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Absorption of Undulator Radiation in Media

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ABSTRACT

Detailed thermal stress analyses of beamline and optical components subject to high heat loads require an accurate determination of the absorbed power profile for accurate prediction of the temperature profile and structural parameters. This is particularly important for high power beams from wigglers and undulators at the third generation synchrotron sources because components must, in general, be designed and maintained with strict mechanical tolerances. The spatial distribution of the power density of an undulator is a rapidly varying function of the energy of the photons suggesting that approximative methods based on a smooth spectral variation may not be valid. In this paper, a fast code for calculating undulator spectra is developed and compared with a wiggler code for approximation of the same spectra. Results from numerical simulations, including the emittance of the stored particle beam, are presented for the absorbed power density in a beryllium window. We find markedly different results for the two models for far off-axis radiation indicating the inadequacy of the wiggler model applied to an undulator spectrum in this case. The wiggler model overestimates the total absorbed power by as much as 82 % for the beryllium window.

1. INTRODUCTION

In the past few years at the Advanced Photon Source (APS), filter and window assemblies such as the carbon/beryllium combination have been studied quite extensively.¹ Today this study also includes novel materials such as diamond² which has favorable thermal and structural properties for use under extreme conditions like those encountered in a high-intensity synchrotron radiation beamline.

The study of the safety and failure modes of such systems has prompted us to perform detailed analytical modeling of the thermal stresses induced by the synchrotron radiation. For a thin structure like the beryllium window assembly, it is necessary to carry out buckling analyses in addition to static analyses. In each case, a detailed knowledge of the absorbed power distribution in the medium is needed, and, consequently, an accurate model describing the emitted synchrotron radiation is necessary.

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Initially, a bending-magnet-type model was used to approximate the emitted power from undulator sources. This model was refined to take into account the softening of the spectrum as the radiation is viewed off-axis of the undulator in the horizontal plane. The refined model, here called the wiggler model or the wiggler approximation, was applied to the calculation of the emitted source power and absorbed power densities in beryllium windows for two proposed wiggler sources at the APS.³ It was found that the absorbed power densities predicted by the two models were significantly different; the wiggler model indicating a relative large absorbed power density for off-axis positions in the horizontal plane.

A third model, the so-called Bessel function approximation, has been utilized in this work for calculation of synchrotron spectra from undulators. This model gives an excellent approximation of the spectrum of an undulator whereas the wiggler model, strictly speaking, only applies to the wiggler regime of the radiation.

It is the purpose of the present work to study the wiggler approximation and the Bessel function approximation as applied to Undulator A at the APS so that we understand their region of validity and their inadequacies, if any. The two models will be discussed first, and then we will give the results and discuss the differences found between them.

2. MODELS

The spectrum of a planar undulator is characterized by sharp peaks at the harmonic energies E_n , given by

$$E_n[keV] = \frac{n \cdot 0.95 E_r^2[GeV]}{(1 + K^2/2) \lambda_u[cm]} \quad (1)$$

for on-axis radiation, where E_r is the energy of the particle beam, K the usual deflection parameter, and λ_u the period length of the undulator. The harmonic energy shifts to lower energies when the radiation is observed off-axis of the undulator. For an ideal undulator and single particle radiation, only the odd harmonics n are observed for on-axis radiation. However, due to the emittance of the stored particle beam and imperfections in the periodic magnetic array, the even harmonics are also seen.

The wiggler spectrum, on the other hand, is a smooth spectrum with no sharp features, and it was anticipated that the absorbed power may not be accurately described because of the large difference between the spectra at energies below ~ 30 keV where the absorption cross-sections are large. When the value of the deflection parameter K increases, the harmonic energies shift to lower energies and the separation of the undulator peaks diminishes so that the wiggler model becomes a better approximation. Actually, one would expect satisfactory agreement with the wiggler model for $K > 5$ and for energies larger than the critical energy as will be discussed below.

The Bessel function approximation^{4,5,6} was used to calculate the undulator spectra, and the wiggler model (Ref. 3) was used to approximate the undulator spectrum by a wiggler spectrum using a suitable characteristic energy. The Bessel function model predicts the spectrum of an undulator with an ideal sinusoidal magnetic field and is more computing intensive than the wiggler model, but provides a more realistic spectrum. The wiggler spectrum is essentially derived from a

bending magnet spectrum by assuming a $\sqrt{1 - \left(\frac{\gamma\theta}{K}\right)^2}$ dependence of the characteristic energy in

the horizontal direction θ , where γ is the relativistic factor. At any given point in space, the spectral distribution of the synchrotron radiation of an undulator is estimated, and the absorption in a medium is then calculated locally from this spectrum. The scattering and absorption cross-sections were taken from the code PHOTON,⁷ which has been used in the past to calculate the absorbed power in media from radiation from bending magnets.

3. RESULTS

The total absorbed power and the absorbed power densities were calculated in a beryllium filter using the two models discussed above for Undulator A at the APS. The APS machine parameters listed in Table I and the undulator specifications listed in Table II were used as input parameters.

Table I. Selected machine parameters for the Advanced Photon Source

Parameter	Value
Ring energy, E_r ^a	7.0 GeV
Current	100 mA
Horizontal beam size, σ_x	0.308 mm
Vertical beam size, σ_y	0.085 mm
Horizontal beam divergence, σ'_x	0.024 mrad
Vertical beam divergence, σ'_y	0.009 mrad

a) Relativistic factor $\gamma \sim 1957 E_r [\text{GeV}]$

Table II. Parameters for the APS Undulator A

Parameter	Value
Period length, λ_u	3.30 cm
Number of periods, N	72
Maximum deflection parameter, K ^a	2.226
Characteristic energy, E_c ^b	23.54 keV
Total emitted power	3.84 kW @ 100 mA
On-axis power density ^c	221 W/mm ² @ 100 mA

a) Value corresponds to a minimum gap of the undulator of approximately 11.5 mm.

b) Calculated from $E_c [\text{keV}] = 0.665 E_r^2 [\text{GeV}] B_0 [\text{T}]$, where $B_0 [\text{T}] = K / (0.934 \lambda_u [\text{cm}])$.

c) Calculated at 24.0 m from the source using the Bessel function approximation. Value shown is reduced about 5 % due to the emittance of the stored particle beam.

Filters of the beryllium window assembly are located at a typical distance of 24 m from the source point (center of the undulator), and the filter parameters used in the calculations are given in Table III.

Table III. Parameters for beryllium

Parameter	Value
Density	1.85 g/cm ³
Thickness	254 μ m (10 mil)
Filter location (distance from the source), D	24.0 m

The emitted power density of Undulator A at the filter location is shown in Fig. 1 (emittance included). The extent of the power envelope shows the expected behavior,⁸ and is approximately DK / γ and D / γ , where D is the distance from the source, in the horizontal and vertical directions, respectively, which is considerably larger than that of the central cone at any given harmonic energy (typically on the order of $D / (\gamma\sqrt{N})$ for low-emittance storage rings).

Fig. 1. Emitted power density of Undulator A at the APS at 24 m from the source. The profile was calculated using the Bessel function approximation.

Similarly, the emitted power density using the wiggler approximation is shown in Fig. 2. This model assumes that there is no radiation beyond $\pm DK / \gamma \sim 3.9$ mm in the horizontal direction. The Bessel function approximation indicates about 50 W/mm² at this position, and we notice that the largest difference between the two models is indeed along the horizontal direction.

Fig. 2. Emitted power density of Undulator A at the APS at 24 m from the source. The profile was calculated using the wiggler approximation.

It is important, however, to consider the *spectral* content of the radiation when calculating the absorbed power in thin media because only the low-energy photons are absorbed. Studying only the frequency-integrated power densities, as shown in Figs. 1 and 2, may be deceiving as will become evident later.

To further examine the spectral properties, we calculated the spatial distribution of the undulator radiation at a few selected energies around the first harmonic energy. As an illustration, Figs. 3 (a) - (c) show the spectral power density at the first harmonic (4,030 eV; red-shifted by 26 eV due to the emittance; cf., Eq. 1), at - 500 eV, and + 50 eV with respect to the energy of the first harmonic. The full width at half maximum of the radiation cone at the energy of the first harmonic at 24 m from the source is only 1.6 mm x 0.75 mm as seen in Fig. 3 (a). We also notice that the second harmonic is discernible at larger angles. At energies below the first harmonic (Fig. 3 (b)), the intensity of the on-axis radiation decreases relatively slowly, and the peak shifts off-axis. However above the harmonic energy, the intensity of the on-axis radiation drops rapidly, and no intensity beyond the central cone is observed except for that of the higher harmonics.

Fig. 3 (a). Emitted spectral power density at the first harmonic of 4,030 eV for Undulator A at 24 m from the source.

Fig. 3 (b). Emitted spectral power density at 500 eV below the energy of the first harmonic for Undulator A at 24 m from the source.

Fig. 3 (c). Emitted spectral power density at 50 eV above the energy of the first harmonic for Undulator A at 24 m from the source.

To understand the difference between the two models, we examined the spectral content of the beam at three positions of the filter: the center of the beam, 2 mm offset in the vertical direction, and 2 mm offset in the horizontal direction. The emitted spectrum at these three positions are compared in Figs. 4 (a) - (c) for the two models. For on-axis radiation and for radiation at an offset of 2 mm in the vertical direction, the wiggler model gives a reasonably good approximation to the undulator spectrum, but for positions offset in the horizontal direction the wiggler model clearly overestimates the flux or the power at energies below 6 keV. This discrepancy becomes even larger for locations further out in the horizontal direction, which is the reason for the large discrepancy between the calculated absorbed power from the two models. The energy-dependent cross-section (primarily the photo-electric cross-section) plays a minor role when calculating the absorbed power because the absorption edges are located at small energies for low-Z materials. However, if an absorption edge coincides with the harmonic energy, the absorbed power may increase substantially.

By integrating the spectra over all energies, we obtain the frequency-integrated power. It is interesting to note that although the wiggler model overestimates the intensity at low energies, the opposite happens at high energies because the frequency-integrated power is smaller for this model at positions far off-axis in the horizontal plane (cf., Figs. 1 and 2).

Fig. 4 (a). Comparison of emitted spectra on-axis for Undulator A at 24 m from the source. The Bessel function approximation shows sharp peaks at the harmonic energies where the wiggler approximation shows a smooth behavior.

Fig. 4 (b). Comparison of emitted spectra at 2 mm offset in the vertical direction for Undulator A at 24 m from the source. Notation same as in Fig. 4 (a).

Fig. 4 (c). Comparison of emitted spectra at 2 mm offset in the horizontal direction for Undulator A at 24 m from the source. Notation same as in Fig. 4 (a).

Fig. 5. Absorbed power densities calculated from the two models for Undulator A at 24 m from the source. The Bessel function approximation (course mesh) gives about a factor of 20 smaller absorbed power density at far-off axis positions in the horizontal direction in comparison with the wiggler approximation (fine mesh).

The results for the absorbed power density from the two models are compared in Fig. 5. The radiation spectrum was calculated typically in the energy range 0.1 keV to 30 keV at each selected mesh point of the filter. We notice a fairly good agreement close to on-axis positions (absorbed power density $\sim 5.4 \text{ W/mm}^2$, in both cases), but considerably different results for radiation far off-axis in the horizontal direction. In particular, the wiggler approximation shows a sharp increase at $\sim \pm DK / \gamma$, indicating an absorbed power density of 11.6 W/mm^2 . The Bessel function model, on the other hand, suggests only $\sim 0.6 \text{ W/mm}^2$ at this position, a factor of 20 smaller! Consequently, the wiggler model overestimates the total absorbed power by 82 % (310 W) in comparison with the undulator model (170 W).

4. DISCUSSION

The Bessel function code has been verified against other undulator codes with excellent agreement of calculated quantities.^{4,9} We have found that there is a significant difference between the undulator model and the wiggler model, and it is worth emphasizing that the deflection parameter K must not be the only parameter considered when approximating an undulator spectrum with a wiggler spectrum. Theory predicts¹⁰ that one can expect good agreement when *both* K is large ($K \gg 1$) and the energy is large ($E \gg E_c$). However, the spectrum should be averaged over an energy interval $\frac{\Delta E}{E} > K^{-3}$ or over an angular width $\Delta\theta > \frac{1}{\gamma\sqrt{K}}$ for excellent agreement. As an example, we compared the angle-integrated flux from the two models (Fig. 6) for Undulator A over an extended energy region. Clearly, the undulator spectrum is reasonably well approximated by the wiggler spectrum, especially in the high energy region, although the value of K is only ~ 2.2 .

Fig. 6. Comparison of angle-integrated flux for Undulator A using the Bessel function approximation and the wiggler approximation. The undulator peaks have been broadened due to the angle integration and the agreement with the smooth spectrum from the wiggler approximation is satisfactory in the high energy region.

One factor that was not taken into account in the Bessel function approximation are the so-called near-field effects. These effects are applicable when the source size, as seen off-axis by an observer, is large compared to the distance D between the observer and the center of the source and the wavelength λ of the radiation. A near-field parameter W can be defined, $W = \frac{(L\Theta)^2}{2\lambda D}$, where L is the length of the undulator and Θ is the angle of observation.¹¹ It can be shown that when $W = 1.0$ there is destructive interference between the light emitted from the center and the ends of the undulator. Here, $W \sim 2.6$ at 2 mm off-axis at the first harmonic energy, and near-field effects should be visible in the spectrum for this and higher harmonics. However, the particle beam emittance tend to smooth out those features, and the near-field effects would not be important. Moreover, if the near-field corrections were taken into account, it can be shown that both the spectral and angular peaks will broaden and diminish in peak intensities, thus making the spectral and angular distributions more featureless. Thus, for the thermal stress analyses, it is more conservative to ignore these corrections.

As a result of the thermal analyses, it was found, that for a window that is actively cooled, the exact absorbed power profile may be secondary to the *total* absorbed power in the medium. The reason is that because the undulator beam gives such a small footprint ($\sim 4 \times 2$ mm at 24 m from the source) and because the thermal conductivity of the window material is generally large, the maximum temperature and the temperature distribution are rather insensitive to the exact power profile. The total absorbed power is more important for a small window, and, in the case of the beryllium window (10 mil thick), the wiggler model grossly overestimated (by 82 %) the total absorbed power (310 W) in comparison with the undulator model (170 W).

The total absorbed power translates almost linearly into the maximum temperature rise in the media with a coefficient of ~ 0.8 depending on the thickness of the media; i.e., if the absorbed power increases by 100 %, the maximum temperature is expected to increase by ~ 80 %. The coefficient diminishes with increasing thickness of the media. Generally, we found that the wiggler model typically overestimated the total absorbed power by ~ 50 % (depending on the material, thickness, geometry, cooling, etc.), and more significant differences for the on-axis absorbed power densities than those seen in Fig. 5 were observed. As an example, for the absorbed power depth-profile of a carbon filter, the wiggler model *underestimated* the on-axis power density for the first few layers, each 100 μ m thick, by about 20 %; the total absorbed power in each layer was overestimated by ~ 50 %, however. The overall shape of the absorbed power profile in the first layer was similar to that shown in Fig. 5.

Thus, our modeling indicates that the wiggler model is not a good model for approximation of an undulator spectrum for the purpose of calculating the absorbed power in media. The Bessel function approximation, on the other hand, seems to give a good estimate of the absorbed power profile and should be adequate for accurate thermal and structural analyses of beamline and optical components. The present undulator code may also be useful for estimating absorbed dose in human tissues, thus providing an important tool for health physicists for radiation protection purposes to determine adequate radiation shielding around undulator beamlines.

DISCLAIMER

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