APPLICATION OF MATRIX FORMALISM TO
PROBLEMS WITH MEMORY

SLAC-PEP-NOTE--317

S. Kheifets

DE89 006338

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

FEB 02 1989

Summary

On an example of the transverse one-dimensional coherent bunch motion in the presence of feedback, we developed a matrix formalism which allows us to find the decrement (or increment) and the tune shift of the motion due to a feedback system which remembers many previous passages of the bunch.

The derived formulae are applied to a particular case of exponential decay of a resonant kicker signal. The possible detuning of the kicker resonator is also considered. The formulae are further applied to the case of a parasitic cavity.

Introduction

There are several reasons, both theoretical and experimental,^{1,4-7} to believe that a feedback system for very big storage rings are not only desirable but also necessary, probably for all three degrees of particle motion freedom. Almost all the storage rings are supplied with some kind of feedback system⁸⁻¹⁵. There is little doubt that if the bandwidth of such a feedback system is large enough, then the dipole coherent motion of each bunch (so-called "rigid bunch" oscillations) can be damped independently. Much work has been done essentially for the case where the signal of the feedback system is faded out by the moment of the passage of the next bunch through the kicker.

Now, it is also clear that the bigger the bandwidth of the feedback system, the more expensive it is. Hence, it is worthwhile to look at the possibility of using systems with narrow bandwidths. Such systems have "long memory" in the sense that they oscillate for many subsequent periods after each bunch passage.

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

yes

The same argument can be applied to a ('parasitic') cavity if one happened to be part of a vacuum chamber. Such a memory binds all bunches in one oscillating system which can be best described by its normal modes.²⁻⁴

For many bunch coupled oscillations all the sources of coherent instability should be considered simultaneously. The only case where the actions of the feedback system and of the cavity can be treated independently is the motion of one bunch in the limit of small kicks produced both by the kicker and by the cavity placed at the same azimuth of the ring.

We develop here a method of solving problems with a memory for one-dimensional dipole coherent oscillations of a single bunch. The bunch oscillation is assumed to be linear, and the perturbations which the feedback system or the cavity brings into the bunch motion are considered as small compared to the focussing of storage ring magnets.

Matrix Formalism for Feedback

Let $x^b(n)$ and $x'^b(n)$ be the displacement and trajectory angle of the bunch center of gravity immediately before the kicker on the n^{th} passage. We assume that the result of the kicker action on the coordinates of the bunch will be:

$$x_k^a(n) = x_k^b(n) \quad (1)$$

$$x_k'^a(n) = x_k'^b(n) + \sum_{m=1}^{\infty} f_m x_D(n-m) \quad (2)$$

where $x_D(n-m)$ stands for the bunch displacement in the detector during the m -th previous passage and f_m are some coefficients which describe the dependence of the kicker signal on the revolution number, m . Expressions (1) and (2) can be written also in matrix form by

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

introducing vector $x_k = \begin{pmatrix} x_k \\ x'_k \end{pmatrix}$:

$$x_k^a(n) = L x_k^b(n) \quad (3)$$

or

$$x_k^b(n) = L^{-1} x_k^a(n) \quad (4)$$

Here, L^{-1} denotes the matrix which satisfies the matrix equation, $L^{-1}L = L L^{-1} = I$, I being the unit matrix.

The transformation from the point immediately after the kicker to the point immediately before it, one revolution later, is described by the usual matrix, M , for one whole revolution (without the kicker):

$$x_k^b(n) = M x_k^a(n-1) \quad (5)$$

or

$$x_k^a(n-1) = M^{-1} x_k^b(n) \quad (6)$$

Combining (3) and (5) we get full matrix for one revolution $T = LM$. On the other hand, from (4) and (6), we get

$$x_k^a(n-1) = M^{-1} L^{-1} x_k^a(n) \quad (7)$$

Let us now introduce the matrix, F , which transforms coordinates of the bunch from the point immediately after the kicker to the point where the detector is placed. We have:

$$x_D(n-1) = F M^{-1} L^{-1} x_k^a(n) \quad (8)$$

and, in general

$$x_D(n-m) = F (M^{-1} L^{-1})^m x_k^a(n) \quad (9)$$

Combining, now, expressions (3) and (9), we get the following matrix equation for the unknown matrix L:

$$L = I + P F \sum_{m=1}^{\infty} f_m (M^{-1} L^{-1})^m \quad (10)$$

Here we introduce the operator, P, which provides for the change in (3) of only the angle but not the coordinate:

$$P = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad (11)$$

To solve equation (10), we use now the smallness of perturbation introduced in the motion by the kicker and solve (10) by the iteration method. In the zeroth order $L_0 = I$. In the next approximation, we substitute L_0 into the right hand side of equation (10) to get:

$$L_1 = I + P F \sum_{m=1}^{\infty} f_m M^{-m} \quad (12)$$

Expression (12) has a very simple meaning.* The m-th term in the sum of it describes the kick experienced by a bunch on a given revolution due to the passage through the detector m revolutions before it, if all the previous kicks are neglected. Hence, the solution (12) of equation (10) uses essentially the assumption of smallness of the

* I owe this comment to A. Chao.

perturbation brought in by the kicker relative to the strength of the focussing system of the ring.

$$L_1 = \begin{pmatrix} 1 & 0 \\ \sqrt{\frac{\beta_D}{\beta_k}} (\Sigma_C - \alpha_k \Sigma_S) & 1 - \sqrt{\beta_D \beta_k} \Sigma_S \end{pmatrix} \quad (13)$$

where the following abbreviations are introduced:

$$\Sigma_S = \sum_{m=1}^{\infty} f_m \sin \left[(m-1) \mu_0 + \mu_{k+1} - \mu_D \right] \quad (14)$$

$$\Sigma_C = \sum_{m=1}^{\infty} f_m \cos \left[(m-1) \mu_0 + \mu_{k+1} - \mu_D \right] \quad (15)$$

β_D , β_k , α_k , μ_D and μ_{k+1} in these expressions are the values of the β -function, the slope ($\alpha = -\beta'/2$) and the phase advance from the arbitrary zero point to the detector and the kicker, respectively. μ_0 denotes the phase advance for the whole revolution around the unperturbed ring.

The values of a damping constant and of a tune shift due to the feedback system can be found now by solving secular equations $T = \lambda I$, where $T = L_1 M$ and $\lambda = e^{i\mu}$. If we substitute

$$\mu = \mu_0 + 2\pi \Delta\nu + i\theta, \quad (16)$$

then

$$\Delta v = - \frac{\sqrt{\beta_k \beta_D}}{4\pi} \Sigma_C \quad (17)$$

$$\theta = \frac{\sqrt{\beta_k \beta_D}}{2} \Sigma_S \quad (18)$$

A Particular Case

We apply now the general formulae (17) and (18) to a particular case of the feedback system, which coefficients f_m have the form

$$f_m = f_0 e^{-\alpha(m-1)} \cos (m-1) \Delta\phi \quad (19)$$

α is the dimensionless decrement of the signal decay with the increase of revolution number. $\Delta\phi$ is proportional to a detuning of the feedback resonant circuit. Performing summations in (17) and (18) we get

$$\Delta v = - \frac{\sqrt{\beta_k \beta_D}}{4\pi} f_0 \phi_V (\alpha, \Delta\phi, \mu_0, \Delta\psi), \quad (20)$$

$$\theta = \frac{\sqrt{\beta_k \beta_D}}{2} f_0 \phi_\theta (\alpha, \Delta\phi, \mu_0, \Delta\psi), \quad (21)$$

where $\Delta\psi = \mu_{k+1} - \mu_D$ is the phase advance from the detector to the subsequent kicker. The form factors ϕ_V and ϕ_θ are:

$$\phi_V = \frac{\cos \Delta\psi e^{-\alpha} \cos(\mu_0 + \Delta\phi - \Delta\psi)}{1 + e^{-2\alpha} - 2e^{-\alpha} \cos(\mu_0 + \Delta\phi)} + \frac{\cos \Delta\psi e^{-\alpha} \cos(\mu_0 - \Delta\phi - \Delta\psi)}{1 + e^{-2\alpha} - 2e^{-\alpha} \cos(\mu_0 - \Delta\phi)} \quad (22)$$

$$\phi_\theta = \frac{\sin \Delta\psi e^{-\alpha} \sin(-\Delta\psi + \mu_0 + \Delta\phi)}{1 + e^{-2\alpha} - 2e^{-\alpha} \cos(\mu_0 + \Delta\phi)} + \frac{\sin \Delta\psi e^{-\alpha} \sin(-\Delta\psi + \mu_0 - \Delta\phi)}{1 + e^{-2\alpha} - 2e^{-\alpha} \cos(\mu_0 - \Delta\phi)} \quad (23)$$

Figures 1-8 and 9-16 present the form factors ϕ_v and ϕ_0 as functions of $x = (\mu_{k+1} - \mu_D)/2\pi$, respectively, for different values of the parameters $\delta v = (\mu_0 - E_n(\mu_0))/2\pi$ (i.e., the fractional part of the tune), α and $\Delta\phi$.

Matrix Formalism for a Cavity

We can apply the same approach to the case of parasitic cavity where all the impedance comes from one short place of the orbit. The action of such a cavity on the bunch motion we will describe by formulae analogous to (1), (2):

$$x_c^a(n) = x_c^b(n), \quad (1')$$

$$x_c^a(n) = x_c^b(n) + \sum_{m=0}^{\infty} C_m x_c^b(n-m). \quad (2')$$

Formula (2') differs from the formula (2) in three aspects. Firstly, the cavity action depends on the bunch displacement at the same place. Secondly, we include also the possibility of the immediate action of the cavity on the bunch, and thirdly, the action is described by another set of coefficients, C_m .

The first two aspects can be easily taken into consideration by substituting in expression (10) the matrix F by a matrix, M . Hence, in this case, we get the same answers as in (17) and (18) with the substitution $\beta_D = \beta_k$, $\mu_D = \mu_{k+1}$ and $f_m \rightarrow C_{m-1}$.

We can further consider in the same way the case of the kicker and the parasitic cavity if they are positioned on the same place in the orbit. The action of both of them results in the appearance of both sums in the formulae analogous to (2') and subsequently in expressions (12), (14) and (15).

The case where the kicker and the cavity are positioned at different places of the ring is much more complicated and needs separate consideration.

Acknowledgements

I am very grateful to Matt Sands and John Rees for their helpful discussions and interest in this work. Matt Sands formulated the problem and attracted my attention to it. I also profited very much from my discussions with Alex Chao, who helped to correct a mistake in an earlier calculation.

References

1. F. Sacharer, IEEE Trans. Nucl. Sci., NS-20, 825 (1973).
2. C. Pellegrini, A. Renieri, Frascati Internal Memo T-64 (1974).
3. A. Renieri, F. Tazzioli, IX Int. Conf. on High Energy Accelerators, Stanford, p.370 (1974).
4. C. Pellegrini, M. Sands, PEP-258, October 1977.
5. F. Amman, IEEE Trans. Nucl. Sci., NS-16, 1073 (1969).
6. R. Branham et al., IX Intl. Conf. on High Energy Accelerators, Stanford, p.359 (1978).
7. E. C. Raka, IEEE Trans. Nucl. Sci., NS-14, 1091 (1967).
8. B. Richter, "Feedback System for Stabilization of Coherent Oscillations in SPEAR," SPEAR-25 (1970).
9. J.-L. Pellegrin, "Beam Feedback Electronics for the SPEAR Storage Ring," SLAC-PUB-1150 (1972).
10. C. Ankenbrand, F. F. Higgins, Jr., R.P. Johnson, IEEE Trans. Nucl. Sci., NS-24, #3, 1698 (1977).
11. G. Carron, S. Myers, L. Thorndahl, IEEE Trans. Nucl. Sci., NS-24, 1833, (1977)
12. J.-L. Pellegrin, J.R. Rees, PEP-Note-315, August 1979.
13. C. H. Pruett, R.A. Otte, F. E. Mills, Proc. V Int. Conf. on High Energy Accelerators, 345 (1955).
14. R. D. Kohaupt, "Feedback Mechanismus bei Transversalen Schwingungen des Bunch-System," Technische Notiz H1-1, (1971).
15. K. Wille, Internal Report, DESY PET-79/01, January 1979.

FIGURE CAPTIONS

- Fig. 1: The function ϕ_θ (see (21)) for feedback with short memory ($\alpha = 10$) and $\Delta\phi = 0$ versus parameter $x = (\mu_{k+1} - \mu_0)/2\pi$. The curves are practically the same for all tunes μ_0 .
- Fig. 2: The function $\phi_\theta(x)$ (21) for $\alpha = 1$, $\Delta\phi = 0$ and different values of μ_0 .
- Fig. 3: The function $\phi_\theta(x)$ (21) for $\alpha = 0.1$ and $\Delta\phi = 0$. The curves correspond to different values of fractional part of $\nu_0 = \mu_0/2\pi$:
1 - $\Delta\nu_0 = 1/8$, 2 - $\Delta\nu_0 = 2/8$, 3 - $\Delta\nu_0 = 3/8$, etc.
- Fig. 4: The same as Fig. 3, but $\alpha = 0.01$.
- Fig. 5: The same as Fig. 1, but $\Delta\phi = 1.234$.
- Fig. 6: The same as Fig. 2, but $\Delta\phi = 1.234$.
- Fig. 7: The same as Fig. 3, but $\Delta\phi = 1.234$.
- Fig. 8: The same as Fig. 4, but $\Delta\phi = 1.234$.
- Figs. 9 - 16: Present the function ϕ_ν (see (20)) for the same parameters as for Figs. 1-8, correspondingly.

Fig. 1: DECREMENT FB ALFA = 0.100D + 02 DPHI = 0.

$$X = (\nu_{k+1} - \nu_D)/2\pi$$

Fig. 2: PHASE ADVANCE
 DECREMENT FB ALFA=0.1000-01 DPHI= 0.

Fig. 3: PHASE ADVANCE
DECREMENT FB ALFA=0.100D+00 DPHI= 0.

Fig. 4: PHASE ADVANCE
 DECREMENT FB ALFA=0.1000-01 DPHI= 0.

Fig. 5: PHASE ADVANCE
DECREMENT FD ALFA=0.100D+02 DPHI= 1.234

Fig. 6: PHASE ADVANCE
DECREMENT FB ALFA=0.100D+01 DPHI= **1.234**

Fig. 7: PHASE ADVANCE

DECREMENT FB ALFA=0.1000+00 DPHI= 1.234

Fig. 8: PHASE ADVANCE
DECREMENT FB ALFA=0.100D-01 DPHI= **1.234**

Fig. 9: PHASE ADVANCE
DECREMENT FB ALFA=0.000+02 DPHI- 0.

Fig. 10: PHASE ADVANCE
 DECREMENT FB $\alpha = 0.1000 \cdot 01$ $\Delta\phi = 0.$

Fig. 11: PHASE ADVANCE
 DECREMENT FB ALFA=0.1000+00 DPHI= 0.

Fig. 12; PHASE ADVANCE
 DECREMENT FB ALFA=0.100D-01 DPHI=0.

Fig. 13: PHASE ADVANCE
DECREMENT FB ALFA=0.100D+02 DPHI- 1.234

Fig. 14: PHASE ADVANCE

DECREMENT FB ALFA=0.100D·01 DPHI= 1.234

Fig. 15: PHASE ADVANCE
DECREMENT FB ALFA=0.1000+00 DPHI= 1.234

Fig. 16: PHASE ADVANCE
 DECREMENT FB ALFA=0.1000-01 DPHI= 1.234