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DEVELOPMENT OF SAMARIUM-COBALT QUADRUPOLE LENSES FOR PARTICLE ACCELERATOR APPLICATIONS

by

N. Saito, E. D. Bush, and D. A. Swenson

ABSTRACT

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A family of samarium-cobalt permanent magnet quadrupole lenses have been designed and fabricated for a new breed of proton linear accelerator under development at the Los Alamos Scientific Laboratory. Four kinds of geometrical cross sections were investigated.

The magnetic field gradients and multipole field harmonics up to $n = 14$ were measured with some compensated measuring coils for all the above models. A Hewlett-Packard Spectrum Analyzer 3580-A was used. Axial and transverse magnetic field surveys were performed with a Hall probe gaussmeter to determine the field gradients and the fringe field distribution. Quadrupole gradients in excess of 10 kG/cm were achieved using samarium-cobalt and certain geometries.

These same quantities were calculated by a new computer code PANDIRA. The comparison between the calculations and the measurements showed excellent agreements on gradients and harmonic components. The field patterns were produced by PANDIRA.

Among the above geometries, it turned out that the flat tip pole geometry is most promising. Grinding the tip decreases the harmonic contents rapidly compared with the slow decrease in the quadrupole field gradients. This is because the effective shape and the magnetization distribution approach to those of a hyperbolic quadrupole magnet. The hyperharmonic components of this geometry is less than 0.5% at $r = 6$ mm. The geometry offers an inexpensive and easy way of the fabrication and the gradient tuning. It satisfies the focal requirements and fits within the limited space of the structure.

MASTER

I. INTRODUCTION

A new breed of proton linear accelerator (PIGMI)¹ is being developed at the Los Alamos Scientific Laboratory (LASL) for medical and industrial applications. The major technical innovations in this program are the use of a higher radio frequency and a higher accelerating gradient, resulting in a substantially smaller and less expensive accelerating structure.²

The space available within the structure for the beam focusing lenses is also substantially reduced, making it impractical to satisfy the focal requirements with the conventional electromagnetic approach.

Figure 1 shows the drift tube profiles for the 64 cells of the PIGMI Prototype. This portion of the linac will accelerate a proton beam from 0.25 MeV to 7.32 MeV in a total length of 2.48 m at a frequency of 450 MHz. The tank is divided into four sections (Fig. 1).

The first section has no quadrupole lenses among its 28 drift tubes, and is entirely dependent on the alternating phase focusing (APF) principle for beam confinement. This section is 61.116 cm long and 46.000 cm in diameter.

Quadrupole focusing is "ramped on" gradually in the second section consisting of 16 cells. Permanent magnet quadrupole lenses are introduced into every other drift tube in a +0 -0 configuration, with quadrupole gradients that vary from -0.95 kG/cm in cell 29 to 7.90 kG/cm in cell 43. This section is 60.684 cm long and 46.000 cm in diameter.

The APF effects are "ramped off" gradually in the third section consisting of 16 cells. The gap phases at the end of the second section are approximately -58, 60, 60, and -58°. They are ramped linearly through the phase ramp section to a constant value of -20°. This section is 94.507 cm long and 45.00 cm in diameter.

The last section, consisting of four cells, is essentially a scaled-down version of a conventional quadrupole-focused drift-tube linac (DTL), and accelerates the beam from 5.87 to 7.32 MeV at an average acceleration rate of 4.6 MeV/m. This section is 31.349 cm long and 44.000 cm in diameter.

Fig. 1.
 Drift tube profiles for the PIGMI Prototype.

II. REQUIREMENTS FOR MAGNETIC FIELD

The calculated quadrupole gradient and the length required for the PiGMI Prototype are given in Table I. The maximum field gradient reaches 7 kG/cm where the length is quite short. The quadrupole strength and the effective length have to be adjusted to less than 1% error. The hyperharmonic components should be less than 1% of the quadrupole field.³

A preliminary study of the permanent quadrupole magnets for this purpose was previously published.⁴ Studies to date have concentrated on short, high-gradient quadrupole of this approximate size: (1) to reduce the harmonic error field, (2) to easily tune the field gradient, and (3) to minimize the fabrication costs.

III. PERMANENT QUADRUPOLE MAGNET MODELS

Figure 2 shows the geometrical cross sections of the four designs which were investigated. The first one has circular poles of permanent magnet material, magnetized along a diameter and set in a soft iron return yoke. The second has trapezoidal poles of permanent magnet material capped with soft iron poles having stepped profiles.* The third and fourth have originally circular

*While at LASL, R. F. Holsinger proposed and executed this design.

TABLE I
QUADRUPOLE FIELD REQUIREMENTS FOR THE PiGMI

Cell Number	Proton Energy Mev	Quadrupole Gradient kG/cm	Quadrupole Length cm
29	1.071	-0.951	3.272
31	1.182	+4.000	1.556
33	1.315	-2.107	3.551
35	1.447	+5.000	1.748
37	1.602	-2.657	3.840
39	1.758	+6.000	1.943
41	1.939	-3.206	4.119
43	2.120	+7.000	2.108
45	2.338	-3.739	4.549
47	2.641	+7.000	2.752
49	2.947	-4.539	4.741
51	3.386	+7.000	3.397
53	3.798	-5.305	4.977
55	4.364	+7.000	4.146
57	4.897	-6.145	5.203
59	5.551	+7.000	4.990
61	6.218	-6.758	5.660
63	6.945	+6.628	5.938

poles and the pole tip and pole sides are flattened, respectively. More than 11 yoke assemblies with $P = R_p/R_b$ (pole radii-to-bore radii ratio) of 0.59 to 1.6 were fabricated to experimentally determine the effects of the ratio for circular pole geometries. For circular poles, the permanent magnet materials investigated are an oriented Alnico-V, a barium ferrite ceramic, and two rare-earth samarium cobalts, HICOREX 18 and HICOREX 22. The circular pole radius and length, which have been fabricated, was unified to 0.904 and 1.5 cm, respectively. Flattening of the poles were made by grinding. For the stepped pole geometry, 16 trapezoidal pole pieces were made of HICOREX 16. The length was 5.0 cm. The quadrupole field gradient, hyperharmonic components, and the leakage field distribution were measured for all the models. Figure 3 shows some of the models.

IV. MULTIPOLE EXPANSION OF THE FIELD WITH PERFECT QUADRUPOLE SYMMETRY

In a current-free region, the magnetic field is expressible as the gradient of a scalar potential ψ ,

$$\vec{B} = \nabla\psi. \quad (1)$$

The scalar potential ψ must satisfy Laplace's equation

$$\nabla^2\psi = 0.$$

Fig. 2.
Geometrical cross sections of the four designs.

Fig. 3.
Permanent magnet quadrupole lens models.

We now assume that the magnetic field inside a quadrupole is independent of z (the axial coordinate), so that, in polar coordinates in Fig. 4, the equation reduces to

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \psi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \theta^2} = 0. \quad (3)$$

The general solution may have the next form because the region includes the origin, eliminating r^{-n} terms,

$$\psi(r, \theta) = \sum_{n=1}^{\infty} \left\{ C_n r^n \sin(n\theta - \alpha_n) \right\}. \quad (4)$$

In the present case, we have a quadrupole symmetry described by the next two conditions,

$$\begin{aligned} \psi(r, \theta) &= -\psi(r, \theta + \frac{\pi}{2}), \\ \psi(r, \theta) &= \psi(r, \frac{\pi}{2} - \theta). \end{aligned} \quad (5)$$

The final form of ψ becomes

$$\psi = \sum_{n=0}^{\infty} A_m \left(\frac{r}{R_B} \right)^m \sin m\theta, \quad m = 2(2n+1), \quad (6)$$

where R_B is the bore radius.

The theoretically ideal pole tip has a hyperbolic cross section ($xy = R_B^2/2$) on which ψ is constant. This means that the first term in Eq. (6) represents the ideal quadrupole field. The second term ($m = 6$) is the lowest order multipole field (dodecapole potential)*. The field components for r, θ can be expressed by:

$$B_r = \sum_{m=2,6,\dots} \frac{m}{r} A_m \left(\frac{r}{R_B} \right)^m \sin m\theta, \quad (7)$$

$$B_\theta = \sum_{m=2,6,\dots} \frac{m}{r} A_m \left(\frac{r}{R_B} \right)^m \cos m\theta. \quad (8)$$

The amplitude of each term at $r = R_N$ (normalized radius, we used $R_N = 6$ mm) is

$$B_m = \frac{m}{R_N} A_m \left(\frac{R_N}{R_B} \right)^m. \quad (9)$$

*In practical cases, the geometrical symmetry and magnetization symmetry are not usually maintained. The sextapole ($m = 3$) is the most easily generated multipole components for asymmetric systems. The ($m = 4$) term corresponds to octupole, ($m = 5$) to decapole. In our experiments, it was possible to suppress those values less than 1%.

We defined the % quadrupole by the expression

$$(\%Q)_m = \frac{B_m}{B_0} = \frac{m}{2} \frac{A_m}{A_0} \left(\frac{R_{Li}}{R_B} \right)^{m-2}. \quad (10)$$

V. HARMONIC MEASUREMENTS WITH ROTATING COILS

The method of quadrupole magnetic spectroscopy employs rotating coils in the aperture of a quadrupole lens.⁴ The coil length (80 mm) was made much longer than the length of the magnets to measure the integrated effect on a particle passing through the lens. Let's consider a loop of n_1 turns, with one bundle of wires at a radial position of r_1 and the other bundle of wires on the axis of rotation, which is made to coincide with the longitudinal magnet axis (Fig. 5). The loop rotates in the magnetic field of Eqs. (7) and (8) at a constant rate of ω radians per second. The voltage induced in the coil per unit length can be expressed as

$$\epsilon_1 = n_1 \omega \sum_{m=2,4,6,\dots} m A_m \left(\frac{r_1}{R_L} \right)^m \sin m\theta = \sum_m e_{1m} \sin m\theta. \quad (11)$$

This output signal ϵ_1 from the coil is fed into a spectrum analyzer, and the voltage amplitude e_{1m} may be measured. But usually higher harmonics decrease rapidly with m , and the accurate measurements of them become difficult because of analyzer saturation of input voltage. To overcome this difficulty, a compensated coil system was used. A second loop of n_2 turns was put on the same plane with the first loop at radial position r_2 . The return wires of the second loops have the same axis as the first one. The induced voltage in the second coil per unit length can be expressed in the same way,

Fig. 4.
Quadrupole lens.

Fig. 5.
Asymmetric coil in a multipole field.

$$\epsilon_2 = n_2 \sum_{m=2,6,\dots} m A_m \left(\frac{r_2}{R_B} \right)^m \sin m\theta. \quad (12)$$

The difference signal $\Delta\epsilon$ is expressed as

$$\begin{aligned} \Delta\epsilon &= \epsilon_1 - \epsilon_2 \\ &= \omega \sum_{m=2,6,\dots} m A_m \left(\frac{n_1 r_1^m - n_2 r_2^m}{R_B^m} \right) \sin m\theta. \end{aligned} \quad (13)$$

To have zero sensitivity of $\Delta\epsilon$ to the biggest component of the quadrupole field, the following condition has to be satisfied,

$$n_1 r_1^2 = n_2 r_2^2. \quad (14)$$

For our system, the relations of $r_2 = r_1/2$, $n_2 = 4n_1$ were selected. For this geometry, $\Delta\epsilon$ is expressed as

$$\Delta\epsilon = \sum_{m=6,10,\dots} K_m e_{1m} \sin m\theta. \quad (15)$$

$$K_m = 1 - 2^{2-m}. \quad (16)$$

It is very interesting to compare the Eq. (15) with Eq. (11). For higher harmonics, the coefficient K_m is close to unity, for example, $K_{10} = 0.996$. This means that the higher harmonics of $\Delta\epsilon$ are almost the same as those of ϵ_1 . In actual design, a 0.05808-mm-diam wire was used. To obtain good stiffness and long-term dimensional stability, the coil system was mounted on a reinforced Fiberglass epoxy cast support body. Fig. 6 shows the series of the coil assemblies. To achieve as high an accuracy of measurements as possible, a large radius coil system was used for each lens. The coil system was driven by a 60-rps, 1/20-hp hysteresis synchronous Bodine motor. The coil rotation center was positioned on axis by minimizing the subharmonic voltage output of ϵ_1 to locate the magnetic center. A Hewlett-Packard Model 3580-A Spectrum Analyzer was used to measure the harmonics amplitude. The ratio e_{1m} to e_{12} is expressed as

$$\frac{e_{1m}}{e_{12}} = \frac{m A_m}{2 A_2} \left(\frac{r_1}{R_B} \right)^{m-2}. \quad (17)$$

The % Quadrupole, therefore, can be expressed by the ratio,

$$(\%Q)_m = \frac{e_{1m}}{e_{12}} \left(\frac{R_N}{r_1} \right)^{m-2}. \quad (18)$$

Figure 7 shows the magnetic field measurement instrumentation. Figure 8 shows the block diagram of the harmonic analyzer for magnetic multipole field measurements.

Fig. 6
Series of the compensated coil assemblies.

Fig. 7.
Magnetic field measurement instrumentation.

Fig. 8.
Block diagram of harmonic analyzer for magnetic multipole field measurements.

VI. HALL PROBE SURVEY

Axial and transverse magnetic field surveys were performed with a Bell Model 240 Hall-type gaussmeter. The probe was positioned and scanned by three mutually perpendicular slides mounted on a black granite surface plate. The field and displacement were plotted by a Hewlett-Packard X-Y recorder. The field gradients obtained by this method showed a good agreement (within a few per cent difference) with the rotating coil method.

VII. COMPUTER CODE "PANDIRA"

A new computer code, "PANDIRA," developed at LASL can aid in the design and analysis of magnetic circuits including permanent magnet materials. The program solves anisotropic magnetostatic circuits by a "direct method" algorithm. The permanent magnet materials are described to the program by the BH curve in the second quadrant, the direction of the "easy axis" for magnetization, and the permeability perpendicular to this "easy axis."

The magnetic field gradients and multipole field harmonics can be calculated by PANDIRA. The field patterns can also be produced by PANDIRA.

These quantities have been calculated by PANDIRA and compared with the experiments for most measured geometries and materials.

VIII. COMPARISON BETWEEN MEASUREMENT AND CALCULATION

The magnetic field gradients and multipole field harmonics were measured for all combinations of the five circular pole geometries and the three magnet materials. These same quantities were calculated by PANDIRA for the same five geometries and two materials (barium ferrite and samarium cobalt).

The stepped pole geometry was studied with PANDIRA, and the step dimensions were chosen so as to eliminate the $N = 6$ and $N = 10$ multipole harmonics. The same quantities were measured for this pole geometry. The pole material was HICOREX 16.

Flat side poles were made to study the effects on the harmonics. Two angles, 60° and 90° between the two flats were fabricated. The flat thickness was 0.0279 cm. The pole materials were HICOREX 18. Table II presents a comparison of these measured and calculated values. The multipole harmonics are expressed as ratios to the quadrupole ($N = 2$) harmonic normalized to the normalized radius ($R_N = 0.6$ cm).

The comparison between the PANDIRA calculations and the measurements shows excellent agreements on gradient and harmonic components.

For circular pole geometry and the latter two materials, the % quadrupoles for $N = 6, 10, 14$ do not have a big dependency on materials. For Alnico-V, all harmonic components are larger than for the latter two materials. This may be caused by the nonlinearity of the B-H curve. The Alnico magnets suffer field reduction or "knockdown" when removed from the yokes during experiments. As a result, symmetry couldn't be kept.

The harmonic components $N = 10, 14$ are negligible for the latter two materials (less than 0.3%). The $N = 6$ component increases from 2.4 to 6.4% as the ratio R_p/R_h increases 1.00 to 1.60. This value exceeds the allowable limits required for the PIGMI Prototype.

Quadrupole gradients, higher than 10 kG/cm have been achieved for the stronger material and the smaller bore radii. This satisfies the requirements of the PIGMI.

TABLE II
MEASURED AND CALCULATED GRADIENTS AND MULTIPOLES
FOR FIVE CIRCULAR POLE, ONE STEP POLE, AND TWO FLAT SIDE POLE
PERMANENT MAGNET QUADRUPOLE LENSES

CIRCULAR POLE

R_p/R_B	1.00	1.15	1.30	1.45	1.60
Bore Radius	.964 cm	.838	.741	.665	.602
	<u>Meas.</u> <u>Calc.</u>	<u>Meas.</u> <u>Calc.</u>	<u>Meas.</u> <u>Calc.</u>	<u>Meas.</u> <u>Calc.</u>	<u>Meas.</u> <u>Calc.</u>
Material - Alnico-V					
Gradient	1.11 kG/cm	1.47	1.69	2.14	2.41
N = 6	.0300	.0373	.0550	.0663	.0987
N = 10	.0011	.0028	.0078	.0184	.0331
N = 14	.0001	.0003	.0015	.0067	.0202
Material - Barium Ferrite	INDOX-5 (Indiana General)				
Gradient	2.49 2.34	3.04 2.89	3.59 3.37	4.28 3.72	4.77 4.27
N = 6	.0213 .0221	.0276 .0309	.0361 .0390	.0477 .0408	.0543 .0572
N = 10	.0003 .0003	.0004 .0008	.0006 .0015	<.0031 .0024	<.0029 .0077
N = 14	.0000 .0000	.0000 .0001	.0001 .0003	<.0026 .0017	<.0019 .0065
Material - Samarium Cobalt	HICOREX 22 (Hitachi Magnetics Corp.)				
Gradient	6.13 5.89	7.42 7.10	8.64 8.28	10.10 9.42	11.10 10.51
N = 6	.0237 .0232	.0310 .0308	.0387 .0388	.0477 .0471	.0641 .0560
N = 10	.0004 .0004	.0006 .0007	.0011 .0012	.0021 .0020	.0029 .0050
N = 14	.0000 .0000	.0000 .0000	.0000 .0001	.0002 .0007	.0006 .0030

STEP POLE $R_B = 0.653$ cm

Material - Samarium Cobalt	HICOREX 16 (Hitachi Magnetics Corp.)
<u>Meas.</u> <u>Calc.</u>	
Gradient	6.42 6.43
N = 6	.0008 .0000
N = 10	.0007 .0000
N = 14	.0055 .0040

FLAT SIDE POLE $R_p/R_B = 1.00$ Grind Thickness = 0.028 cm

Material - Samarium Cobalt	HICOREX 18 (Hitachi Magnetics Corp.)
Angle between two Flats	60° 90°
<u>Meas.</u> <u>Calc.</u> <u>Meas.</u> <u>Calc.</u>	
Gradient	5.61 5.72
N = 6	0.0278 0.0250
N = 10	0.0007 0.0004
N = 14	0.0000 0.0000

Note: Gradient $\equiv B'$ (calculated) and $\int B' dl/L_{\text{iron}}$ (measured).

For step pole geometry, the harmonic components N = 6, 10 are nearly equal to zero as expected by PANDIRA calculations, but N = 14 harmonics has about 0.6%. Actually, this geometry was selected to eliminate the N = 6 and N = 10 multipole harmonics. To diminish N = 14 harmonics, one more step on iron poles may be needed. The structure will be complicated.

The typical example of the field pattern produced by PANDIRA is shown in Fig. 9 for a circular pole geometry and a step pole geometry.

For the flat side pole, the $N = 6$ multipole has a larger value than for the circular pole geometry. For the smaller angle (60°) the harmonic content is greater than for the larger angle (90°). This indicates that sharpening the pole tip of the circle will increase the $N = 6$ multipole.

Next, flat tip poles were shaped by grinding. Two representative plots of the variation of the quadrupole field gradient ($\dot{B}_2$) and $N = 6, 10$ harmonic contents with flat depth are shown in Fig. 10 for $P = R_p/R_b = 1.00$ and in Fig. 11 for $P = 1.45$. The normalized radius is 6 mm and $n = t/0.279$ where t is the height of the segment removed in millimeters (Fig. 12). The lines in Figs. 10 and 11 show the PANDIRA calculation results. Black circles and open circles are the measured values for $N = 6$ harmonics and $N = 10$ harmonics, respectively. The $N = 14$ harmonic content was very small in both the calculations and experiments (at most, 0.3%). Again, the calculations and the measurements show very good agreement to each other. One of the most remarkable features is the rapid decrease of $N = 6$ components with t . For $P = 1.45$, it has the minimum value at some specific thickness. On the contrary, the quadrupole field gradients decrease slowly and monotonically with t . When P is larger than 1.45 (small bore), $(\%Q)_6$ is very sensitive to t . Sometimes $(\%Q)_{10}$ becomes larger than $(\%Q)_6$. In the present application for PIGMI, the required maximum field is 7 kG/cm. If we use the samarium-cobalt materials, this field strength (7 kG/cm) can be obtained even if the ratio P is kept smaller than 1.30. And we can keep all the harmonic components less than 0.5%. This flat tip pole can offer a very easy and inexpensive way of tuning. Beginning with circular poles having a field gradient in excess of that required, field tuning can be done to the required field by grinding a flat on the tip of each of the poles perpendicular to the pole center-to-bore center line. The PANDIRA plot of the flux lines for this example is shown in Fig. 13.

IX. QUALITATIVE EXPLANATION OF FLAT TIP POLE CHARACTERISTICS

Figure 14 shows the general properties of the theoretically ideal quadrupole magnet and our circular pole quadrupole magnet. The ideal quadrupole magnet has a hyperbolic cross section and the surface magnetization is distributed uniformly. In the circular pole quadrupole magnet,

Fig. 9.

PANDIRA plot of circular pole and step pole quadrupole magnet field.

Fig. 10.

Plot of quadrupole field and harmonics (6th & 10th) as a function of flat depth.

Fig. 11.

Plot of quadrupole field and harmonics (6th & 10th) as a function of flat depth.

Fig. 12.

Flat tip pole.

Fig. 13.

FANDIRA plot of flat tip pole quadrupole magnet field.

Fig. 14.

The general properties of the ideal quadrupole magnet and circular pole quadrupole magnet.

the cross section is circular and the surface magnetization has the maximum value at the tip and decreases gradually to zero at the right-angle sides.

It is expected that the grinding of the circular pole tip will make the surface magnetization approximately uniform and its effective shape will approach the hyperbola. The suitable grind thickness makes the $(\%Q)_0$ component minimum value. Then, the more grind will increase $(\%Q)_0$ because the apparent pole edges are distorted from quadrupole lens. But excessive grinding will decrease the $(\%Q)_0$ also because of the distance effects. This may be the reason why the $(\%Q)_0$ has the minimum value at some grind thickness t .

X. CONCLUSION

For a new breed of proton linear accelerator, short, high-gradient, and high-field quality permanent quadrupole magnets were presented.

The small signals of hyperharmonic components were measured by a new compensated coil system, which has zero sensitivity to the quadrupole signal. Four kinds of quadrupole magnet with different geometrical cross sections were investigated. The permanent magnet materials investigated were an oriented Alnico-V, a barium ferrite ceramic, and three kinds of rare-earth samarium cobalts. The magnetic field gradients and multipole field harmonics were measured for all the magnets. These same qualities were calculated and the field plots were produced by the new computer code PANDIRA. The comparison showed excellent agreements. Quadrupole gradients in excess of 10 kG/cm were achieved for samarium-cobalts and certain geometries. An easy way to decrease harmonic contents and to adjust the field gradient was originated. By making a flat on pole tips, all the multipole harmonics become less than 0.5%. Now the concrete designs of permanent quadrupole lenses with flat tips are in progress for the application to the PIGMI Prototype at LASL.

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