MASTER

PRELIMINARY MEASUREMENTS OF PLASMA DENSITY

J. T. Keane
April 11, 1978

ACCELERATOR DEPARTMENT
BROOKHAVEN NATIONAL LABORATORY
ASSOCIATED UNIVERSITIES, INC.
UPTON, NEW YORK 11973

under contract No. EY-76-C-02-0016 with the
UNITED STATES DEPARTMENT OF ENERGY

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

PRELIMINARY MEASUREMENTS OF PLASMA DENSITY

J. T. Keane

April 11, 1978

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Department of Energy, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

ACCELERATOR DEPARTMENT

BROOKHAVEN NATIONAL LABORATORY
ASSOCIATED UNIVERSITIES, INC.
UPTON, NEW YORK 11973

under contract No. EY-76-C-02-0016 with the
UNITED STATES DEPARTMENT OF ENERGY

Summary:

The phase velocity of a microwave signal propagating in a plasma is increased providing that its frequency is much higher than the plasma frequency

$$V\phi = c \left[\frac{1}{1 - \frac{f_p^2}{f^2}} \right]^{\frac{1}{2}}$$

where $V\phi$ = phase velocity

c = velocity of light

f_p = frequency of plasma

f = frequency of signal

It follows that by forming a resonator around the plasma and noting the resonant frequency shift plasma frequency and thus plasma density may be measured.

To check the feasibility of this type of measurement, a section of vacuum pipe in the HIF beam transport line was resonated and the frequency shift resulting when beam was turned on was noted.

I. Modes in Cylindrical Cavity

From the Waveguide Handbook of the MIT Radiation Laboratory Series, the cutoff wavelength (free space) for cylindrical waveguide is given by

for TM modes $\lambda_c = \frac{2\pi}{X_{mn}} a$; for TE modes $\lambda_c = \frac{2\pi}{X'_{mn}} a$

where a = radius of guide

$$X_{10} = 2.405$$

$$X'_{11} = 1.841$$

$$X_{11} = 3.832$$

$$X'_{21} = 3.054$$

$$X'_{01} = 3.832$$

For a vacuum pipe whose diameter is $5\frac{5}{8}$ " the frequency of the lowest three modes that can propagate is as follows

Mode	$\frac{2\pi}{X_{mn}} a$ (inches)	frequency (KMC)
TE ₁₁	9.6	1.23
TM ₀₁	7.35	1.61
TE ₂₁	5.8	2.04

Our interest was only in the two lower frequency modes. By placing two shorting planes approximately 40 inches apart a resonance is set up. The wave length is given by

$$\lambda_{TE_{11P}} = \frac{2}{\left[\left(\frac{P}{L} \right)^2 + \left(\frac{1}{1.706a} \right)^2 \right]^{1/2}} \quad \text{where } a = \text{radius}$$

$$L = \text{length}$$

$$\text{since } \frac{P}{L} \ll \frac{1}{1.706a} \quad \text{since } L = 30'' \gg a = 2.8''$$

$$\lambda_{TE_{111}} \cong \lambda_{\text{Cutoff}}$$

The TE₁₁₀ cannot exist because of the shorting plane but there will be numerous TE_{11P} modes starting near the cutoff frequency of the guide (1.23 KMHZ). Similarly there will be numerous TM_{01P} modes starting near the cutoff frequency of the TM₀₁ mode (1.6 KMHZ).

II. Resonance in Beam Pipe

There existed a section of pipe in the beam line bounded by a collimator and beam scraper. The conducting planes at these terminated point reduced the cylindrical diameter to 2". It was hoped these end walls would sufficiently terminate the transverse electric field to set up resonance in the TE_{11P} and TM_{01P} modes. This turned out to be the case. As it turned out there also was a BNC type connector from air to vacuum that could be used as an rf coupler. It was close to the collimator end of the pipe. An inch length of #10 wire was soldered on to the BNC connector on the vacuum side forming an electric field coupler.

A HP 8410 network analyzer was used to find resonances. (See Fig. 1). This device measures the forward and reflected power going to the "E" field coupler and then provides a (Smith Chart) polar coordinate plot of the reflector coefficient, Γ , where:

$$\bar{\Gamma} = \frac{\bar{V}_{\text{reflected}}}{\bar{V}_{\text{incident}}} \quad \bar{\Gamma}, \bar{V} \text{ are complex quantities}$$

It can be shown that the universal resonance curve plots as a circle on the Γ plane. (See Fig. 2).

The frequency for a TE_{11p} resonance that was nearly matched was set up. A pulsed beam was sent through the line and the resultant Γ change was noted.

The trace observed reasonable approximated the ray of the circle expected. (See Fig. 3).

Deviation from this ideal circle is possible due to changes in "Q" introduced into the circuit due to dielectric loss. This would cause a smaller circle. Since a long length of cable was used between cavity and measuring equipment by changing frequency the ideal circle would tend to rotate. This may cause point outside the ideal circle.

By changing frequency to simulate the maximum shift on the " Γ " plane the effective frequency shift was found. For this case it was approximately 3 MHz for a resonant frequency of 1250 MHz.

In addition to the polar plot, video signal giving the X and Y co-ordinates of the Γ plane plot were taken off the unit. It was noted that the maximum frequency shift too 50 microseconds to occur.

III. Plasma Measurements

For low density plasmas ($f_p \ll f$) the change in frequency is given as follows

$$\frac{\Delta f}{f} \approx \left[\frac{0.5}{1 + \left(\frac{r}{f}\right)^2} \right] \frac{\int_v \left(\frac{f_p}{f}\right)^2 E_0^2 dv}{\int_v E_0^2 dv}$$

where r - is collision frequency

For $r \ll f_p$ and uniform plasma density ($\frac{f_p}{f} = \text{constant}$)

$$\frac{\Delta f}{f} \approx \frac{1}{2} \left(\frac{f_p}{f}\right)^2$$

$$\therefore f_p = (2 \times \Delta f \times f)^{\frac{1}{2}} = (6 \times 1250)^{\frac{1}{2}} \times 10^6$$

$$f_p \approx 87 \text{ megahertz}$$

but ³

$$f_p = 8.979 n^{\frac{1}{2}}$$

where n = number electrons/meter³

$$n = \left(\frac{f_p}{8.979}\right)^2 = \left(\frac{87 \times 10^6}{8.979}\right)^2 = 9.4 \times 10^{13} \frac{\text{electrons}}{\text{meter}}$$

This calculated electron density is somewhat higher than the density of the proton beam ($\approx 3 \times 10^{13} \frac{\text{proton}}{\text{meter}}$). Since the proton beam was neutralized. This is the results expected.

IV. Conclusions

This seems to be a viable method and will be pursued further. A shorter resonator should be used to separate modes a little further and more careful "Q" measurements should be made. Cable length should be shortened from the cavity to the Network Analyzer. It would help to sweep frequency during beam time in order to get a frequency Vs r circle display.

1. Heald and Wharton, "Plasma Diagnostics With Microwaves", Eq. (1.39), p. 5; John Wiley & Sons, Inc., New York, 1965

2. Ibid, Eq. (5.1.8), p. 159

3. Ibid, Eq. (1.2.8), p. 4

Acknowledgment

The work of E. Meier and K. Riker and the members of the Brookhaven Heavy Ion Fusion Group was essential for the realization of a working system.

TEST SET UP USING SINGLE PROBE TO DETERMINE
RESONANT FREQUENCY

FIG. 1

UNIVERSAL RESONANCE CURVE

INNER CIRCLE IS PLOT OF Γ VS. FREQ. FOR A MATCHED INPUT TO RESONATOR

FIG. 2

NO BEAM

TRACE OBSERVED WITH BEAM

FIG. 3

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the U.S. nor the U.S. Department of Energy, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

PRINTED IN THE UNITED STATES OF AMERICA
Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road
Springfield, VA 22161

Price: Domestic \$ 4.00

Microfiche \$ 3.00

Printed: April 1978

745 Copies