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AN INVESTIGATION OF
SINGLE-SUBSTANCE
HORIZONTAL TWO-PHASE FLOW

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HORIZONTAL TWO-PHASE FLOW

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NOTE

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ABSTRACT

Despite the abundance of work in the field of two-phase flow, it seems as though a consensus has not been reached on some of the fundamental points. Although exceptions exist, adequate physical interpretation of the flow seems to be hindered either by complexity of analysis or, in the opposite extreme, the trend toward limited-range analysis and correlations. The dissertation presents the derivation of basic conservation equations for the phases. The combined equations are used to examine the phenomenon of "slip" and its practical limitations, the Fanno line for single-substance flow and the effect of slip on choking. Equations for critical mass flux in the presence of slip are derived. The Mach, Reynolds and Froude numbers based on conditions at flashing are introduced as the characteristic parameters, and the importance of compressibility in single-substance two-phase flow is discussed. Experimental measurements of pressure change and void fraction for flow in the highly compressible range ($.5 < Ma < 1$) are presented. The working fluid is Refrigerant R-114, at room temperature, in a test section of diameter 5cm and length 8m. The effect of the Froude and Mach numbers is examined. The experimental facility is operated intermittently with running times of approximately two minutes and is instrumented for rapid measurements using a computer data acquisition and control system. A description of the facility and procedure is provided.

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I. INTRODUCTION

Two-phase flow, as a branch of fluid mechanics, has commanded a great deal of attention for the last forty years. A recent selective literature guide is that of Ref. [1]. An earlier, very complete survey was compiled by Gouse [2]. Originally motivation for research came from the two-phase flow aspect of many chemical engineering processes. More recently work has been concentrated on specific problems involved with production of electricity from geothermal energy and design of the basic power cycle and safety systems for nuclear plants. However, the number of approaches taken to two-phase flow is nearly as large as the number of investigators and it seems as though a consensus has not been reached on some of the most basic questions.

One approach taken during the earliest work on the subject was that of direct correlation of experimental data for pressure change and void fraction, most of it for two-substance flow such as air-water. This is represented by a series of papers by Martinelli and co-workers in the 1940's [3,4]. Despite an abundance of work over the next twenty years, the Lockhart-Martinelli correlation [3] was found in 1964 [5] to give generally the best results when compared to other correlations and the growing body of experimental data. This was true even for single-substance flow (e.g. steam-water), although with considerable error. Recent correlations have been shown to give more accurate results [6,7] but that of Martinelli is still among the most widely used. The theoretical basis has been examined in detail and various extensions and modifications have been proposed [8,9,10,11].

The early correlations were independent of the flow configuration or "pattern" existing within the duct. The observation of different types of patterns ("bubbly," stratified, etc.) sparked an alternate approach that originated at approximately the same time. This was the flow map, which sought to determine when the different patterns would exist and when transitions would occur. One of the earliest diagrams was given by Kosterin in 1949 [12], but that of Baker [13] in 1954 was more widely accepted and has provided the basis for later versions. The need to determine flow patterns came about because of the direct effect that it was observed to have in various two-phase processes. For example, the "holdup" or liquid void fraction, related to production in oil-gas wells, was clearly different for different flow types.

It was recognized that the general methods, like Martinelli's, gave more accurate predictions for some flow patterns than others. This resulted in pattern-dependent correlations, applicable only over the range of conditions where a particular flow existed. The beginning of this trend is especially clear in a 1959 paper by Hoogendoorn [14]. It was determined that particular flow patterns could be analyzed based on investigations such as that of Taylor [15] for bubbles. Specific "pattern models" introduced during the 1960's are the "slug" flow of Griffith-Wallis [16] and "annular" flow of Levy [17]. The development of correlations and models which depend on flow type has provided impetus to the development of the flow map for accurate prediction of pattern

transitions, and work has continued right up to the present [18,19]. Some of this has sought to eliminate the uncertainty of visual observation by the measurement of pressure fluctuation as pattern identification [20].

A number of investigators have attempted to break away from the restrictions inherent in some of the approaches just described. Examples are the similarity analysis of Dukler [21], the mixing length approach of Levy [22,23] and that of Huey and Bryant [24] based on gas dynamics. Perhaps the simplest approach, similar to the latter, is to consider the flow as a "homogeneous" substance which behaves as a simple compressible fluid [25]. This allows for some simplifications, including straightforward determination of compressibility and hence choking characteristics [26]. A typical problem involves the definition of a mixture viscosity and a number of combination rules (of questionable physical significance) using the phase viscosities have appeared. Of course, the homogeneous flow actually represents a pattern-model of sorts and so it works well in some cases. Bankoff [27] considered density and velocity distributions within the cross section while Levy [28] considered the phases individually. The latter technique ultimately requires closures for phase interactions, such as interface shear stress. The use of closures based on phase configuration (e.g. drag on droplets or bubbles) once again restricts applicability of the general equations to certain ranges of flow conditions.

Recent work has demonstrated a renewed interest in determining the

proper basic parameters to characterize a two-phase flow, such as that of Chesters [29] and Friedel and Mayinger [30], and increasing recognition of the importance of compressibility in determining the behavior of a single-substance flow [31]. It is becoming increasingly clear that the long-held postulate [4] that friction is the same with or without phase change, and that acceleration can be merely superimposed, is not valid. Recent texts [32,33,34] provide a good summary and references of more recent work. Within this, two opposing trends are apparent. The first is a realization of the need for better basic understanding of the flow, while the second seems to be an insistence on developing methods which by nature tend to restrict this understanding. For example, renewed interest in the flow map and limited-range correlations, in some cases, may indicate that two-phase flow research is about to come full circle instead of moving ahead. The range of applicability of the theory needs to be expanded, not shortened. Present interpretations of the state of the research range from the feeling that the problem has been completely solved to one of frustration over the inadequacy of present knowledge. The former is contradicted by the amount of work still being done. The latter was summarized well in a very recent paper [7] which mentioned the lack of "satisfactory methods" and the need for "firmer theoretical bases." The frustration is underscored by comparing these comments to some written over twenty years earlier [35], which are essentially the same. What is needed is a better understanding of the basic physical processes, which will lead to more general and more accurate predictions over the full range of

two-phase conditions.

The statements above describe the goal of the research presented here. It is intended to provide the first steps in a unified approach to the two-phase flow problem.

II. DERIVATION OF A BASIC SET OF GOVERNING EQUATIONS FOR TWO-PHASE FLOW

1. Remarks

The equations that are presented describe the behavior of a flow involving two phases (gas and liquid) in an adiabatic duct of constant cross-sectional area, and horizontal orientation. The equations will be written, in general, for a single-substance flow (liquid and vapor of a single substance) where mass transfer between phases is possible due to evaporation or condensation. Using the general formulation, different flows can be considered as special cases. An example is the flow of air and water (two-substance) where the amount of mass in each phase remains constant and mass exchange does not occur as the phases move through the duct. Only flow under steady-state conditions will be considered. That is, mass flow rate, stagnation conditions and duct exit conditions remain constant, and no time derivatives appear.

A total of six equations will be used: energy and momentum conservation and continuity (mass conservation), with each equation written for both phases. The flow at a given cross section will be described by individual variables for each phase (density, enthalpy, velocity, etc.). These variables represent time averages of the same fluctuating quantities in the turbulent two-phase flow. Each variable is taken to be uniform throughout the portion of the cross section containing the phase that it describes, and only change in a variable along the flow will be considered. Hence, the equations describe an

arrangement which is uniform and one-dimensional in each phase. The difference between mass, energy and momentum flux calculated based on uniform variables and those obtained by integration over the actual spatial distribution will be assumed secondary and negligible for present purposes. Where a distribution results from a primary effect within the flow, an equivalent mechanism may be substituted so that the effect is accounted for in the equations. Such is the case of interfacial shear stress, representing the effect of velocity gradients present in the shear layer between phases.

Thermodynamic equilibrium is assumed between phases, with equal temperature and pressure at a given cross section. The meaning of equilibrium in this type of flow will be discussed in more detail and the origin and effect of pressure and temperature differences between phases will be examined.

The description of the flow just presented is best compared to "separated flow models" or "two-fluid models," since the phases are considered individually. Unfortunately both labels tend to imply a lack of interaction between the phases, which is not the case. The present treatment is different from the "homogeneous flow model," which treats the two-phase fluid as a simple compressible fluid with a single uniform velocity and set of properties. It is also distinct from the various "flow pattern models" in which analysis is based on the assumption of a particular geometric arrangement of the phases. The present analysis allows for an understanding of the flow which is not possible with the two latter approaches and which can be lost in a

more complex approach, as would be required for obtaining a detailed description of local behavior. However, it is necessary that its limitations be remembered clearly at each step.

2. Continuity Equation and Basic Definitions

The coordinate z will be used to specify position along the horizontal duct, with its value increasing in the direction of flow. Each phase conservation equation will be written by considering a duct segment of differential length dz . A differential of a phase variable will denote change in that variable as the flow moves from a cross-section at position z to one at $z + dz$. With this approach established, the mass balance for a phase is expressed by the equation

$$\rho_p w_p A_p + d(\dot{m}_p) = (\rho_p + d\rho_p)(w_p + dw_p)(A_p + dA_p). \quad (1)$$

The quantities ρ_p, w_p and $\dot{m}_p$ denote density, velocity and mass flow rate of phase "p" at position z . The variable A_p is the area occupied by the phase within the total cross-section area A at this position. For two-phase flow the subscript will be replaced by "f" or "g", corresponding to liquid or vapor phases, respectively, and following the usual convention in thermodynamics. When this equation is expanded and simplified by neglecting terms of second order magnitude the result is

$$d(\dot{m}_p) = d(\rho_p w_p A_p). \quad (2)$$

When integrated this gives

$$\dot{m}_p = \rho_p w_p A_p. \quad (3)$$

It is convenient to introduce the mass flux ψ :

$$\psi = \dot{m}/A, \quad (4)$$

where $\dot{m}$ is the total mass flow rate. The mass and area fractions of the individual phases will be introduced as

$$x_p = \dot{m}_p/\dot{m}, \quad \alpha_p = A_p/A. \quad (5a,b)$$

The following equations must be satisfied, based on the above definitions:

$$\sum_p \dot{m}_p = \dot{m}, \quad \sum_p x_p = 1 \quad (6a,b)$$

$$\sum_p A_p = A, \quad \sum_p \alpha_p = 1. \quad (7a,b)$$

The differentials of x_p and α_p must satisfy the equations

$$\sum_p dx_p = 0, \quad \sum_p d\alpha_p = 0 \quad (8a,b)$$

as obtained from Eqs. (6b) and (7b). All of the equations being developed may be generalized to more than two phases by applying the summation formalism to the appropriate terms. This will be done where it is particularly convenient or lends clarity to the equations, although the present analysis is concerned basically with only two phases.

Note that the mass flux of a phase may be defined and expressed in a number of ways:

$$\psi_p = \dot{m}_p / A_p = \rho_p w_p = \frac{x_p}{\alpha_p} \psi. \quad (9)$$

Since no mass enters or leaves the flow through the duct walls, the quantity $d(\dot{m}_p)$ refers to mass transfer between phases only. Hence, the requirement

$$\sum_p d(\dot{m}_p) = 0 \quad \text{or} \quad d(\dot{m}) = 0 \quad (10a,b)$$

must be satisfied. This allows $d(\dot{m}_p)$ to be written

$$d(\dot{m}_p) = \dot{m} dx_p. \quad (11)$$

Similarly, the cross-section area A is constant along the duct and

$$\sum_p d(A_p) = 0 \quad \text{or} \quad d(A) = 0. \quad (12a,b)$$

The phase continuity equation is obtained by combining Eq. (2) with the above expressions for $d(\dot{m}_p)$ and $d(A)$, and the definitions of x_p, α_p and ψ :

$$\psi dx_p = d(\rho_p w_p \alpha_p). \quad (13)$$

When integrated, this gives

$$\psi x_p = \rho_p w_p \alpha_p. \quad (14)$$

The sum of Eq. (13) for each phase results in the total flow continuity equation,

$$d \left(\sum_p \rho_p w_p \alpha_p \right) = 0 \quad \text{or} \quad d(\psi) = 0 \quad (15a,b)$$

where the conditions (8a) have been applied.

For two-phase flow, it is necessary to specify only a single mass fraction and single void fraction, since the other in each case can be obtained from Eqs. (6b) and (7b), respectively. Based on the usual convention in thermodynamics, the mass fraction x will be defined as that of the vapor:

$$x = x_g; x_f = 1 - x. \quad (16a,b)$$

Similarly the void fraction will be defined as

$$\alpha = \alpha_g; \alpha_f = 1 - \alpha. \quad (17a,b)$$

With these definitions, the mass flux in Eq. (15b) may be expressed as

$$\psi = \rho_f w_f (1 - \alpha) + \rho_g w_g \alpha. \quad (18)$$

It will be convenient to introduce the velocity and density ratios of the phases:

$$\sigma = w_g/w_f, \quad \gamma = \rho_f/\rho_g. \quad (19a,b)$$

Hence, "no slip" refers to $\sigma = 1$. When Eq. (14) is written for both phases and ψ is eliminated the result is

$$\frac{\rho_g w_g \alpha}{x_g} = \frac{\rho_f w_f \alpha_f}{x_f}. \quad (20)$$

With the definition of σ, γ, α and x this can be written

$$\sigma/\gamma = \left(\frac{x}{1-x} \right) \left(\frac{1-\alpha}{\alpha} \right). \quad (21)$$

This will yield an expression for void fraction which is useful:

$$\alpha = 1/\left\{ \frac{\sigma}{\gamma} \left(\frac{1-x}{x} \right) + 1 \right\}. \quad (22)$$

Another quantity frequently referred to with respect to two-phase flow is the "holdup", or liquid volume fraction $\alpha_f = 1 - \alpha$:

$$(1-\alpha) = 1/\left\{ \frac{\gamma}{\sigma} \left(\frac{x}{1-x} \right) + 1 \right\}. \quad (23)$$

Before closing this section, the difference between "in-place" and "flowing" quantities should be noted. The variable x is described as a "flowing" quantity since it is the ratio of vapor mass flow rate to total flow rate. The variable α is an "in-place" quantity since it gives the ratio of the vapor volume to the total volume at a given cross section. (The area ratio is equivalent to a volume ratio if one imagines an infinitesimal length of duct δz over which α is constant.) The mass fraction x may be written in the form

$$x = \rho_g w_g A_g / \{ \rho_g w_g A_g + \rho_f w_f A_f \} \quad (24a)$$

or

$$x = 1/\left\{ 1 + \frac{\gamma}{\sigma} \left(\frac{1-\alpha}{\alpha} \right) \right\}. \quad (24b)$$

An "in-place" mass fraction x_0 may be defined:

$$x_0 = \rho_g A_g / \{\rho_g A_g + \rho_f A_f\} \quad (25a)$$

or

$$x_0 = 1 / \{1 + \gamma(\frac{1-\alpha}{\alpha})\}. \quad (25b)$$

The quantities x and x_0 differ only by σ , and are identical when $\sigma = 1$. This illustrates clearly the importance of accounting for phase velocities properly when defining properties of the two-phase fluid. For calculating thermodynamic properties in the equations that follow, x is the proper parameter. This will become obvious when the energy equation is developed.

The term "slip" used in the above discussion is not intended to imply that slip actually occurs at the interface. Rather it is used, as is conventional in the two-phase flow literature, to signify that the average phase velocities are different.

A sketch of the duct segment dz is given in Fig. 1, showing some of the quantities (w_p, A_p , etc.) that have been introduced in this section and some that will be used for derivation of the energy and momentum equations. It must be emphasized that for clarity a very simple phase arrangement has been shown. The equations are general and apply to more complex phase structures (annular or bubble flow, etc.).

3. Energy Equation

The energy balance for a phase, written for duct length dz is expressed by the equation

$$\begin{aligned} \dot{m}(x_p + dx_p)(h_p + dh_p + w_{p/2}^2 + d(w_{p/2}^2)) \\ - \dot{m}_p(h_p + w_{p/2}^2) - d\dot{m}_p(h_i + w_{i/2}^2) = \dot{q}_p L_i dz - \tau_{ip} L_i w_i dz \end{aligned} \quad (26)$$

where h_p denotes the phase enthalpy and $w_p^2/2$ its kinetic energy. The left hand side expresses the energy change of mass crossing the phase control volume surface. The energy supplied by heat transfer and work interaction with the other phase is given on the right-hand side. The quantity $\dot{q}_p$ is the heat flux, defined as positive when heat flows into the phase. Since the duct is adiabatic, heat transfer occurs only internally. If L_i is the length of contact between phases in a given cross section, heat transfer occurs through an area $L_i dz$. The adiabatic restriction results in the requirement

$$\sum_p \dot{q}_p = 0. \quad (27)$$

The quantity τ_{ip} is the shear stress at the interface between phases. The area affected by the stress is the same as that for the heat flux, $L_i dz$. The point of action of the resulting force $\tau_{ip} L_i dz$ moves at an "interface velocity" w_i and work is performed at the rate $\tau_{ip} L_i w_i dz$.

Subscripts i and p placed in the order τ_{ip} indicate stress

exerted on the interface by the phase, taken as positive in the flow direction. A reversal of the order causes a change of sign:

$$\tau_{ip} = -\tau_{pi}. \quad (28)$$

This convention has been introduced since the ability to change sign is convenient in developing the different equations. In the energy equation, the stress exerted by the phase is important, whereas in the momentum equation the stress exerted on the phase is important.

Stress exerted at the interface must be matched between phases, resulting in the requirement

$$\tau_{ig} = \tau_{fi}. \quad (29)$$

The last term on the left-hand side of Eq. (26) gives the energy contribution due to a net mass transfer between phases. Its simple form results since mass entering and leaving the phase has been assigned the same properties h_i and w_i . Equivalently, each quantity of mass passes through the same intermediate state as it changes phase. The selection of a single velocity w_i is necessary because of the effect of the interface shear layer. Thus, the change in kinetic energy which is experienced by the transferred mass is apportioned properly to each phase. It is clear that this interface velocity must be the same as that appearing in the τ_{ip} term.

When Eq. (26) is expanded and simplified the result is

$$\psi d\{x_p(h_p + w_p^2/2)\} = \dot{q}_p \frac{L_i}{A} dz - \tau_{ip} \frac{L_i}{A} w_i dz + \psi dx_p(h_i + w_i^2/2). \quad (30)$$

It is useful to introduce the following normalized quantities, where C is the interior pipe circumference and R_h the hydraulic radius:

$$\tilde{L}_i = L_i/C, R_h = A/C, \tilde{z} = z/R_h \quad (31a,b,c)$$

$$\bar{q}_p = \dot{q}_p \tilde{L}_i, \bar{\tau}_{ip} = \tau_{ip} \tilde{L}_i. \quad (31d,e)$$

Equation (30) then becomes

$$\psi \frac{d}{d\tilde{z}} \{x_p (h_p + w_{p/2}^2)\} = \bar{q}_p - \bar{\tau}_{ip} w_i + \psi \frac{dx_p}{d\tilde{z}} (h_i + w_{i/2}^2). \quad (32)$$

Note that $d(R_h) = 0$ has been used, implying constant cross-section configuration.

The sum of Eq. (32) for both phases, along with the conditions (27, 28, 29) results in the total flow energy conservation equation:

$$d\{h_g x + h_f(1-x) + w_{g/2}^2 x + w_{f/2}^2(1-x)\} = 0 \quad (33)$$

or

$$d\{h_0\} = 0 \quad (34)$$

where the sum of enthalpy and kinetic energy of the two-phase flow (its stagnation enthalpy) has been denoted by h_0 . In a non-adiabatic flow a heat transfer term would appear on the right-hand sides of Eqs. (33) and (34) expressing the net heat added from outside the duct.

Equation (33) can be written in the more general form

$$d \left\{ \sum_p x_p (h_p + w_p^2/2) \right\} = 0. \quad (35)$$

Note that the enthalpy of the two-phase fluid is defined here as

$$h = h_g x + h_f (1-x) \quad (36)$$

where x , based on mass flow rate, is the proper thermodynamic variable. The same holds true for specifying other properties, including entropy (s) and specific volume (v). For a single substance flow the individual phase properties correspond to properties at saturation conditions (since equilibrium is assumed) and only pressure or temperature is required for them to be determined. The mass fraction x is then an additional variable sufficient to determine the state of the two-phase fluid. In a two-substance flow temperature and pressure may vary independently, and each must be known for specification of the phase properties.

4. Momentum Equation

The momentum balance for an individual phase is expressed by the equation

$$\begin{aligned} \dot{m}(x_p + dx_p)(w_p + dw_p) - \dot{m}x_p w_p - w_i d\dot{m}_p &= \tau_{pw} L_p dz + \tau_{pi} L_i dz \\ &+ P A_p + P dA_p - (P + dP)(A_p + dA_p) \end{aligned} \quad (37)$$

where P is the uniform pressure at the cross section. The left-hand side gives the net momentum crossing the phase control volume surface,

and the right-hand side is a sum of forces.

The wall shear stress τ_{pw} follows the same sign convention on direction and subscript order as τ_{pi} :

$$\tau_{pw} = -\tau_{wp} \quad (38)$$

The quantity L_p is the length of interior duct circumference contacted by the phase at a given cross section, and $L_p dz$ the area on which the stress acts. Note that the momentum contribution due to mass transfer is given in terms of the interface velocity w_i .

When Eq. (37) is expanded and simplified the result is

$$\psi \frac{d}{dz} \{x_p w_p\} = -\alpha_p \frac{dP}{dz} + \bar{\tau}_{pw} + \bar{\tau}_{pi} + \psi \frac{dx_p}{dz} w_i \quad (39)$$

where the additional normalized quantities

$$\tilde{L}_p = L_p/C, \quad \bar{\tau}_{pw} = \tau_{pw} \tilde{L}_p \quad (40)$$

have been introduced. When Eq. (39) is written for the phases and summed, the result is

$$\psi \frac{d}{dz} \{x w_g + (1-x) w_f\} = - \frac{dP}{dz} + \bar{\tau}_{gw} + \bar{\tau}_{fw} \quad (41)$$

The left-hand side is the differential of the total momentum. The right-hand side is the sum of shear and pressure forces acting on the total flow.

Equation (41) can be written in the more general form

$$\psi \frac{d}{d\tilde{z}} \left\{ \sum_p x_p w_p \right\} = - \frac{dP}{d\tilde{z}} + \sum_p \bar{\tau}_{pw}. \quad (42)$$

5. Gibbs Equation and Dissipation Equation

The void fraction α_p can be replaced in the phase momentum equation using Eq. (14) and the resulting equation multiplied by w_p . Then subtracting it from the phase energy equation, Eq. (32), yields

$$\begin{aligned} \psi x_p (dh_p - \frac{dP}{\rho_p}) / d\tilde{z} &= \bar{q}_p + \bar{\tau}_{wp} w_p + \bar{\tau}_{ip} (w_p - w_i) \\ &+ \psi \frac{dx_p}{d\tilde{z}} (h_i - h_p) \\ &+ \psi \frac{dx_p}{d\tilde{z}} (w_i^2/2 - w_p^2/2) \\ &+ \psi \frac{dx_p}{d\tilde{z}} (w_p - w_i) w_p. \end{aligned} \quad (43)$$

The phase Gibbs equation is

$$dh_p = T ds_p + \frac{dP}{\rho_p}. \quad (44)$$

With this substitution, the left-hand side of Eq. (43) becomes

$$\psi x_p (dh_p - \frac{dP}{\rho_p}) / d\tilde{z} = \psi x_p T ds_p / d\tilde{z}. \quad (45)$$

Hence each of the terms on the right-hand side of Eq. (43) can be identified as a contribution to the phase entropy change. In addition to the heat transfer and shear stress terms, the enthalpy and kinetic energy of mass entering from the interface also contribute to this change in entropy. The last term on the right-hand side represents work dissipated by an apparent shear stress on the phase due to mass entering at a different velocity. The velocity terms may be combined in an alternate manner in terms of a kinetic energy relative to the interface. When this is done, and Eq. (45) is used, Eq. (43) becomes

$$\begin{aligned} \psi x_p T \frac{ds}{dz} = & \bar{q}_p + \bar{\tau}_{wp} w_p + \bar{\tau}_{ip} (w_p - w_i) \\ & + \psi \frac{dx_p}{dz} (h_i - h_p) + \psi \frac{dx_p}{dz} (w_p - w_i)^2 / 2. \end{aligned} \quad (46)$$

The last term on the right-hand side is the kinetic energy relative to the moving interface frame of reference.

The phase Gibbs free energy is defined as

$$g_p = h_p - Ts_p. \quad (47)$$

When this definition is used and Eq. (46) is summed for both phases, the result is

$$\begin{aligned} \psi T \frac{ds}{dz} = & \bar{\tau}_{wg} w_g + \bar{\tau}_i (w_g - w_f) + \bar{\tau}_{wf} w_f \\ & + \psi \frac{dx}{dz} (g_f^o - g_g^o) + \psi \frac{dx}{dz} (w_g - w_i) w_g + \psi \frac{dx}{dz} (w_i - w_f) w_f. \end{aligned} \quad (48)$$

The interface shear stress $\bar{\tau}_i$ has been defined as $\bar{\tau}_i = \bar{\tau}_{ig}$. The left-hand side expresses the differential of the total entropy, where

$$s = s_g x + (1-x)s_f. \quad (49)$$

In addition, the g_p^0 terms have been defined, in general, as

$$g_p^0 = g_p + w_p^2/2. \quad (50)$$

The apparent shear stress terms may be combined with the g_p^0 terms with the result

$$\psi T \frac{ds}{dz} = \bar{\tau}_{wg} w_g + \bar{\tau}_i (w_g - w_f) + \bar{\tau}_{wf} w_f + \psi \frac{dx}{dz} (g_f' - g_g') \quad (51)$$

where, in general,

$$g_p' = g_p - (w_p - w_i)^2/2. \quad (52)$$

The physical significance of such terms is discussed in more detail in Appendix A.

The interface velocity w_i will be within the range between the individual phase velocities w_f and w_g ; its actual value will be determined within the flow by the details of the velocity profile near the interface, the required matching of the stress τ_i for each phase at this point, and the details of the interface itself. Within the

limits of the present approach, the interface velocity w_i may be set to the average of the phase velocities. The phase free energies g_p will be set to the corresponding phase properties at saturation at the uniform flow temperature T . Hence

$$w_i = (w_f + w_g)/2 = \bar{w} \quad (53a,b)$$

$$g_f = g_g.$$

The last term in Eq. (51) may be written, in general, in the form

$$\psi \frac{dx}{dz} \{ (g_f - g_g) + (w_g - w_f)(\bar{w} - w_i) \}. \quad (54)$$

With Eqs. (53a,b), each term vanishes and the total flow dissipation equation (Eq. (51)) becomes

$$\psi T \frac{ds}{dz} = \bar{\tau}_{wg} w_g + \bar{\tau}_i (w_g - w_f) + \bar{\tau}_{wf} w_f. \quad (55)$$

6. Summary of Equations and Additional Definitions

(i) Continuity Equation

The continuity equation was derived previously (Eq. 15a) as

$$\frac{d}{dz} (\sum_p \rho_p w_p \alpha_p) = 0. \quad (56)$$

This can be written in convenient form by defining the "volumetric velocity" (or "volumetric flux density" [46]) w_v :

$$w_v = \dot{V}/A \quad (57a)$$

where $\dot{V}$ is the volume flow rate. This can be expressed in terms of the phase velocities as

$$w_v = (w_f A_f + w_g A_g)/A = w_f(1-\alpha) + w_g \alpha \quad (57b)$$

or, in general

$$w_v = \sum_p w_p \alpha_p. \quad (57c)$$

In terms of the mass flux, w_v can be written

$$w_v = \frac{\rho \dot{V}}{\rho A} = \frac{\dot{m}}{\rho A} = \psi/\rho \quad (58a)$$

where

$$\rho = \frac{1}{v} = 1/\{v_f(1-x) + v_g x\}. \quad (58b)$$

Clearly, w_v is a "center of volume" velocity. It corresponds to the velocity the two-phase flow would have with the condition $\sigma = 1$.

From the above definitions the continuity equation will be written

$$\frac{d}{d\tilde{z}} (\rho w_v) = 0. \quad (59)$$

(ii) Momentum equation

The momentum equation was presented in Eq. (39) as

$$\psi \frac{d}{d\tilde{z}} \{xw_g + (1-x)w_f\} = -\frac{dP}{d\tilde{z}} + \bar{\tau}_{gw} + \bar{\tau}_{fw}. \quad (60)$$

This can be written in the form

$$\psi \frac{d}{d\tilde{z}} w_{cm} = -\frac{dP}{d\tilde{z}} + \bar{\tau}_{gw} + \bar{\tau}_{fw} \quad (61)$$

if one defines the "center of mass velocity"

$$w_{cm} = \sum_p x_p w_p. \quad (62)$$

(iii) Energy equation

The energy equation was given in Eq. (33) as

$$d\{xh_g + (1-x)h_f + w_{g/2}^2 + (1-x)w_{f/2}^2\} = 0. \quad (63)$$

This can be written in the concise form

$$\psi \frac{d}{d\tilde{z}} \left\{ h + \frac{w_{rms}^2}{2} \right\} = 0 \quad (64)$$

by introducing a "root mean square velocity" defined as

$$w_{rms} = \left(\sum_p x_p w_p^2 \right)^{1/2} = \{ w_{cm}^2 + \sum_p x_p (w_p - w_{cm})^2 \}^{1/2}. \quad (65)$$

This equation is easily integrated:

$$h + \frac{w_{rms}^2}{2} = h_0. \quad (66)$$

The quantity h_0 denotes the stagnation enthalpy as before.

(iv) Entropy equation

The entropy equation was expressed in Eq. (55) as:

$$\psi T \frac{ds}{dz} = \bar{\tau}_i (w_g - w_f) + \bar{\tau}_{wg} w_g + \bar{\tau}_{wf} w_f. \quad (67)$$

Equations (59), (61), (64) and (67) describe the two-phase flow system and can be jointly solved provided that the shear stresses are known. This can only be achieved via appropriate closure assumptions for the three shear stresses $(\bar{\tau}_i, \bar{\tau}_{wg}, \bar{\tau}_{wf})$.

It should be possible to place these in a form that is general enough not to limit the applicability of the equations. This is part of the research that is presently continuing.

III. PHYSICAL INTERPRETATION OF FLOW PHENOMENA

1. The Mechanics of Slip

The "slip" or difference between phase velocities which exists under many conditions is a fundamental characteristic of two-phase flow. It is a result of mechanical interaction of the phases with the flow boundary and at the phase interface. It depends upon flow compressibility and effects of gravity, such as stratification. The slip is intimately related to the geometric configuration or "pattern" assumed by the phases, the dissipation of the flow, and conditions at choking such as critical mass flux. Development of an understanding of the mechanics of slip is a logical first step toward understanding many of these associated phenomena.

Consider the phase momentum equation, given by Eq. (39). This can be written in the equivalent form

$$\rho_p x_p \frac{d}{dz} \{w_p\} = -\alpha_p \frac{dp}{dz} + \bar{\tau}_{pw} + \bar{\tau}_{pi} + \psi \frac{dx_p}{dz} (w_i - w_p) \quad (68)$$

where the left-hand side gives the rate of change of velocity and the right-hand side is a sum of forces, including the effective shear force on the phase due to mass exchange ψdx_p . When the phase continuity equation is used (Eq. (14)) the above expression becomes

$$\rho_p w_p \frac{dw_p}{dz} = \frac{-dp}{dz} + \frac{\bar{\tau}_{pw} + \bar{\tau}_{pi}}{\alpha_p} + \psi \frac{dx_p}{dz} \left(\frac{w_i - w_p}{\alpha_p} \right). \quad (69)$$

Beginning with this equation a number of simplifications will be introduced to allow identification of the primary mechanisms while the

secondary effects are neglected.

At the speeds normally encountered in two phase flow, the phases may be considered as incompressible individually. In that case,

$$\frac{d\rho_p}{dz} \approx 0 \quad (70)$$

and Eq. (69) may be written

$$\frac{d(Q_p)}{dz} = \frac{-dP}{dz} + \frac{\bar{\tau}_{pw} + \bar{\tau}_{pi}}{\alpha_p} + \psi \frac{dx_p}{dz} \frac{(w_i - w_p)}{\alpha_p} \quad (71)$$

The phase dynamic pressure has been introduced as

$$Q_p = \rho_p w_p^2 / 2 \quad (72)$$

Although the phases are considered as incompressible, single-substance flow taken as a whole is extremely compressible due to the transfer of mass and the difference of phase densities. In such a case, specific volume will change by a factor of ten with a change in mass fraction on the order of one percent. In this respect, a two-substance flow is basically different from a single-substance flow. When Eq. (71) is written for both phases and the dP/dz term eliminated by subtracting one from the other, using $w_i = \bar{w}$ from Eq. (53a), the result is

$$\frac{d(\Delta Q)}{dz} = \frac{\bar{\tau}_{wg}}{\alpha_g} - \frac{\bar{\tau}_{wf}}{\alpha_f} + \frac{\bar{\tau}_i}{\alpha_g \alpha_f} + \psi \frac{dx}{dz} \frac{1}{2} (w_g - w_f) \left(\frac{1}{\alpha_g} - \frac{1}{\alpha_f} \right) \quad (73)$$

where the difference between the phase dynamic pressures has been introduced:

$$\Delta Q = Q_f - Q_g. \quad (74)$$

When phase velocities are equal, ΔQ is positive since the density of the liquid is larger than that of the gas. If the gas velocity increases with respect to the liquid ΔQ will decrease, perhaps eventually reaching $\Delta Q = 0$ or even $\Delta Q < 0$.

It is reasonable to consider a phase arrangement within the cross-section such that the fraction of wall contacted by the phase is the same as its area fraction α_p . In such cases the normalized shear stress is given by

$$\bar{\tau}_{wp} = \tau_{wp} \alpha_p, \quad (75)$$

based on the definition of $\bar{\tau}_{wp}$ in Eq. (40).

The normalized interface shear stress is

$$\bar{\tau}_i = \tau_i \tilde{L}_i \quad (\tilde{L}_i = L_i/C) \quad (76)$$

as defined by Eq. (31e).

The wall shear stress τ_{wp} can be expressed in terms of the phase dynamic pressure,

$$\tau_{wp} = f_p Q_p \quad (77)$$

where f_p is a friction factor for the phase. This is given in the Blasius form for turbulent flow as

$$f_p = C_B / \text{Re}_p^{1/4} \quad (78)$$

where C_B is a constant. If both phases are assumed turbulent, then from the definition of Reynolds number the friction factor ratio is given approximately by

$$f_g/f_f \approx \left(\frac{\rho_g w_g \mu_f}{\rho_f w_f \mu_g} \right)^{-1/4} = \left(\sigma \frac{v_f}{v_g} \right)^{-1/4} \quad (79)$$

The length parameters required for the Reynolds numbers have been taken as equal. The quantities μ_p and v_p are the phase dynamic and kinematic viscosities, respectively. For many substances of interest in two-phase flow applications, the kinematic viscosity ratio of vapor and liquid is of order unity.* As a result, the phase friction factors may be taken as approximately equal for a fairly wide range of slip (σ). A single friction factor f will be defined as

$$f = f_g = f_f \quad (80)$$

When Eqs. (80), (75) and (77) are used in Eq. (73) the result is

$$\frac{d(\Delta Q)}{d\tilde{z}} = -f(\Delta Q) + \frac{\tau_i \tilde{L}_i}{\alpha_f \alpha_g} + \psi \frac{dx}{d\tilde{z}} \frac{1}{2} (w_g - w_f) \left(\frac{1}{\alpha_g} - \frac{1}{\alpha_f} \right) \quad (81)$$

This will be written in terms of the void fraction α as

$$\frac{d(\Delta Q)}{d\tilde{z}} = -f(\Delta Q) + \frac{\tau_i \tilde{L}_i}{\alpha(1-\alpha)} + \psi \frac{dx}{d\tilde{z}} (w_g - w_f) \frac{(\frac{1}{2} - \alpha)}{\alpha(1-\alpha)} \quad (82)$$

* For R-114 at 25°C, $v_f/v_g \approx 3.0$.

This equation expresses the change of the dynamic pressure difference ΔQ as the flow moves along the duct. The rate of change of ΔQ with $\tilde{z}$ is basically proportional to the local ΔQ , suggesting an exponential profile. This profile is modified by the terms involving τ_i and $dx/d\tilde{z}$, corresponding to the interaction between phases at the interface and the exchange of mass. The first of these can be treated as constant since $\alpha(1-\alpha)$ will hold a nearly fixed value over a moderate range and τ_i and $\tilde{L}_i$ will remain nearly constant for a given type of flow. Equation (82) may then be considered for two particular cases, based on whether the quantity $dx/d\tilde{z}$ is nonzero or zero (i.e., whether or not mass exchange occurs).

Consider first the case when $dx/d\tilde{z} = 0$. Equation (82) can then be written in the form

$$\frac{d}{d\tilde{z}} \left\{ \Delta Q - \frac{1}{f} \frac{\tau_i \tilde{L}_i}{\alpha(1-\alpha)} \right\} = -f \left\{ \Delta Q - \frac{1}{f} \frac{\tau_i \tilde{L}_i}{\alpha(1-\alpha)} \right\}. \quad (83)$$

The general solution of this differential equation is

$$\Delta Q - \frac{1}{f} \frac{\tau_i \tilde{L}_i}{\alpha(1-\alpha)} = c_0 \exp\{-f\tilde{z}\} \quad (84)$$

where c_0 is a constant dependent on initial conditions for ΔQ and $\tilde{z}$. If the dynamic pressure difference at the entrance to the duct ($\tilde{z} = 0$) is specified as $\Delta Q = \Delta Q_i$ then the particular solution is

$$\frac{\Delta Q - \frac{1}{f} \frac{\tau_i \tilde{L}_i}{\alpha(1-\alpha)}}{\Delta Q_i - \frac{1}{f} \frac{\tau_i \tilde{L}_i}{\alpha(1-\alpha)}} = \exp\{-f\tilde{z}\}. \quad (85)$$

Two important facts are apparent from this equation; both are obtained by noting that as $\tilde{z}$ becomes large (i.e. the flow moves downstream), the difference ΔQ approaches a value given by

$$\Delta Q = \tau_i \tilde{L}_i / f\alpha(1-\alpha). \quad (86)$$

First, the characteristic value of $\tilde{z}$ for this approach is $1/f$. For flow well within the turbulent regime, f may be taken from the Moody diagram as $f \approx 0.005$. This yields $\tilde{z} = 200$ or, recalling that $\tilde{z} = z/R_h$, a length 200 times the hydraulic radius. For a circular cross section $R_h = D/4$ and the above value corresponds to approximately 50 diameters. For two-phase flow, f would likely be larger than 0.005 tending to shorten the characteristic $\tilde{z}$. Hence, the length required for a flow to become fully developed after entering a duct is very short.

The second, more important result is that in a flow without mass exchange between phases ΔQ approaches a constant value. This value depends directly upon the shear stress $\overline{\tau}_i$. Without a change in the way the phases interact at the interface there is no mechanism for further change in the phase velocities, or the slip. A possible cause for a change in the interaction would be, for example, a change in flow pattern caused by stratification.

It should be noted at this point that the situation is different in vertical flow. In that case an additional term involving the gravitational force appears on the right-hand side of Eq. (82), which then takes the form (for $dx = 0$),

$$\frac{d}{dz}(\Delta Q) = -f(\Delta Q) + \frac{\tau_i \tilde{L}_i}{\alpha(1-\alpha)} - g(\rho_f - \rho_g)R_h \quad (87)$$

where g is the gravitational acceleration. Since the additional term is a constant, it can be combined with the shear stress term in solving the differential equation; its effect, then, is to alter the stable value of ΔQ to which the flow tends. This value is given by

$$\Delta Q = \left[\frac{\tau_i \tilde{L}_i}{\alpha(1-\alpha)} - g(\rho_f - \rho_g)R_h \right] \frac{1}{f} \quad (88)$$

as opposed to Eq. (86) for horizontal flow. For vertical flow upward, the gravitational term $g(\rho_f - \rho_g)R_h$ is positive, indicating that ΔQ will be smaller than for horizontal flow or, equivalently that the gas will move faster with respect to the liquid (larger slip) than in the horizontal case. It is clear that vertical flow is basically different as a direct result of gravity, as would have been expected. The difference in slip suggests also a difference in void fraction and "hold up" between the two cases.

The interfacial shear stress in the above equations is most easily interpreted as a combination of two effects. The first is the velocity difference between phases. By analogy to the form for wall shear stress in terms of velocity (Eq. (77)), the interfacial stress can be

expressed as proportional to the square of the difference between phase velocities:

$$\tau_i \sim (w_g - w_f) |w_g - w_f| \quad (89)$$

where the absolute value is used to ensure a proper sign on τ_i . The second effect involves the strength of coupling between phases. For a given difference in velocities, a flow with the phases interspersed, as with bubbles in liquid, will exhibit more interface "drag" than one with phases separated as in a stratified flow. This suggests that a number of different closure equations for shear stress are possible, depending on the type of flow pattern which exists. Both interface and wall stresses appearing in the momentum equation are independent of mass transfer and must be specified based only on the dynamic characteristics of each phase. For present purposes the strength of coupling between phases will be expressed by a proportionality coefficient k :

$$\tau_i = k(w_g - w_f) |w_g - w_f|. \quad (90)$$

It was demonstrated above that the dynamic pressure difference reaches a value given by

$$\Delta Q = \frac{\tau_i \tilde{L}_i}{\alpha(1-\alpha)} \frac{1}{f}. \quad (91)$$

When τ_i is eliminated between these equations, the result is

$$k = \frac{f \Delta Q}{(w_g - w_f) |w_g - w_f|} \frac{\alpha(1-\alpha)}{\tilde{L}_i}. \quad (92)$$

A weak coupling corresponds to a small value of k . The limiting case is $k = 0$ for which $\tau_i = 0$, from Eq. (90). This corresponds to a value $\Delta Q = 0$ from Eq. (91), and also from Eq. (92) for a finite velocity difference. Hence a weak coupling between phases results in the dynamic pressures being equalized:

$$\rho_g w_g^2 = \rho_f w_f^2. \quad (93)$$

In terms of the slip parameter σ this gives

$$\sigma = \gamma^{1/2}. \quad (94)$$

A strong coupling corresponds to a large value of k . As $k \rightarrow \infty$, Eq. (90) requires that the phase velocities become equal in order for τ_i to remain finite, or equivalently for ΔQ to remain finite, from Eq. (92). Hence for a strong coupling

$$w_g = w_f \quad (95)$$

or, in terms of σ ,

$$\sigma = 1. \quad (96)$$

This result would be expected based simply on physical insight for this case.

The above analysis indicates the practical limits of $\sigma = 1$ and $\sigma = \gamma^{1/2+}$ for two-phase flow. These limits place corresponding limits on void fraction. A general expression for α was given in Eq. (22). Substitution of the limiting values of σ gives

⁺ For R-114 at 25°C, $\gamma^{1/2} \approx 10$.

the expressions

$$\alpha = 1/\{(\frac{1-x}{x})/\gamma + 1\} \quad (\sigma = 1) \quad (97a)$$

$$\alpha = 1/\{(\frac{1-x}{x})/\gamma^{1/2} + 1\} \quad (\sigma = \gamma^{1/2}). \quad (97b)$$

Note that for vertical flow with a weak interaction at the interface, the flow will tend toward a stable value of ΔQ given by Eq. (88) with $\tau_i = 0$:

$$\Delta Q = -g(\rho_f - \rho_g)R_h/f. \quad (98)$$

Hence $\Delta Q < 0$, as opposed to $\Delta Q = 0$ for horizontal flow. In vertical flow the dynamic pressure of the vapor may exceed that of the liquid, at least in theory. For horizontal flow the weak coupling would exist in a completely stratified flow, with a smooth liquid surface. It is difficult to imagine a configuration that would provide such a weak coupling in the vertical case.

Recall that the above limits were derived for the case $dx/d\tilde{z} = 0$, and as a result are primarily valid for flows without mass exchange (two-substance flow). The effect of the $dx/d\tilde{z}$ term is clear from Eq. (82). Its magnitude depends on the velocity difference and the void fraction, and its sign changes depending on whether $\alpha > 1/2$ or $\alpha < 1/2$. The effect of this term is best seen by imagining a flow with $dx/d\tilde{z} = 0$ which has reached its stable value of ΔQ . If mass transfer occurs, its effect is to upset the equilibrium which has been reached and provide a continuously changing, driving force for further

change in ΔQ . The continuous change is a characteristic of single-substance flow.

The $dx/d\tilde{z}$ term under consideration results solely from the fact that mass entering or leaving a phase at a velocity different than the phase velocity produces a force on that phase. The α portion of the term becomes very large for $\alpha \rightarrow 0$ or $\alpha \rightarrow 1$, although it is expected physically that the velocities will become equal in these limits. This ensures a finite value for the entire term. For $\alpha = 1/2$ the entire term becomes zero. This suggests that, at least over the middle range of void fraction, the phase velocities may change in such a way that ΔQ remains constant at the value determined by the wall and interface stresses. It must be noted that the influence of the $dx/d\tilde{z}$ term on ΔQ is actually a secondary effect of the mass exchange. The primary effect is that for the single-substance case the flow taken as a whole must accelerate as mass transfer occurs because of the resulting change in specific volume. This is true regardless of other relationships between phase velocities, such as constant ΔQ or constant σ .

The basic influence on slip in two-phase flow can be identified clearly as the wall and interfacial stresses and acceleration, the latter due primarily to phase change. In the limit of zero slip ($\sigma = 1$) and small vapor content ($x \rightarrow 0$) the only relevant parameter becomes the wall shear stress corresponding to the entire mass flow rate in the liquid state. Hence the corresponding liquid flow Reynolds number (or pressure change) will be an important reference parameter. As flashing occurs, slip will be promoted by the

existence of unequal wall stresses, but is strongly dependent on the strength of coupling between phases. With weak coupling, slip will increase until wall stresses are equalized. With strong coupling, this effect is opposed, and phase velocities tend to be equalized. It is clear that the coupling between phases will be influenced greatly by stratification of the flow. For example, vapor in the form of dispersed bubbles will be affected much more by the liquid phase ("feel" more drag due to that phase) than when it is present as a separate entity, e.g., the bubbles having migrated and formed a free surface at the top of the duct. Hence, the Froude number will be important. A high Froude number will correspond to a smaller amount of stratification and, it is expected, a higher overall pressure change since the fluid will behave more as a homogeneous mixture. The heavier liquid will move with the vapor, creating a larger net momentum flux when accelerated than if the vapor were allowed to "slip" through. Acceleration will modify and perhaps dominate the other basic effects, and will be determined by a suitably defined Mach number.

2. Flow States. The Fanno Line

The previous section examined the relationship between phase velocities and the way these velocities will change as the flow proceeds along the duct. By combining the phase momentum equations it was possible to obtain physical limitations on the velocity ratio σ . By using the combined energy and continuity equations, the present section will examine the range of states through which the flow will pass in terms of the properties enthalpy and entropy, or the Fanno line for the flow process. The Fanno line will depend on σ since this parameter determines the kinetic energy.

For two-substance flow the amount of mass present in each phase at the duct entrance remains constant as the flow proceeds and negligible acceleration will occur. Then since the flow is adiabatic enthalpy will remain constant. Entropy will increase due to dissipation. For single-substance flow the phase change produces an acceleration and a corresponding decrease in enthalpy. This accompanies the entropy increase. While a two-substance flow will retain a fixed mass fraction, the single substance flow could conceivably produce a range of conditions from completely liquid to completely vapor, all developing successively downstream from the same entrance conditions. When the proper conditions exist at the duct exit, choking will occur at this point.

Single-substance flows for three different values of mass flux ψ are shown in Fig. 2(a). The complete range of states is shown from the stagnation condition to choking. In the first case, liquid is

accelerated at constant entropy to the duct inlet, with mass flux ψ_1 . From there, a pressure loss occurs because of friction and the entropy increases until the pressure has been reduced to the saturation pressure. The process occurs at constant enthalpy since the liquid density is essentially constant and no acceleration occurs. At saturation, evaporation (or "flashing") occurs and the remainder of the process is in the two-phase regime, with the mass of vapor present steadily increasing until choking occurs. For a larger mass flux ψ_2 with the same stagnation conditions, the change in enthalpy before entering the duct is larger because of the higher velocity (or kinetic energy) at the duct entrance. This brings the flow closer to the saturation line, such that a smaller entropy increase is required in the liquid flow to produce flashing. The two-phase portion of the process is similar to that for ψ_1 . The different slopes of the lines for different ψ will be examined in more detail. Under certain conditions the behavior shown for ψ_3 may occur such that the flow never becomes two-phase but chokes as a saturated liquid at the end of the duct.

A calculation of the Fanno line is possible since the combined mass and energy conservation equations can be integrated directly and used in conjunction with an equation of state for the particular substance. The momentum equation would have to be solved, with proper closures for the shear stresses, to specify the location along the duct at which each state occurs and whether choking is reached at the exit. Note that this is not a problem for flow in the liquid portion

of the process before flashing since single phase flow is well understood and the location of the flash point is easily calculated.

The two basic equations to be used in analyzing the two-phase portion of the flow process are Eqs. (14) and (66):

$$\psi = \rho_g w_g \alpha / x, \quad \psi = \rho_f w_f (1-\alpha) / (1-x) \quad (99a,b)$$

and

$$h_0 - h = x w_g^2 / 2 + (1-x) w_f^2 / 2 \quad (100)$$

where h_0 is the stagnation enthalpy and is constant for the adiabatic flow. Equation (100) will be written using the definition of σ as:

$$h_0 - h = \frac{1}{2} w_f^2 \{ x \sigma^2 + (1-x) \}. \quad (101)$$

It will be convenient to denote the kinetic energy as KE. The above equation is then

$$h_0 - h = KE = \frac{1}{2} w_{rms}^2 \quad (102)$$

where h is the enthalpy, given by

$$h = h_f(1-x) + h_g x = h_f + x h_{fg}. \quad (103)$$

The liquid phase velocity w_f can be obtained from Eq. (99b), and Eq. (101) becomes

$$h_0 - h = \frac{1}{2} \left(\frac{\psi}{\rho_f} \right)^2 \left(\frac{1-x}{1-\alpha} \right)^2 \{x(\sigma^2-1) + 1\}. \quad (104)$$

By combining Eqs. (99a) and (99b), eliminating ψ and using the definitions of σ and γ the following expression is obtained:

$$\left(\frac{1-x}{1-\alpha} \right) = x \left(\frac{\gamma}{\sigma} - 1 \right) + 1 \quad (105)$$

and hence Eq. (104) is

$$h_0 - h = \frac{1}{2} \left(\frac{\psi}{\rho_f} \right)^2 \{x(\sigma^2-1) + 1\} \{x \left(\frac{\gamma}{\sigma} - 1 \right) + 1\}^2. \quad (106)$$

Although the temperature changes from the flash point downstream, the density of the liquid phase is approximately constant. Then the quantity ψ/ρ_f is constant and can be written as

$$\psi/\rho_f = w^* \quad (107a)$$

where w^* is the velocity at the flash point.

As a result, Eq. (106) becomes

$$h_0 - h = \frac{1}{2} w^{*2} \{x(\sigma^2-1) + 1\} \{x \left(\frac{\gamma}{\sigma} - 1 \right) + 1\}^2. \quad (107b)$$

Alternatively, this can be written

$$\left(\frac{w_{rms}}{w^*} \right)^2 = \{x(\sigma^2-1) + 1\} \{x \left(\frac{\gamma}{\sigma} - 1 \right) + 1\}^2 \quad (108)$$

where the left-hand side is the ratio of the two-phase kinetic energy to the kinetic energy of the flow when flashing just begins.

For liquid flow ($x = 0$) between the duct inlet and the flash

point, Eq. (107b) becomes

$$h_0 - h = \frac{1}{2} w^{*2}. \quad (109)$$

Both temperature and density are approximately constant between these two points, and the single phase liquid velocity will be equal to the velocity at the flash point. An asterisk will be used in all of the work which follows to denote properties at flashing.

Once stagnation conditions and mass flux (h_0 and ψ) have been specified, the quantity x may be calculated and the state of the fluid completely determined from Eq. (106) for any point downstream of flashing ($T < T^*$). The entropy and specific volume are then

$$s = (1-x)s_f + xs_g \quad (110a,b)$$

$$v = \frac{1}{\rho} = (1-x)v_f + xv_g.$$

The mass fraction x is a state parameter when the flow is two-phase. A convenient choice for the other is temperature, since this specifies the saturation properties of the individual phases, and allows for all properties to be expressed directly as functions of (x,T) in the form of Eqs. (110a,b). Temperature can be measured accurately in practice and allows the state of the flow to be easily calculated. Stagnation enthalpy may be determined in practice by measuring stagnation temperature and pressure in a settling chamber where velocity is essentially zero. For performing theoretical calculations for different cases it is most convenient to specify the temperature at the flash point and calculate stagnation enthalpy from the enthalpy h_f

and kinetic energy at that location.

It is convenient to treat σ as a parameter for fixed ψ as illustrated in Fig. 2(b). The Fanno lines have been shown including only the portion from flashing through two-phase flow to choking. In an actual flow the slip will not remain constant, but will vary beginning from $\sigma = 1$ at flashing. Hence the flow will traverse a number of slip conditions and will follow a path similar to that indicated in Fig. 2(b). The choking point in all cases is characterized by a point of maximum entropy ($ds = 0$) where the curve has a vertical tangent. Figure 2(b) illustrates that slip shifts the choking point to higher values of entropy, thus in a sense delaying the onset of choking for a given mass flux. This effect is even more pronounced when σ increases along the flow, as shown by the variable slip curve. (The effect of slip and its rate of change on choking is discussed in Section III.5). The primary effect of slip, illustrated in Fig. 2 (b) is that the slope of the Fanno curve changes in such a way that dissipation (Δs) is more significant relative to acceleration (Δh) for larger values of σ . This is equivalent to saying that the pressure change due to friction represents a greater portion of the overall pressure change.

The effects of slip on the position and slope of the Fanno curves is limited to a particular range. The curves are shifted upward for larger σ only until a specific limit is reached. Larger values of σ then shift the curve progressively downward. This suggests the existence of a minimum kinetic energy, since curves positioned

higher correspond to smaller changes in enthalpy from the flash point. The extreme case is a Fanno line extending horizontally from the flash point, and corresponds to a constant enthalpy flow process.

The kinetic energy is given in terms of x, σ and γ by the right-hand side of Eq. (106):

$$KE = \frac{1}{2} \left(\frac{\psi}{\rho_f} \right)^2 \{x(\sigma^2 - 1) + 1\} \{x(\frac{\gamma}{\sigma} - 1) + 1\}^2. \quad (111)$$

An extremum can be located by differentiation with respect to σ with the other parameters fixed. By setting the expression equal to zero and solving for σ , the value of σ for minimum KE can be determined. It is given by

$$\sigma = \gamma^{1/3}. \quad (112)$$

As a result, no Fanno curve will have less kinetic energy than that for $\sigma = \gamma^{1/3}$. Equivalently, no Fanno curve will be located above that for $\sigma = \gamma^{1/3}$.

In the last section, it was demonstrated that a practical upper limit on the value of σ in horizontal flow was $\sigma = \gamma^{1/2}$, the lower limit being $\sigma = 1$. Logically, the next question is the location of the Fanno curve for $\sigma = \gamma^{1/2}$. Since $\sigma = \gamma^{1/3}$ represents the minimum possible kinetic energy, the larger kinetic energy for $\sigma = \gamma^{1/2}$ will place the corresponding Fanno curve below that for $\gamma^{1/3}$. Whether it is large enough to place it below the $\sigma = 1$ curve is not immediately apparent.

The kinetic energy for $\sigma = 1$ can be obtained directly from the

right-hand side of Eq. (106) as

$$KE|_{\sigma=1} = \frac{1}{2} \left(\frac{\psi}{\rho_f} \right)^2 \{x(\gamma-1) + 1\}^2 \quad (113)$$

and that for $\sigma = \gamma^{1/2}$ as

$$KE|_{\sigma=\gamma^{1/2}} = \frac{1}{2} \left(\frac{\psi}{\rho_f} \right)^2 \{x(\gamma-1) + 1\} \{x(\gamma^{1/2}-1) + 1\}^2. \quad (114)$$

Location of the $\sigma = 1$ curve below that for $\sigma = \gamma^{1/2}$ requires that ,

$$KE|_{\sigma=1} - KE|_{\sigma=\gamma^{1/2}} > 0. \quad (115)$$

By substituting Eqs. (113) and (114) into Eq. (115) and simplifying, the requirement becomes

$$(\gamma^{1/2}-1)^2 > 0 \quad (116)$$

which must always be true for $\gamma > 1$. If $\gamma = 1$, then $\sigma = \gamma^{1/2}$ gives simply $\sigma = 1$ and the two Fanno curves coincide. Hence, the curve for $\sigma = \gamma^{1/2}$ must always fall between that for $\sigma = 1$ and $\sigma = \gamma^{1/3}$. Since $\gamma^{1/2}$ represents an upper limit on σ , it is clear that all curves for horizontal flow will fall in this range.

It should be noted that the slip for vertical flow may be larger than $\gamma^{1/2}$, as shown in the last section, and the Fanno curves may approach or fall below that for $\sigma = 1$. A slip σ approximately twice as large as $\gamma^{1/2}$ is required to approach the curve $\sigma = 1$.

Figure 3 is a plot of Fanno lines for R-114 with a flash temperature $T^* = 25^\circ\text{C}$ and mass flux of $2500 \text{ kg/m}^2\text{-s}$ (these conditions

compare closely to average conditions selected for a large portion of the experimental work). In this diagram, enthalpy and entropy are calculated relative to the respective conditions at flashing, and only the two-phase portion of the curve is shown. The entropy change has been multiplied by the flash temperature T^* in order that the axes be dimensionally identical, and to reflect the slope Tds/dh , which gives the ratio of energy dissipation to increase in kinetic energy due to acceleration at a given point. The curves demonstrate the effects discussed above. The Fanno lines are shown for the range $\sigma = 1$ to $\sigma = 5$. The latter case corresponds approximately to $\sigma = \gamma^{1/3}$ for R-114 at 25°C. All curves for this substance then must fall within the range shown.

Sample values of mass fraction x and reduced specific volume v_R are shown in Fig. (3). The reduced specific volume is defined as

$$v_R = \frac{v}{v_f} \quad (117)$$

From Eq. (58b) and the definition of γ , an equivalent form is

$$v_R = x(\gamma+1) + 1. \quad (118a)$$

Hence v_R depends on x and temperature (γ is a ratio of phase properties, which depend on T), and is a measure of the amount of vapor generated due to flashing. It is also a measure of the acceleration which would take place in a given flow if phase velocities remained equal from the flash point onward, since v_R is just the ratio of the two-phase velocity to the flash point velocity for that

case ($\sigma = 1$):

$$v_R = w_V/w^* \quad (118b)$$

where w_V is the volumetric velocity defined by Eq. (58a).

Values of x and v_R have been shown on the isotherm $T = 18^\circ\text{C}$ in Fig. 3. Average values $\bar{x}$ and $\bar{v}_R$ over the range of σ are shown for other constant temperature lines. The mass fraction x (and hence v_R) are nearly constant along the isotherms. This suggests that lines of constant x and those of constant T nearly coincide. This has important implications for the calculation of x . First, it will be useful to examine the slopes of the lines, in general, in the coordinates (h,s) . The slope of a constant temperature line is given by

$$\left. \frac{\partial s}{\partial h} \right|_T = \left[\left. \frac{\partial s}{\partial x} \right|_T \right] / \left[\left. \frac{\partial h}{\partial x} \right|_T \right]. \quad (119)$$

Enthalpy and entropy are expressed in terms of x as

$$\begin{aligned} h &= h_f + xh_{fg} \\ s &= s_f + xs_{fg} \end{aligned} \quad (120a,b)$$

and hence

$$\left. \frac{\partial s}{\partial h} \right|_T = s_{fg}/h_{fg}. \quad (121)$$

The quantity s_{fg} is given by

$$s_{fg} = h_{fg}/T \quad (122)$$

and the slope becomes simply

$$\left. \frac{\partial s}{\partial h} \right|_T = \frac{1}{T} \quad (123)$$

Slope of the constant x line is given by

$$\left. \frac{\partial s}{\partial h} \right|_x = \left(\left. \frac{\partial s}{\partial T} \right|_x \right) / \left(\left. \frac{\partial h}{\partial T} \right|_x \right) \quad (124)$$

From the definitions of s and s_{fg} in Eqs. (120b) and (122) the numerator can be written

$$\left. \frac{\partial s}{\partial T} \right|_x = \frac{ds_f}{dT} + x \left(\left. \frac{\partial \left(\frac{h_{fg}}{T} \right)}{\partial T} \right|_x \right) \quad (125)$$

The latent heat will be treated as constant. Equation (125) then gives

$$\left. \frac{\partial s}{\partial T} \right|_x = \frac{ds_f}{dT} - \frac{xh_{fg}}{T^2} \quad (126)$$

The Gibbs equation for the liquid phase gives

$$\frac{ds_f}{dT} = \frac{1}{T} \left(\frac{dh_f}{dT} - v_f \frac{dp}{dT} \right) \quad (127)$$

The differential of h_f is the rate of change of the liquid enthalpy with temperature along the saturation line. The property C_f will be

defined by the equation

$$C_f = dh_f/dT. \quad (128)$$

The quantity dP/dT is the rate of change of saturation pressure with temperature. From the Clausius-Clapeyron equation this is given by

$$dP/dT = s_{fg}/v_{fg} = \left(\frac{h_{fg}}{T} \right) / v_{fg}. \quad (129)$$

With these substitutions, Eq. (127) becomes

$$\frac{ds_f}{dT} = \frac{1}{T} \left\{ C_f - \frac{v_f}{v_{fg}} \frac{h_{fg}}{T} \right\}. \quad (130)$$

Along with Eq. (126), this gives

$$\left. \frac{\partial s}{\partial T} \right|_x = \frac{1}{T} \left\{ C_f - \frac{h_{fg}}{T} \left(\frac{v_f}{v_{fg}} + x \right) \right\}. \quad (131)$$

The denominator in Eq. (124) is obtained by differentiating the expression for h in Eq. (120a), with latent heat h_{fg} treated as constant. Hence,

$$\left. \frac{\partial h}{\partial T} \right|_x = \frac{dh_f}{dT} = C_f. \quad (132)$$

The slope of the constant x line is obtained by combining Eqs. (131) and (132) as

$$\left. \frac{\partial s}{\partial h} \right|_x = \frac{1}{T} \left\{ 1 - \frac{h_{fg}}{C_f T} \left(\frac{v_f}{v_{fg}} + x \right) \right\}. \quad (133)$$

The fractional difference in slope between the constant x and T

lines will be defined as Δ :

$$\Delta = \left\{ \left. \frac{\partial s}{\partial h} \right|_x - \left. \frac{\partial s}{\partial h} \right|_T \right\} / \left\{ \left. \frac{\partial s}{\partial h} \right|_T \right\}. \quad (134)$$

From Eqs. (123) and (133) this is

$$\Delta = \frac{-h_{fg}}{C_f T} \left\{ \frac{v_f}{v_{fg}} + x \right\}. \quad (135)$$

The specific volume of the liquid is much smaller than that of the vapor, and hence

$$v_{fg} \simeq v_g. \quad (136)$$

Equation (135) then becomes

$$\Delta \simeq \frac{-h_{fg}}{C_f T} \left\{ \frac{1}{\gamma} + x \right\}. \quad (137)$$

For many fluids⁺, under conditions experienced in two-phase flow, γ is of the order 100, while h_{fg} and $C_f T$ will be comparable in magnitude. The difference in slope, then, ranges from approximately 1% to over 100%, depending on x . This is clear from the shape of the two-phase region in (h,s) coordinates. Constant temperature lines

⁺For R-114 at 25°C ($T = 298K$):

$$h_{fg} \simeq 128 \text{ kJ/kg}, C_f \simeq 1 \text{ kJ/kg}, \gamma \simeq 92.$$

This gives

$$\Delta \simeq -0.43\{.01+x\}.$$

are nearly parallel to the saturated liquid line (where $x = \text{constant} = 0$) but intersect the saturated vapor line ($x = 1$) at a much larger angle.

The difference in slope given by Eq. (137) produces a very small change in x along an isotherm over the range of kinetic energy given in Fig. 3. This means that x can be accurately determined from the temperature alone, and in this respect the difference in slope over this range is negligible. The latter is true when the kinetic energy change is small compared to the change in enthalpy of the phases caused by evaporation. Consider the expression for x obtained by rearrangement of Eq. (103):

$$x = (h - h_f) / h_{fg}. \quad (138)$$

Notice that x is a function of the variables (h, T) since h_f and h_{fg} depend only on temperature. Its differential can be written in general form as

$$dx = \left. \frac{\partial x}{\partial h} \right|_T dh + \left. \frac{\partial x}{\partial T} \right|_h dT. \quad (139)$$

For adiabatic flow the change in enthalpy must balance the change in kinetic energy as expressed by Eq. (102). Hence,

$$dh = -d(KE) \quad (140)$$

and Eq. (139) becomes

$$dx = \left. \frac{\partial x}{\partial T} \right|_h dT - \left. \frac{\partial x}{\partial h} \right|_T d(KE). \quad (141)$$

If the constant x and T lines were to coincide exactly the last partial derivative on the right-hand side would be zero and the effect of the kinetic energy would disappear completely. The actual value of this derivative is obtained by differentiating Eq. (138):

$$\left. \frac{\partial x}{\partial h} \right|_T = \frac{1}{h_{fg}} \quad (142)$$

The other derivative on the right-hand side can be similarly obtained:

$$\left. \frac{\partial x}{\partial T} \right|_h = \frac{-dh_f}{dT} \frac{1}{h_{fg}} = -C_f/h_{fg} \quad (143)$$

Since h_{fg} has been assumed constant, dh_f/dT represents the change of enthalpy of both phases with temperature. With the above expression, Eq. (141) becomes

$$dx = \frac{-1}{h_{fg}} \{C_f dT + d(KE)\} \quad (144)$$

The equation can be integrated and written in the form of a function of T alone and a small additional term that depends on kinetic energy:

$$x = x(T) - \delta \quad (145a)$$

where

$$\delta = \Delta KE/h_{fg} \quad (145b)$$

and

$$x(T) = C_f(T^* - T)/h_{fg}. \quad (146)$$

The increase in kinetic energy from the flashpoint has been denoted by ΔKE .

The requirement $x = 0$ at $T = T^*$ has been used in obtaining Eq. (146), where T^* is the temperature at the flash point and T the downstream temperature. The KE term can be neglected when the kinetic energy is small compared to the enthalpy change of the phases, given by $C_f(T^* - T)$. Then x is given in terms of temperature by Eq. (146). The actual value of the deviation δ caused by kinetic energy is given by the ratio $\Delta KE/h_{fg}$.

The process of reaching the downstream state at temperature T from the flash point can be considered as a sequence of two processes. The first, at constant h , involves the formation of a quantity of vapor along with a decrease in temperature. The second process, at constant x , involves acceleration to a higher velocity because of the decreased density. Since constant x implies nearly constant T , the temperature change (or enthalpy change) of the phases due to the acceleration process is small compared to that for the evaporation process.

The calculation of x from T alone is not useful to produce Fanno lines since the position of the individual curves depends on the small term δ in Eq. (145a) resulting from the kinetic energy. However, it is useful to be able to determine x accurately enough to calculate associated quantities like v_R without having to consider details of the flow such as σ , which is not generally known.

3. Kinetic and Potential Energy. Froude Number

In the last section the two-phase flow process was examined on the basis of the flow states or the Fanno line. The kinetic energy of the flow and its effect on the Fanno line were analyzed by considering the velocities, or kinetic energies, of the individual phases in terms of the velocity ratio σ . Potential energy was not considered since the flow was horizontal, that is, perpendicular to the direction of the gravitational field. In fact, change in potential energy can be important. Although the duct is horizontal, the phases may be arranged within a given cross section in such a way that more mass passes through the upper or lower portion. This arrangement is free to change as the flow proceeds along the duct, depending on the interaction between phases, and the center of mass for the flow can change. At the flash point for example the center of mass corresponds to the duct centerline. If stratification develops after flashing, the center of mass will be different. The phase arrangement determines the potential energy, while the location of a cross section along the flow is not important. This is in contrast to vertical flow, which moves parallel to the gravitational field. In this case, a change in potential energy will occur from cross section to cross section which depends solely on location along the duct and is independent of the internal configuration. Hence, for vertical flow the position along the duct must be known to determine the flow state since changes in both kinetic and potential energy combine to produce the change in enthalpy. The change in pressure (or temperature) with length involves the momentum

equation and all three equations for mass, energy and momentum conservation are required. For horizontal flow this direct coupling between potential energy and position along the flow does not exist and the momentum equation is not required to determine the Fanno line. The kinetic energy (KE) and potential energy (PE) must still combine to balance the enthalpy change. The analog of Eq. (140) is then

$$dh = -d(KE+PE). \quad (147)$$

The KE and PE terms are given by

$$KE = xw_g^2/2 + (1-x)w_f^2/2 \quad (= w_{rms}^2/2) \quad (148a)$$

$$PE = g(xy_g + (1-x)y_f) \quad (= gy_{cm}) \quad (148b)$$

both of which arise from combination of the phase energy equations. The phase equations with the potential energy terms included are presented in detail in Appendix A. The quantities y_g and y_f are heights of the center of mass of the gas and liquid phases, relative to the duct centerline or other convenient datum; g is the gravitational acceleration and y_{cm} the center of mass for the combined flow.

If the enthalpy change from the flash point due to the combined kinetic and potential energy is small compared to that for the phase change, then x can be calculated from the temperature alone, as in the previous section. The mass fraction can be expressed as in Eq. (145a) but where δ has the modified definition

$$\delta = (\Delta KE + \Delta PE) / h_{fg} \quad (149)$$

as compared to Eq. (145b). The δ term, although negligible for a basic calculation of x , is responsible for determining the position and slope of the Fanno line. It is equally clear from Eq. (147) that both kinetic and potential energy will determine the flow state depending on their relative magnitudes. While the magnitudes individually will determine the Fanno line, their values are interdependent in a given flow. For example, a stratified flow will not only have a different potential energy than a flow that is more intermixed but will also tend to cause a different slip, as discussed in Section III.1, and thus a different kinetic energy. Comparison of the kinetic versus potential energy provides a means of predicting the physical interaction between phases, which ultimately determines the flow configuration and its thermodynamic state.

When ΔKE is large compared to ΔPE the effect of gravity will disappear and the flow will be intermixed. When ΔPE is large the flow will be stratified with various configurations possible, including waves moving on a free liquid surface. Comparing the ratio KE/PE at the flash point provides a convenient means of predicting downstream behavior. It also provides a means for comparing flows under different conditions. The flash point kinetic energy is $w^2/2$. The potential energy change available when the fluid flashes depends on the pipe diameter, D . The ratio of kinetic energy at the flash point and potential energy corresponding to the pipe diameter represents a characteristic Froude number for the flow:

$$Fr^* = w^{*2}/gD. \quad (150)$$

When this number is much larger than unity, as for flow in a very small diameter tube, gravity will not play an important role in any part of the flow from the flash point onward and stratification or other flow patterns will not be discernible. When Fr^* is of order unity, various flow patterns will develop when the flow first flashes, but once sufficient vapor has been generated and the velocity has increased to the point where kinetic energy is the dominant factor the patterns will disappear. Hence, a local Froude number, given by

$$Fr = w_{rms}^2/gD \quad (151)$$

will determine local conditions in the two-phase regime.

The value of Fr^* is constrained to be within a definite range. The maximum velocity at the flash point is the critical speed a^* of the saturated liquid at the temperature T^* . The maximum value of Fr^* is then given by

$$Fr_{max}^* = a^{*2}/gD \quad (152)$$

and comparison of Fr^* for different flows should always be made with respect to the maximum range available, Fr_{max}^* .

An important aspect of this discussion concerns the similarity of flows involving different substances. On the basis of Eq. (149), both kinetic energy and potential energy should be scaled according to the latent heat. Hence, pipe diameter is scaled

in direct proportion to h_{fg} . Since pipe length divided by diameter is a proper dimensionless length, the length is also scaled in direct proportion to h_{fg} . Velocity should be scaled as the square root of h_{fg} . An important comparison for the present work is that between R-114 and water at typical geothermal well conditions or those experienced in similar power production systems. The ratio of h_{fg} for water at 200°C and that for R-114 at 25°C is approximately 15. Hence, results with R-114 in a 5 cm diameter cross section, 8 m long test section will compare to a water flow in 75 cm diameter and 120 m length. The ratio of the critical speed a^* for water to R-114 is approximately 4, and hence the proportion for the range of w^* for either fluid corresponds nicely to the square root of the latent heat ratio. Some dimensionless parameters for comparing flows of different substances have been proposed in a recent paper [36] and the further development of this topic is continuing. The comparison in length and velocity scales for different substances mentioned above has been observed and described, at least qualitatively, by Friedel and Mayinger [31].

4. Acceleration and Dissipation. Mach Number

Fanno curves are shown in Fig. 2(a) for a number of mass flux values ψ . For each value of ψ a family of curves is possible, as shown in Fig. 2(b), depending on the amount of slip or stratification that develops within the flow. However, the slope of the family of curves for given ψ is determined solely by the compressibility, as in single phase compressible flow. The starting point for two-phase flow is determined by conditions at flashing. The slope at that point represents the minimum acceleration that will occur and is characteristic of the importance of compressibility for the remainder of the process.

It is indicated in Fig. 2(a) that the slope of the Fanno curve at flashing depends on the mass flux ψ , with the steeper slope occurring when ψ , or equivalently w^* , is larger. For low velocity w^* as shown for ψ_1 the slope Tds/dh approaches $-\infty$ as w^* approaches zero. In the limit, the velocity is $w^* = 0$ and the slope is horizontal. For larger w^* , the slope is steeper and the choking point is shifted closer to the flash point. The two-phase flow process is "shortened" in the sense that less vapor is generated between flashing and choking. For high velocity the slope Tds/dh approaches zero as w^* approaches its upper limit, which is the choking speed, a^* . In the limit $w^* = a^*$, as shown for ψ_3 , and the slope is vertical at flashing. The choking point has been shifted to the extent that it coincides with the flash point. The process has been shortened to the extent that no vapor is generated and no two-phase flow process

exists. The slope of the curve covers its full range as the ratio w^*/a^* varies from 0 to 1. The characteristic parameter for assessing the importance of compressibility is appropriately the Mach number defined for two-phase flow on the basis of conditions at flashing:

$$Ma^* = w^*/a^*. \quad (153)$$

Figure 4 shows the effect of Ma^* at flashing for R-114. This figure is essentially an enlargement of Fig. 3 in the region close to flashing, and curves for different ψ have been added. An isotherm has been shown for reference. It was noted previously that the phase velocities must be equal at the flash point and the slope of the line will correspond to that for $\sigma = 1$. One curve with different σ has been shown in Fig. 4 for comparison. As the flow proceeds downstream slip and stratification may develop and will determine the flow state. These effects are characterized by the Froude number Fr^* as discussed in the last section. Any such downstream development must modify the direction of the process established by conditions at the flash point, as characterized by Ma^* . For low Ma^* , a certain amount of vapor must be generated before the acceleration that is characteristic of choking begins to dominate the flow. Slip or stratification may develop that will determine the flow process in the intermediate stages. For high Ma^* , acceleration will dominate the entire process, since the flow is close to choking when flashing occurs, and the other effects will be suppressed. In either case, slip and stratification are expected to be nonexistent close to

choking. For a two-substance flow like air and water, compressibility is unimportant and these other effects alone govern the flow.

The pressure change that occurs in a flow will consist of a portion due to dissipation and a portion due to acceleration. The ratio of these is the same as the slope of the Fanno line:

$$\frac{dP_d}{dP_a} = \frac{-Tds}{dh} \quad (154a)$$

For a general compressible flow in a constant cross-sectional area and with a velocity that is uniform throughout the entire cross section, the above ratio can be written in terms of the local Mach number as

$$\frac{-Tds}{dh} = (1-M^2)/\bar{\gamma}M^2 \quad (154b)$$

For convenience Mach number Ma will be denoted by M when it appears in an equation. The quantity $\bar{\gamma}$ is a property of the fluid given by

$$\bar{\gamma} = \left(\frac{\partial v}{\partial P} \right)_h \bigg/ \left(\frac{\partial v}{\partial P} \right)_s \quad (155)$$

If the fluid were an ideal gas, $\bar{\gamma}$ would be the ratio of isothermal to isentropic compressibilities, and also the ratio of specific heats c_p and c_v . For a fluid consisting of two phases of a single substance, it is given by

$$\bar{\gamma} = 1 + \frac{v_{fg}}{v} \frac{a^2}{h_{fg}} \quad (156)$$

Derivations of these expressions are given in Appendix A. At the flash point a two-phase flow satisfies the requirements stated for Eq. (154b). Hence, the slope at flashing will be given by β^* , defined as

$$\beta^* = (1-M^{*2})/\bar{\gamma}^* M^{*2} \quad (157)$$

where

$$\bar{\gamma}^* = 1 + \frac{v_{fg}}{v_f} \frac{a^{*2}}{h_{fg}}. \quad (158)$$

If the approximation $v_{fg} \simeq v_g$ is used, $\bar{\gamma}^*$ can be expressed as

$$\bar{\gamma}^* \simeq 1 + \gamma \frac{a^{*2}}{h_{fg}}. \quad (159)$$

For comparing flows under different conditions, either Ma^* or β^* may be used. The latter gives the ratio of dissipation to acceleration energy directly, and in that sense is more meaningful.

5. The Influence of Slip on Critical Mass Flux

Fluid entering the duct as compressed liquid will reach its flash point at some intermediate position and flashing will continue, with the two-phase flow traversing a number of flow patterns as it progresses along the channel. If the reservoir pressure at the channel exit is low enough, the flow will choke at this position. Depending on details of the flow development, a variety of conditions will be possible as the flow reaches this point. The exact specification of the state depends on solution of the set of six conservation equations for the phases with the proper closures. It is likely that the flow will not be of "homogeneous" composition and that a slip will exist between the phases; from the previous analysis, prediction of this slip depends on knowledge of the coupling between phases at the choking point. However, it is possible to determine the influence that slip, when present, will have on the critical mass flux.

The procedure for calculating the choking velocity in a two-phase mixture, when the mixture is treated as a simple compressible substance, is straightforward. The phase change that occurs causes the compressibility to be large, and the choking velocity is quite low. The theory has been presented in a previous report [26] along with tabulated results for choking velocity, a , and critical mass flux, $\psi_c = \rho a$ for water substance and R-114. These results predict ψ_c for two-phase flow only when the phase velocities are equal ($\sigma = 1$) and remain that way at choking. In general, ψ_c can be quite different if slip is present. In the work that follows, the notation ψ_{c0} will be

used to denote ψ_c for the case $\sigma = 1$. A numerical example from calculations presented previously demonstrates the effect of σ on ψ_c for given x and T . For R-114, with $\psi = 2500 \text{ kg/m}^2\text{-s}$ a number of Fanno lines are shown in Fig. 3 for different σ . For $\sigma = 1$, the choking point is reached at $x = 7.8\%$, $T = 14.5^\circ\text{C}$. At this state (given by x and T), ψ_{c0} from the previous work is $2500 \text{ kg/m}^2\text{-s}$ and agrees with the specified ψ . For a slip $\sigma = 2$ choking is reached at $x = 15\%$, $T = 3.5^\circ\text{C}$. (This point has not been shown in Fig. 3.) At this state the corresponding ψ_{c0} is $1589 \text{ kg/m}^2\text{-s}$, and hence $\psi_c/\psi_{c0} = 1.6$. The calculation supports the expectation that ψ_c will depend on slip. The Fanno curves in Fig. 3 are for constant σ , that is σ is specified as remaining constant along the flow. If the curves were calculated for σ changing in a specified way, a different ψ_c/ψ_{c0} would be obtained. From a practical standpoint, the state of an actual two-phase flow, determined from a temperature measurement, could show the flow to be at or even beyond choking by comparison of ψ to ψ_{c0} , when in reality this were not the case because of the presence of slip. A method for determining ψ_c based on slip and the rate of change of slip is outlined below.

Consider first the flow of a simple compressible fluid of uniform velocity w . The energy equation is expressed by

$$-dh = d(w^2/2) = wdw \quad (160)$$

and mass conservation by

$$d(\psi) = 0, \quad \psi = \rho w. \quad (161)$$

Combination yields the result

$$-dh = -w^2 d\rho \frac{1}{\rho} . \quad (162)$$

From the Gibbs equation,

$$dh = Tds - \frac{dP}{\rho} . \quad (163)$$

Combination of Eqs. (162) and (163) yields

$$T \frac{ds}{dP} = \frac{1}{\rho} (w^2 \frac{d\rho}{dP} - 1) . \quad (164)$$

At the point where choking occurs,

$$ds = 0 \quad \text{and} \quad \frac{d\rho}{dP} = \frac{\partial \rho}{\partial P} \bigg|_s = \frac{1}{a^2} . \quad (165a,b)$$

and the critical velocity and mass flux are

$$w_c = a, \quad \psi_c = \rho a. \quad (166a,b)$$

The task at hand involves deriving an equation for two-phase flow analogous to Eq. (164). In this case the kinetic energy is expressed in terms of the mass fractions and phase velocities:

$$-dh = d(x_g w_{g/2}^2 + x_f w_{f/2}^2) . \quad (167)$$

Mass conservation is expressed for either phase:

$$\rho_p w_p \alpha_p = \psi x_p, \quad d(\psi) = 0. \quad (168a,b)$$

Combination and expansion of Eqs. (167) and (168) results in the expression

$$dh = \frac{x_g w_g^2}{\rho_g} d\rho_g + \frac{x_f w_f^2}{\rho_f} d\rho_f - \frac{3}{2} dx(w_g^2 - w_f^2) + d\alpha \left(\frac{x_g w_g^2}{\alpha_g} - \frac{x_f w_f^2}{\alpha_f} \right). \quad (169)$$

If Eq. (163) is introduced with $x \equiv x_g$, $\alpha \equiv \alpha_g$, then

$$T \frac{ds}{dP} = \frac{x_g}{\rho_g} \left(w_g^2 \frac{d\rho_g}{dP} - 1 \right) + \frac{x_f}{\rho_f} \left(w_f^2 \frac{d\rho_f}{dP} - 1 \right) - \frac{3}{2} \frac{dx}{dP} (w_g^2 - w_f^2) + \frac{d\alpha}{dP} \left(\frac{x_g w_g^2}{\alpha_g} - \frac{x_f w_f^2}{\alpha_f} \right). \quad (170)$$

The first two terms are analogous to Eq. (164). The differentials of density in these terms are properties of the individual phases, since both phases remain at saturation as the flow moves along the duct.

Note that in contrast to Eq. (164) for single phase flow, these derivatives do not correspond to $\frac{1}{2}$ for the individual phases when

the flow chokes. They may be expressed in terms of temperature in the general form

$$\frac{d\rho_p}{dP} = \frac{d\rho_p}{dT} \frac{1}{P'_s} \quad (171)$$

where P'_s is the slope of the vapor pressure curve, given by

Eq. (129). The additional terms in Eq. (170) represent the effects of slip, mass transfer and rate of change of void fraction. Eq. (170)

will be expressed in terms of the slip σ and density ratio γ :

$$Tds = \left\{ \left(\frac{\psi x_f}{\rho_f \alpha_f} \right)^2 \{K\} - \frac{1}{\rho} \right\} dP \quad (= dh - \frac{dP}{\rho}) \quad (172a,b)$$

where the term K is given by

$$K \equiv \left[\frac{1}{\rho_f} \left(x_g \frac{d\rho_g}{dP} \gamma \sigma^2 + x_f \frac{d\rho_f}{dP} \right) \right] - \frac{3}{2} \frac{dx}{dP} (\sigma^2 - 1) + \frac{d\alpha}{dP} \frac{x}{\alpha} \frac{1}{\sigma} (\sigma^3 - \gamma). \quad (173)$$

By comparing Eqs. (172a) and (172b) it is clear that K simply contains all the differential quantities obtained from dh . Note that the last term in Eq. (173) becomes zero when $\sigma = \gamma^{1/3}$. It was shown in Sec. III.2 that this condition corresponds to the minimum kinetic energy with respect to σ . At the choking point, Eq. (165) applies and the expression for critical mass flux is obtained from Eq. (172a),

$$\psi_c^2 = \left\{ \rho \left(\frac{x_f}{\rho_f \alpha_f} \right)^2 K_c \right\}^{-1} \quad (174)$$

where K_c signifies that K must be evaluated at choking.

The differentials of x and α contained in K_c will be evaluated separately. For mass fraction,

$$h = h_f + x h_{fg} \quad (175)$$

and hence

$$dx = \{dh - dh_f - x(dh_g - dh_f)\} / h_{fg}. \quad (176)$$

At choking conditions $dh = \frac{dP}{\rho}$ so that

$$\left(\frac{dx}{dP}\right)_c = \left[\frac{1}{\rho} - \frac{dh_f}{dP} - x \left(\frac{dh_g}{dP} - \frac{dh_f}{dP} \right) \right] / h_{fg} \quad (177)$$

where the differentials of enthalpy are phase properties. By analogy to Eq. (171) these may be written, in general, as

$$\frac{dh_p}{dP} = \frac{1}{P'} \frac{dh_p}{dT} \quad (178)$$

Equation (177) can be expressed in the simpler form,

$$\left(\frac{dx}{dP}\right)_c = \frac{1}{P' h_{fg}} \left[\frac{P'}{\rho} - \frac{dh_f}{dT} - x \frac{dh_{fg}}{dT} \right] \quad (179)$$

The differential of void fraction α contained in K_c will be obtained by differentiating the expression for σ given by Eq. (21):

$$\sigma = \gamma \frac{1-\alpha}{\alpha} \frac{x}{1-x} \quad (180)$$

The result is

$$\frac{d\sigma}{\sigma} = \frac{d\gamma}{\gamma} + \frac{dx}{x(1-x)} - \frac{d\alpha}{\alpha(1-\alpha)} \quad (181)$$

This can be rearranged for $d\alpha$ as

$$\left(\frac{d\alpha}{dP}\right)_c = \alpha(1-\alpha) \left[\frac{1}{\gamma} \frac{d\gamma}{dP} + \frac{1}{x(1-x)} \left(\frac{dx}{dP}\right)_c - \frac{1}{\sigma} \left(\frac{d\sigma}{dP}\right)_c \right] \quad (182)$$

where a subscript c has been added to denote conditions at choking.

The term $\frac{dy}{dP}$ is a combination of the phase properties given in Eq. (171) and $(dx/dP)_c$ was determined above.

Equation (174) together with Eqs. (177) and (182) allow ψ_c to be calculated for a given x and T . In contrast to calculating ψ_{c0} , the quantities σ and $d\sigma$ must be specified. That is, it is necessary to specify the slip and the way that it changes as the flow proceeds. For given σ , two particular cases of interest are constant slip ($d\sigma = 0$) and constant void fraction ($d\alpha = 0$). The latter corresponds to the maximum $d\sigma$, as seen from Eq. (181). As stated previously, the rate of change of slip is actually determined within a flow by local conditions, specifically interaction between the phases (shear and mass transfer) and shear at the walls, but these are conveniently bracketed within the limiting cases above.

A convenient normalized form of ψ_c is obtained by dividing by the critical mass flux ψ_{c0} at the same state (x, T) . This dimensionless quantity is shown in Figs. 5 and 6 for $d\sigma = 0$ and $d\alpha = 0$, respectively. The fluid is R-114 at 25°C.

It will be noted from these figures that for the case of $\sigma = 1$ and slip constant ($d\sigma = 0$) the critical mass flux reduces to ψ_{c0} for the same state. At other values of slip, ψ_c is greater than ψ_{c0} (up to twice as large). The difference between ψ_c and ψ_{c0} is most pronounced at small values of x , that is when the flow is mostly liquid. It can be seen from the definition of K in Eq. (173) that this occurs because $d\rho_g/dP$ is large compared to $d\rho_f/dP$. When the flow is mostly vapor and x is large the vapor term $d\rho_g$ dominates

the expression. When x is small and the flow is mostly liquid, both contributions due to dp_p become small and the dx and $d\sigma$ terms (which involve σ) dominate so that the effect of $\sigma > 1$ is greater in the range of small x . Hence, just as the increase in compressibility due to phase change is greatest when a two phase system is predominantly liquid, the same is true of the effective change in compressibility of the flowing system due to the presence of slip.

The ratio ψ_c/ψ_{c0} is largest for a value of slip $\sigma = \gamma^{1/3}$. This is shown approximately as $\sigma = 5$ for R-114 in Fig. 5. Curves for $\sigma > \gamma^{1/3}$ are shifted successively downward. This is similar to the effect of σ on kinetic energy. It was shown previously that $\sigma = \gamma^{1/3}$ corresponded to minimum kinetic energy.

A check may be made on the calculations in this section by referring back to the Fanno line calculation for $\sigma = 2$, $\psi = 2500 \text{ kg/m}^2\text{-s}$. It was shown that choking occurred at $x = 15\%$, $T = 3.5^\circ\text{C}$. For this value of x and T , with $d\sigma = 0$, the analysis of this section yields the ratio $\psi_c/\psi_{c0} = 1.6$, identical to that obtained previously.

An important aspect of this work is the fact that a single characteristic "choking velocity" does not appear in the equations when slip is present, but mass flux becomes the important quantity. This also indicates that the ratio ψ/ψ_c may be analogous to the Mach number for this case. Of course it is identical to the Mach number when $\sigma = 1$.

IV. PRESENTATION OF EXPERIMENTAL RESULTS

1. Remarks

A program of measurement has been carried out in two-phase flow. The approach taken during this program is based on the analytical work presented previously; the quantities measured and the manner of their presentation will reflect this fact. The working substance that has been used is R-114, in a circular test section of diameter 2 inches. (A description of the experimental facility is given in Appendix B.) Measurements of pressure change and void fraction have been performed over a range of flow conditions, from liquid flow at saturation to high speed two-phase flow with large amounts of vapor, very close to choking.

Three series of measurements have been performed (I, II, III). Each series of measurement corresponds to a particular set of conditions at the flash point, specified in terms of Fr^* , Ma^* , Re^* . For each set of flash point conditions, measurements were taken over a range of downstream two-phase flow conditions. Comparison of the individual series allows the effect of the various flash point parameters to be examined.

The range of two-phase flow conditions is specified most conveniently by specifying the range of the reduced specific volume, $v_R = v/v_f$. An alternative parameter useful for reference is the Mach number Ma_0 that would describe the flow if the phase velocities were equal ($\sigma=1$), at the same state (x,t) .

A list of the important parameters for each of the series of runs is given in Table 1. Froude number Fr^* has been varied over the

approximate range 1 to 8. This amount of variation will be adequate to demonstrate the effect of stratification due to gravity. Froude number depends only on velocity for a given pipe diameter, and as a result a change in mass flow rate will always correspond to a change in this parameter.

Mach number Ma^* has been varied over the range 0.2 to 0.8; flows that are essentially incompressible will be compared to flows where compressibility dominates. Mach number depends on both velocity and temperature, since critical speed a^* is a property of the saturated liquid. Hence Ma^* and Fr^* may be adjusted independently by suitable choice of the flash point temperature T^* .

Reynolds number of the liquid at the flash point, Re^* , has been selected such that it falls in the mid-range of turbulent flow, for all runs conducted. Values are between 10^5 and 10^6 .

Determination of the state of the two-phase flow is made based on measurement of temperature at the point of interest. This temperature, in conjunction with measured flow rate, and temperature and pressure measured in the settling chamber (equivalently, ψ and h_0), allows calculation of x . Kinetic and potential energy can be neglected and x calculated accurately as discussed in Section III.2.

The calculation of x is based on the assumption that thermodynamic equilibrium exists between the phases, with each phase at saturation conditions everywhere in the measuring portion of the test section. Comparison of the measured dP/dT along the test section with the known $P'_s (= dP/dT|_{sat})$ for the substance indicates consistent agreement to

within $\pm 5\%$; this deviation corresponds to a difference of approximately 0.2°C between measured temperature changes and those expected assuming equilibrium. It is consistent with the accuracy of the temperature probes.

The determination of x allows calculation of all properties for the flow, including v_R and Ma_0 . The quantity v_R is the ratio of velocity of the two-phase flow, with $\sigma = 1$, to the velocity at the flash point, w^* . It provides a measure of the vapor content of the flow. For the runs conducted, values of v_R have ranged from 1 to 50. This quantity will be treated as the basic independent variable for the presentation of other measurements. The Mach number Ma_0 provides an indication of the proximity of the flow to choking (neglecting the effect of slip) and has ranged from 0.5 to 0.95.

The primary quantity that has been measured is the reduced pressure derivative, defined as

$$\Delta P_R = \left(\frac{dP}{dz} \right)_{TP} / \left(\frac{dP}{dz} \right)_L \quad (183a)$$

where $(dP/dz)_{TP}$ is the directly measured rate of change of pressure with pipe length z for the two-phase flow. The quantity $(dP/dz)_L$ is the pressure change that would exist if the flow were entirely in the liquid state (the parameter Re^* is implicit in the calculation of the latter quantity) with properties of the liquid at temperature T^* .

The second measured parameter is the void fraction α defined as previously:

$$\alpha = A_g/A. \quad (183b)$$

This has been examined using two techniques. The first involved quick-closing valves to allow direct measurement of the liquid and vapor volumes. The second technique involves the use of a "dynamic pressure probe" for void measurement, and was employed on a trial basis. The technique will be described and modifications suggested for further measurements using this type of probe.

Relatively few data points were taken for Series I and it can be seen from Table 1 that these were performed at essentially fixed Fr^* and Ma^* , but over a wide range of v_R compared to Series II and III. The runs for Series I were preliminary, intended to establish the trend of ΔP_R over the full v_R range and to verify various aspects of the procedure. No void fraction measurements were performed for these runs. Series II and III runs were performed over a limited range of v_R but with a variation of the parameters Fr^* and Ma^* . Measurements of both ΔP_R and α were made for runs in Series II and III. Hence these latter two series present the basic data of the experimental investigation.

The thermodynamic properties of R-114 that have been used for the interpretation of experimental measurements and in the calculations thus far are based on the data of Oguchi [42]. Sufficiently accurate correlations have been developed which give the properties at saturation as functions of temperature. For example, saturated liquid enthalpy h_f has been correlated with a standard deviation corresponding to $.008^\circ\text{C}$. Standard deviation for specific volume v_f is 0.2%. The phase

viscosities at saturation have been correlated using data given by ASHRAE [43]. The latter was compiled based on an evaluation and review of the literature. Standard deviations are 0.3% for the liquid and 0.6% for the vapor. It should be noted that data for thermodynamic properties has been supplied by ASHRAE in Ref. [43] and agrees with Oguchi within the correlation accuracies stated above. Previous viscosity data from this source has also shown good agreement with other measurements [44]. Details of the correlations are given in Ref. [26].

2. Measurement of Pressure Change

Calculations

The test section configuration consists of five measuring stations, each with a temperature probe and differential pressure transducer. This arrangement allows for four pairs of differential temperature and pressure measurements. After checking for consistency in the trend of each measurement from station to station, the first and last pairs are discarded. The two intermediate differential pressure measurements are used to evaluate the two-phase pressure derivative $(dp/dz)_{TP}$ at the center station by calculating the average of $\Delta P/\Delta z$ for the two measurements. Since the lengths of the pipe segments are identical, this is equivalent to calculating the slope of the best linear fit of pressure vs. length for the three points, and is also the same as evaluating the derivative at the midpoint of a smooth quadratic curve passing precisely through each point. The pressure derivative is assumed to correspond to the flow state (in terms of v_R) calculated from the temperature measured at the center station.

The two-phase pressure derivative is normalized by dividing it by the pressure derivative that would be expected if the entire mass flow rate were in liquid phase, calculated from the Moody diagram. (Measurements of pressure change with liquid flow only indicate that the pipe may be assumed "smooth.") The normalized quantity is then the reduced pressure ΔP_R defined by Eq. (183a). Measurements of ΔP_R are presented as a function of v_R , with the corresponding Fr^* and Ma^* as parameters.

The quantity β^* (as defined by Eq. (157)) will be shown in addition to Ma^* for the different sets of data. The property $\bar{\gamma}^*$ (which appears in β^*) for saturated liquid R-114 at 25°C is 1.002 and varies only slightly along the saturation line. As a result all calculations of β^* are based on $\bar{\gamma}^* = 1.0$.

It is informative to compare the measured reduced pressure change to predictions of the same quantity based on three simple interpretations of the two-phase flow. These models predict the frictional pressure change and will be used only as references for comparing data.

Model 1. The two-phase flow pressure change may be predicted by attempting an analogy to single phase incompressible flow in the form

$$\left(\frac{dP}{dz}\right)_{TP} = f_{TP} \rho \frac{w^2}{2} \frac{1}{D} \quad (184)$$

where f_{TP} is a two-phase friction factor, and ρ and w are density and velocity. If the entire mass flow rate were liquid (properties denoted by subscript L) the pressure change would be

$$\left(\frac{dP}{dz}\right)_L = f_L \rho_f \frac{w_L^2}{2} \frac{1}{D} \quad (185)$$

and, hence

$$\Delta P_R = \frac{f_{TP}}{f_L} \frac{v_f}{v} \left(\frac{w}{w_L}\right)^2. \quad (186)$$

If the two-phase flow is assumed homogeneous, with the same friction factor as the liquid flow, then

$$f_{TP} = f_L, \quad \frac{w}{w_L} = \frac{v}{v_f} \quad (187)$$

and the result (given also by Owens [25]) is

$$\Delta P_R = \frac{v}{v_f} = v_R. \quad (188)$$

Model 2. The pressure change of each phase within the two-phase flow must be the same as that of the two-phase flow itself. Hence the latter can be predicted by calculating the pressure change of one of the phases. The phase is assumed to behave in the mixture as it would if it were flowing through a pipe of cross-sectional area equal to the cross-sectional area that it occupies in the two-phase mixture. If the liquid phase is selected (subscript f denotes properties of the phase in the mixture), the pressure change is expressed as

$$\left(\frac{dP}{dz}\right)_{TP} = f_f \rho_f \frac{w_f^2}{2} \frac{1}{D_f}. \quad (189)$$

This will be written in the alternative form

$$\left(\frac{dP}{dz}\right)_{TP} = f_f \frac{1}{2\rho_f} \left(\frac{\dot{m}_f}{A_f}\right)^2 \frac{1}{D_f}. \quad (190)$$

Similarly, for liquid flow only:

$$\left(\frac{dP}{dz}\right)_L = f_L \frac{1}{2\rho_f} \left(\frac{\dot{m}}{A}\right)^2 \frac{1}{D}. \quad (191)$$

If the friction factors are taken to be equal ($f_L = f_f$) and also that $D_f = D$, combination of the equations above gives

$$\Delta P_R = (\dot{m}_f/\dot{m})^2 / (A_f/A)^2 = \{(1-x)/(1-\alpha)\}^2 \quad (192)$$

where the definitions of mass and void fractions have been used. The ratio R will be defined as

$$R = \{(1-x)/(1-\alpha)\}. \quad (193)$$

The assumption $D_f = D$ is valid for a model of the flow which assumes only liquid to flow at the walls (with friction factor $f_f = f_L$) and vapor to move through the center of the pipe. The concept of calculating the pressure change for the liquid phase within the two-phase mixture is also most valid for this model. The form of ΔP_R in Eq. (192) is the same as that proposed by Lottes and Flinn [37].

The relationship between x and α in the equation above depends on the velocities of the individual phases. When the continuity equations for each phase are combined, the following general relationship is obtained, in terms of the density and velocity ratios:

$$R = \left\{ 1 + x \left(\frac{\gamma}{\sigma} - 1 \right) \right\}. \quad (194)$$

When x is eliminated using v_R the result is

$$R = \left\{ 1 + \left(\frac{v_R - 1}{\gamma - 1} \right) \left(\frac{\gamma}{\sigma} - 1 \right) \right\}. \quad (195)$$

For the case $\sigma = 1$ we have $R = v_R$ and hence

$$\Delta P_R = v_R^2. \quad (196)$$

Model 3. The derivation of Model 2 is used except the case of equal dynamic pressures for the phases is used to calculate the slip ratio. Hence

$$\sigma = \gamma^{1/2} \quad (197)$$

and

$$\Delta P_R = \left\{ 1 + \left(\frac{v_R^3 - 1}{\gamma - 1} \right) \left(\gamma^{1/2} - 1 \right) \right\}^2. \quad (198)$$

The density ratio γ depends only slightly on temperature, and will be calculated at the average downstream temperature when this model is used as a reference for a group of runs.

Results

The results of pressure change measurements (ΔP_R vs. v_R) for Series I are shown in Figs. 7 and 8.

Figure 7 shows measurements obtained for two runs using Procedure 1, as described in Appendix B. This procedure involves opening or closing the throttle valve continuously during a run and points taken under each condition have been denoted by different symbols. The direction of the process, in terms of increasing or decreasing v_R , has been denoted by arrows for each run. The pressure change for a valve continuously opening is seen to be shifted slightly higher, in general, than that for a continuous closing. This corresponds to the effect that would be expected to appear due to a continuous acceleration versus deceleration, respectively. Figure 8 shows points taken over the same range of v_R with a fixed valve (Procedure 2). Superposition of Figs. 7 and 8 shows

the fixed valve measurements to fall between those obtained while changing the valve setting. The small difference between all measurements indicate that the flow stabilizes quickly and that steady, developed flow is readily achieved during normal operation. The points shown in Fig. 8 correspond to the true measurement of ΔP_R over this range of v_R .

When the Series I measurements were taken, a test was made to determine the effect of the temperature probes on the ΔP_R measurements, since these protrude into the test section as far as the center line. One of the runs shown in Fig. 8 was made with these probes withdrawn to the wall. This run has been identified in the figure. No significant effect is evident, when compared to the standard deviations of the measurements. Sample standard deviations are shown for two points.

Figure 8 demonstrates a value of ΔP_R as high as 20 in the range of v_R between 10 and 50. The corresponding average Froude and Mach numbers are 1.2 and 0.29, respectively. Reference lines have been shown which correspond to the three models discussed previously. All points fall between the equal dynamic pressure line (EDP) and the homogeneous flow (HMG). The no-slip (NS) reference is much higher than either of the other models or the measured data. Hence the pressure change prediction based on liquid flow at the walls with equal phase velocities is clearly unrealistic.

Results of pressure change measurements for Series II are shown in Figs. 9 and 10. For this series of measurements, Ma^* has been increased to an average value of 0.64, from an average value of 0.29 for Series I. Froude number was increased from 1 to an average value of 5.8 for

Series II. Figure 9 shows all points that have been taken. The range of v_R is much smaller because of the higher Ma^* (the fluid is much closer to choking when it flashes, and less vapor is generated in approaching two-phase choking). The measured ΔP_R has increased, with points now crossing the reference line corresponding to homogeneous flow.

In Fig. 9 a change in the slope of ΔP_R with v_R is apparent for v_R between 5.0 and 6.0. The same will be seen with the data of Series III. For each run performed, a description of the visual appearance of the flow in the test section was recorded. These descriptions indicate that a change in the basic structure of the flow occurs between these values of v_R . For v_R less than approximately 3, the flow was completely stratified, with a free, wavy liquid surface. The void fraction appeared to be approximately 1/3 to 1/2. For v_R between 3 and about 4.5, regimes were characterized by a higher velocity, with waves on the liquid surface reaching the top of the tube, and breaking as they were swept along in the flow direction by the vapor moving at higher speed. The waves and vapor regions were not clearly defined because of the froth created by the high speed. For v_R greater than 4.5, but less than approximately 6, stratification was clearly visible in the tube, but with both phases moving at high speed and without a wavy or frothy interface area. For v_R larger than 6, the flow appeared "homogeneous," with a high speed. The individual phases were indistinguishable and no stratification was visible at all. The change in slope of ΔP_R in Fig. 9 corresponds to development of this "homogeneous" flow structure.

Figure 9 shows a large amount of scatter between measured points. This corresponds to the wide range of Ma^* (0.59-0.72). Figure 10 gives the same points, but shows them grouped according to Mach number. The parameter β^* is also shown. Points with a common value of Ma^* which cover a sufficiently large range of v_R have been connected by labeled, solid lines. Points with miscellaneous values of Ma^* or those appearing in a cluster have been distinguished by individual symbols. The effect of Mach number on pressure change for a given value of v_R is clear when comparing Series I to Series II, and individual points within Series II. The increased compressibility of the mixture at higher Ma^* , unique to single-substance flows, results in a higher pressure change. This trend is followed most exactly by points above $v_R = 4.0$. Below this value of v_R , the effect of Ma^* is still predominant, but a high degree of scatter exists for all points due to the difficulty of an accurate measurement of ΔP_R at small values of v_R . This is reflected by the sample standard deviations, which have been shown for a number of points over the full v_R range.

Results of pressure change measurements for Series III are shown in Figs. 11 and 12. The presentation is identical to that for Series II. Figure 11 shows all points measured and the reference lines corresponding to each of the flow models. For this series of measurement, the Mach number has been decreased to an average value of 0.56, from 0.64 for Series II, and Froude number has been increased to an average value of 7.0 from 5.8. Figure 12 shows measured points grouped according to Ma^* as before; the same effect due to Mach number is evident, and consistent

for all measured points except one, for which the standard deviation is excessive.

Comparison of measurements for Series II and III indicate the same range of ΔP_R , although the Mach number range has been shifted downward for Series III measurements. It was originally expected that a change in Froude number over a narrow range of approximately 5 to 8 would not be visible, but this is precisely the effect demonstrated between these sets of measurements. A change in Froude number from an average of 5.8 to one of 7.0 has been sufficient to offset the decrease in Mach number. From Table 1, Fr_{max}^* for Series II is 13, while that for Series III is 22. The seemingly small change in Fr^* represents a significant portion of the range available, given by Fr_{max}^* . (It should be noted at this point that $Fr^* = Fr_{max}^*$ implies $Ma^* = Ma_{max}^* = 1$.) The effect of Fr^* is in agreement with that predicted previously, namely an increased pressure change resulting from a larger net momentum flux when the liquid phase is accelerated along with the vapor. This occurs when Fr^* is greater and hence the flow is less stratified.

3. Measurement of Void Fraction

Calculations

Void fraction was measured for all of the runs in Series II and Series III for which Procedure 2 was used. The measurements were obtained by using the test section isolation valves as quick-closing valves. A direct measurement of the vapor volume fraction is then possible by measuring the level of the liquid surface. The proper corrections are applied for the quantity of fluid that drains into the test section from the settling chamber and that which drains into the diffuser from the test section after the isolation valves are closed. A correction is also required to account for the phase change that occurs as the fluid is brought to rest. This latter effect is negligible because of the magnitude of the kinetic energy, as discussed earlier. A correction is instead applied to account for the temperature change due to warming of the fluid from the run temperature to room temperature after the valves are closed.

The measurement represents an average void fraction over the entire test section, between throttle plate and gate valve, including the region of non-developed flow immediately following the throttle plate. Due entirely to this region, the actual void fraction may be less than the measured value by an estimated maximum 7% of its value, and hence represents a rather crude determination. The measured void fraction is assumed to correspond to the value of v_R calculated at the center measuring station.

The experimental determination of void fraction will be compared to two reference limits: phase velocities corresponding to no slip ($\sigma = 1$) and equal phase dynamic pressures ($\sigma = \gamma^{1/2}$). Expressions for void fraction α were given in Eqs. (22,97a,b). Mass fraction x may be eliminated using v_R . The general expression for α is then

$$\alpha = \left\{ \left(\frac{\sigma}{\gamma} \frac{\gamma - v_R}{v_R - 1} \right) + 1 \right\}^{-1} \quad (199)$$

and the special cases become

$$\alpha = \left\{ \left(\frac{\gamma - v_R}{v_R - 1} \right) / \gamma^{1/2} + 1 \right\}^{-1} \quad (\sigma = \gamma^{1/2}) \quad (200)$$

$$\alpha = \left\{ \left(\frac{\gamma - v_R}{v_R - 1} \right) / \gamma + 1 \right\}^{-1} \quad (\sigma = 1). \quad (201)$$

When reference values of α are shown for a set of runs, γ is calculated at an average temperature for the set.

Results

Results for void fraction are given in Fig. 13 for Series II and III. The individual series have been denoted by different symbols. All measurements fall within the reference limits. A reference line for slip $\sigma = 2.8$ has also been shown. Although all points show nearly the same slip, they tend toward the no-slip line at smaller values of v_R , as would be expected. The runs shown were performed at an average Fr^* of approximately 6.0. However, two points corresponding to the

extremes of the range have been denoted, namely $Fr^* = 4.9$ and $Fr^* = 7.1$. A higher Fr^* (less stratification) should correspond to a flow closer to no-slip, the opposite being true for a lower Fr^* . Although the labeled points are essentially identical to the others within the accuracy of the measurement, the expected behavior is apparent.

A Measurement Technique Based on Dynamic Pressure

A technique for measuring void fraction will be proposed which relies on an obstruction placed in the flow to create an additional pressure change that is measurable. The additional pressure change will depend on the force exerted on the obstruction due to each of the phases. The portion of the total force due to a particular phase is directly related to the phase dynamic pressure by the relation

$$F_p = C_{Dp} \rho_w \frac{v_p^2}{2} A_{pp} \quad (202)$$

where C_{Dp} is a phase drag coefficient and A_{pp} is the projected area of the obstruction exposed to the particular phase. The configuration chosen for this obstruction or dynamic pressure probe is that shown in Fig. 14. A separated phase configuration is shown for illustration only and is not required for this technique. The probe shape selected is cylindrical, since in that case the drag coefficient is well known, as a function of Reynolds number based on probe diameter. A common coefficient of constant value C_D will be used for both phases. This step will be justified in the sample calculations. The total force on the probe is then given by

$$F = C_D \left\{ \rho_f w_f^2 \frac{\ell_f d}{2} + \rho_g w_g^2 \frac{\ell_g d}{2} \right\} \quad (203)$$

where d is the probe diameter and ℓ_p is the portion of the probe length exposed to a particular phase. In general, $A_{pp} = \ell_p d$. The approximation may be made that

$$\ell_p = \alpha D. \quad (204)$$

The above equation of force is then

$$F = C_D \frac{Dd}{2} \left(\rho_f w_f^2 (1-\alpha) + \alpha \rho_g w_g^2 \right). \quad (205)$$

The total force may be written as

$$F = (\Delta P) A \quad (206)$$

where ΔP is pressure difference induced in the flow due to the frictional effect of the probe and A is the pipe cross section area.

When the phase continuity equations are used along with the definition of γ , Eq. (205) may be written in the form

$$\tilde{F} \equiv \frac{2\Delta P \rho_f}{\psi^2 C_D \tilde{A}} = \left\{ \frac{(1-x)^2}{(1-\alpha)} + \gamma \frac{x^2}{\alpha} \right\}. \quad (207)$$

The quantity $\tilde{A}$ is the ratio of projected probe area to pipe area,

$$\tilde{A} = Dd/A.$$

The pressure difference ΔP is determined based on a measured pressure difference, which will be denoted ΔP_m . This is the difference between

a pressure change over a given length of pipe with the probe in place and the pressure change over the same length of pipe without the probe under identical flow conditions. Hence the quantity ΔP_m is a total pressure difference, containing components due to both friction and acceleration. Two corrections must be applied to ΔP_m in order to obtain the value ΔP appearing in Eq. (207).

It was demonstrated previously (Eq. (154a,b)) that the ratio of frictional pressure change to accelerational pressure change for a simple compressible flow is given by

$$\frac{\Delta P_d}{\Delta P_a} = \frac{1-M^2}{\gamma M^2} \equiv \beta. \quad (208)$$

As a result, the frictional part of the total measured pressure change is obtained by applying the acceleration correction

$$\Delta P_d = \Delta P_m / \left(\frac{1+\beta}{\beta} \right). \quad (209)$$

When the probe is inserted into a finite cross section, the effective flow area is decreased, and the flow velocity increases as it passes the obstruction. This increases the dynamic pressure from that of the undisturbed flow. Hence, a blockage correction must be applied. This is easily derived in terms of the Mach number, with the familiar form

$$\Delta P = \Delta P_d (1 - \tilde{A})^{\left(\frac{2-M^2}{1-M^2} \right)}. \quad (210)$$

Hence, ΔP is the frictional pressure change due to the true dynamic pressure of the two-phase flow, and is the same quantity which appears

in Eq. (207). In terms of the measured total pressure change, ΔP is then given by

$$\Delta P = \Delta P_m / \left(\frac{1+\beta}{\beta} \right) (1-\tilde{A})^{\frac{M^2-2}{1-M^2}}. \quad (211)$$

This will be written in terms of correction factors for acceleration and blockage C_a and C_b , respectively in the form

$$\Delta P = \Delta P_m / C_a C_b. \quad (212)$$

The correction for blockage was obtained assuming that the effect of the area reduction would be the same throughout the cross section. In reality the velocity increase at the obstruction is much less, since the streamlines close to the wall are the ones most displaced from the undisturbed flow. For a cylinder in a two-dimensional duct of width D' , the cylinder may be simulated by a doublet, and the effect of the walls accounted for by an infinite series of doublets of equal strength. The technique is summarized in Ref. [38]. The blockage correction is then

$$C_b = 1 + \frac{2-M^2}{(1-M^2)^{3/2}} \epsilon \quad (213)$$

where ϵ depends on the cylinder and duct dimensions, and hence the doublet strength. For the present case it is given by

$$\epsilon = (\pi^2/12) (d/D')^2. \quad (214)$$

The two-dimensional diameter D'^2 is replaced by $D'^2 = D^2/2$, the average of the diameter squared for the circular cross section.

The drag coefficient for a cylinder introduced in the above equations has been determined by Prandtl [39], as a function of Reynolds number based on cylinder diameter. The corresponding chart is reproduced in Fig. 15.

Reynolds numbers of the individual phases have been calculated for the two limiting cases $\sigma = 1$ and $\sigma = \gamma^{1/2}$, for the range of mass flux ψ which has been used. Both Reynolds numbers vary only between 6.4×10^4 and 2.5×10^5 . Thus a common drag coefficient can then be used for both phases, with a constant value $C_D = 1.2$.

Measurements of ΔP_m were obtained using a modified version of Procedure 2. After starting the run and taking a set of measurement scans with undisturbed two-phase flow in the test section, measurements were halted for a period of 5 seconds. During this time the cylindrical probe was inserted manually into the pipe cross section and locked in place without halting the flow. Another set of measurements were then taken. The fitting used for this purpose is identical to that used to hold the thermistor probe, except modified to accept cylinders of 1/8 inch to 1/4 inch. By using the procedure just described, ΔP_m can be measured with and without the probe in place under identical flow conditions.

The quantity $\tilde{F}$ of Eq. (207) is shown in Fig. 16 for 5 runs, repeated with the same conditions in the test section. Measurements were made with probe diameters of 1/4, 3/16 and 1/8 inch. The calculation of $\tilde{F}$ for the first set of points in the figure, labeled as "uncorrected data," was made using the directly measured value ΔP_m . The measurement depends strongly and systematically on probe diameter as a result of the

different amounts of blockage. A best straight line fit for these points is shown, extrapolated to $d = 0$. Also shown in the figure is $\tilde{F}$ calculated based on $\Delta P_m/C_b$, that is with the blockage correction applied. Sets of points for both the doublet and the direct area correction are shown. The former does not provide the correction required. For the latter, the points are seen to be independent of probe diameter, as required for $\tilde{F}$, but slightly lower than the extrapolated value. The final set of points is calculated from $\Delta P/C_a C_b$, that is with the direct area blockage correction and the acceleration corrections applied. The corrected pressure difference remains independent of probe diameter but is shifted to a much lower value because of the high Mach number; the frictional pressure change represents a small portion of the total.

A sample calculation for a particular run will be shown. The profile of α vs. $\tilde{F}$ from the right-hand side of Eq. (207) is given as the curve in Fig. 17. The values of α corresponding to $\sigma = 1$ and $\sigma = \gamma^{1/2}$ are shown. The measured value of α is expected to fall within this range. The void fraction measured using quick-closing valves for this run is also shown as $\alpha' = 68\%$. Values of $\tilde{F}$ from the left-hand side of Eq. (207) are shown based on the basic measurement. The values correspond to an uncorrected pressure change (ΔP_m), blockage correction only ($\Delta P_m/C_b$) and finally both corrections ($\Delta P_m/C_a C_b$). The extrapolated value from Fig. 16 is also shown. It can be seen that the blockage correction renders a value of $\tilde{F}$ which is higher, as it should be, than that required to match α' . But when the acceleration correction is applied (to either the extrapolated value or that corrected for

blockage), the resulting value not only falls below that for α' but also out of the range of solutions given by the parabola.

The calculations above are representative of all runs for which this technique has been applied. The accelerational pressure change, predicted based on homogeneous flow, is much higher than that which actually occurs. This is consistent with the effects of slip as discussed in Section III; the presence of slip causes friction to represent a greater portion of the overall pressure change.

In order to apply the technique just described, it is necessary to measure the drag on the cylinder directly, eliminating the correction for acceleration. This has been a standard type of measurement in wind tunnel testing, and a proper technique could be adapted to the present case. Alternatively, a different technique must be developed for measuring the phase dynamic pressures.

APPENDIX A. Supplementary Derivations

Fanno Line Slope

Consider a compressible flow of uniform velocity, in an adiabatic duct of constant cross-sectional area. The energy and continuity equations are

$$d(h + w^2/2) = 0 \quad \text{or} \quad dh = -wdw \quad (\text{A1a,b})$$

$$\frac{dw}{w} = - \frac{d\rho}{\rho} \quad (\text{A1c})$$

In addition, the Gibbs equation is

$$dh = Tds + v dP \quad \text{or} \quad dP = \rho(dh - Tds). \quad (\text{A2a,b})$$

The process can be considered as a sequence of two processes between the same end states: a throttling process at constant enthalpy, followed by an acceleration process at constant entropy. The pressure differential then can be written as the sum of contributions due to each process:

$$dP = (dP)_s + (dP)_h \quad (\text{A3})$$

where the subscript denotes the property being held constant. Comparison of Eq. (A3) with Eq. (A2b) gives the relation

$$dP_s = \rho dh. \quad (\text{A4a})$$

Substitution for dh from Eqs. (A1b) and (A1c) yields

$$dP_s = w^2 d\rho. \quad (\text{A4b})$$

The density differential can also be written in terms of two contributions. Equation (A4b) is then

$$w^2 \left(\frac{d\rho_s}{dP_s} + \frac{d\rho_h}{dP_s} \right) = 1. \quad (A5)$$

The speed of sound a is given by

$$\frac{1}{a^2} = \left. \frac{\partial \rho}{\partial P} \right|_s = \frac{d\rho_s}{dP_s} \quad (A6)$$

and Eq. (A5) becomes

$$1 - M^2 = \frac{d\rho_h}{dP_s} w^2 \quad (A7)$$

where $M = w/a$.

The quantity $d\rho_h$ may be expanded:

$$d\rho_h = \left. \frac{\partial \rho}{\partial P} \right|_h dP_h = \left(-\frac{1}{v^2} \left. \frac{\partial v}{\partial P} \right|_h \right) dP_h.$$

Hence, Eq. (A7) becomes

$$1 - M^2 = -\frac{1}{v^2} \left. \frac{\partial v}{\partial P} \right|_h \frac{dP_h}{dP_s}. \quad (A8)$$

If both sides of this equation are multiplied by $\partial v / \partial P|_s$ the result is

$$\frac{1-M^2}{M^2} = \left(\left. \frac{\partial v}{\partial P} \right|_h / \left. \frac{\partial v}{\partial P} \right|_s \right) \frac{dP_h}{dP_s}. \quad (A9)$$

The quantity $\bar{\gamma}$ will be defined

$$\bar{\gamma} = \left. \frac{\partial v}{\partial P} \right|_h / \left. \frac{\partial v}{\partial P} \right|_s \quad (A10)$$

and hence the ratio of the pressure change due to dissipation (dP_h) and that due to acceleration (dP_s), from Eq. (A9), becomes

$$\frac{dP_h}{dP_s} = \frac{1-M^2}{\gamma M^2} \quad (A11)$$

From Eqs. (A2a) and (A3) the following relationship is obtained directly

$$\frac{dP_h}{dP_s} = - \frac{Tds}{dh} \quad (A12)$$

which gives the Fanno line slope:

$$\frac{Tds}{dh} = - \left(\frac{1-M^2}{\gamma M^2} \right) \quad (A13)$$

1. Special case: Ideal gas

Equation (A10) is equivalent to the form

$$\bar{\gamma} = \left. \frac{\partial \rho}{\partial P} \right|_h \bigg/ \left. \frac{\partial \rho}{\partial P} \right|_s \quad (A14)$$

The denominator is just $1/a^2$, where

$$a = \sqrt{\frac{c_p}{c_v} RT} \quad (A15)$$

for an ideal gas. The numerator may be obtained from the ideal gas equation of state,

$$P = \rho RT \quad (A15a)$$

since constant enthalpy implies constant temperatures in this case.

Hence:

$$\left. \frac{\partial \rho}{\partial P} \right|_h = \left. \frac{\partial \rho}{\partial P} \right|_T = \frac{1}{RT} \quad (A15b)$$

and Eq. (A14) becomes

$$\bar{\gamma} = \frac{c_p}{c_v}. \quad (\text{A16})$$

2. Special case: Homogeneous two-phase mixture

The quantity $\bar{\gamma}$ is given by Eq. (A10) in the general form

$$\bar{\gamma} = \left. \frac{\partial v}{\partial p} \right|_h \bigg/ \left. \frac{\partial v}{\partial p} \right|_s. \quad (\text{A17})$$

Transformation of the numerator, using Jacobians, to independent variables (P, s) results in

$$\bar{\gamma} = \left\{ \left. \frac{\partial v}{\partial p} \right|_s \left. \frac{\partial h}{\partial s} \right|_p - \left. \frac{\partial v}{\partial s} \right|_p \left. \frac{\partial h}{\partial p} \right|_s \right\} \bigg/ \left\{ \left. \frac{\partial h}{\partial s} \right|_p \left. \frac{\partial v}{\partial p} \right|_s \right\} \quad (\text{A17a})$$

$$\bar{\gamma} = 1 - \left\{ \left. \frac{\partial v}{\partial h} \right|_p \left. \frac{\partial h}{\partial p} \right|_s \right\} \bigg/ \left\{ \left. \frac{\partial v}{\partial p} \right|_s \right\}. \quad (\text{A17b})$$

The additional expressions will be used:

$$\left. \frac{\partial h}{\partial p} \right|_s = v, \quad (\text{A18a})$$

$$\left. \frac{\partial v}{\partial p} \right|_s = -v^2 \frac{1}{a^2} = -\frac{1}{\psi_c^2} \quad (\text{A18b})$$

$$\left. \frac{\partial v}{\partial h} \right|_p = v_{fg}/h_{fg}. \quad (\text{A18c})$$

The quantity $\bar{\gamma}$ is then expressed in the convenient form:

$$\bar{\gamma} = 1 + v \psi_c^2 \frac{v_{fg}}{h_{fg}} = 1 + \frac{v_{fg}}{v} \frac{a^2}{h_{fg}}. \quad (\text{A19})$$

Governing Equations Including Phase Potential Energy

The governing equations for two-phase flow derived in Section II will be modified to include the potential energy of the horizontal flow. This was discussed very briefly in the text as a basis for introducing the Froude number.

Potential energy is important in two-phase flow when the average heights of each phase, measured from a convenient datum, are not equal in a given cross section of pipe. In a homogeneous phase arrangement irrespective of slip, these heights are equal and correspond to the center line of the pipe. They are not equal in a non-homogeneous arrangement with one phase predominantly present in the upper or lower portion of the cross-section. In the most general case, this is represented by a continuous density gradient. The simplest case is a completely stratified flow with a separate vapor layer above the liquid, each phase moving at a different velocity. The equations will be written, in general, by introducing the average phase heights, y_p . In cases when the flow is not stratified the potential energy terms become negligible compared to the kinetic energy and the equations properly describe the flow as given previously.

The difference in potential energy between phases resulting from flow stratification also gives rise to a difference between the average pressures of each phase. The assumption of uniform temperature will be maintained at this point, but will be discussed in more detail once the basic equations have been derived.

The effect of surface tension will also produce a pressure difference between phases. Such an effect occurs on a length scale much smaller

than that experienced with the type of flow being considered, except in the case of extremely small droplets and bubbles, and will not be considered.

Based on the remarks above, the phase conservation equations will be rewritten. The continuity equation remains unchanged and is given by

$$\psi d(x_p) = d(\rho_p w_p \alpha_p) \quad (A20)$$

or

$$\psi x_p = \rho_p w_p \alpha_p. \quad (A21)$$

The energy balance is given, in a form analogous to Eq. (9),

$$\dot{m}(x_p + dx_p) \{ (h_p + dh_p + w_{p/2}^2 + d(w_{p/2}^2) + g(y_p + dy_p) \} \quad (A22)$$

$$- \dot{m}x_p \{ h_p + w_{p/2}^2 + gy_p \} - \dot{m}dx_p \{ h_i + w_{i/2}^2 + gy_i \}$$

$$= \dot{q}_p L_i dz - \tau_{ip} L_i dz w_i$$

where g is the gravitational acceleration and y_p and y_i are the average height of the phase and the height of the interface, respectively. The introduction of y_i reflects the fact that mass crosses the phase boundary with a potential energy corresponding to the actual height of the interface. The total change in potential energy occurring with the transferred mass is hence apportioned properly to each phase, as with the interface velocity and corresponding kinetic energy.

When Eq. (A22) is expanded and simplified the result is

$$\psi \frac{d}{dz} \{x_p (h_p + w_{p/2}^2 + gy_p)\} = \bar{q}_p - \bar{\tau}_{ip} w_i + \psi \frac{dx_p}{dz} \{h_i + w_{i/2}^2 + gy_i\}. \quad (A23)$$

The sum of the equation for the phases yields

$$d\{x(h_g + w_{g/2}^2 + gy_g) + (1-x)(h_f + w_{f/2}^2 + gy_f)\} = 0 \quad (A24)$$

or

$$d(h_0) = 0. \quad (A25)$$

A center of mass height may be defined

$$y_{cm} = xy_g + (1-x)y_f. \quad (A26)$$

The stagnation enthalpy is then

$$h_0 = h + w_{rms/2}^2 + gy_{cm}. \quad (A27)$$

The phase momentum equation is given by

$$\psi \frac{d}{dz} (x_p w_p) = -\alpha_p \frac{dp}{dz} + \bar{\tau}_{pw} + \bar{\tau}_{pi} + \psi \frac{dx_p}{dz} w_i \quad (A28)$$

which is derived in exactly the same way as Eq. (39) and is identical except that pressures of the individual phases will be distinguished by a subscript, since in general they will not be equal. Summation of Eq. (A28) for both phases yields

$$\psi \frac{d}{dz} \{xw_g + (1-x)w_f\} = -\alpha \frac{dp}{dz} - (1-\alpha) \frac{dp_f}{dz} + \bar{\tau}_{gw} + \bar{\tau}_{fw}. \quad (A29)$$

When the phase pressures are the same, this equation becomes

$$\psi \frac{d}{dz} \{w_{cm}\} = -\alpha \frac{dP}{dz} + \bar{\tau}_{gw} + \bar{\tau}_{fw} \quad (A30)$$

as obtained previously.

The phase Gibbs equation is just

$$dh_p = T ds_p + \frac{dP}{\rho_p} \quad (A31)$$

When this is combined with the phase momentum and energy equations (Eq. (A28) and (A23)) the result is

$$\begin{aligned} \psi x_p \left\{ T \frac{ds_p}{dz} \right\} &= \bar{q}_p + \bar{\tau}_{ip} (w_p - w_i) + \bar{\tau}_{wp} w_p \\ &+ \psi \frac{dx_p}{dz} (h_i - h_p) \\ &+ \psi \frac{dx_p}{dz} (w_{i/2}^2 - w_{p/2}^2) \\ &+ \psi \frac{dx_p}{dz} g(y_i - y_p) \\ &+ \psi \frac{dx_p}{dz} (w_p - w_i) w_p \\ &- \psi x_p g \frac{dy_p}{dz} \end{aligned} \quad (A32a)$$

This equation is analogous to Eq. (43), and includes the difference in potential energy between the mass entering and that of the phase. The last

term expresses the change in height of the phase and the resulting contribution of the potential energy change to the phase entropy change.

The velocity terms may be combined here also to produce the term

$$(w_p - w_i)^2/2, \quad (\text{A32b})$$

and hence an equation analogous to Eq. (46).

When the phase Gibbs free energy g_p is introduced and Eq. (A32a) is summed for both phases, the result is

$$\psi T \frac{ds}{dz} = \bar{\tau}_w w_g + \bar{\tau}_i (w_g - w_f) + \bar{\tau}_{wf} w_f \quad (\text{A33})$$

$$+ \psi \frac{dx}{dz} \{g_f^o - g_g^o\}$$

$$+ \psi \frac{dx}{dz} (w_g - w_i) w_g + \psi \frac{dx}{dz} (w_i - w_f) w_f$$

$$- \psi x g \frac{dy_g}{dz} - \psi (1-x) g \frac{dy_f}{dz}$$

where, in general,

$$g_p^o = g_p + w_{p/2}^2 + g y_p. \quad (\text{A34})$$

Equation (A33) is analogous to Eq. (48). The additional terms $g dy_p$ represent the changes in potential energy of the individual phases. By combining all velocity terms on the right-hand side of (Eq. A33), the equation may be written as

$$\begin{aligned} \psi T \frac{ds}{dz} = & \bar{\tau}_w w_g + \bar{\tau}_i (w_g - w_f) + \bar{\tau}_w w_f \\ & + \psi \frac{dx}{dz} \{g_f' - g_g'\} - \psi x g \frac{dy_g}{dz} - \psi (1-x) g \frac{dy_f}{dz} \end{aligned} \quad (A35)$$

where

$$g_p' = g_p - (w_p - w_i)^2/2 + g y_p. \quad (A36)$$

The definitions of g_p^0 and g_p' may be modified by introducing the interface property g_i , in addition to w_i and y_i . In this case the former definitions become

$$g_p^0 = (g_p - g_i) + (w_p^2 - w_i^2)/2 + g(y_p - y_i) \quad (A37a,b)$$

$$g_p' = (g_p - g_i) - (w_p - w_i)^2/2 + g(y_p - y_i).$$

The additional terms simply cancel in Eqs. (A35) and (A33). The quantity g_i represents the free energy of mass entering a phase at the intermediate interface state. It is also equal to the free energy of either individual phase at the interface, since equilibrium and saturation conditions exist at this point. It will be demonstrated shortly that the phases at other locations may not be at saturation conditions.

In order to understand the role of the kinetic and potential energy terms in relation to the free energy, consider the special case of a two-phase flow which is completely stratified, with vapor alone in the top portion of the cross section, liquid alone in the bottom. The

difference between free energy of either phase and free energy at the interface may be calculated from the equation

$$dg = -sdT + \frac{dP}{\rho} \quad (A38)$$

The difference in pressure between the interface and the average height of the phase is given by

$$P_p - P_i = \int dP = - \int \rho_p g dy = \rho_p g(y_i - y_p). \quad (A39)$$

Hence, for a uniform temperature we have

$$g_p - g_i = g(y_i - y_p). \quad (A40)$$

The gravitational terms in Eqs. (A37a,b) correspond directly to a difference between interface and phase free energies due to a difference in pressure. By analogy the kinetic energy terms may correspond to a free energy difference due to a temperature difference, although it is not clear whether the relative or absolute difference is the proper quantity. For present purposes, the former will be assumed, since this results from the combination of all terms involving ψdx . Hence, the following condition may be written:

$$g_p - g_i = (w_p - w_i)^2/2 - g(y_p - y_i) \quad (A41)$$

which corresponds to phase "equilibrium" in the presence of differences in kinetic and potential energy, since it results in the elimination of

entropy production associated with the mass transfer process. It also indicates the existence of a temperature difference between phases in a two-phase system whenever a velocity difference exists, in addition to the pressure difference due to individual phase heights. When the condition of Eq. (A41) is satisfied the dissipation equation becomes

$$\psi T \frac{ds}{dz} = \bar{\tau}_w w_g + \bar{\tau}_i (w_g - w_i) + \bar{\tau}_w w_f - \psi x g \frac{dy_g}{dz} - \psi (1-x) g \frac{dy_f}{dz} \quad (A42)$$

If the differences in pressure and temperature are neglected, the phase free energies may be set equal:

$$g_g = g_f \quad (A43a)$$

If, in addition, the interface velocity is assumed to be the average of the phase velocities

$$w_i = \bar{w} \quad (A43b)$$

then the dissipation equation becomes

$$\psi T \frac{ds}{dz} = \bar{\tau}_w w_g + \bar{\tau}_i (w_g - w_f) + \bar{\tau}_w w_f - g \frac{dy_{cm}}{dz} \quad (A44)$$

where y_{cm} is the center of mass of the flow defined by Eq. (A26). The above equation is most convenient for use with the one-dimensional, two-fluid equations.

APPENDIX B. Description of the Experimental Facility, Calibration, and Measurement Procedure

The research facility which has been constructed is a highly versatile one, allowing the study and analysis of a wide variety of phenomena involving the flow of a fluid at or near its saturation point as well as in the two-phase region. The design and construction of the facility required approximately two years. The detailed decisions involved in that process have been presented in a previous report [40], which should be considered as part of the present dissertation. The object of this appendix is to provide a summary description of the final configuration of the apparatus and the techniques used for the basic measurements presented in Section IV. Both apparatus and techniques were finalized after a period of trial runs. The versatility of the facility will allow many measurements in the future such as those appropriate for investigations of turbulence or choking in two-phase flow, and vertical flow.

An important feature of the facility, setting it apart from others, is that it is dedicated to two-phase flow of a single substance as opposed to the case of two-substance, two-phase flow (i.e., the flow of air and water). The fluid used is dichlorotetrafluoroethane (designated Refrigerant-114 or simply, R-114); this allows two-phase experiments to be carried out at manageable temperatures and pressures, namely, between 0 and 60°C and up to 10 bar, with the proper similarity relationships to various other cases of practical interest involving different fluids.

Physical Arrangement and Operation

A simplified schematic diagram of the system is given in Fig. B1,

which shows only the main fluid path; auxiliary piping and control systems have been omitted for the sake of clarity. The main circuit consists of the vessels A, B, C and D, and the piping indicated by solid lines. The components are arranged in a closed loop containing only R-114 in either its liquid or vapor state. When the system is not in operation, the bulk of the R-114 is stored in tank (D) as a saturated liquid at room temperature, with vapor occupying the rest of the system.

The system operates in a "blowdown," or intermittent mode rather than as a continuous, steady-state loop. A typical experimental run lasts approximately three minutes, with about three hours required for preparation between runs.

At the beginning of a run, liquid is pumped from the storage tank (D) to the accumulator tank (A) and the booster tank (B), and is heated to the desired initial conditions by means of electric immersion heaters (H). Since the tanks contain saturated liquid and vapor, the pressure is adjusted solely by setting the fluid temperature by means of the heaters. The run is started by opening the isolation valves (IV). Liquid then flows from the accumulator (A) through a venturi flow meter (V) and enters the test section (T). The function of the booster tank (B) is to provide superheated vapor to the accumulator (A) through a pressure regulating valve (PR). This will condense very slowly and maintain a predetermined accumulator pressure and thus compressed liquid in the accumulator. Measurements are carried out in the test section (T) as the flow develops from single-phase to two-phase because of throttling and pressure losses. The mixture is converted back to the liquid phase in the condenser (C) and finally collects in the storage (or "dump") tank (D). The condenser is cooled by a methanol-water solution (shown in the

diagram by dashed lines) from a cold storage tank (CS) which is maintained at a predetermined temperature (approximately -20°C) by a standard refrigeration chiller (CH). The chiller uses city water as its heat sink.

The compressed liquid working fluid, close to saturation, enters the 2 inch diameter test section from a larger diameter settling chamber. Two-phase flow is produced in the test section by the natural decrease in pressure due to friction, or by an orifice plate if more throttling is required. Temperature and differential pressure measurements are taken at successive stations along the test section. The entire 2 inch diameter portion is transparent Pyrex glass, thus allowing direct observation of the flow. The flow rate and conditions in the test section are controlled by the size of the orifice and the setting of the gate valve. Finally, the fluid passes through a diffuser and into a return pipe which leads to the condenser.

The facility occupies portions of the basement and first floor of the laboratory. Plan views and a section are given in Fig. B2(a) and B2(b).

Test Section

A schematic of the test section is shown in Fig. B3. The test section proper is located between two isolation valves. With the exception of mass flow rate, all experimental measurements and observations are made between these two valves. They are of the butterfly type and are pneumatically controlled for fast operation either by hand or automatically by the computer. The valves are always either fully open or closed and are not used to condition the flow in the test section in any

way. The upstream valve isolates the settling chamber from the pipe leading from the venturi flow meter, whereas the downstream valve isolates the diffuser from the return pipe and the condenser.

The settling chamber has a 5 inch diameter. An insert of machined plastic (Delrin), provides a smooth transition to the 2 inch diameter glass portion of the test section, consisting of a series of tubes of various lengths connected to one another by means of clamps fitted at flanged ends. Their lengths range from 1 ft to 4 ft, with 4 ft between the primary measuring stations.

At the end of the test section a gate valve and diffuser are connected to the condenser by means of the return pipe. The diffuser provides an expansion in diameter from 2 inches to 5 inches.

The settling chamber and the fixed support mentioned above are mounted on a 6 inch square aluminum I-beam which runs the full length of the test section. The glass sections are supported on this beam by adjustable stands which clamp onto the beam and allow alignment of the glass. The test section is supported separately and connected properly to other piping to eliminate vibration. The total length of the transparent section is 24 ft.

Pressure and temperature transducers are accommodated along the test section in special fittings: Delrin inserts fit between the ends of the glass sections and have an inside diameter matching that of the glass pipe.

Pressure transducers are connected by plastic tubing to a brass fitting which screws into a well in the insert, with a small hole connecting the well to the inside wall of the insert. Hence, static pressure is measured at the wall. Both absolute and differential transducers

are connected using this arrangement.

Temperature probes are mounted in a brass fitting which screws into the insert and allows the probe to be positioned radially.

A temperature and absolute pressure transducer are connected similarly at the settling chamber, with fittings screwed into the steel pipe wall.

Flow control for the test section is accomplished by four devices: a throttle plate, a throttle valve, and two isolation valves. The latter are used only as shut-off valves, as mentioned above.

The throttle plate is an aluminum disc with a number of holes drilled to provide a specified total flow area. The disc is clamped in an aluminum holder which fits between two glass sections, in the same manner as the Delrin inserts. Control of the flow is accomplished by causing the liquid to choke at the plate. In this way, the flow rate can be held nearly constant during the run, regardless of changes which occur in the flow further downstream. The downstream gate valve is adjusted to control the amount of flashing and the type of flow which occurs in the test section, without changing the mass flow rate. Also, the pressure in the diffuser is maintained sufficiently low (by the operation of the condenser) so that the two-phase flow in the test section is choked at the gate valve. Thus, conditions in the test section can be held constant during a run, despite changes which occur in other parts of the system, since the flow occurs between two choked cross sections.

With a throttle plate of a given flow area the mass flow rate can be adjusted over a limited range by adjusting the pressure in the accumulator tank. If a larger variation is required, a plate with a different flow area is used. The applied boost is the pressure applied to the

accumulator above the saturation pressure in that tank. The principle of choked liquid flow has been examined in detail and is the subject of a forthcoming report [41].

Some specific characteristics of the test section are illustrated in Fig. B3 and should be noted.

The throttling plate has a total flow area which is determined as outlined above. This flow area is composed of 41 small holes arranged symmetrically in the plate, with a total area corresponding to a single opening of diameter 0.5 inch. It was determined that this number of holes, in conjunction with the 6 ft length before the first measuring station, produced a jet which was dispersed sufficiently to not affect flow in the measuring portion of the test section. A fully developed flow is thus produced before the first measuring point.

The orifice essentially simulates the friction effect which a much longer section of constant area pipe would have, and allows conditions to be produced in the measuring section which would exist at various locations along a pipe of much greater length. The various "locations" are produced by changing the setting of the gate valve to produce more or less flashing immediately following the orifice plate.

Temperature measurements are taken directly by the temperature probes placed as shown in Fig. B3. Differential pressure measurements are taken across each of the 4 ft sections by actually placing differential pressure transducers between the test section and a reference pressure line filled with hydraulic oil, and then calculating the difference between these measurements. This technique is used since connecting the differential transducers directly between stations created long sections of tubing, filled with freon which could change phase during a run,

creating arbitrary, indeterminable pressure fluctuations due to gravity. The reference system allows use of an incompressible liquid in the long sections of tubing, with very short sections connected to the test section. The pressure in the reference line adjusts automatically to a proper value (assuring that all ΔP transducers will be within range) since it is exposed to a pressure at the first measuring station by means of an interface system. The interface consists of a small "U" tube with mercury in its base and oil and freon vapor on opposite sides. The U tube (or "surge pot") is heated to prevent freon inside from changing phase as the pressure fluctuates in the test section. The mass of mercury in the U tube, along with a small air bubble placed at each transducer on the reference side produces a damping effect sufficient to render the reference pressure essentially constant during a run.

Monitoring and Control Instrumentation

During the time the facility is idle, conditions throughout the system are continuously monitored by the computer-controller, a Hewlett-Packard HP-85 desk-top model. This machine is programmed to take action to maintain certain conditions, e.g., operation of the chiller to maintain the cold storage tank at the desired temperature.

During the time the system is being prepared for a test run, the preparation procedure is monitored and controlled by the same computer so that once the desired starting time for the run has been entered into the computer by the operator, all of the required steps will be initiated and controlled automatically (operation of valves, heaters, etc.) assuring that the system will be ready for a run at the prescribed time.

The link between the controller (HP-85) and the physical system is

provided by the HP-IB "Interface Bus" with the Hewlett-Packard Multi-Programmer. These devices allow the computer to read a number of instruments and take action, i.e., operate equipment, by switching relays.

There are two types of measurements taken for automatic control and monitoring purposes: tank temperatures and liquid levels. Temperatures are measured using thermistors attached to the exterior wall of the tank, beneath the insulation. These are located on each of the tanks in the system, including the methanol cold storage tank. Liquid levels are determined by reading differential pressure transducers connected across the top and bottom of the tank to sense hydrostatic pressure, and hence liquid level.

In addition to the above, visual monitoring of temperatures, pressures, and liquid levels is possible, when needed, using standard mercury-in-glass thermometers, dial pressure gauges, and sight glasses.

Data Acquisition Instrumentation

The instrumentation for the experimental measurements consists of three types of sensors: temperature, absolute pressure, and differential pressure. The sensors are of the same basic types as used for the monitoring measurements, but are calibrated and mounted so as to produce the highest accuracy possible.

Although not located in the test section proper, the venturi meter allows measurement of flow rate by means of a differential pressure transducer connected between its inlet and throat sections. Details on the test section instrumentation and the venturi are given later in this appendix.

When preparation for a test run has been completed, measurements are taken in the test section using the controller (HP-85). The computer scans all instruments in the test section, and the venturi, at predetermined time intervals (approximately 1 sec). It is also programmed to control the test run by operating the isolation valves, gate valve, pressure regulating valve and condenser.

Data collected during a test run are stored in the memory of the HP-85 in real time. At the end of the run, the data are transcribed on tape and can be printed in raw form directly, i.e., voltage measurements from each instrument as read in each scan, or in reduced form in units of temperature, pressure, and mass flow rate.

Lastly, a set of programs has been developed to provide additional data reduction and graphic display for calculations made from the measurements, such as values for density ratios, Mach numbers, etc.

Since the facility was designed as an intermittent one with running times of one-half to three minutes, it is clear that for all operations of the facility including the acquisition of data and the running of the experiment, automation was highly desirable, if not an absolute necessity. After investigating a number of options, the Hewlett-Packard HP-85 computer was chosen as the basic controller for the system. This not only satisfied requirements for control and data acquisition, but also made available a reasonably powerful computer for data evaluation and theoretical studies.

Measurement for Experimental Data

1. Temperature Measurement

The sensors selected for measuring temperature were thermistors, a semiconductor material having a resistance which is sensitive to

temperature. The dependence of the resistance on temperature is given by

$$\phi(T) = \alpha e^{-\beta/T}, \quad (B1)$$

where T is the absolute temperature and the constants α and β depend on the material. A known current pulse is passed through the thermistor and ϕ is a voltage measured as a function of temperature.

The specific thermistors were selected based on their sensitivity over the required temperature range and an output which is compatible with the A to D voltage conversion equipment. The manufacturer's specifications were used for the selection process. A method for mounting was designed and implemented, and calibrations were then carried out for each thermistor.

Because of the way the thermistor had to be mounted (i.e., in a fluid flow, and along the radius of a circular cross section), the type selected was a glass-coated bead with adjacent leads. This allowed the thermistor to be mounted as shown in Fig. B4, with the leads passing through a sealed tube. The tube is held in a fitting attached to a Delrin pipe insert and allows the tube to be positioned radially and locked in place.

Thermistors with resistance values at 25°C of either 1 k Ω or 2 k Ω are presently in use. With an applied measuring current of 200 μ A, the resulting voltage is between 0 and 1 V over the calibration temperature range of 0 to 30°C. The 1 k Ω thermistors allow temperatures lower than 0°C to be measured when needed, without exceeding 1 V output of the A-D converter.

Calibration of the temperature transducers was carried out using a heated, thermostatically-controlled water bath. Points were taken at approximately 3°C intervals over the calibration range. A computer program has been written which allows the HP-85 to read the transducer output voltage at each point. The corresponding temperatures, read on a precision mercury-in-glass thermometer with increments of 0.1°C, are provided by the operator. The computer then correlates the data for $\phi(T)$ using a least-squares analysis. The form of the correlating equation is obtained from the resistance equation above as

$$\frac{1}{T} = A + B \ln \phi, \quad (B2)$$

where

$$A = \frac{\ln \alpha}{\beta} \quad \text{and} \quad B = -\frac{1}{\beta}. \quad (B3)$$

A sample correlation is given in Fig. B5, showing the input data, resulting correlation equation, standard deviation, and deviation of data from the smooth curve.

2. Absolute and Differential Pressure Measurement

The type of probe selected for measuring absolute and differential pressure is an integrated circuit pressure transducer[†] which provides a voltage output that is linear with the applied pressure or pressure difference. The transducer consists of a single-crystal silicon sensor and circuit elements mounted in a ceramic housing. Attached directly to the

[†]Supplier: SenSyn Company, Sunnyvale, California. Formerly National Semiconductor.

housing are ports of brass tubing for connection to the pressure source, and electrical connections for a 15-volt excitation signal and output voltage. For this application, it was decided to mount the transducer in a metal box, as shown in Fig. B6, which allows the transducer to be isolated from stress created by the external tubing and electrical connections.

The differential pressure transducers selected have a range of $\Delta P = -5$ to $+5$ lbf/in² (-30 to $+30$ kPa) with a voltage output in the range 0 to 12 volts, and an offset of approximately 7 V when $\Delta P = 0$.

Calibration is performed over the range -30 to 0 kPa by the use of a program on the HP-85 similar to that used in the temperature probe calibrations. In this case, the low pressure port of the transducer is left open to the atmosphere, and a controlled vacuum is applied to the high pressure port in increments of approximately 7 kPa, as read by the operator on a mercury manometer, accurate to 0.1 mm Hg. Thus, the transducer output is kept in the range $\phi = 0 - 7$ V. The function $P(\phi)$ is correlated by a least-squares analysis to an equation of the form

$$P(\phi) = A + B\phi. \quad (B4)$$

A sample correlation is shown in Fig. B7. The units of pressure are mbar, converted from mm Hg by the computer.

The absolute pressure transducers used have a range of $0-60$ lbf/in² ($0-4$ bar). Calibration is accomplished by applying a known pressure to the transducer, as read using a precision bourdon type pressure gauge with increments of 0.1 lbf/in². The data is correlated using the same computer program as the differential transducer, and the form of the correlating equation is the same.

3. Mass Flow Rate Measurement

The flow rate is measured by means of a venturi meter which determines mass flow rate by measuring the change in pressure as a flow of liquid passes through a contraction, or throat section. Because of a number of special requirements of this meter, since the fluid is close to saturation, it was decided to design and fabricate the venturi rather than purchase a standard instrument.

The meter consists of an outer tube and flanges made of stainless steel which fit between adjacent pipe flanges in a conventional manner. The throat section is machined from Delrin, and slides into the outer tube. It is held in place by means of two end plates. Sealing, both internally around the throat pressure chamber and externally at the flanges, is accomplished using O-rings. A drawing of the outer tube and insert is given in Fig. B8. There are three interchangeable throat sections, allowing a flow measurement range of about 1 to 21 kg/s, corresponding to a test section mass flux of roughly 400 to 10,000 kg/m²·s, with throat diameters of 3/4, 1-1/4 and 2 inches.

Measurement of ΔP at the meter is accomplished using a differential pressure transducer of the type discussed earlier, except mounted in a zinc housing (as supplied). It is bolted to a vibration isolator at the venturi and connected by the necessary fittings and valves. The inlet pressure tap is located in the upstream pipe section rather than the venturi unit itself. A temperature probe, of the type discussed earlier, is located at the same position for accurate determination of the fluid density.

Mass flow rate $\dot{m}$ is calculated from the equation

$$\dot{m} = (2\rho\Delta P)^{1/2} A_i \left/ \left(\frac{A_i^2}{A_t^2} - 1 \right) \right. ^{1/2}, \quad (B5)$$

as derived from Bernoulli's equation. The quantities ρ , ΔP , A_i , A_t are the fluid density, measured pressure difference between inlet and throat, inlet area and throat area, respectively.

The accuracy of the venturi was checked by directly measuring the quantity of fluid which leaves the accumulator tank in a given time period and comparing this to the flow rate measured by the venturi. Calibration data for venturi throat sections 1 and 2, which cover the range of mass flow rate used in the experiments, are given in Figs. B9(a) and B9(b). Mass flow rate measured by the venturi is plotted against the known calculated flow rate. The straight reference lines are lines of equality between these two quantities.

No correction factor on the standard venturi flow equation above has been found necessary, to an accuracy of approximately 2%.

Facility Operation and Measurement Procedure

The process of preparing for an experimental run is always the same, and is controlled by computer program which allows a set of predetermined starting conditions to be attained in the system without manual intervention. Once these have been reached, the system will automatically maintain such conditions until the operator decides to begin the run, when he enters a run "control program." Using the control program, the computer operates the system and takes data based on a set of predetermined, timed actions for controlling the flow and reading the instruments. The waiting period between run preparation and actually starting the test section flow allows very stable conditions to be reached in each part of the system and steady conditions are achieved quickly. The fact that both run preparation and execution are automated allows procedures to be developed and optimized beforehand, and then assures their precise performance. This eliminates operating inconsistencies and possible errors resulting from manual operation. It also allows the operator to monitor measuring systems and observe the flow without being concerned about the instantaneous mechanics of the process.

The run control program has been written in three versions. The type of data required determines the version which is used. A few common features of these programs will be presented before considering their individual details.

The gate valve is first opened to a position selected according to the amount of flashing desired for the run and the accumulator pressure is increased using the booster. Liquid is then allowed to enter the test section slowly, through small diameter tubing which bypasses the inlet isolation valve, until the test section is half full. At this time the

liquid level just covers the thermistor beads and is just below the pressure transducer ports. After a waiting period for equilibrium to be reached, a set of eight scans are taken. A "scan" consists of one measurement from each instrument. With uniform pressure at saturation conditions in the test section, the temperature at the liquid surface will be uniform. Similarly, the difference between test section pressure and reference line pressure will be the same at each measuring station. Pressure difference across the venturi pressure transducer will be zero since no flow exists. These measurements are used for checking consistency between individual temperature and pressure probes, and to provide a reference or "zero" for each probe under known conditions. A correction is applied to all later measurements in a run based on comparison of average readings over four scans. A total of eight are taken to detect any trend which may be present. Absolute pressure and temperature are checked against the known relationship for the fluid at saturation. In addition to calibration, these procedures allow for detection of a problem with any instrument before another run is performed.

Following the referencing procedure, liquid again is allowed to enter the test section slowly until it is full. The methanol pump is then started to reduce the dump tank pressure. At that time flow through the test section is started by opening both isolation valves simultaneously. After a waiting time for the flow to stabilize, a number of measurement scans are carried out, the exact procedure depending on the particular control program. A description of the procedures is given on the pages that follow.

Procedure 1

This program initially sets the gate valve to a position nearly fully open or fully closed. The valve is then opened or closed continuously while a block of scans is taken. In this way a sequence of two-phase flow conditions is produced in the test section in a single run. When measurements are completed, the inlet isolation valve is closed. Once the test section has emptied, the outlet isolation valve is closed and the gate valve reset to closed position.

Procedure 2

The second version of the control program sets the gate valve to a particular position, and this position remains fixed while measurements are taken. A range of flow regimes is examined by setting the valve differently for successive runs. Measurements for versions 1 and 2 showed no difference, indicating that the flow stabilizes quickly under all conditions. For some runs version 2 was modified slightly to include a measurement of void fraction. After the main block of scans were taken, a void fraction probe was inserted, and another set of scans made. Comparison between the two sets give a measurement of void fraction. Details are given in Section III. At the end of the run, the isolation valves are closed simultaneously, and the average void fraction is measured directly by measuring the liquid level, and hence the vapor volume fraction, after applying the proper corrections.

Procedure 3

This version of the control program is a variation of version 1, producing a set of different test section conditions in a single run.

However, the gate valve setting is not changed while measurements are taken. Instead, four blocks of scans are taken. Between each block the isolation valves are closed simultaneously and the gate valve is repositioned. Each time, a waiting period is incorporated between opening the valves (starting the flow) and the start of the measurements.

For runs conducted on the basis of each of the control programs outlined above, subsequent calculations are based on time-averaged data, that is on data averaged over a number of scans. The exception is Procedure 1, where individual scans must be used since conditions vary continuously.

Three series of experimental data were taken. A list of the runs and additional data are provided in the tables. The experimental results are discussed in Section IV.

APPENDIX C. Correlation and Measurement Comparison

The correlation proposed by Lockhart and Martinelli [3] was discussed briefly in the Introduction with relation to other work, past and present, in two-phase flow. It will be compared here to measurements using R-114 from this investigation and with some using R-12 performed by Friedel and Mayinger [31]. The correlation predicts friction pressure change and void fraction. It was obtained based on measurements with two-substance flows using mixtures of air with various liquids, including water and oil. The friction and total pressure change are identical in most such cases since significant acceleration does not occur. The technique was extended to single-substance, evaporating flows in a paper by Martinelli and Nelson [4]. It was assumed that the friction pressure change would be essentially the same as for non-evaporating flow and the total could be obtained by adding the acceleration or "momentum" pressure change required if evaporation occurs. Calculation of this effect requires additional information for slip, or equivalently void fraction. In [4] the phase velocities were calculated both by assuming "no slip" and from the correlation for void fraction. Of course the latter is then also assumed to be the same for an evaporating flow. For present purposes experimental data will be compared to the correlated friction pressure change alone, since this is the basis of both papers mentioned above and represents the lower limit for predicted pressure gradient.

The independent variable selected for the correlation and denoted by X is the square root of the ratio of the liquid "superficial"

pressure gradient to that of the vapor. Each is calculated for the phase flowing alone in the pipe at the same mass flow rate as that phase in the two-phase mixture. The corresponding velocity has since been labeled in the literature as the "superficial" phase velocity. It will be denoted here, in general, for phase "p" as

$$w_{sp} = x_p \psi / \rho_p \quad (C1)$$

where ψ is the total mass flux or "mass velocity" of the two-phase flow. The variable X will be denoted by X_M so that it is distinct from the dryness fraction x . The ratio is then

$$X_M^2 = (dP/dz)_{sf} / (dP/dz)_{sg} \quad (C2)$$

If the friction factor is expressed in the Blasius form (Eq. 78) then in general

$$(dP/dz)_{sp} = \frac{C_B}{Re^n} \rho_p \frac{w_{sp}^2}{2} \frac{1}{D} \quad (C3a)$$

where

$$Re = \rho_p w_{sp} D / \mu_p \quad (C3b)$$

The constants C_B and n were determined in [3] depending whether

each phase flowed viscously or turbulently in the mixture ($n = 1.0$ or 0.2 , respectively). Hence four different forms of X_M were possible. In [4] only the case with both phases turbulent was considered, with the exponent modified to $n = 0.25$. For this case, with $\gamma = \rho_f/\rho_g$,

$$X_M^2 = \frac{1}{\gamma} \left(\frac{\mu_f}{\mu_g} \right)^{0.25} \left(\frac{1-x}{x} \right)^{1.75} \quad (C4)$$

which is the form that will be used here.

The independent variable for pressure change is the ratio of the two-phase friction to the superficial liquid pressure gradient,

$$\phi_L^2 = (dP/dz)_{TPF} / (dP/dz)_{sf} \quad (C5)$$

Values of ϕ_L are given in [3] as a function of X_M , in tabular form. An alternate independent variable ϕ_{L0}^2 introduced in [4] is the same as ϕ_L^2 from Eq. (C5) but with the denominator replaced by the pressure gradient for liquid flow at the total two-phase mass flow rate. Then in terms of ϕ_L ,

$$\phi_{L0}^2 = \phi_L^2 (1-x)^{1.75} \quad (C6)$$

The term $(1-x)$ may be expressed in terms of X_M and phase properties using Eq. (C4).

If the two-phase friction pressure gradient is replaced by the total, then ϕ_{L0}^2 is essentially the same as ΔP_R introduced for the

presentation of experimental data in Sec. IV. The only difference is that for ΔP_R the liquid pressure gradient in the denominator is calculated using liquid properties μ_f and ρ_f at the flash temperature T^* while for ϕ_{LO} properties at the two-phase temperature are used. For the present experimental data with R-114, ϕ_{LO}^2 has been obtained from ΔP_R by correcting for temperature based on the Blasius form of friction factor. The correction becomes

$$\phi_{LO}^2 = \Delta P_R \left\{ \left(\frac{\mu_f^*}{\mu_f} \right)^{.25} \frac{\rho_f}{\rho_f^*} \right\}. \quad (C7)$$

The factor varies only between 1.0 and 0.97 over the experimental temperature range. Hence the correction is less than 3.0%

Experimentally determined values of ϕ_{LO}^2 using the total pressure change measurements from Series I, II, III are given in Fig. C1. The Martinelli prediction [3] of friction alone for each series has been shown as a solid line. The measurements for Series II and III exhibit the same change in slope mentioned in Sec. IV.2. The slope increases greatly as the flow pattern changes from stratified to "homogeneous". This transition phenomenon was observed also in data for air-water presented in the discussion at the end of Ref. [3]. In that case, the position of the change in slope varied if the water flow rate were changed. This implies a change of Froude number. Data for Series I and that for Series II and III exhibit a definite symmetry. Each appears to originate from a common line that is nearly horizontal

close to $\phi_{LO}^2 \approx 2$ but becomes steeper for smaller values of X_M . The position of the change in slope of the line in this case depends on both Froude and Mach numbers. For the air-water data just mentioned, larger Fr moved the change in slope to higher values of ϕ_{LO} , but the data continued to follow closely along the correlation curve. The horizontal shift from Series II and III ($Ma \approx 0.6$) to Series I ($Ma \approx 0.3$) may be basically a Mach number effect. Plotting this data as a function of X_M is essentially the same as plotting versus v_R , except X_M decreases as v_R (or x) increases. Hence, the influence of Mach number identified in Figs. 10 and 12 will be similar in Fig. C1 for Series II and III, and corresponds to the same type of horizontal shift.

Shown in Fig. C1 is the range of friction pressure change calculated by Friedel and Mayinger [31] based on total pressure change measurements using R-12 in a vertical 1 in. diameter tube. (Details of the calculation were not provided in the paper.) The ϕ_{LO} for friction is larger for this data than for the total pressure change measurements taken using R-114 with stratified flow at the same X_M . This observation agrees, at least qualitatively, with the discussion of Sec. III.1. It was noted there that horizontal stratified flow would be expected to give a much weaker friction force at the interface than would be possible in vertical flow. Calculation of ϕ_{LO}^2 for friction alone from the Martinelli correlation for R-12 has been shown in Fig. C1 as a dashed line, and for R-114 as just mentioned. The correlation in each case exceeds nearly all of the experimental

measurements, including the total pressure change from this investigation. The same has been observed [31] when steam-water data (also single-substance) is compared to the correlation.

The void fraction measurements for Series II and III are shown in Fig. C2. This figure is essentially the same as Fig. 13 except the coordinates are X_M and "holdup" R_L . In terms of void fraction α ,

$$R_L = 1 - \alpha. \quad (C8)$$

The correlation for R_L is given in tabular form as a function of X_M in Ref. [3], and is shown in Fig. C2 as a solid line. The reference lines for equal phase dynamic pressures (EDP) and equal phase velocities (no slip, NS) are also shown. The measurements correspond quite closely to the Martinelli prediction, but in general tend to give higher values of R_L (lower α). Within the accuracy of the measurement, all points exceed the correlation. This would be expected for an evaporating versus non-evaporating flow. In the presence of an overall acceleration the denser liquid phase will tend to be accelerated less than the vapor, resulting in greater slip and hence smaller void fraction. This effect seems to be greater for smaller x (larger X_M). The description of the Froude number effect given previously for Fig. 13 also applies to Fig. C2.

A number of other correlations for void fraction have been compared and discussed in detail in a recent report [45].

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FIGURE 1

FIGURE 2(a)

FIGURE 2 (b)

FIGURE 3

FIGURE 4

FIGURE 5

FIGURE 6

FIGURE 7

FIGURE 8

FIGURE 9

FIGURE 10

FIGURE II

FIGURE 12

FIGURE 13

FIGURE 14

FIGURE 15

FIGURE 16

FIGURE 17

FIGURE B1

FIGURE B2(a)

FIGURE B4

Input Data

.2	.614
3.6	.524
6.3	.465
9.3	.409
12.3	.36
15.4	.314
18.4	.276
21.4	.246
24.3	.219
27.3	.195
30.3	.174
33.6	.154

Calibration for Transducer No
10

DesK/T =
 $3.799E-03 + 2.877E-04 * \text{LOG}(I/V)$
 $\sigma \text{DesK} = 8.17E-07$

Error of T-Transducer 10

+1 DesC

-1 DesC

FIGURE B5

scale: 3/4

FIGURE B6

Input data :

P/mBar	I/V
0	7.05
-69.4323677411	6.04
-139.130251229	5.04
-207.102282364	4.06
-276.269134358	3.07
-347.294596579	2.04

Calibration of P-Transducer 5

$$P/\text{mBar} = -000488.99 + 00069.40 * I/V$$

$$\sigma/\text{mBar} = .305$$

Error of P-Transducer 5

+5 mBar

-5 mBar

FIGURE B7

FIGURE B8

FIGURE B9(a)

FIGURE B9(b)

FIGURE C1.

FIGURE C2.

TABLE 1 BASIC PARAMETERS - DATA SERIES I, II, III

	Fr^* max.	$\bar{m}(\text{kg/s})$	$\bar{Re}^*/10^5$	$\bar{Fr}^*$	$\bar{Ma}^*$	v_R Range
		Range	Range	Range	Range	Ma_o Range
Series I $T_0 = 25^\circ\text{C}$	13	2.28	1.70	1.2	.29	19 - 45
		2.23 - 2.34	1.63 - 1.74	1.1 - 1.3	.27 - .30	.50 - .84
Series II $T_0 = 25^\circ\text{C}$	13	5.00	3.72	5.8	.64	2 - 9
		4.64 - 5.53	3.45 - 4.08	5.0 - 7.1	.59 - .72	.63 - .91
Series III $T_0 = 35^\circ\text{C}$	22	5.33	4.35	6.9	.56	3 - 10
		4.50 - 6.02	3.70 - 4.90	4.9 - 8.7	.46 - .63	.55 - .86

Table B1 - Valve Positions, Series I

Throttle Valve Position (% Open)											
0	10	20	30	40	50	60	70	80	90	100	Run
-----											6/29A
-----											7/06A
Δ											6/18A
Δ											6/28A
Δ											7/09B
Δ											7/12A
Δ											7/13A
Δ											7/13B
Δ											7/14A
Δ											7/15B
Δ											7/16A

--- Procedure 1, No Time Average

Δ Procedure 2, 12 Scan Time Average

Table B2 - Valve Positions, Series II

Throttle Valve Position (% Open)												Run
0	10	20	30	40	50	60	70	80	90	100		
			o		o			o				9/03A
				o		o		o		o		9/07A
			o		o		o		o			9/08A
			•	o		o						9/17A
			•	o		o						9/20A
				•		o		o				9/21A
			o	o	o	o						9/28B
										Δ		9/22A
										Δ		9/23A
										Δ		9/23B
										Δ		9/24A
						Δ						9/27A
				Δ								9/27B
						Δ						9/27C
							Δ					9/28A
									Δ			9/28C
						Δ						10/05A
						Δ						10/05B

- o Procedure 3, 6 Scan Time Average
- Δ Procedure 2, 6 Scan Time Average
- Block Repeated

Table B3 - Valve Positions, Series III

Throttle Valve Position (% Open)											Run
0	10	20	30	40	50	60	70	80	90	100	
			o		o		o		o		9/29A
			o		o		o		o		10/03A
			o		o		o		o		10/03B
		o		o		o		o			10/03C
			•	o	o						10/04A
									Δ		10/04B
						Δ					10/04C
				Δ							10/04D

o Procedure 3, 6 Scan Time Average

Δ Procedure 2, 6 Scan Time Average

• Block Repeated

Table B4 - Data Series I

Run	$\dot{m}$ (kg/s)	$T_o(^{\circ}\text{C})$	$P_o - P_s(T_o)$ (kPa)	$Re^*/10^5$	Fr^*	Ma^*	$T(^{\circ}\text{C})$	%x	v_R	Ma_o	ΔP_R	
6/18A	2.31	27.2	86.5	1.73	1.2	.29	0.0	19.2	43.7	.84	21.5	---
6/28A	2.23	27.3	83.5	1.67	1.2	.28	6.7	15.1	27.1	.61	12.2	---
7/09B	2.16	27.8	83.8	1.63	1.1	.27	11.2	12.5	19.3	.50	7.70	---
7/12A	2.27	26.3	93.2	1.68	1.2	.29	5.3	15.2	28.7	.66	12.6	---
7/13A	2.31	26.4	94.0	1.72	1.2	.30	1.6	17.7	38.1	.78	17.2	---
7/13B	2.30	27.7	82.4	1.73	1.2	.28	0.0	19.6	44.6	.84	22.9	---
7/14A	2.27	27.3	89.0	1.71	1.2	.28	5.4	15.9	30.0	.66	12.6	---
7/15B	2.34	26.2	100.1	1.73	1.3	.30	5.2	15.3	29.0	.68	12.0	---
7/16A	2.29	26.0	94.8	1.69	1.2	.30	5.0	15.2	29.1	.67	13.2	---

Table B5 - Data Series II

Run	$\dot{m}$ (kg/s)	T_o (°C)	$P_o - P_s(T_o)$ (kPa)	$Re^* / 10^5$	Fr^*	Ma^*	T (°C)	%x	v_R	Ma_o	ΔP_R	α (%)
9/22A	5.31	26.5	31.6	3.95	6.5	.68	19.1	5.69	7.34	.89	9.95	---
9/23A	5.45	26.4	38.9	4.05	6.9	.70	19.5	5.27	6.77	.90	9.02	67.6
9/23B	5.38	26.3	35.7	3.99	6.7	.69	19.2	5.46	7.05	.90	9.71	67.9
9/24A	5.39	26.4	35.6	4.00	6.7	.69	19.3	5.46	7.03	.90	9.70	68.1
9/27A	4.87	27.2	29.7	3.65	5.5	.61	22.9	3.37	4.29	.71	2.55	58.5
9/27B	4.99	25.7	39.2	3.68	5.7	.65	23.7	1.66	2.58	.70	2.27	31.3
9/27C	5.12	26.9	31.8	3.83	6.1	.65	21.1	4.55	5.73	.80	5.47	64.6
9/28A	5.36	26.0	37.0	3.96	6.6	.69	19.8	4.75	6.15	.87	7.53	66.1
9/28C	5.53	25.7	41.5	4.08	7.1	.72	19.5	4.80	6.27	.91	8.69	70.9
10/05A	4.98	27.1	34.3	3.73	5.8	.63	23.4	2.93	3.81	.72	2.54	---
10/05B	4.85	27.5	28.2	3.65	5.5	.60	22.9	3.58	4.50	.71	2.61	61.8

Table B5 - Data Series II (cont.)

Run	$\dot{m}$ (kg/s)	T_o (°C)	$P_o - P_s(T_o)$ (kPa)	$Re^*/10^5$	Fr^*	Ma^*	T (°C)	%x	v_R	Ma_o	ΔP_R	
9/03A a												
b	4.77	25.6	31.6	3.51	5.2	.62	23.0	2.10	3.04	.69	2.73	---
c	5.02	25.8	29.1	3.71	5.8	.65	20.4	4.20	5.47	.80	5.86	---
d	5.07	25.9	26.6	3.75	5.9	.66	18.8	5.44	7.11	.86	9.56	---
9/07A a												
b	4.94	26.4	30.4	3.67	5.6	.63	22.2	3.31	4.31	.74	3.19	---
c	5.09	26.5	28.6	3.79	6.0	.65	20.0	4.97	6.34	.83	8.00	---
d	5.11	26.5	26.4	3.80	6.0	.65	19.0	5.77	7.43	.86	10.62	---
9/08A a												
b	4.94	26.6	29.9	3.68	5.7	.63	22.0	3.64	4.66	.75	3.50	---
c	5.09	26.8	26.7	3.80	6.0	.64	19.7	5.53	7.03	.84	8.24	---
d	5.08	26.9	23.9	3.80	6.0	.64	18.5	6.48	8.36	.88	11.19	---

Table B5 - Data Series II (cont.)

	$\dot{m}(\text{kg/s})$	$T_o(^{\circ}\text{C})$	$P_o - P_s(T_o)$ (kPa)	$Re^*/10^5$	Fr^*	Ma^*	$T(^{\circ}\text{C})$	$\%x$	v_R	Ma_o	ΔP_R	
9/17A a												
b	4.68	25.9	33.1	3.46	5.0	.60	24.4	1.23	2.14	.64	1.62	---
c	4.65	26.2	30.4	3.45	5.0	.60	23.8	1.90	2.80	.65	2.12	---
d	4.79	26.3	27.1	3.56	5.3	.61	22.3	3.11	4.10	.71	2.19	---
9/20A a												
b	4.64	26.4	34.4	3.45	5.0	.59	24.8	1.33	2.22	.63	1.68	---
c	4.67	26.6	31.4	3.48	5.1	.59	24.1	1.95	2.83	.65	2.20	---
d	4.81	26.6	28.5	3.58	5.4	.61	22.6	3.16	4.12	.71	2.29	---
9/21A a												
b												
c	4.88	27.1	29.1	3.66	5.5	.61	23.3	3.01	3.90	.71	2.49	---
d	5.02	27.4	23.8	3.77	5.9	.62	20.7	5.25	6.52	.80	7.25	---

Table B5 - Data Series II (cont.)

Run	$\dot{m}(\text{kg/s})$	$T_o(^{\circ}\text{C})$	$P_o - P_s(T_o)$ (kPa)	$Re^*/10^5$	Fr^*	Ma^*	$T(^{\circ}\text{C})$	%x	v_R	Ma_o	ΔP_R	
9/28B a												
b	4.81	26.5	34.9	3.57	5.3	.61	23.9	2.01	2.90	.67	1.89	---
c	4.87	26.7	33.3	3.63	5.5	.62	23.2	2.72	3.63	.70	2.13	---
d	4.92	26.8	30.3	3.67	5.6	.62	22.4	3.45	4.42	.73	2.48	---

Table B6 - Data Series III

Run	$\dot{m}$ (kg/s)	T_o (°C)	$P_o - P_s(T_o)$ (kPa)	$Re^* / 10^5$	Fr^*	Ma^*	T (°C)	%x	v_R	Ma_o	ΔP_R	α (%)
9/29A a												
b	5.85	34.4	45.4	4.75	8.2	.62	29.5	4.00	4.14	.73	3.98	---
c	6.00	34.6	41.9	4.88	8.7	.63	27.2	5.97	6.06	.82	7.99	---
d	6.02	34.7	39.4	4.90	8.7	.63	26.0	6.96	7.15	.86	10.27	---
10/03A a												
b	5.55	34.4	35.2	4.50	7.4	.59	28.6	4.69	4.80	.72	4.42	---
c	5.72	34.6	31.7	4.66	7.9	.60	26.2	6.72	6.89	.81	9.00	---
d	5.74	34.8	28.2	4.68	7.9	.60	25.1	7.73	8.03	.84	12.14	---
10/03B a												
b	5.10	34.6	16.0	4.15	6.2	.54	27.1	6.04	6.13	.70	5.55	---
c	5.28	34.9	11.0	4.31	6.7	.55	24.6	8.15	8.52	.79	11.23	---
d	5.31	35.0	8.2	4.34	6.8	.55	23.3	9.23	9.90	.83	13.88	---

Table B6 - Data Series III (cont.)

Run	$\dot{m}$ (kg/s)	T_o (°C)	$P_o - P_s(T_o)$ (kPa)	$Re^*/10^5$	Fr^*	Ma^*	T (°C)	%x	v_R	Ma_o	ΔP_R	α (%)
10/03C a												
b	4.87	35.1	16.2	3.99	5.7	.51	29.2	4.75	4.76	.62	3.19	---
c	5.22	35.5	9.85	4.29	6.6	.54	26.0	7.59	7.68	.75	9.15	---
d	5.33	35.8	6.22	4.39	6.9	.54	24.1	9.20	9.63	.82	12.90	---
10/04A a												---
b	4.74	34.2	28.6	3.84	5.4	.50	30.8	2.74	3.06	.56	3.06	---
c	4.88	34.4	26.4	3.96	5.7	.51	30.1	3.49	3.69	.60	2.38	---
d	5.14	34.5	22.5	4.18	6.3	.54	28.1	5.19	5.28	.68	4.41	---
10/04B	5.54	34.9	18.1	4.52	7.4	.58	23.8	8.74	9.29	.85	12.8	73.8
10/04C	5.21	35.7	11.6	4.29	6.6	.53	26.1	7.68	7.76	.75	8.25	73.9
10/04D	4.50	35.7	13.4	3.70	4.9	.46	30.5	4.25	4.24	.55	2.27	55.6

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