{\mathbf{k}}, \quad (16)$$

$$\int d^3v J_0 g_{\mathbf{k}}^{(3)} = -\sum_{\mathbf{k}'} (C_S \rho_S \mathbf{k} \times \mathbf{k}' \cdot \mathbf{b})^2 \langle L_{i\mathbf{k}} L_{i\mathbf{k}'} \rangle \{ (\omega_{*e}/\omega_{\mathbf{k}}) - (\omega_{*e}'/\omega_{\mathbf{k}'}) \} \{ 1 + (u^2 - 3) \eta_i / 2 \} J_0^2 J_0'^2 |\Phi_{\mathbf{k}'}|^2 \Phi_{\mathbf{k}}. \quad (17)$$

Here we have kept only the resonances from the (driven) beat mode particle interactions, $L_{e\mathbf{k}} = i(\omega'' - \omega_{de}'' + i0^+)^{-1}$ and $L_{i\mathbf{k}} = i(\omega'' - k_{\parallel}'' v_{i\parallel} + i0^+)^{-1}$. Other possible wave-particle resonant interactions at $\mathbf{k}$ or $\mathbf{k}'$ are neglected in the nonlinear terms since only the high energy trapped electrons or high energy ions participate. The real part of Eq. (14) determines the eigenmode structure and the real frequency. By multiplying Eq. (14) with $\Phi_{\mathbf{k}}^*$ and taking the imaginary part, we obtain the wave-kinetic equation which describes the time evolution of the spectral intensity. The wave-kinetic equation obtained from Eq. (14) is,

$$\begin{aligned} & \{ 1/2 (\partial \text{Re} \epsilon^{(1)} / \partial \omega) \partial / \partial t + \text{Im} \epsilon^{(1)} \} |\Phi_{\mathbf{k}}|^2 \\ &= -\text{Im} \sum_{\mathbf{k}'} (C_S \rho_S \mathbf{k} \times \mathbf{k}' \cdot \mathbf{b})^2 \int d^3v (\omega'' - \omega_{de}'' + i0^+)^{-1} \omega^{-1} \{ (\omega_{*e}/\omega_{\mathbf{k}}) - (\omega_{*e}'/\omega_{\mathbf{k}'}) \} \\ & \{ 1 + (E/T_e - 3/2) \eta_e \} F_m |\Phi_{\mathbf{k}'}|^2 |\Phi_{\mathbf{k}}|^2 \\ & + \text{Im} \sum_{\mathbf{k}'} (C_S \rho_S \mathbf{k} \times \mathbf{k}' \cdot \mathbf{b})^2 \int d^3v (\omega'' - k_{\parallel}'' v_{i\parallel} + i0^+)^{-1} \omega^{-1} \{ (\omega_{*e}/\omega_{\mathbf{k}}) - (\omega_{*e}'/\omega_{\mathbf{k}'}) \} \\ & \{ 1 + (u^2 - 3) \eta_i / 2 \} J_0^2 J_0'^2 F_m |\Phi_{\mathbf{k}'}|^2 |\Phi_{\mathbf{k}}|^2, \end{aligned} \quad (18)$$

where the expression on the right hand side can be factored into a form,

$M^{\text{TES}}_{\mathbf{k}\mathbf{k}'} |\Phi_{\mathbf{k}'}|^2 |\Phi_{\mathbf{k}}|^2 + M^{\text{ICS}}_{\mathbf{k}\mathbf{k}'} |\Phi_{\mathbf{k}'}|^2 |\Phi_{\mathbf{k}}|^2$. For $k_{\theta} < k'_{\theta}$, we can show that $M^{\text{TES}}_{\mathbf{k}\mathbf{k}'} < 0$ and $M^{\text{ICS}}_{\mathbf{k}\mathbf{k}'} > 0$; i.e., trapped electron scattering ($M^{\text{TES}}_{\mathbf{k}\mathbf{k}'}$) transfers

fluctuation energy to the shorter wavelength modes while ion Compton scattering ($\text{MICS}_{\mathbf{k}\mathbf{k}'}$) transfers fluctuation energy to the longer wavelength modes in agreement with the local theory prediction.¹³

In analytically performing the mode summation for various nonlinear interaction channels (i.e., the summation over all the background modes which satisfy $\mathbf{k}' = \mathbf{k} - \mathbf{k}''$), we adopt the following continuum approximation.³³ By changing the independent variables from n', m' to k_ϕ' and x' , $\sum_{\mathbf{k}'} = \sum_{n', m'} \equiv \int dn' dm' = \int |\partial(n', m')/\partial(k_\phi', x')| dk_\phi' dx' = R^2 (dq/dr) \int dk_\phi' |k_\phi'| \int dx' \equiv (rS/q) \int dk_\theta' |k_\theta'| \int dx'$.

We will also consider only a radial-mode-width averaged spectral intensity in order to make the wave-kinetic equation analytically tractable; i.e., we assume $|\Phi_{\mathbf{k}}|^2 = I(k_\theta) \exp(-x'^2/w_n^2)$, where $I_{\mathbf{k}} = I(k_\theta)$ is the (radial-mode-width averaged) spectral intensity. Consequently, the right hand side can be evaluated explicitly at the rational surface of the test mode ($x=0$) after using the appropriate radially nonlocal expressions, $(\mathbf{k} \times \mathbf{k}' \cdot \mathbf{b})^2 = k_\theta'^2 x'^2/w_n^4$, and $k_{||}'' v_{||} = k_\theta' x' v_{||}/L_s$. Then, the wave-kinetic equation simplifies to

$$\begin{aligned} & \{1/2(\partial \text{Re}\epsilon^{(1)}/\partial\omega)\partial/\partial t + \text{Im}\epsilon^{(1)}\} I_{\mathbf{k}} = \\ & -(C_s \rho_s)^2 (rS/q) \int dk_\theta' |k_\theta'| \int dx' (k_\theta'^2 x'^2/w_n^4) \exp(-x'^2/w_n^2) \omega_{\mathbf{k}}^{-1} \\ & \{(\omega_{*e}/\omega_{\mathbf{k}}) - (\omega_{*e}'/\omega_{\mathbf{k}'})\} \text{Im} \int_t d^3v (\omega'' - \omega_{de}'' + i0^+)^{-1} \{1 + (E/T_e - 3/2)\eta_e\} F_m I_{\mathbf{k}'} I_{\mathbf{k}} \\ & + (C_s \rho_s)^2 (rS/q) \int dk_\theta' |k_\theta'| \int dx' (k_\theta'^2 x'^2/w_n^4) \exp(-x'^2/w_n^2) \omega_{\mathbf{k}}^{-1} \\ & \{(\omega_{*e}/\omega_{\mathbf{k}}) - (\omega_{*e}'/\omega_{\mathbf{k}'})\} \text{Im} \int d^3v (\omega'' - k_\theta' x' v_{||}/L_s + i0^+)^{-1} \{1 + (u^2 - 3)\eta_i/2\} J_0^2 J_0'^2 F_m I_{\mathbf{k}'} I_{\mathbf{k}}. \quad (19) \end{aligned}$$

The dominant contribution to trapped electron scattering comes from the "distant" k -space interactions which are due to the beating between low k_θ and high k_θ' fluctuations with similar frequencies; i.e., $\omega - \omega' \ll \omega \equiv \omega'$, but $\omega_{*e} \ll \omega_{*e}'$. The "close" k -space interactions between similar wavelength fluctuations are weak due to the fact that $\omega_{de}' - \omega_{de} \ll \omega' - \omega$, since $\omega_{de} \ll \omega$ and $\omega_{de}' \ll \omega'$. Therefore, we keep only the distant interaction contributions for the k_θ' sum. To facilitate the explicit evaluation of the distant interactions, we assume a specific k_θ -dependence of the linear frequency given in Eq. (2). This yields a simple relation between k_θ

and its short wavelength counterpart k_θ^t with the same frequency [i.e., $\omega(k_\theta) = \omega(k_\theta^t)$, $k_\theta \neq k_\theta^t$ with $\omega(k_\theta) = v^* k_\theta / (1 + k_\theta^2 / \kappa_M^2)$, where $\kappa_M^{-2} \equiv \rho_s^2 / (1 + \tau^{-1}(1 + \eta_i))$]. For this simplified dispersion relation, the reciprocal relation for the distant interaction is $k_\theta^t / k_M \equiv k_M / k_\theta$. We also note that this specific form of the dispersion relation does not affect our main conclusions about the long wavelength spectrum at saturation. For a given k_θ , the distant interaction occurs mainly with k_θ' such that $\omega'' / \omega_{de} < 1$. Therefore, the integration is simplified by expanding ω'' around $k_\theta' \equiv k_\theta^t$; i.e., $\omega'' = \omega(k_\theta) - \omega(k_\theta') \equiv -(k_\theta' - k_\theta^t)(\partial\omega/\partial p)_{k_t}$, where we use $\omega(p)$ to indicate the k_θ dependence of ω and to avoid possible confusion with the test mode k_θ . Also $I(k_\theta')$ is expanded around k_θ^t , $I(k_\theta') \equiv I(k_\theta^t) + (k_\theta' - k_\theta^t)(\partial I(p)/\partial p)_{k_t}$. The velocity space resonant integral and the radial integration over the background mode width are straightforward. The k_θ' -integration is facilitated by changing the integration variable to ω'' / ω_{de} using $\omega'' / \omega_{de} \equiv -(k_\theta' - k_\theta^t)(k_\theta - k_\theta^t)^{-1}(\partial\omega/\partial p)_{k_t}(eBR/cT_eG)$. Finally, the trapped electron scattering contribution [the first term on the right side of Eq. (19)] becomes

$$- (\pi/2)(C_s \rho_s)^2 (rS/q)(2\pi\epsilon)^{1/2} \kappa_M^2 (k_\theta^2 - k_\theta^t{}^2) \omega(k_\theta)^{-1} |\partial\omega/\partial p|_{k_t}^{-1} \{ I(k_\theta^t) - 3(1 + \eta_e) 2^{-1} \omega''_{de} (\partial\omega/\partial p)_{k_t}^{-1} \partial I/\partial p|_{k_t} \}. \quad (20)$$

For the ion Compton scattering, the dominant contribution comes from the close interactions between modes with similar frequencies and similar k 's. This is because the distant interaction which involves high- k' contributions is reduced by the gyro-orbit average factor J_0^2 . For close interactions between low k modes, we can approximate the Bessel functions by 1. After carrying out the resonant velocity space integration, the ion Compton scattering term in Eq. (19) becomes

$$(C_s \rho_s)^2 (rS/q) \int dk_\theta' |k_\theta'| \int dx' k_\theta'^2 x'^2 w_n'^{-4} \exp(-x'^2/w_n'^2) \omega_k^{-1} \{ (\omega_{*e}/\omega_k) - (\omega_{*e}'/\omega_k') \} (\pi/2)^{1/2} |k_\theta' x' v_{ti}/L_s|^{-1} \{ 1 + (|k_\theta' x' v_{ti}/\omega'' L_s|^{-2} - 1) \eta_i/2 \} \exp\{-|k_\theta' x' v_{ti}/\omega'' L_s|^{-2}/2\} I_{k'} I_k. \quad (21)$$

The last exponential factor indicates that the ion Compton scattering amplitude increases with $k_{||}'' \propto x'$, while the extent of x' is limited by a radial mode structure factor, $\exp(-x'^2/w_n'^2)$. Next, the radial integration, which accounts for a part of the mode summation, can be approximately

evaluated by the saddle point method observing that the controlling factors are the two aforementioned exponential dependencies. The integration yields

$$\begin{aligned} & (C_s \rho_s)^2 (rS/q) \int dk_\theta' |k_\theta'| k_\theta'^2 w_{n'}^{-4} \omega_k^{-1} \\ & \{ (\omega_{*e}/\omega_k) - (\omega_{*e}'/\omega_{k'}) \} \pi^{2-5/4} (k_\theta' S)^{-3} qR/v_{ti} (|\omega''| qR/v_{ti})^{1/2} \\ & \{ 1 + (\sqrt{2} |\omega''| qR/v_{ti} - 1) \eta_i/2 \} \exp \{ -\sqrt{2} |\omega''| qR/v_{ti} \} I_{k'} I_k. \end{aligned} \quad (22)$$

The equation above clearly indicates that only the low frequency beat waves with $|\omega''| < v_{ti}/qR$ strongly interact with the ion distribution function and contribute significantly to the spectrum evolution. We then perform the k_θ' -integration using the following differential approximation for close interactions,^{27,33} $I(k'_\theta) \equiv I(k_\theta) + (k'_\theta - k_\theta)(\partial I(p)/\partial p)_k$. Finally combining with Eq. (20), the RHS of the wave-kinetic equation simplifies to

$$\begin{aligned} & 15 \pi^{3/2} 2^{-4} (C_s \rho_s)^2 (rS^2/q) (v_{ti}/qR)^2 (1 + 5\eta_i/4) k_\theta^3 \omega(k_\theta)^{-1} |\partial \omega / \partial k_\theta|^{-3} k_M^{-2} \\ & I(k_\theta) \partial / \partial k_\theta \{ k_\theta^2 I(k_\theta) \} \\ & - (\pi/2) (C_s \rho_s)^2 (rS^2/q) (2\pi \epsilon)^{1/2} k_M^2 (k_\theta^2 - k_\theta'^2) \omega(k_\theta)^{-1} |\partial \omega / \partial p|_k t^{-1} \\ & \{ I(k_\theta') - 3(1 + \eta_e) 2^{-1} \omega''_{de} (\partial \omega / \partial p)_k t^{-1} \partial I / \partial p|_k t \}. \end{aligned} \quad (23)$$

III. Fluctuation Spectrum at Saturation

In the preceding section, we have derived a wave-kinetic equation which describes the nonlinear evolution of the fluctuation spectrum. In this section we solve that equation at nonlinear saturation. Since $\partial I(k_\theta)/\partial t = 0$ at saturation, we can write Eq. (23) in the following form to emphasize the k_θ -dependences,

$$\begin{aligned}
 & -\beta L F(\Omega(k_\theta)) = \\
 & \beta_{ICS} k_\theta^3 \omega(k_\theta)^{-1} |\partial \omega / \partial k_\theta|^{-3} \partial \{k_\theta^2 I(k_\theta)\} / \partial k_\theta \\
 & - \beta_1 TES (k_\theta^2 - k_{t\theta}^2) \omega(k_\theta)^{-1} |\partial \omega / \partial p|_{k_t}^{-1} \\
 & (I(k_{t\theta}) - \beta_2 TES (k_\theta - k_{t\theta}) (\partial \omega / \partial p)_{k_t}^{-1} \partial I / \partial p|_{k_t}), \quad (24) \\
 & \text{where } \beta L = 2(2\pi\epsilon)^{1/2} (R/GL_n)^{3/2}, \quad \Omega = 1/(1 + k_\theta^2/k_M^2), \\
 & F(\Omega(k_\theta)) = \exp(-R\Omega/GL_n) \Omega^{1/2} \{1 - \Omega + \eta_e (R\Omega/GL_n - 3/2)\}, \\
 & \beta_{ICS} = 15 \pi^{3/2} 2^{-4} (C_s \rho_s)^2 (rS^2/q) (v_{ti}/qR)^2 (1 + 5\eta_i/4) k_M^{-2}, \\
 & \beta_1 TES = (\pi/2) (C_s \rho_s)^2 (rS^2/q) (2\pi\epsilon)^{1/2} k_M^2, \\
 & \beta_2 TES = 3(1 + \eta_e) GL_n (C_s \rho_s) / 2R.
 \end{aligned}$$

First, we consider wavelengths in the range, $k_\theta < k_M$, which are mainly responsible for the induced transport. Observing that the ion Compton scattering term in Eq. (24) is proportional to $|\partial \omega / \partial k_\theta|^{-3}$ and that the trapped electron scattering term has a factor $(k_\theta^2 - k_{t\theta}^2) \omega(k_\theta)^{-1} |\partial \omega / \partial p|_{k_t}^{-1}$, we expect that as $k_\theta \Rightarrow k_M$ (then, $k_{t\theta} \Rightarrow k_M$) the ion Compton scattering becomes stronger while the trapped electron scattering becomes weaker. Therefore, for $k_L < k_\theta < k_M$, where k_L is to be determined later, we can ignore the trapped electron scattering and get a spectrum at saturation by balancing the linear drive [the LHS of Eq. (24)] against the ion Compton scattering (the 1st term on the RHS). Then, Eq. (24) simplifies to the following first order inhomogeneous differential equation for the spectral intensity $I(k_\theta)$,

$$\begin{aligned}
 & -\beta L F(\Omega(k_\theta)) = \\
 & \beta_{ICS} (C_s \rho_s / L_n)^{-4} k_\theta^2 (1 + k_\theta^2/k_M^2)^{7/2} (1 - k_\theta^2/k_M^2)^{-3} \partial / \partial k_\theta \{k_\theta^2 I(k_\theta)\}. \quad (25)
 \end{aligned}$$

Taking an inhomogeneous solution which decays to zero as k_θ becomes larger, we get

$$I(k_\theta) = (\beta L / \beta_{ICS}) (C_s \rho_s / L_n)^4 k_\theta^{-2} \int_{k_\theta} dk_\theta k_\theta^{-2} (1 + k_\theta^2 / k_M^2)^{-7/2} (1 - k_\theta^2 / k_M^2)^{3/2} F(\Omega). \quad (26)$$

Here, we note that $I(k_\theta)$ vanishes as $k_\theta \Rightarrow k_M$. However, this is only an artifact of our approximate treatment of mode summation with $\omega'' \equiv -(k'_\theta - k_\theta)(\partial\omega/\partial p)_k$. Since our approximation becomes poorer as $k_\theta \Rightarrow k_M$, we cannot predict a detailed shape of the spectrum in that wavelength regime. Nevertheless, it is clear that $I(k_\theta)$ decays faster than k_θ^{-3} . For $k_\theta \ll k_M$, Eq. (26) can be further simplified to

$$I(k_\theta) \equiv (2^5 (2\epsilon)^{1/2} / 15\pi) (q^3 / S^2) (T_e / T_i) (1 + 5\eta_i / 4)^{-1} (k_M \rho_s)^2 (R / L_n)^2 (R / GL_n)^{3/2} (R / GL_n - 3/2) \exp(-R / GL_n) r^{-1} L_{Te}^{-1} L_n^{-1} k_\theta^{-3}. \quad (27)$$

Next, we consider the range $k_\theta < k_L$, where the ion Compton scattering is negligible. Keeping only the linear term and the trapped electron scattering term from Eq. (24), we have

$$\begin{aligned} -\beta L F(\Omega(k_\theta)) = \\ -\beta_1 TES(k_\theta^2 - k_{t_\theta}^2) \omega(k_\theta)^{-1} |\partial\omega/\partial p|_{k_t}^{-1} \\ (I(k_{t_\theta}) - \beta_2 TES(k_\theta - k_{t_\theta})(\partial\omega/\partial p)_{k_t}^{-1} \partial I / \partial p|_{k_t}). \end{aligned} \quad (28)$$

Since we want to obtain the spectrum in the very long wavelength regime, it is convenient to get a solution $I(k_{t_\theta})$ for $k_{t_\theta} < k_M < k_\theta$. The right side of this equation can be simplified by introducing an integrating factor $S(\phi)$,

$$dS/d\phi = (\beta L / \beta_1 TES \beta_2 TES) (C_s \rho_s / L_n)^3 / k_M \exp(-(C_s \rho_s / 2L_n \beta_2 TES) \phi^2) \phi^4 f(\Omega), \quad (29)$$

where $S(\phi) = \exp(-(C_s \rho_s / 2L_n \beta_2 TES) \phi^2) I(\phi)$, and $\phi = k_{t_\theta} / k_M$. Since the trapped electron scattering transfers energy to shorter wavelengths, and the linear ion Landau damping becomes effective at very long wavelengths, we take a solution which vanishes as $k_\theta / k_M \Rightarrow 0$. Then, a simple integration yields

$$\begin{aligned} I(k_\theta) = (\beta L / \beta_1 TES \beta_2 TES) (C_s \rho_s / L_n)^3 / k_M \exp((C_s \rho_s / 2L_n \beta_2 TES) (k_{t_\theta} / k_M)^2) \\ \int_{k_\theta} dk_\theta d\phi F(\Omega(k_\theta)) \exp(-(C_s \rho_s / 2L_n \beta_2 TES) \phi^2) \phi^4. \end{aligned} \quad (30)$$

Since $\phi < k_\theta / k_M \ll 1$, the integral in Eq. (30) can be further simplified to

$$\int k_t d\phi \exp(-(1+1/3(1+\eta_e))R\phi^2/GL_n)\phi^5(1+\eta_e(R\phi^2/GL_n-3/2)). \quad (31)$$

If $\eta_e > 2/3$, the lower limit of the integral should be modified to $\{GL_{Te}(3\eta_e/2-1)/R\}^{1/2}$. However, it will be shown later that in practice this detailed refinement is not really required. This is because the trapped electron nonlinear scattering makes the spectrum in this region very small, regardless of the detailed treatment of the kinetic ion linear response. Effectively, the nonlinear trapped electron response determines the low-k spectrum cutoff. This is in contrast to most previous theories where either the three-wave decay²⁷ or the linear damping^{15,18} has been responsible for the low-k spectrum cutoff. For $(k_\theta/k_M)^2 \ll L_n/R$, the integral increases very sharply with k_θ (according to k_θ^8), although the magnitude stays very small. For $(k_\theta/k_M)^2 > L_n/R$, the integrand begins to decrease due to the exponential factor and the integral asymptotes to a constant value as k_θ is increased. The constant is easily found to be $C^T = (R(4+3\eta_e)/L_n (3+3\eta_e))^{-4} (R/2L_{Te}) \{(3\eta_e^2+4\eta_e+8/3)/(\eta_e^2+\eta_e)\}$. Thus, $I(k_\theta)$ is extremely small for $(k_\theta/k_M)^2 < L_n/R$ and monotonically increasing. For $(k_L/k_M)^2 > (k_\theta/k_M)^2 > L_n/R$, $I(k_\theta)$ is approximately given by

$$C^T (qR/S^2 r) \eta_e (1+\eta_e)^{-1} (k_M L_n)^{-3} (R/GL_n)^{-3/2} \exp((C_s \rho_s / 2 L_n \beta_2^{TES}) (k_\theta^t / k_M)^2). \quad (32)$$

Since Eq. (26) predicts a monotonically decreasing $I(k_\theta)$ for $k_L < k_\theta < k_M$, it is clear that the spectral intensity peaks at k_L . Now, we can determine k_L by equating Eq. (27) with Eq. (32) at $k_\theta = k_L$. After some algebraic arrangement, we obtain

$$(k_L/k_M)^3 \exp[(R/3GL_n)(1+\eta_e)^{-1}(k_L/k_M)^2] = (2^5(2\epsilon)^{1/2}/15\pi) q^2 (T_e/T_i) \eta_e (1+\eta_e)^{-1} (1+5\eta_i/4)^{-1} (k_M \rho_s)^2 (R/L_n) (R/GL_n)^3 (R/GL_n-3/2) \exp(-R/GL_n) / C^T. \quad (33)$$

Since Eq. (33) is a transcendental equation, it does not yield a simple formula for k_L . Nevertheless, considering typical parameters for large tokamaks, the right side of this equation is of order unity, and we have $(k_L/k_M)^2 < 3(1+\eta_e) GL_n/R$. The expected shape of the spectral intensity, $I(k_\theta)$, is plotted in Fig. 1, signifying three different asymptotic regions in k_θ . The region "a" corresponds to an extremely long wavelength regime where fluctuations are

suppressed due to the trapped electron scattering. The region "b" corresponds to a long wavelength regime where most of the fluctuation energy is populated and $I(k_\theta)$ decays according to a power law, k_θ^{-3} . Finally, region "c" corresponds to a relatively shorter wavelength regime, $k_\theta > k_M$, where $I(k_\theta)$ decays more steeply. We note that the saturated amplitude of the fluctuations is smaller than the usual rough mixing length estimates, $I(k_\theta) \equiv 1/k_\theta^2 L_n^2$. Also, the spectral shape exhibits a k_θ -dependence which is steeper than the mixing length prediction. These features are in rough agreement with most microwave scattering measurement results from tokamaks.³⁴⁻³⁷

IV. Fluctuation-induced Transport

In this Section, we calculate the anomalous particle and thermal fluxes of both electrons and ions induced by the nonlinearly saturated trapped electron driven drift wave fluctuations. Since the particle transport obeys the ambipolarity constraint, it suffices to consider only the electron particle flux. The untrapped electron transport is very small because the drift wave fluctuation resonates only with the extremely low velocity part of the distribution function, due to its slow phase velocity along the magnetic field line. Therefore, the trapped electrons are transported preferentially. Since the flux is induced by the radial component of the $E \times B$ velocity involving the fluctuating E_θ and the density fluctuation, the electron particle flux is given by

$$\Gamma_e = \text{Re} \sum_{\mathbf{k}} i k_\theta c T_e \Phi_{-\mathbf{k}} \int d^3v (h_{\mathbf{k}}^{(1)} + h_{\mathbf{k}}^{(3)}) / e B_0. \quad (34)$$

First, we note that the contribution from $h_{\mathbf{k}}^{(3)}$ is proportional to $\text{Im} \sum_{\mathbf{k}'} (C_s \rho_s \mathbf{k} \times \mathbf{k}' \cdot \mathbf{b})^2 \int d^3v (\omega'' - \omega_{de}'' + i0^+)^{-1} \omega^{-1} \{ (\omega_{*e}/\omega_{\mathbf{k}}) - (\omega_{*e}'/\omega_{\mathbf{k}'}) \} (1 + (E/T_e - 3/2) \eta_e) F_m I_{\mathbf{k}}' I_{\mathbf{k}}$; i.e., approximately the $\mathbf{k}$ -sum of the product of the intensity at k_θ and the intensity at the conjugate

wave number k_θ , $I(k_\theta)$. However, for low k_θ , $I(k_\theta)$ is extremely small and vice-versa for high k_θ . As a consequence, the product $I(k_\theta)I(k_\theta')$ is always small. Therefore, the dominant contribution is from $h_k^{(1)}$, which can be written in an integral form using a continuum approximation and straight-forward velocity space integration; i.e.,

$$\begin{aligned}\Gamma_e &= \text{Re} \sum_{\mathbf{k}} i k_\theta (cT_e / eB_0) \Phi_{-\mathbf{k}} \int d^3v h_{\mathbf{k}}^{(1)} \\ &= (cT_e / eB_0) (rS/q) 2(2\pi\epsilon)^{1/2} (R/GL_n)^{3/2} \int dk_\theta k_\theta^2 I(k_\theta) F(\Omega(k_\theta)) \int dx \exp(-x^2/w_n^2).\end{aligned}\quad (35)$$

Using Eq. (25) and after the radial integration over the mode width, the k_θ -integration becomes

$$\begin{aligned}&\pi^{1/2} (\beta L / S \beta_{ICS}) (C_s \rho_g / L_n)^4 k_M^{-1} \int d\theta \theta^{-2(1+\theta^2)-15/2} (1-\theta^2)^3 \\ &\exp(-R/GL_n(1+\theta^2)) \{1-(1+\theta^2)^{-1/2} + \eta_e (R(1+\theta^2)^{-1/2} / GL_n - 3/2)\} \\ &\int d\phi \phi^{-1(1+\phi^2)-1/2} \exp(-R/GL_n(1+\phi^2)) \{1-(1+\phi^2)^{-1/2} + \eta_e (R(1+\phi^2)^{-1/2} / GL_n - 3/2)\},\end{aligned}\quad (36)$$

where $\theta = k_\theta'/k_M$, $\phi = k_\theta/k_M$, and we have changed the order of integration. Since $\phi < \theta < 1$ and the integrand of θ -integration vanishes rapidly as $\theta \Rightarrow 0$, for realistic finite values of η_e , the ϕ -integration can be approximated by ignoring ϕ^2 in comparison to 1 without significantly influencing the final result. Then, the last factor of Eq. (36) becomes, after an elementary integration, $\exp(-R/GL_n) \eta_e (R/GL_n - 3/2) (-\ln(k_L/k_M) + \ln\theta)$. Now, it is straightforward to perform the θ -integration which is dominated by the small- θ behavior. Finally the expression for the particle flux is

$$\begin{aligned}\Gamma_e &\equiv -D^{\text{eff}} dn_0/dx \\ &= -(2^7 \epsilon / 15) (q^2/S^2) (T_e/T_i) \eta_e (1+5\eta_i/4)^{-1} [k_M/k_L - \ln(k_M/k_L) - 1] (k_M \rho_s) \\ &\quad (R/L_n)^2 (R/GL_n)^3 (R/GL_n - 3/2)^2 \exp(-2R/GL_n) \rho_s L T_e^{-1} (cT_e/eB) dn_0/dx.\end{aligned}\quad (37)$$

The electron thermal flux is calculated by the same method. Since the contribution from the nonlinear trapped electron scattering is negligible due to a small fluctuation level at high k_θ ($k_\theta > k_M$), the trapped electron thermal flux is given by

$$Q_e = \text{Re} \sum_{\mathbf{k}} i k_\theta (cT_e / eB_0) \Phi_{-\mathbf{k}} \int d^3v E h_{\mathbf{k}}^{(1)}.\quad (38)$$

Since the resonant trapped electrons are transported preferentially, the procedure of calculating

Q_e is very similar to the one used for the particle flux calculation. The only difference is that the energy moment rather than the density moment is involved in the integration here. The resonant energy of the trapped electron is given by $E_{res} = (R/GL_n(1+k_0^2/k_M^2))T_e$. Thus, there exists the following relation between Q_e and Γ_e ,

$$Q_e = (R/GL_n)T_e\Gamma_e. \quad (39)$$

The resulting trapped electron thermal flux can be explicitly written as

$$\begin{aligned} Q_e &\equiv -\chi_e^{eff} n_0 dT_e/dx \\ &= -(27\epsilon/15)(q^2/S^2)(T_e/T_i)(1+5\eta_i/4)^{-1}[k_M/k_L - \ln(k_M/k_L) - 1](k_M\rho_s) \\ &\quad (R/L_n)^2(R/GL_n)^4(R/GL_n - 3/2)^2 \exp(-2R/GL_n)\rho_s L T_e^{-1}(cT_e/eB)n_0 dT_e/dx. \end{aligned} \quad (40)$$

The ion thermal flux is also calculated in this Section. As mentioned before, for most of the k_0 -range of interest where the fluctuation is appreciable, the magnetic shear-induced ion Landau damping does not affect the nature of fluctuation structure. Hence, the linear ion Landau damping has no significant direct effect on the ion thermal transport, and the ion Compton scattering plays a dominant role in determining the amount of ion heat flux driven by the fluctuations. Keeping only the dominant nonlinear (ion Compton scattering) contribution gives,

$$Q_i = -\sum_{\mathbf{k}} k_0 (cT_e/eB_0) \Phi_{-\mathbf{k}} \text{Im} \int d^3v E J_0 g_{\mathbf{k}}^{(3)}, \quad (41)$$

where the expression for $g^{(3)}$ is available from Eq. (13). For relatively long wavelength fluctuations, the Bessel function, J_0 , can be approximated by 1, and the expression for the velocity space resonant integral for the Maxwellian back-ground distribution function becomes

$(2\pi)^{-1/2} \int dv_{\parallel} \int dv_{\perp} v_{\perp} \{ (v_{\parallel}^2 + v_{\perp}^2)/2 \} \text{Im}(\omega - k_{\parallel} v_{\parallel} + i0^+)^{-1} \{ 1 + ((v_{\parallel}^2 + v_{\perp}^2)/2 - 3/2)\eta_i \} \exp\{ -(v_{\parallel}^2 + v_{\perp}^2)/2 \}$, where $v_{\parallel}$ and $v_{\perp}$ have been normalized to v_{Ti} . Using Premelji's formula for the wave-particle resonance, it is straight-forward to evaluate the integral which yields $(\pi/2)^{1/2} (|k_{\parallel}| v_{Ti})^{-1} \{ \eta_i (|\omega|^2/2 (|k_{\parallel}| v_{Ti})^2)^2 + (1 + |\omega|^2/2 (|k_{\parallel}| v_{Ti})^2) (1 + \eta_i/2) \} \exp\{ -|\omega|^2/2 (|k_{\parallel}| v_{Ti})^2 \}$. The remaining dx and dk_0 integrals for mode summations are similar

to the previous calculations for Γ_e and Q_e , and are not repeated here. The final expression for Q_i is given by

$$\begin{aligned} Q_i &\equiv -\chi_i^{\text{eff}} n_0 dT_i/dx \\ &= 2.75 (1 + 1.93\eta_i) (1 + 1.25\eta_i)^{-1} T_i \Gamma_e. \end{aligned} \quad (42)$$

Finally, the anomalous heat exchange rate between two species in the presence of electrostatic fluctuations can be calculated using

$$\partial/\partial t W_j = e_j \text{Re} \sum_{\mathbf{k}} \Phi_{-\mathbf{k}} \partial/\partial t \int d^3v (h_{\mathbf{k}}^{(1)} + h_{\mathbf{k}}^{(3)})_j, \quad (43)$$

with $\sum_j \partial/\partial t W_j = 0$. For electrons, the anomalous "cooling" rate is then roughly estimated to be given by $Q_{ei} \equiv -T_e \Gamma_e / L_n$. This expression, which is similar to the result for dissipative trapped electron modes,³⁸ shows that the electrons lose energy to the fluctuation (the wave) while the ions gain energy from the fluctuation at the same rate thereby keeping the fluctuation amplitude at the saturated level.

V. Discussions

In this paper, we have developed a weak turbulence theory of CTEM in toroidal geometry in the presence of both ion and electron temperature gradients treating ion Compton scattering and the trapped electron scattering on an equal footing. It is found that both nonlinearities are important in determining the spectral shape of the saturated potential fluctuations. This shape is given by an approximate power law, k_θ^{-3} , and extends from $k_\theta \rho_s \equiv (L_n/R)^{1/2}$ to $k_\theta \rho_s \equiv 1$. The corresponding fluctuation amplitude is found to be smaller than the mixing length estimate. Fluctuation-induced anomalous particle and heat fluxes have also been obtained and are again found to be smaller than the mixing length estimates. As indicated by Eqs. (37), (40) and (42), various fluxes obey similar scalings. The plasma confinement is

predicted to improve with i) higher T_i/T_e , ii) more peaked density profile, iii) larger aspect ratio, and iv) higher plasma current. In addition, the results indicate a significant dependence of transport on the electron temperature gradient. This might be relevant to the rigidity of the electron temperature profiles¹⁸ observed in many tokamak plasmas.¹⁹

The favorable dependence on T_i/T_e is a consequence of ion Compton scattering (which is obviously stronger for higher v_{Ti}) being an important nonlinear saturation mechanism. This favorable dependence on T_i/T_e is almost universally obtained in hot-ion mode operation of the large tokamaks^{6,39,40} where T_i/T_e can be as large as 3-4.^{6,39} In contrast, many previous theories of drift wave turbulence were developed using the familiar cold ion ($T_i/T_e \ll 1$) assumption.

The calculated confinement improvement with density peaking⁴¹ is a consequence of the particular destabilizing mechanism⁸ considered in this paper; i.e., the precession drift resonance of the trapped electrons with waves. As the density profile gets more peaked, the wave frequency ($\approx \omega_{*e}$) increases and detunes from the precession drift frequency of thermal trapped electrons. From the experimental side, the confinement improvement with density peaking has been a familiar trend observed in a number of pellet experiments⁴²⁻⁴⁴ and has frequently been quoted as evidence supporting the relevance of ITG-modes. However, it should be remembered that the present analysis is valid in a low collisionality (CTEM) regime. In this sense, it is more meaningful to compare the present results to TFTR auxiliarily heated plasmas where the confinement improvement from L-mode to Supershot can be characterized by the density peakedness.^{6,45} It is also interesting to note that for these shots, the dependence of ITG-mode threshold conditions on the density gradient scale length (L_n) is rather weak due to high values of T_i/T_e .^{46,47}

The aspect ratio (ϵ^{-1}) dependence found in the present calculations comes from the fact that the fraction of trapped electrons is proportional to $\epsilon^{1/2}$. Our weak turbulence analysis

indicates that the anomalous particle and heat fluxes are proportional to ϵ , while simple mixing length estimates³⁻⁵ predict an $\epsilon^{1/2}$ dependence. Recent results from TFTR⁴⁸ tend to be in somewhat better agreement with our result.

The absence of a significant scaling of local transport with current (I_p) and/or the edge safety factor ($q(a)$) in most drift-type microinstability theories has been a persistent and challenging problem. This is a very relevant issue because a strong scaling of this kind has in fact been found in empirically-deduced global energy confinement times^{49,50} for auxiliary-heated tokamak plasmas. It is, therefore, important to note that the results of our present analysis of collisionless trapped-electron modes does indeed exhibit a significant local $q(r)$ dependence. As pointed out in Ref. 21, such behavior can be explained in part by the fact that as $q(a)$ (and the associated local value of S) increases, the equilibrium value for the trapped-electron orbit-averaged magnetic precession drifts ($\propto G$) becomes increasingly unfavorable for $q(a)$ values up to 7 or so. Moreover, the favorable transit ion nonlinear (as well as linear) Landau damping effects are likewise enhanced for lower $q(r)$ values. While the present results are suggestive of the proper trend, care should be exercised in applying them to specific experimental scenarios. For example, although the confinement zone of TFTR supershots falls well within the CTEM regime, no appreciable current dependence is observed. This could be due to the fact that the density profiles in these discharges were observed to steepen more as $q(a)$ increases (i.e., I_p decreases). Hence, the unfavorable influence of decreasing I_p might be offset by the aforementioned predicted improvement of confinement with density peaking.

In the present study a significant dependence of transport on the electron temperature gradient (∇T_e) is found (i.e. ; Q_e , Γ_e , and $Q_i \propto L_{Te}^{-2}$). Such a trend is naturally expected since the electron temperature gradient can often be a more effective source of free energy than the density gradient in driving microinstabilities. It is well known⁵¹ that if the electron

temperature gradient is ignored for these modes, A much weaker version of these instabilities (requiring finite ion gyroradius reduction of the drift frequency) results. Accordingly, it is important to exercise caution in the application of those drift wave turbulence theories which ignore the electron temperature gradient to the interpretation of confinement trends observed in tokamak plasmas.

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Figures

FIG. 1. Plot of spectral intensity, $I(k_\theta)$, in logarithmic scale. Three different asymptotic regions in k_θ are identified. The region "a" corresponds to an extremely long wavelength regime where fluctuations are suppressed due to the trapped electron scattering. The region "b" corresponds to a long wavelength regime where most of the fluctuation energy is populated and $I(k_\theta)$ decays according to a power law, k_θ^{-3} , given in Eq. (27). Finally, region "c" corresponds to a relatively shorter wavelength regime, $k_\theta > k_M$, where $I(k_\theta)$ decays more steeply.

Fig. 1

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