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USING IONS TO PROBE THE TRANSVERSE SIZE OF A BUNCH*

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1. INTRODUCTION

The electric field carried along by a SLC bunch is very intense at the surface of the bunch because of the bunch's tiny transverse dimensions and its high charge density. For a given bunch population, the maximum electric field—which occurs at the surface—is inversely proportional to the bunch radius for a round bunch. The smaller the radius, the higher the peak field. A charged particle such as an ion or an electron which is placed at rest in the path of the oncoming bunch will be accelerated by the field as the bunch has passed having sampled the field of the bunch. Thus by placing a swarm of stationary charged particles in the path of the bunch and measuring their momentum distribution when they emerge, we can hope to infer the bunch's transverse size. We are using the terms "size" and "surface" in a qualitative way, of course, expecting that their meaning will be reasonably clear to the reader. In our calculations we shall use a cylindrical model for the bunch in which their meanings are precise.

Since the electric field is zero on the axis of the bunch, it rises to its peak at the surface, and falls off eventually inversely with distance, our charged particles must not move very much radially during the passage of the bunch. If they do,

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they will average over a range of radii and their momentum will therefore represent a poor sample of the peak field. We need particles which will move only small fraction of the bunch radius during the passage. Electrons do not satisfy this criterion for SLC bunches. Ions of hydrogen and helium, on the other hand, do satisfy it for radii of 25 microns and up, and they provide probes of some promise. The fact that they move very little during the passage, guarantees that the measurement of bunch radius is nearly independent of bunch length, a distinct benefit. Very heavy ions can satisfy the criterion even for the one-micron radii sought at the interaction region.

An apparatus to realize this scheme for bunch-size measurement consists of an ion source which projects a stream of low-energy ions across the SLC beam path and some sort of spectrometer to measure the momentum distribution of the outgoing ions. The arrangement is shown conceptually in Figure 1. It takes advantage of the fact that all of the ions expelled by this process come out radially—they have no axial or azimuthal velocity.

2. THE ELECTRIC FIELD OF THE BUNCH

Our model of the SLC bunch will be a uniformly populated cylinder of charge (electrons or positrons) moving in the direction of its axis. The cylinder's radius, a , may be identified approximately with the standard deviation of a cylindrical Gaussian distribution; its length, ℓ_b , should however be identified with twice the standard deviation of a longitudinal Gaussian distribution.

As a function of the axial coordinate, the electric field of the model bunch is constant over the length of the bunch and zero outside the bunch. The sharp rise at the head and fall at the tail are consequences of the extreme relativistic motion of the particles. The electric field is shown in Fig. 2 as a function of the radial coordinate. Only the radial component need be plotted, since the others are zero.

$$E_r(r) = \begin{cases} E_a(\frac{r}{a}) & , \quad r \leq a \\ E_a(\frac{a}{r}) & , \quad r > a \end{cases} \quad (1)$$

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where

$$E_a = E(a) = \frac{2 N_b e}{a \ell_b} \quad (\text{cgs unit}) \quad (2)$$

The charge in the bunch is the product of the population, N_b , and the charge, e , of the electron or positron as appropriate.

The bunch carries a magnetic field as well, of course, which is everywhere orthogonal to the electric field and, in cgs units, equal to it; however since the ions we shall be interested in are slow moving, we shall be justified to ignore the magnetic field.

3. ION MOTION

We require that the ion move but little in comparison with the bunch radius in the time during which the bunch field is present and acting on it. For the field, radii and lengths of SLC bunches, this requirement automatically ensures the conditions for the validity of non-relativistic mechanics for treating the ion motion. Assuming these conditions are satisfied we may take the electric field accelerating the ion radially to be constant at a value determined by the ion's initial radial position—its position before the arrival of the bunch. Call that radial coordinate b . The ion receives an impulse which transfers to it momentum.

$$p(b) = \begin{cases} p_{max} \left(\frac{b}{a} \right) & , \quad b \leq a \\ p_{max} \left(\frac{a}{b} \right) & , \quad b > a \end{cases} \quad (3)$$

where

$$p_{max} = \frac{Q_{ion} E_a \ell_b}{c} \quad (4)$$

is the maximum momentum transfer for any value of b . Q_{ion} is the charge of the ion. The maximum excursion the ion can make during bunch passage occurs when $b = a$, and it is

$$\Delta r = \frac{1}{2} \left(\frac{Q_{ion} E_a}{M_{ion}} \right) \left(\frac{\ell_b}{c} \right)^2$$

This excursion must be small compared to the bunch radius, so our criterion for the ion to be useful as a probe of the surface field can be written as follows.

$$M_{ion} c^2 \gg \frac{Q_{ion} E_a \ell_b^2}{2a} \quad (5)$$

Using Eq. (2) and specializing to singly charged ions, the criterion can be written in the convenient form

$$\frac{M_{ion}}{M} \gg \frac{N_b r_p \ell_b}{a^2} \quad \text{Criterion} \quad (6)$$

where r_p is the proton's classical radius and M is its rest mass. The right-hand member of Eq. (6) is tabulated below for the full SLC bunch population, $N_b = 5 \times 10^{10}$, and a bunch length $\ell_b = 0.2$ cm.

a	$\left(\frac{N_b r_p \ell_b}{a^2}\right)$
$10^{-4} \text{ cm } (1 \mu)$	153.
$10^{-3} \text{ cm } (10 \mu)$	1.5
$10^{-2} \text{ cm } (100 \mu)$	0.015

Inspection of this table reveals that hydrogen and helium satisfy our criterion very nicely for 100-micron beams—the size we shall be dealing with in the arcs. We emphasize helium not only because, with $(M_{ion}/M) = 4$, it satisfies the criterion better than hydrogen, but also because it is a far more benign as with which to work. for 10-micron beams even helium is marginal, but it will probably work well enough. however to measure the one-to two-micron beam sizes we are trying to achieve at the interacting region we shall need a very heavy gas indeed.

We conclude that there are ions—singly charged ions—which will do th job for us of probing the SLC bunch's peak field and thus its radius, if we can measure the momentum spectrum of the outcoming ions. In the next section we shall derive the spectrum, but we already know that the information we seek is concentrated in its end-point and that its endpoint is at p_{max} what values of p_{max} shall we be dealing with?

Combining Eq. (2) and Eq. (4) for singly charged ions we get

$$p_{max} = \frac{2 N_b e^2}{a c} \quad (\text{cgs}) \quad (7)$$

The maximum momentum is (a) independent of the bunch length, a result we anticipated, and (b) independent of the mass of the ion. Another form of Eq. (7) which is convenient is the following.

$$p_{max} = \frac{2 N_b r_p}{a} M c$$

($r_p = 1.535 \times 10^{-16}$ cm and $Mc = 938$ Mev/c.) For the full SLC bunch intensity then

$$p_{max} = \frac{144}{a_\mu} \text{ MeV/c} \quad (8)$$

where a_μ is the bunch radius in microns. A 100-micron throws out ions of momenta up to 1.44 Mev/c.

The corresponding kinetic energies are, of course, not independent of ion mass. The heavier the ion, the lower the kinetic energy.

$$T_{max} = \frac{c^2 p_{max}^2}{2 M_{ion} c^2} = \frac{10^4}{a_\mu^2 M_{ion} c^2} \text{ MeV} \quad (9)$$

If helium ions were used as probes of the 100-micron beam, their maximum kinetic energy would be only 250 eV, while that of protons would be 1 keV.

4. THE MOMENTUM SPECTRUM

Suppose the ion source projects a sheet of nearly-zero -velocity ions across the path of the SLC bunch (as crudely depicted in Fig. 1), the thin dimension of the sheet, L , coinciding with the direction of the bunch's motion. Let the ion sheet contain a uniform density of ions per cubic centimeter ρ_{ion} . In other words, each particle of the bunch passes through an ion target of density ($L\rho_{ion}$) ions per square centimeter.

Now consider an annulus of the ion sheet of radius b and width db . It contains ($2\pi L \rho_{ion} b db$) ions, all of which are projected out radially by the passing bunch with the momentum $p(b)$ given by Eq. (3). Taking a look at Fig. 3, in which b is plotted as a function of p , admonishes us however that, while the momentum

is a single-valued function of the initial position of the ion, the converse is not true; b is a double-valued function of p . Thus each element of the momentum spectrum gets contributions from two regions of b .

$$dN(p) = 2\pi L\rho_{ion}(|b_1 db_1| + |b_2 db_2|)$$

where

$$b_1 = a\left(\frac{p}{p_{max}}\right)$$

and

$$b_2 = a\left(\frac{p_{max}}{p}\right)$$

so the momentum spectrum is

$$\frac{dN}{dp}(p) = \frac{2\pi a^2 L\rho_{ion}}{p_{max}} \left[\left(\frac{p}{p_{max}}\right) + \left(\frac{p_{max}}{p}\right)^3 \right] \quad (10)$$

This is a compact form, but we must not forget that p_{max} is inversely proportional to a , — Eq. (7) — and therefore the spectral density at the end-point actually varies as a -cubed. Figure 4 displays two representative spectra for bunches with radii differing by 25%. The difference in radii is reflected primarily in the momenta of the end-points which differ, according to Eq. (7), reciprocally with the radii. The larger bunch has the lower end-point ($p_{max} = 0.8$) and the spectral density at the end-point is about twice as great as that of the smaller bunch.

These spectra, produced by projecting sheets of ions across the beam, have the valuable feature that their sharp end-points are very sensitive to bunch radius. If that feature can be reliably detected by a spectrometer, it can be used to measure the radius. On the other hand, the spectra will display a reciprocally cubic rise at lower momenta; floods of low-momentum ions will enter the spectrometer and will have to be dealt with—prevented from “piling up” and obscuring the end point.

5. BEAM-PRODUCED IONS — THEIR SPECTRA

The rising low-momentum tails of the spectra shown in Fig. 4 are entirely due to ions which lay outside of the bunch initially — radially outside its skin. We have no use for those ions; they contribute nothing valuable to our measurement of the radius. If they were all absent, i.e., if our ion beam were exactly the lateral size of the SLC bunch and aimed perfectly at the bunch, the spectrum would be given by Eq. (10) with the second term in the brackets eliminated. It would be a simple linear rise from the origin, reaching its peak and terminating at p_{max} : a triangular spectrum which would be almost ideal for determining the bunch radius. Unfortunately we cannot make and aim an ion beam with these happy properties.

The thoughtful reader will have already realized that there is an obvious and convenient way of creating ions only within the bunch: project across the SLC beam a jet of gas and let the bunch do the ionizing. Ions will not be formed outside the bunch, so those undesirables are eliminated. But there is a significant drawback to this method. An ion is equally likely to be formed anywhere along the length of the bunch, and the impulse it receives is proportional to only the fraction of the bunch which passes it after it is formed. The spectra of beam-produced ions will not have steps for end-points. They will have end points however — the same end-points as the spectra we have already studied — and if these end-points are readily identifiable, they can be made to serve for measuring beam radii. Let us derive these spectra.

Consider a batch of ions formed in a longitudinal element, ds , of the bunch located a distance s in front of the trailing edge of the bunch. The momentum spectrum of those ions is a triangle. See Fig. 5.

$$\Delta[dN(p)] = \begin{cases} \frac{2\lambda_{ion} \ell_b^2}{p_{max} s^2} p dp ds & , \quad p < p_{max} \frac{s}{\ell_b} \\ 0 & , \quad p > p_{max} \frac{s}{\ell_b} \end{cases} \quad (11)$$

where λ_{ion} is the number of ions formed per centimeter along the beam. The total spectrum is the sum of the contributions of all of the elementary spectra from

all of the segments. The total spectral density at momentum p is the integral

$$\frac{2\lambda_{ion}\ell_b^2}{p_{max}^2} p dp \int_{\ell_b \frac{p}{p_{max}}}^{\ell_b} \frac{ds}{s^2}$$

The lower limit of the integral accounts for the fact that no contributions at p are made from the part of the bunch that lies behind $s = \ell_b p/p_{max}$. The value of the integral is

$$dN(p) = 2\lambda_{ion}\ell_b \left(1 - \frac{p}{p_{max}}\right) \frac{dp}{p_{max}} \quad (12)$$

The spectrum is a decending straight line with an intercept at p_{max} . There is no inverse cubic flood of low-momentum ions and the end-point should be easy to establish. Fig. 6 and Fig. 7 show two spectra of beam-produced ions generated by computer simulation for different bunch lengths, other bunch parameters being the same. The spectra are almost identical, as they should be since we expect the use of ions as opposed to electrons to obviate any dependence of the spectra on bunch length. Moreover the end-points coincide quite accurately with the computed value of p_{max} . Finally Fig. 8 and Fig. 9 show spectra for different values of bunch population and radius; they verify the dependence of p_{max} on those variables given by Eq. (7).

6. CONCLUSIONS

Ions offer promising probes of bunch size in the SLC. Helium ions are massive enough to stand still while the bunch passes (if the bunch is large enough radially) and thus to sample the electric field at a single radius. The momentum spectrum of the ions will display an end-point which will reveal the maximum field and, through it, the bunch radius, provided the bunch population is known. The results presented here have been based on a simple cylindrical model of the bunch, but extension to more realistic, sophisticated models is quite possible, and we expect that the salient features of the spectra and their applicability to measuring bunch radii to be preserved.

Fig. 1. Schematic arrangement of apparatus for measuring bunch radius using an ion gun.

Fig. 2. The radial electric field of the bunch as a function of the radial coordinate.

Fig. 3. Radius of origin as a function of (p/p_{max}) .

Fig. 4. Spectral shapes for bunches with radii differing by 25%.

Fig. 5. The momentum spectrum of ions formed by the bunch in the segment ds .

Fig. 6. Spectrum of beam - produced *He* ions.

Fig. 7. Spectrum of beam - produced *He* ions.

IO= 3

NBUNCH = 5.00E+10 RADIUS = 1.25E+02 LENGTH = 2.00E+01

Fig. 8. Spectrum of beam - produced He ions.

ID= 3

NBUNCH = 4.00E+10 RADIUS = 1.25E-02 LENGTH = 2.00E-01

Fig. 9. Spectrum of beam - produced *He* ions.

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