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Error Analysis of a New Asymmetric Abel-Inversion Method

Hyeon K. Park

Princeton Plasma Physics Laboratory, Princeton University

Princeton, NJ 08543

Abstract

Propagation of the various errors in a new asymmetric Abel-inversion method¹ is investigated. In order to provide a meaningful error analysis of the inversion process, both the effects of geometrical factors and measuremental errors are studied. The dominant source of uncertainty in the determination of local electron density is due to the spline fitting of the interferometric measurements with uneven spacings. The propagation of these errors is highly spatially localized and heavily damped in this inversion method. The absolute range of error in the local electron density is $\pm 1.5 \times 10^{12} \text{ cm}^{-3}$.

I. Introduction

The uncertainties in the determination of local electron density from chordal interferometric measurements in a tokamak plasma are dependent upon the propagation of errors in the inversion method. The sources of errors can be divided into uncertainties in the input data (measurements or fitted data) and the geometrical factor (path length). If the path length estimation is incorrect or the assumptions on the geometry are not realistic, the estimated errors in the inversion process are meaningless. A new asymmetric Abel-inversion method¹ developed for the TFTR interferometry system² has produced reliable inverted electron density profiles. A simultaneous estimation of flux geometry and local density allow calculation of various electron density and pressure parameters.³ In this paper, the detailed study of the propagation of errors in this method is presented. In Sec. II, the effects of uncertainties in the flux geometry used in this inversion method are examined in detail. these include effects of magnetically determining the major radius (R_0) and minor radius (a_0), the radial dependence of the Shafranov shift, and the flux surface elongation. In Sec. III, the effect of boundary conditions and spline fitting of unevenly spaced interferometric measurements is examined for various density profile shapes (e.g. hollow, parabolic and highly peaked profiles). The dominant source of error is the propagation of the fitting uncertainties in the inversion process. It is also notable that the errors in this inversion method are generally spatially localized where the uncertainty is

large and the propagation of these errors are quickly damped. The asymmetries of the electron density on a flux surface within $r/a \leq 0.5$ are caused by this type of error. Within the estimated error, the electron density seems to be constant on a magnetic flux surface.

II. Errors due to uncertainties in flux geometry

The flux geometry used in this inversion method is approximated by employing the magnetically determined R_0 and a_0 which have small errors ($\leq 2\%$). Therefore, the variation of the inverted profiles due to these uncertainties in R_0 and a_0 is relatively small. Similarly, the error on the Shafranov shift and the surface elongation profile shapes are also a small contribution, as discussed in Ref. 1. Although, the uncertainties in the center of the Shafranov shift (ϵ_0) do not effect the global shape of the inverted profile as shown in Fig. 3 in Ref. 1, asymmetries of the electron density on a flux surface are very sensitive to this parameter in the inversion process. As demonstrated in Fig. 1, the ϵ_0 determined self-consistently with the assumption that the asymmetry in the innermost flux surface is less than 5%, provides the smallest values of asymmetry across the profile. In general, the errors due to the geometrical factors in this inversion method are not a significant factor.

III. Errors due to measurements and data preparation

The uncertainties in the input data to the inversion are classified in two categories; these are measuremental errors and errors due to data preparation. The errors due to various cross talk problems in an interferometer system are eliminated using a half wave technique.⁴ Among the measuremental errors, the systematic error is due to the slow motion of the interferometer structure during a discharge period. The range of this type of error is less than 1/10 th of a fringe (one fringe is equivalent to $1 \times 10^{15} \text{ cm}^{-2}$ for a wavelength of $119 \mu\text{m}$). This type of error can be easily corrected by adjusting the baseline. The other type of error is caused by the magnetic field effect on the lasers and the fast response of the structure. These errors are even smaller by a factor of two compared to the first type of error and the propagation of these errors in the inversion is negligible.

Uncertainties due to data preparation are mainly boundary estimation and spline fitting errors. Since the outermost flux surface is very close to the inner bumper limiter, the innermost boundary point is chosen to be a linear extrapolation of two measured points (the first and second data points from the inner belt limiter as shown in Fig. 2a). On the other hand, the outermost boundary point can be determined with a better accuracy, since the last measured point is beyond the scrape-off layer. In order to test the effects of boundary conditions, three line-integral density profiles are constructed using

different inner boundary conditions as shown in Fig. 4a in Ref. 1. The three extrapolation points are: higher than the linear extrapolation value, zero density at the limiter position, and a linear extrapolation point. Since the geometry employed for the inversion is fixed, errors in the inverted profile are purely those in the density. The error propagation is suppressed very quickly as shown in Fig. 4b in Ref. 1. The linearly extrapolated results had the best agreement with the measurements made by Thomson scattering.

In the TFTR interferometry system,² the line density profile is based on a ten-channel measurement with uneven chord spacing, as shown in Fig. 2a, a cubic spline fit is therefore introduced. Among the ten chords of the interferometry system, the greatest distance is between the 6th (268 cm) and 7th chord (300 cm) as shown in Fig. 2a. The effect of the spline fitting of uneven chords can easily be demonstrated here. First, the data are prepared with a standard cubic spline fit of the ten chordal measurements and the inversion is performed. The inverted result will serve as a reference. Second, the line-integral density of the spline-fitted data at 285 cm is perturbed by adding and subtracting $1 \times 10^{14} \text{ cm}^{-2}$. Including the perturbed data points, the data are re-splined and inverted. These results are compared with the reference profile. In order to test the sensitivity of the spline fitting to the shape of the profile, three different types of profiles are tested; these are ohmic, hollow, and highly shifted and peaked profiles.

In an ohmic plasma where the Shafranov shift is relatively small ($\epsilon_0 \leq 10 \text{ cm}$) and the shape is parabolic, one might expect that the spline fitting

would be reasonable. As shown in Fig. 2b, the perturbation made at 285 cm is confined within half of the profile. The deviation from the reference profile shown in this figure is as expected. If the electron density is nearly constant on a magnetic flux surface, the reference profile is the best results. A similar example is given for a hollow profile in Fig. 3a. It is notable that if one of the peaks of the hollow profile is located near 285 cm where the perturbation is made, the reference profile may underestimate one side of the hollowness based on the symmetry argument. Since the geometry is similar to the ohmic case (small ϵ_0), the propagation of errors is expected to be similar to the ohmic case. For the a highly peaked profile with a large Shafranov shift (ϵ_0 is up to 30 cm), the propagation of the perturbation is no longer confined within half of the profile as shown in Fig. 3b.

Similar tests were also made throughout the profile with the same perturbation made at 285 cm. The absolute error and its propagation were smaller than the results demonstrated at 285 cm. Together with the assumption of symmetry of the electron density on a flux surface, the range of absolute uncertainty in local electron density determined with this inversion method is $\pm 1.5 \times 10^{12} \text{ cm}^{-3}$.

IV. Summary

In this paper, the error analysis of a new asymmetric Abel-inversion method is presented. The major uncertainties in inverted profiles are due

to spline fitting of uneven chordal measurements. Propagation of the errors are heavily damped and localized where the perturbation is made. The range of absolute error in an inverted electron density is $\pm 1.5 \times 10^{12} \text{cm}^{-3}$.

Acknowledgments

This work was supported by U.S. Department of Energy contract No. DE-ACO2-76-CHO-3073.

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Figures

FIG. 1. Electron density asymmetry factor (ξ) as a function of the normalized radius for three different values of Shafranov shift center ($\epsilon_0 = 22$ cm, 26 cm and 30 cm). All other parameters are kept constant in the inversion process.

FIG. 2. (a) Example of the spline fitting for line-integrated electron density profile for an ohmic discharge and arrangement of interferometric chords in TFTR. (b) The inverted profile of (a) is compared with the inverted profiles of perturbed line integral density at 285 cm.

FIG. 3. The inverted profile without perturbation is compared with profiles of perturbed line integral density at 285 cm for different types of the profile shapes. (a) Hollow profile (b) Highly peaked and shifted profile.

Figure 1

Figure 2a

Figure 2b

Figure 3a

Figure 3b

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