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Methodology for Physical Modeling of Melter Electrode Power Plug

W. O. Heath

September 1984

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**Pacific Northwest Laboratory
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SUMMARY

This report presents a method for building and testing a one-third scale model of an electrode power plug used to supply up to 3000 amperes to a liquid fed ceramic melter. The method describes how a one-third scale model can be used to verify the ampacity of the power plug, the effectiveness of the power plug cooling system and the effect of the high amperage current on eddy current heating of rebar in the cell wall. Scale-up of the test data, including cooling air flow rate and pressure drop, temperature profiles, melter water jacket heat duty and electrical resistance is covered. The materials required to build the scale model are specified as well as scale surface finish and dimensions. Prints developed for the model and support equipment are presented in the Appendix.

The method for designing and testing a model power plug involves developing a way to recreate the thermal conditions including heat sources, sinks and boundary temperatures on a scale basis. The major heat sources are the molten glass in contact with the electrode, joule heat generation within the power plug, and eddy current heating of the wall rebar. The melting cavity heat source is modelled using a plate heater to provide radiant heat transfer to a geometrically similar, one-third scale electrode housed in a scale model of a melting cavity having a thermally and geometrically similar wall and floor. The joule heat generation within the power plug is simulated by passing electricity through the model power plug with geometrically similar rebar positioned to simulate the eddy heating phenomenon.

The proposed model also features two forced air cooling circuits similar to those on the full design. The interaction of convective, natural and radiant heat transfer in the wall cooling circuit are considered. The cell environment and a melter water jacket, along with the air cooling circuits, constitute the heat sinks and are also simulated.

The complexity of the heat transfer phenomena to be modelled prohibits the analysis from being rigorous but where possible, the limitations of the method are described, as well as whether they result in "worst case" or "best case" simulations, and under what conditions. It is felt that in the process of

testing, the operating range of the power plug can be determined by tweaking the variables and the result will be conservative design recommendations.

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1.0 INTRODUCTION

The purpose of this work is to develop a method of verifying, using a physical model, the ampacity of a single phase electrical bus, the effectiveness of the bus cooling system, and the flux density of the magnetic field caused by the alternating current. Little work has been documented on the use of physical models in the verification of large-scale equipment viability. The melter power plug concept was developed to provide up to 3000 amperes needed by a joule-heated glass melter that will immobilize radioactive waste in a remote cell. This report describes the test method that can be used, analyzes the errors associated with the methodology, and defines how the test data will be interpreted. This work was done in support of the Department of Energy's (DOE) Hanford Waste Vitrification Program at the Pacific Northwest Laboratory (PNL) operated by Battelle Memorial Institute.

1.1 RADIOACTIVE WASTE IMMOBILIZATION

Military and civilian applications of nuclear technology in the U.S. and throughout the world have generated an inventory of high-level radioactive wastes. This material is accumulated in temporary storage facilities as solutions and sludges where it is isolated from the environment. Since the liquids are not an acceptable form for permanent storage of high level wastes, environmentally acceptable methods of reducing the volume of, immobilizing, and permanently storing high-level waste have received the support of the Department of Energy.

Vitrification, the process of containing radioactive particles in a stable glass product, is being developed into a technically feasible and environmentally acceptable method of immobilizing high-level wastes. The glass product is relatively insoluble, resistant to chemical attack, physically durable and well suited for dissipating decay heat and resisting radiation damage.

PNL has demonstrated the vitrification technology using non-radioactive simulated waste, and has conducted limited demonstration of the vitrification of liquid high level radioactive waste by the In-Can Melting process. A current task of PNL is to demonstrate the vitrification of high level radioactive

waste using the Liquid Fed Ceramic Melter technology. The first stage, adaptation of process equipment designs to remote service operation, is nearing completion, and the construction of a pilot-scale remote facility (under another DOE program) is under way. In this manner the technical feasibility of vitrification is being demonstrated. In addition, a recent study (Larson et al. 1983) indicates that beyond its potential to provide excellent waste volume reduction and safe immobilization, vitrification will be economically competitive with current, less desirable forms of radioactive waste volume reduction and immobilization. Figure 1 shows an assembly view of the melter, canister and power plug components of the proposed Hanford Waste Vitrification process. In this figure, two of the four melter electrode power plugs are visible.

1.2 MELTER POWER PLUG

A bus bar, or power plug, will be needed to provide energy for vitrification. The process requires electricity to generate the heat needed to maintain the molten glass temperature at 1150°C, melt raw glass-forming feed components, and evaporate water and other volatiles in the feed slurry. Due to the radioactivity of the process materials, high-level radioactive waste vitrification will be performed in a sealed, no-personnel-access facility. Such a remote facility consists of shielded, cement-encased rooms (cells) wherein process equipment is located. The entire process (which includes waste handling, system operation, equipment maintenance, and product handling) will be performed using radio-controlled cranes and manipulators with closed-circuit monitoring and direct window viewing. All process services, e.g., air, water, electricity, must be passed through the cement wall separating the operation gallery from the remote cell. Service lines are passed through cell walls within wall plugs designed to allow services in, but prevent radiation from escaping. The melter power plug is a through-wall electrical service line that is currently in the initial, conceptual design stage. Figure 2 illustrates the melter power plug concept. The electrode portion is contained on three sides within the melter refractory brick with the fourth side in direct contact with molten glass.

FIGURE 1. Power Plug/Melter Assembly View

1.3 VERIFICATION OF MELTER POWER PLUG CONCEPTUAL DESIGN

The melter power plug will be installed and operated in a remote cell and, if necessary, remotely replaced. The associated design criteria are discussed next.

1.3.1 Operability

Whether or not a through-wall 3000-Ampere power plug can be operated for the lifetime of the melter without loss of effectiveness or without causing damage to the cell or cell equipment depends on the method of electrical contact, whether cell equipment can make incidental contact with the power plug,

FIGURE 2. Melter Power Plug Conceptual Design (side electrode shown)

and whether any wall-contacting components of the power plug operate at a temperature high enough to damage the wall structure.

First, with essentially a one-piece electrode/power plug, contact maintenance problems are eliminated. The electrical contact between the power plug and the melter will be a full-penetration weld, precluding contact failure by oxidation or corrosion. Since the power plug and electrode will each be 6 in. dia at the point of the weld, it is unlikely that the contact could be broken through incidental physical damage.

Second, the ~2 ft portion of the power plug extending from the cell wall interior to the melter exterior will be electrically isolated and protected with a vented metal shroud. The shroud will consist of an inner metal cylinder, a middle layer of dielectric material and an outer metal cylinder. This shroud will be strong enough to resist deformation by falling objects or a swinging crane hook. The damage potential to both cell equipment and the power plug is thus minimal.

Third, hot spots ($T > 50^{\circ}\text{C}$) on the surface of power plug components in direct contact with the wall cement will tend to damage the contacting cement, leading to possible structural weakening within the wall. Three distinct heat sources must be considered in predicting possible hot spots: conductive heat transfer from the melter interior through the electrode and along the power plug within the wall; joule heating of the power plug due to the up to 3000 A current passed through it; and induced eddy current heating in the wall rebar.

Eddy currents will be induced in the rebar by the magnetic field emanating from the single-phase power plug. The magnitude of the magnetic field at a given distance from any conductor depends on the frequency and amount of current passed through the conductor and the presence and proximity of other conductors. Usually, when alternating current is passed, two conductors of opposite phase are placed close enough together that their magnetic fields cancel to a net of zero. However, this will not be the case with the melter power plugs, which will be of different phase but separated enough to allow only partial cancellation of their magnetic fields. Thus eddy current heating in the wall rebar is a concern.

Whether hot spots along the power plug wall sleeve will occur due to the single or combined effects of conduction from the melter, joule heating within the power plug or eddy current rebar heating can be determined using an operational 1/3-scale-model power plug. Such a model would eliminate the need for completely understanding the physics and chemistry of the construction materials as would be the case if computer modeling were used.

1.3.2 Remotability

The ability to remotely replace a melter containing power plugs can be evaluated at PNL using a remote operations testing facility. The facility is equipped with mock cell walls, a radio controlled crane with remote cell type swivel hook, remote two-camera closed-circuit monitoring, and mock cell equipment having the same external dimensions and arranged as planned in-cell. To perform the power plug remotability evaluation, an existing mock melter shell can be fitted with protruding mock power plugs of the correct geometry. A mock wall can be fitted with geometrically correct wall sleeves and the remote installation of the mock melter power plugs into mock wall sleeve can be performed. This standard method of evaluating remotability can be used to determine the feasibility of remote melter installation.

1.4 PHYSICAL MODELING METHODOLOGY

The general methods of physical modeling are discussed as well as their application toward designing a 1/3-scale physical model of the power plug. Physical modeling involves the development of criteria for similarity between scale model and full scale equipment. These criteria are used to assess how representative the physical model is, how to fabricate a scale model, what scale should be used, what errors are involved, and how test data can be used to predict the behavior of a full scale power plug.

Prior physical modeling done by PNL includes the modeling of the flow behavior and electrical distribution within glass melts using scale models of HLW vitrification process melters (Quigley and Kreid 1979).

2.0 GENERAL PHYSICAL MODELING AND SCALE-UP METHODS

Physical modeling is the process of recreating hydrodynamic, thermal, and other characteristic operating conditions on a scale model basis and predicting the full-scale behavior from the model data. Intuitively, the physical model must be similar in some respect to the equipment being modeled. To correlate the model data with full scale quantities, a similarity analysis is performed. The similarity analysis establishes scale ratios between model and full-scale quantities.

2.1 TYPES OF SIMILARITIES

Seven types of similarity are relevant to physical modeling (Johnstone and Thring 1957): 1) geometric, 2) kinematic, 3) dynamic, 4) thermal, 5) physical/chemical, 6) mechanical, and 7) static.

Objects with similar shape are geometrically similar. Mechanical similarity is important when moving parts such as spring/mass systems are modeled. Static similarity refers to the similar deformation, e.g., creep behavior of materials under static stresses. Kinematic similarity generally means similarity in forced fluid flow. Dynamic similarity refers to similar gravity-induced flow caused by density gradients (as occurs in natural convection). Thermal similarity involves temperature profile, boundary conditions and time response similarity. Physical/chemical similarity involves chemical, surface and physical properties such as pH, surface tension, and thermal conductivity respectively.

- Geometric similarity

In a 1/3-scale-model power plug, geometric similarity can be achieved to the extent possible and necessary. The cylindrical coordinates z (axis), and r (radius) are scaled down by a factor of 1/3. This includes reduction of material thicknesses by a factor of 1/3 (except where considered unnecessary) in addition to general scale down.

- Kinematic similarity

Kinematic similarity can be achieved by appropriately scaling cooling air flow rates within the model power plug.

- Dynamic similarity

Dynamic similarity is not considered a requirement in modeling a power plug, and would be difficult, although not impossible, to achieve (as discussed in Section 5.0).

- Thermal similarity

Thermal similarity would be achieved to the extent possible and is the fundamental goal of the modeling effort.

- Physical/chemical similarity

Physical/chemical similarity can be achieved by using the same materials in the model as used in full scale.

- Mechanical and static similarity

Mechanical and static similarity will not be achieved since it is not a requirement of the modeling effort. (There are no moving parts and static stresses are not a determining factor in the initial power plug conceptual design.)

2.2 METHODS FOR DEDUCING SIMILARITY CRITERIA

There are several methods for deducing model similarity criteria that involve either dimensional analysis or analysis of existing mathematical models (differential equations). The goal of both techniques is to derive independent dimensionless groups that describe the dependence of a phenomena on each variable known to affect it. The dimensionless groups arrived at by either technique establish the criteria for similarity and are thus called "similarity parameters."

2.2.1 Dimensional Analysis: Buckingham-Pi Theorem

Dimensional analysis involves considering all the parameters that are known to affect the phenomenon being modeled. As an example, fluid flow through a pipe is known to depend on; fluid viscosity, μ ; fluid velocity, $\vec{V}$; fluid density, ρ ; and pipe hydraulic diameter, D . If these parameters are arranged as:

$$\left[\frac{\rho \vec{V} D}{\mu} \right] = \Pi$$

the result is a dimensionless group. This particular group is known as the Reynolds number. The criterion for similarity in fluid flow through pipes is that the dimensionless Reynolds numbers be equal. The Reynolds number is therefore a pipe-flow similarity parameter.

At the turn of the century, Buckingham proposed a theorem to predict the number of independent dimensionless groups, Π_i , that can be written for a set of variables known to affect a phenomenon. This theorem, known as the Buckingham-Pi theorem predicts that the number of independent dimensionless groups required is equal to the number of variables minus the number of primary quantities used to define their dimensions.

Buckingham-Pi Theorem

Dimensionless groups, $\pi_1, \pi_2, \dots, \pi_i$

such that $i = n-m$

where $n \equiv$ variables, e.g., $\vec{V}, \rho, \mu, \sigma$

$m \equiv$ primary quantities, e.g., mass, length, time.

A problem with the Buckingham-Pi theorem is that trial and effort is required to construct the $n-m$ independent dimensionless groups.

2.2.2 Rayleigh's Method of Indices

Rayleigh's method of indices (Johnstone and Thring 1957) is an improvement of the Buckingham-Pi theorem in that the independent dimensionless groups are

easily constructed using matrix algebra (where the matrix is $m \times n$). Therefore, once the list of important variables is arrived at, it is strongly recommended that the dimensional analysis be done using Rayleigh's method.

As with all methods of dimensional analysis, a thorough understanding of the phenomena being modeled is prerequisite. This prior knowledge is required to both arrive at a complete list of variables and to decide which, if any, variables are of negligible importance, as is most often the case.

2.2.3 Differential Equations

Similarity criteria can be deduced from differential equations known to describe the phenomena being modeled. If the differential equation is complete, no other a priori knowledge is required, making this method at once preferable to all dimensional analysis methods. In addition, the mechanics of deducing similarity criteria from differential equations is relatively simple.

First, the appropriate differential equation is selected. If a phenomenon has previously been studied, chances are good that a differential equation describing it exists and therefore the first step is usually possible. (If not, either dimensional analysis must be performed or a differential equation developed.)

The differential equation is then written in dimensional form, removing the differential terms. Since all terms in the equation have the same units, dimensionless groups can be formed merely by dividing one term by another and so on. This process is illustrated next for the case of heat conduction in isotropic solids.

2.2.4 Deducing Similarity Parameters from Differential Equations

The differential equation describing the steady state conduction of heat in one dimension through solids is Fourier's law (Kreith 1973):

$$H = -kA \frac{dT}{dx} \quad (2.5)$$

where A = cross sectional area of conductor

H = heat input

$\frac{dT}{dx}$ = temperature gradient along x

$\frac{H}{A}$ = heat flux.

For the following discussion, refer to Figure 3. In Figure 3a is represented an ideal (one-dimensional) thermal conductor consisting of a metal block with well insulated sides (adiabatic), an isothermal face maintained at T_C and a hot face exposed to a heat source of magnitude W (watts or BTU/h). With the dimensions shown, the area of the conductor is $z \cdot y$ and the temperature gradient is $-\frac{dT}{dx} = \frac{T_C - T_H}{x}$. For this situation, Equation (2.5) is written as:

$$W = k (z \cdot y) \left(\frac{T_H - T_C}{x} \right) \quad (2.6a)$$

Figure 3b represents the case where the same block is subjected to triple the original heat load, or, $H = 3W$. Consequently, the hot surface temperature is increased from T_H to $T_{H'}$ and Equation (2.5) becomes:

$$3W = k (z \cdot y) \left(\frac{T_{H'} - T_C}{x} \right) \quad (2.6b)$$

Comparison of (2.6a) and (2.6b) leads to the conclusion that

$$\left(\frac{T_{H'} - T_C}{x} \right) = 3 \left(\frac{T_H - T_C}{x} \right) \quad (2.6c)$$

or, that by tripling the heat input, the steady state temperature gradient is tripled. Figure 3c represents a 1/3 scale model of the insulated block, using the same materials so that thermal conductivity, k , is unchanged. If we want to model the case where the hot face is maintained at $T_{H'}$, Equation 2.5 can be used with equation (2.6b) to solve for H in terms of W :

(a)

$$H = W$$

$$\frac{dT}{dX} = \frac{T_H - T_C}{X}$$

$$A = Zy$$

(b)

$$H = 3W$$

$$A = Zy$$

$$\frac{dT}{dX} = \frac{T_{H'} - T_C}{X} = \frac{3(T_H - T_C)}{X}$$

(c)

$$\frac{dT}{dX} = \frac{T_{H'} - T_C}{X/3}$$

$$A = \frac{Z}{3} \cdot \frac{y}{3}$$

$$T_H = W$$

(d)

$$\frac{dT}{dX} = \frac{T_{H'} - T_C}{X}$$

$$A = \frac{Z}{3} \cdot \frac{y}{3}$$

$$H = \frac{W}{3}$$

FIGURE 3. Similarity Relations

$$H = k \left(\frac{z}{3} \cdot \frac{y}{3} \right) \left(\frac{T_{H'} - T_C}{x/3} \right) \quad (2.6d)$$

$$= \frac{1}{3} k (z \cdot y) \left(\frac{T_{H'} - T_C}{x} \right)$$

$$= \frac{1}{3} (3W)$$

and,

$$H = W.$$

The conclusion here is that in a 1/3 scale model, to maintain the same hot face temperature requires 1/3 the heat input. Note that although the difference in temperature between hot and cold faces is the same, the temperature gradient in the model is three times greater. Also, the heat flux in the model is 3 times greater since the cross sectional area is 9 times smaller. Finally, Figure 3d is of an insulated block with only cross sectional area scaled down, and with temperature gradient, $-\frac{dT}{dx} = \frac{T_{H'} - T_C}{x}$, the same as in case 2, Figure 3b. Equation (2.5) for this case is

$$H = k \left(\frac{z}{3} \cdot \frac{y}{3} \right) \left(\frac{T_{H'} - T_C}{x} \right) \quad (2.6e)$$

$$= \frac{1}{9} (3W)$$

$$= \frac{W}{3}$$

In this last case, the length dimensions were not scaled down uniformly so that this model is not geometrically similar.

Assuming uniform geometric scale-down will be used, Equation (2.5) can be written in dimensional form as:

$$H = \frac{-kL^2T}{L} \quad (2.7a)$$

or,

$$H = -kLT \quad (2.7b)$$

Note that the T in Equation 2.7b now refers a temperature difference, or ΔT , between two points in a conductor separated by a distance L . Thermal similarity means that between two points in a scale model, the temperature difference will be the same as between two corresponding points in the full scale conductor. However, since the distance between two corresponding points in the scale model is smaller (by a factor of 3 for a 1/3 scale model), the temperature gradient will be greater (by a factor of three for a 1/3 scale model). Therefore, when size is scaled down, temperature gradients are scaled up, if thermal similarity is to be achieved. In addition, identical boundary temperatures are required for temperature equivalence between corresponding points.

When the left hand side of Equation (2.7) is divided by the right hand side, the following dimensionless group results:

$$\left[\frac{H}{-kLT} \right] = \pi \quad (2.8)$$

The criteria for similar steady state heat conduction can be written as:

$$\phi \left[\frac{H}{kLT} \right] = \text{constant} \quad (2.9)$$

where the coefficient, ϕ , absorbs the minus sign.

2.2.5 Use of Similarity Parameter

A convention for distinguishing full-scale quantities from model-scale quantities must now be introduced. First, model quantities will be indicated with primes, full-scale quantities without primes and the ratio of a model-to-scale quantities in boldface type. For example:

L' = model scale length quantity

L = full scale length quantity

L = model scale length/full scale length.

For a 1/3 scale geometrically similar model, $L = 1/3$. With this convention, equation (2.9) for one-dimensional steady-state heat conduction in isotropic solids can be used to evaluate the criteria for the similar behavior of this heat transfer phenomena in a 1/3-scale model.

As derived in the preceding section,

$$\phi \left[\frac{H}{kLT} \right] = \text{constant.} \quad (2.9a)$$

If this is true then

$$\phi' \left[\frac{H'}{k'L'T'} \right] = \text{constant} \quad (2.9b)$$

is also true. Note that ϕ may be unequal to ϕ' . Combining (2.9a) and (2.9b):

$$\phi' \left[\frac{H'}{k'L'T'} \right] = \phi \left[\frac{H}{kLT} \right] \quad (2.9c)$$

and

$$\frac{\left[\frac{H'}{k'L'T'} \right]}{\left[\frac{H}{kLT} \right]} = \frac{\phi}{\phi'} \quad (2.9d)$$

and

$$\left[\frac{\frac{H}{H}}{\frac{k}{k} \cdot \frac{L}{L} \cdot \frac{T}{T}} \right] = \left[\frac{H}{kLT} \right] = \frac{\phi}{\phi} \quad (2.9e)$$

The criterion for physical behavior similarity is that the term

$$\frac{\phi}{\phi} = 1 \quad (\text{required for similarity}) . \quad (2.10)$$

In other words, the numerator and denominator in expression (2.9d) must be equivalent for similarity in conductive heat transfer. This is completely analogous to the fact that flow in two different pipes will be similar if their dimensionless Reynolds numbers are identical. While the independent variables, such as pipe diameter, may be different, the dimensionless group of these variables must have the same value in both situations for the two flow behaviors to be similar.

To use expressions (2.9e) and (2.10) to evaluate heat conduction similarity in a 1/3 scale model power plug (assuming one-dimensional conduction for the time being), first note that $k = 1$ can be achieved if the model is constructed of similar materials. We will assume this is the case. Also, as mentioned earlier, $L = 1/3$ uniformly. For complete thermal similarity (which we are trying to achieve) temperatures measured at various points in the model must be identical to those that would be measured at corresponding points (separated by a factor of three greater distance) in the full-scale power plug. Thus T/L , the temperature gradient, is equal to 3 since $L = 1/3$. We thus arrive at the expression:

$$\frac{H}{kLT} = \frac{H}{kL \cdot T/L=1} = \frac{H}{\frac{1}{9} \cdot 3} = \frac{\phi}{\phi} \quad (2.11a)$$

or

$$3H = 1 \quad (2.11b)$$

and thus,

$$H = \frac{1}{3}. \quad (2.11c)$$

Note that the requirement of 1/3 heat input in the 1/3 scale, thermally similar model is consistent with the conclusion made in the discussion in Section 2.2.4.

Therefore, having started with geometric and physical/chemical similarity and wanting to achieve thermal similarity ($T = 1$), we must operate the model so that the heat inputs (and losses) are 1/3 the magnitude of those in the full scale equipment. (Equivalently, the temperature gradient, $\frac{T}{L} = 3$ and the heat flux $\frac{H}{L} = 3$.) This is done by correctly controlling, to the extent possible, the boundary temperatures so that $T = 1$ and $H = 1/3$ are achieved.

The similarity criteria for the heat sources and sinks, involving a complex mixture of heat transfer phenomena in more than one dimension, are discussed in the following chapters.

3.0 SIMILARITY CRITERIA FOR MELTER POWER PLUG PHYSICAL MODELING

The goal of the physical modelling effort is to achieve thermal similarity in a 1/3 scale model to the degree possible given the complexity of the task. As mentioned, material and geometric similarity are, in general, prerequisites. Thermal similarity is then achieved by controlling the heat inputs and losses such that the temperature gradients within the 1/3 scale model are maintained at three times those in the full scale. (This is usually done by assuming that the boundary temperatures in the model are the same as in full scale.)

The two main heat sources to simulate are the molten glass in the melting cavity in contact with the electrode portion of the power plug, and joule heat generation within the power plug.

In addition to temperature gradient control by controlling heat input and heat loss, the magnitude of the temperatures within the model must be controlled by controlling the boundary temperature at the melting cavity end of the power plug. With heat flux and boundary temperatures controlled to the extent possible, temperatures at every point within the model will approach being identical with temperatures at corresponding points within a full scale power plug.

Achieving thermal similarity therefore requires characterizing and controlling heat flows and boundary conditions by considering the heat transfer phenomena associated with the power plug. The phenomena that must be considered and, if possible, simulated are:

- conductive heat transfer
- melter water jacket heat sink
- melting cavity heat source
- forced convective heat transfer
- radiant heat transfer
- joule heat generation within power plug
- induced eddy current heating in wall rebar
- natural convective heat transfer

Discussed next for each of these phenomena is where it occurs in the power plug system, what its relative importance is, and how its similarity parameters are

derived and used. (Induced eddy current heating is discussed in Chapter 4.0 and natural convection in Chapter 5.0.) For reference, the proposed design of a 1/3-scale model apparatus is detailed in Figures 10 through 16 in the Appendix. These figures are called out when appropriate in the ensuing discussions.

3.1 CONDUCTIVE HEAT TRANSFER IN THREE DIMENSIONS

Steady-state conductive heat transfer occurs at virtually all points within the power plug, electrode, melter, and the cell wall. Since all materials within the power plug system are isotropic (at least with respect to thermal conductivity, k), equation (2.9e) is applicable as rewritten:

$$\left[\frac{H}{kLT} \right] = 1, \quad (3.1)$$

with the assumption that this criteria can be used additively in each dimension to controll each source or sink. As discussed earlier, $k = 1$, $L = 1/3$ and $T = 1$ for material, geometric and thermal similarity, respectively. This then forces the steady state heat input/loss criterion, $H = 1/3$.

Within the scale-model power plug, there will be three important areas of material and geometric dissimilarity that may result in different H criteria. These three are in the simulated melter water jacket, the melting cavity and the cell wall. The effect of dissimilarity in each case is discussed in Sections 3.2, 3.3 and 4.0.

3.2 MELTER WATER JACKET HEAT SINK

The sides of the melter are cooled using water jackets located on the melter exterior. Since the power plug electrode assembly passes through the melter wall refractory into the melting cavity, it also passes through the melter exterior cooling jackets. The jacket will thus tend to cool that portion of the power plug passing through it.

A seal is located at the point where the power plug penetrates the melter exterior. The function of the seal is to prevent air inleakage at the point of penetration and to electrically isolate the power plug from the melter exterior. The seal is a clamping device containing a deformable gasket surrounded

by a layer of dielectric material. For cooling to occur, heat must pass from the power plug surface through the gasket and dielectric, along the melter exterior wall and into the cooling water within the jacket. The conduction path is therefore very complex, and accurate modeling would require a very precise scale version of the sealing mechanism and gasket.

On the other hand, what is important to know from a design standpoint is how much if any cooling is required from the jacket. This can be easily done in the scale model by building a simplified water jacket located at the correct point along the power plug axis, varying the water flow rate and noting the effect on temperature profiles along the power plug. To provide the largest variability of cooling rate, the simulated jacket should be in direct contact with the model power plug. From Equation (3.1), we know that the heat flux ratio will be $H = 1/3$, assuming one dimensional, radial heat transfer (temperatures invariant in the axial and angular dimension). Therefore, for each temperature profile observed along the model power plug, the amount of cooling required of the full-scale jacket will be three times that measured using the model water jacket. The recommended model water jacket design is detailed in Figure 12 of the Appendix.

3.3 MELTING CAVITY HEAT SOURCE

In a full-scale melter, molten glass is in direct contact with the electrodes and refractory walls within the melting cavity. Heat is transferred through natural convection from the molten glass to the refractory walls and to the electrode surface. Some of the heat transferred from the molten glass to the electrode is carried away by cooling air ducts within the electrode. The remainder of the heat is conducted out of the melter along the power plug. This heat conduction source is the major component of the melter power plug overall temperature profile.

From a modeling standpoint, the most straightforward method of simulating the melting cavity heat source would be to build a 1/3 scale operational model of a melter having complete geometrical and material similarity. This would, however, be an unnecessarily difficult task. From Equation (3.1), the criteria for thermal similarity is that the model heat input ratio be $H = 1/3$. This

means that any source of heat can be substituted for molten glass as long as the heat input achieved in the model is one-third that of full scale. Were a full-scale heat input known, e.g., 9 kW per power plug, a heat input (3 kW for this example) would be maintained in the most convenient manner. In the model a one-third-scale electrode with geometric and material equivalence can be placed in a small open cavity heated by a plate heater. The model cavity would contain air rather than molten glass and heat transfer from the plate heater to the model electrode surface would be mainly through radiation. At a certain plate heater temperature and distance, the heat input to the model electrode surface would be exactly one-third of that to a full-scale electrode from molten glass with the model electrode surface temperature (boundary condition) at 1150°C as in full scale.

The plate heater temperature and distance required to maintain $H = 1/3$ in the model cavity could be calculated if H were known. The full-scale heat input H , can be estimated using computer-aided numerical techniques, as can the plate heater temperature in the model melting cavity required for thermal equivalence. In this manner the melting cavity heat source can be estimated and simulated. Identically the $H = 1/3$ criteria will be achieved when the electrode surface boundary condition is met, i.e. $T = 1$.

As mentioned, the heat transferred to the electrode will be dissipated via three routes: forced convection electrode air cooling, conduction into the refractory, and conduction into the portions of the power plug external to the melter.

Geometrically similar air-cooling ducts in the model electrode can be used to simulate the effect of electrode cooling. The cooling will be through forced convection which can be well modeled, as will be discussed in Section 3.4. To simulate heat loss to the refractory, the model melting cavity refractory can be made thermally and geometrically equivalent in the locality of the electrode. For the side electrode being modeled, this means that one wall and portions of the refractory floor can be made thermally and geometrically similar. To do this, a castable refractory material would be poured to the correct shape and thickness and encased as required with insulative board. The criterion for refractory similarity is that the resistance of the

composite wall to conductive heat transfer, R_H , will be $R_H = 1/3$. This is written using standard convention as:

$$3 \cdot R' = 3 \cdot \left[\underbrace{\frac{k_1 A_R'}{L_1}}_I + \underbrace{\frac{k_2 A_R'}{L_2}}_{II} \right] = \left[\frac{k_1 A_R}{L_1} + \frac{k_2 A_R}{L_2} + \frac{k_3 A_R}{L_3} + \dots \right] = R, \quad (3.2)$$

where A_R refers to a unit surface area of the refractory wall with dimensions of length squared. Term I in (3.2) has only two terms since only two refractory components (castable and board) are used, while term II reflects the complexity of actual melter refractories which have typically more than three components. To use Equation (3.2), the k_i and L_i variables in term II will be estimated using melter refractory design prints. The thermal conductivity, k_i , is a function of the refractory material while L_i is a function of the material geometry as determined by the design prints. The A_R terms can be removed by noting that for a given melting cavity surface,

$$A_R = \frac{A_R'}{A_R} = \frac{(L')^2}{(L)^2} = \frac{(\frac{1}{3})^2}{1} = \frac{1}{9} \quad (3.3)$$

and terms I and II in (3.2) can be rewritten as:

$$\frac{1}{3} \left[\frac{k_1}{L_1} + \frac{k_2}{L_2} \right] = \left[\frac{k_1}{L_1} + \frac{k_2}{L_2} + \frac{k_3}{L_3} + \dots \right] \quad (3.4)$$

With all other terms known, L_1' and L_2' (thickness of castable and board refractory layers in model) can be calculated. Thus similarity in the melter refractory can be easily achieved. The recommended model melting cavity design is detailed in Figure 13 in the Appendix.

The third component of heat loss from the electrode is through conduction to the power plug extending out of the melter. This heat is dissipated primarily through forced convection air cooling at the exterior surface of the power plug and through radiant heat transfer to the inner surface of the power plug shroud, as discussed in the following two sections.

3.4 FORCED CONVECTIVE HEAT TRANSFER

There are two forced-air cooling systems in the melter power plug/electrode assembly. The first is an electrode air-cooling circuit and the second is an annular power plug cooling circuit. The annulus in this second cooling circuit is formed between the exterior surface of the power plug and the interior surfaces of the wall sleeve and shroud, all of which are cylindrical. This cooling results in a radial heat loss.

For the geometrically similar model, the criteria for similarity in forced convective heat transfer is the equivalence of: the Nusselt modulus, N_{NU} ; the Reynolds number, N_{RE} ; and the Prandtl number, N_{PR} . All of those are dimensionless groups derived from a differential equation describing forced convective heat transfer (Bird, Stewart, and Lightfoot 1960). The three similarity criteria are

$$\frac{N_{NU}}{N_{NU}} = N_{NU} = \frac{H}{KL\dot{T}} = 1 \quad (3.5)$$

$$\frac{N_{RE}}{N_{RE}} = N_{RE} = \frac{\rho L \dot{V}}{\mu} = 1 \quad (3.6)$$

$$\frac{N_{PR}}{N_{PR}} = N_{PR} = \frac{C_p \mu}{k} = 1 \quad (3.7)$$

where the boldface symbols are ratios of model to full scale quantities.

The Nusselt modulus criterion is equivalent to the criteria for thermal conduction through solids; this can be met, as previously discussed, if $H=1/3$. The Reynolds number contains two fluid properties, ρ and μ , a length quantity, L , and fluid velocity. Air will be used as the cooling medium in

both the model and full scale so that ρ and μ similarity is met. Equation (3.6) then becomes:

$$\frac{\rho L \vec{v}}{\mu} = \frac{1 \cdot \frac{1}{3} \cdot \vec{v}}{1} = 1 \quad (3.8)$$

and $\vec{v}=3$ is necessary for similarity. This means that for similarity, air coolant velocities in the model must be maintained three times higher than full scale in both the electrode and annular power plug air cooling circuits. The Prandtl number contains only fluid properties and is therefore automatically simulated when air is used. It might be pointed out that fluid properties are temperature dependent; the temperature dependency is not particularly strong and, assuming thermal similarity, the fluid temperatures in the model will be similar, further minimizing the effect of temperature dependencies.

3.5 RADIANT HEAT TRANSFER

Some of the heat conducted from the melting cavity to the power plug is transferred by radiation to the interior surfaces of the shroud and wall sleeve.

For the geometrically similar model, the criteria for similarity can be alternatively described by the following dimensionless parameters (Johnstone and Thring 1957):

$$N_R = \frac{H}{\sigma \epsilon L^2 (T_1^4 - T_2^4)} = \frac{\text{conductive heat transfer}}{\text{radiant heat transfer}} \quad (3.9)$$

or,

$$N_{TH} = \frac{\rho C_p \vec{v}}{\sigma \epsilon T^3} = \frac{\text{convective heat transfer}}{\text{radiant heat transfer}} \quad (3.10)$$

In (3.9) and (3.10), σ is the Stefan-Boltzmann constant and e is the surface emissivity. Since σ is a constant $\sigma = 1$, and, as discussed in previous sections, $H = 1/3$, $L = 1/3$, $\rho = 1$, $\nabla = 3$, and $C_p = 1$. For thermal similarity, $(T_1^4 - T_2^4) = 1$ and $T^3 = 1$. This reduces equations (3.9) and (3.10) to:

$$\left[\frac{\frac{1}{3}}{1 \cdot e \cdot \frac{1}{9} \cdot 1} \right] = 1, \text{ or, } e=3 \quad (3.11)$$

and

$$\left[\frac{1 \cdot 1 \cdot 3}{1 \cdot e \cdot 1} \right] = 1, \text{ or, } e=3. \quad (3.12)$$

Thus, both criteria can be satisfied if the ratio of model to full-scale emissivities, $\frac{e}{e}$, is maintained at $e = 3$.

Emissivity is dependent mainly on surface roughness, color, and degree of oxidation of the power plug, shroud and wall sleeve metal surfaces. During operation, the degree of oxidation will increase with time to a steady-state value. Both the steady-state value and the time required to reach it are difficult to estimate. The emissivity values used for this analysis were taken from Table 10-17 in Chemical Engineer's Handbook (Perry and Chilton, et al 1973). For metal surfaces initially they are 0.10 to 0.30; after oxidation at 600°C they are 0.80; and for flat black lacquer surfaces they are 0.97.

If the radiant heat transfer surfaces in the model are painted with flat black lacquer, an initial value range of $e = 9.7$ to $e = 3.2$ is achieved. As time goes on, the full-scale surfaces become oxidized and a final value of $e = 1.2$ is maintained. While a constant value of $e = 3$ is not maintained, a range of $1.2 < e < 9.7$ will be achieved. This can be considered a good enough approximation since radiant heat transfer is anticipated as being negligible over most of the power plug. Radiant heat transfer would become important only

if the power plug surface temperatures exceeded ~250°C over the length of the plug. This could probably occur only if the power plug cooling system were shut off.

3.6 JOULE HEAT GENERATION WITHIN POWER PLUG

When current is passed through any conductor, heat is generated in the conductor as described by Joule's power loss equation:

$$\text{power loss} = H_J = i^2 R . \quad (3.13)$$

The similarity criteria is the same for all sources and sinks and is the Nusselt modulus, N_{NU} :

$$\left[\frac{H_J}{kLT} \right] = 1 . \quad (3.14)$$

Equation (3.13) is then substituted for H_J , leaving:

$$\left[\frac{i^2 R}{kLT} \right] = 1 . \quad (3.15)$$

Electrical resistance, R , is an extensive property related to electrical resistivity, σ_e , by

$$R = \frac{\sigma_e \cdot L}{L^2} , \quad (3.16)$$

which is substituted into (3.15) to give the joule-heating similarity criteria for the 1/3 scale model having geometric and material similarity:

$$\left[\frac{i^2 \frac{\sigma_e \cdot L}{L^2}}{kLT} \right] = \left[\frac{i^2 \sigma_e}{kL^2 T} \right] = 1 . \quad (3.17)$$

For material similarity, $k = \sigma_e = 1$, for thermal similarity, $\Delta T = 1$, and for geometric similarity, $L = 1/3$. This leaves

$$\frac{i^2 \cdot 1}{1 \cdot (\frac{1}{3})^2 \cdot 1} = 1, \text{ or, } i = \frac{1}{3} . \quad (3.18)$$

The criteria for joule-heating similarity is therefore reduced to $i = 1/3$.

4.0. INDUCED EDDY CURRENT HEATING

The alternating current in the melter power plug will create an alternating magnetic field, B , with a magnetic flux Φ_B . This magnetic field will permeate the space between the power plug and the wall sleeve and further into the wall rebar. The alternating magnetic field will induce eddy currents in the wall sleeve and in the rebar, causing them to heat up. Heating of either the sleeve or rebar past 50°C will damage the contacting cement, resulting in loss of adherence and thus weakening the cell wall structure next to the wall plug. It is, therefore, important to know the magnitude of the eddy current heating in the wall sleeve and rebar.

The following derivation is based on the Biot-Savart law and Faraday's law of induction (Halliday and Resnick 1981), with nomenclature as indicated in Figure 4.

4.1 CALCULATION OF ALTERNATING MAGNETIC FIELD, $B(t)$

From the Biot-Savart law:

$$B(t) = \frac{\mu_0 i(t)}{2\pi r_p} \quad [=] \text{ T} \quad (4.1)$$

FIGURE 4. Eddy Current Heating Nomenclature

where r_p is the perpendicular distance from the power plug and μ_0 is the permeability constant with units T·M/A. Equation (4.1) assumes the power plug is a conductor of infinite length. (Preliminary calculations indicate the plug's finite length has a negligible effect on the calculation of $B(t)$.)

4.2 CALCULATION OF MAGNETIC FLUX, $\Phi_B(t)$

From the definition of magnetic flux:

$$\Phi_B(t) = \int B(t) \cdot ds \text{ [=] } T \cdot M^2, \quad (4.2)$$

where ds is a differential surface area element of a surface oriented perpendicular to the magnetic field. The geometry of the power plug, showing the direction of the magnetic field, B , is presented in Figure 5.

As Figure 5 shows, the cell wall rebar is located in a plane perpendicular to the current and parallel with the magnetic field. Only the cross-sectional

FIGURE 5. Orientation of Rebar and Magnetic Field

area of the rebar is oriented perpendicular to the magnetic field and is therefore the only surface subject to the magnetic flux, Φ_B . The differential surface element, ds , must therefore be defined based on the cross section of the rebar. The definition of ds is illustrated in Figure 6.

The finite surface element, Δs can be idealized as the annular disk contained by the radii r and $r + \Delta r$. This area is:

$$\Delta s = \pi (r + \Delta r)^2 - \pi(r)^2 \quad (4.3a)$$

$$= \pi[(r + \Delta r)^2 - r^2] \quad (4.3b)$$

$$= \pi [r^2 + 2r\Delta r + \Delta r^2 - r^2] . \quad (4.3c)$$

Since Δr is itself a small finite quantity, Δr^2 is a second-order-small quantity and can be neglected. This leaves:

$$\Delta s = \pi 2r\Delta r . \quad (4.3d)$$

FIGURE 6. Cross Section of Rebar

As finite elements Δs and Δr approach zero,

$$ds = 2\pi r dr \text{ .} \quad (4.3e)$$

Equation (4.2) is then rewritten in terms of r :

$$\phi_B(t) = \int_0^r B(t) \cdot 2\pi r dr \quad (4.4a)$$

or

$$\phi_B(t) = B(t) \int_0^r 2\pi r dr \text{ .} \quad (4.4b)$$

Here the assumption is made that the magnetic field, $B(t)$, is not a function of r . While $B(t)$ is a function of perpendicular distance, r_p , between the rebar and the power plug, r_p varies less than $\pm 10\%$ above and below the centerline of the rebar. The r_p corresponding to the centerline of rebar will be chosen and considered constant with respect to r . Therefore, $B(t)$ is considered independent of r .

4.3 CALCULATION OF INDUCED EDDY VOLTAGE, $\epsilon(t)$

From Faraday's law, the induced eddy voltage (emf) is equal to the rate of change of the magnetic flux, or

$$\epsilon(t) = \frac{-d\phi_B}{dt} [=] \frac{I \cdot M^2}{S} [=] \text{ volts .} \quad (4.5)$$

Differentiating Equation (4.4b) and inserting,

$$\epsilon(t) = \frac{-dB}{dt} \int_0^r 2\pi r dr \text{ .} \quad (4.6)$$

Considering the definition of $B(t)$ and differentiating Equation (4.1) with respect to time,

$$\frac{dB}{dt} = \frac{\mu_0}{2\pi r_p} \frac{di}{dt} \text{ .} \quad (4.7)$$

The amplitude of alternating current varies with time sinusoidally and is described by

$$i(t) = i_m \sin (wt) , \quad (4.8a)$$

where i_m is the maximum amplitude of $i(t)$ and w is the angular frequency of the current, which is related to the time frequency ν , in cycles per second (Hz) by

$$w = 2\pi\nu .$$

Taking the time derivative of the current,

$$\frac{di}{dt} = i_m w \cos(wt) \quad (4.8b)$$

or

$$\frac{di}{dt} = i_m 2\pi\nu \cos(2\pi\nu t) . \quad (4.8c)$$

It should be noted that di/dt takes on its maximum value of $\pm i_m 2\pi\nu$ at $t = (2n+1)/4\nu$ seconds, where $n = \{0,1,2,3,\dots\}$.

Since the root mean square current, i_{rms} , is more commonly employed, the following relation will be used:

$$\sqrt{2} i_{rms} = i_m \cos (2\pi\nu t) \quad (4.8d)$$

at values of time such that $t=(2n+1)/4\nu$ seconds. Substituting Equations (4.8c) and (4.8d) into Equation (4.7),

$$\frac{dB}{dt} = \frac{\mu_0}{2\pi r_p} i_{rms} 2\pi\sqrt{2} \nu . \quad (4.9)$$

(Here an average (rms) value of dB/dt is calculated.)

Substituting Equation (4.9) into (4.6), we obtain the following expression for the induced eddy current voltage, $\epsilon(t)_{rms}$:

$$\frac{dB}{dt} = \frac{-\mu_0}{2\pi r_p} i_{rms} 2\pi \sqrt{2} \nu \int_0^r r 2\pi r dr . \quad (4.10)$$

The minus sign in Equation (4.10) indicates that $\epsilon(t)$ varies in the inverse direction of $i(t)$ in the power plug. Figure 7 is a qualitative plot comparing $i(t)$ and $\epsilon(t)$.

4.4 CALCULATION OF ϵ_{rms} IN WALL REBAR

For a wall plug diameter of 4 in. and a 16-in.-dia loop of 1 in. rebar concentric with the cross section of the wall plug, calculate the induced eddy current voltage in the rebar if 3000 amperes (rms) at 60 Hz is passed through the wall plug to a melter electrode.

• Solution

Using Equation (4.10) ϵ_{rms} can be calculated directly:

$$\epsilon_{rms} = \frac{-(4\pi \times 10^{-7}) \frac{T \cdot M}{A} (3000)A 2\pi \sqrt{2} (60) \int_0^{\frac{1}{2} \text{ in.}} 2\pi r dr}{2\pi \left(\frac{16}{2} - \frac{4}{2} \right) \text{ in.} \cdot s} \quad (4.10a)$$

$$= -(0.3350) \left[\frac{r^2}{2} \right]_0^{1/2} \frac{T \cdot M \cdot A \cdot \text{in}^2}{s \cdot A \cdot \text{in}} \quad (4.10b)$$

FIGURE 7. Comparison of $\epsilon(t)$ and $i(t)$ Versus Time, $1/4 \nu \text{ sec}$

$$= -(0.0418) \left(2.54 \frac{\text{cm}}{\text{in}}\right) \left(\frac{1\text{m}}{100\text{cm}}\right) \frac{\text{T} \cdot \text{M} \cdot \text{in}}{\text{s}} \quad (4.10c)$$

$$= -0.0011 \frac{\text{T} \cdot \text{M}^2}{\text{s}} \quad (4.10d)$$

and, since the units of Tesla meter squared over seconds are equal to volts,

$$\epsilon_{\text{rms}} = -0.0011 \text{ V} \quad (4.10e)$$

Since 3000 amps is anticipated as the maximum power plug current, and the minimum perpendicular distance between the power plug and the rebar is 6 in., the illustration represents a worst case situation. Therefore, the voltage induced in the wall rebar will always be less than 0.0011 V, as predicted by the Biot-Savart law and Faraday's law of induction.

4.5 QUALITATIVE DERIVATION OF EDDY CURRENT ELECTRICAL RESISTANCE

Joule's law for heat generation in a conductor can be written for our use as:

$$H_r = \frac{\epsilon_{\text{rms}}^2}{R} [=] \text{ watts} . \quad (4.11)$$

The electrical resistance (R) corresponding to a given geometry can be calculated by the relation,

$$R = \frac{\sigma_e L}{A_c} \quad (4.12)$$

where σ_e is the electrical resistivity of the rebar material, L is the length of the current path and A_c is the cross sectional area of the current path with all terms in consistent units. For carbon steel $\sigma_e = 10 \times 10^{-6}$ ohm-cm.

To evaluate the resistance in Equation (4.10), the current path in the rebar must be considered. Figure 8 shows the correct current path as given by Lenz's law (the right-hand rule). The current paths form concentric circles

FIGURE 8. Direction of Current in Rebar

about the centerline of the rebar. For a given path between r and $r + \Delta r$ with thickness Δr , the path geometry is defined by a finite cross-sectional area, ΔA_C , where $\Delta A_C = 1\Delta r$ and an average path length, $\bar{L}$:

$$\bar{L} = \frac{2\pi(r + \Delta r) + 2\pi r}{2} = \pi(2r + \Delta r) \quad (4.13a)$$

For the resistance, ΔR , of the finite path, Equation (4.12) becomes

$$\Delta R = \frac{\sigma_e \pi (2r + \Delta r)}{1\Delta r} \quad (4.13b)$$

If Equation (4.13b) is written in dimensional form it becomes

$$R = \frac{\sigma_e}{l} \quad (4.14)$$

Equation (4.14) implies that the eddy current resistance is inversely dependent on the length of the rebar and not on the radius of the rebar. The nondependency of rebar radius on eddy-current resistance can be rationalized

somewhat by noting that while an increase in rebar radius will result in a factor of r increase in $A_c = 1r$ and thus decrease in R , it will also result in a factor of r increase in $L = 2\pi r$ (for a given path), resulting in an increase in R . Therefore, the dependencies on r cancel.

Equation (4.11) can now be written, in dimensional form, as

$$H = \frac{\epsilon_{rms}^2 l}{\sigma_e} , \quad (4.15)$$

where l is the length of the rebar. Equation (4.10) can be written in dimensional form as

$$\epsilon_{rms} = \frac{\mu_o i_{rms} \Delta r_r^2}{r_p} . \quad (4.16)$$

The dimensionless parameter for similarity between model and full scale power plug can now be written as;

$$\left[\frac{\mu_o^2 i_{rms}^2 v^2 r_r^4 l}{r_p^2 \sigma_e H} \right] = 1 , \quad (4.17a)$$

where each symbol in boldface represents the ratio of the full-scale and model parameters. If model parameters are noted with primes, the working similarity parameter is written:

$$\left[\frac{\left(\frac{\mu_o'}{\mu_o}\right)^2 \left(\frac{i_{rms}'}{i_{rms}}\right)^2 \left(\frac{v'}{v}\right)^2 \left(\frac{r_r'}{r_r}\right)^2 \left(\frac{l'}{l}\right)}{\left(\frac{r_p'}{r_p}\right)^2 \left(\frac{\sigma_e'}{\sigma_e}\right) \left(\frac{H'}{H}\right)} \right] = 1 . \quad (4.17b)$$

In the physical model, we will be interested in the surface temperature gradient in the cement near the rebar, which is given by the equation for steady-state one-dimensional heat conduction in solids:

$$H = k A_s \frac{\Delta T}{\Delta L} , \quad (4.18a)$$

where A_s is the surface area of the rebar, k is the thermal conductivity of the cement, and $\Delta T/\Delta L$ is the temperature gradient.

In the physical model, a wall cement temperature gradient similar to that in a full-scale power plug can be achieved if the wall cement material in the model is the same as full scale and the rebar surface watt density, H/A_s , is made similar since

$$\frac{H}{A_s} = k \frac{\Delta T}{\Delta L} . \quad (4.18b)$$

The surface of area of the rebar is proportional to its radius by;

$$A_s \propto 2\pi r_r , \quad (4.19a)$$

and the watt density $H_d = H/A_s$ is thus proportional to r_r by

$$H_d \propto \frac{H}{r} \quad (4.19b)$$

or, for similar watt densities

$$H_d' = H_d \quad (4.19c)$$

and

$$\frac{H'}{r_r'} = \frac{H}{r_r} \quad (4.19d)$$

This similarity criteria can be rewritten:

$$\frac{\frac{H'}{r_r'}}{\frac{H}{r_r}} = \frac{H_d'}{H_d} \quad (4.19e)$$

or

$$\frac{H'}{H} = \frac{H_d' r_r'}{H_d r_r} \quad (4.19f)$$

This watt density similarity criteria can now be written into Equation (4.17b) which becomes the final similarity parameter:

$$\left[\frac{\left(\frac{\mu_o'}{\mu_o}\right)^2 \left(\frac{i'_{rms}}{i_{rms}}\right)^2 \left(\frac{v'}{v}\right)^2 \left(\frac{r_r'}{r_r}\right) \left(\frac{l'}{l}\right)}{\left(\frac{r_p'}{r_p}\right)^2 \left(\frac{\sigma_e'}{\sigma_e}\right) \left(\frac{H_d'}{H_d}\right)} \right] = 1 \quad (4.20)$$

Equation (4.20) is the criterion for induced-eddy-current heating similarity between scale model and full scale.

If Equation (4.20) is satisfied for $H_d = H_d'$, then the temperature gradients in the cement in both model and full scale will be identical provided there are no heat sources within the wall other than the rebar heating.

4.6 APPLICATION OF SIMILARITY PARAMETER FOR EDDY-CURRENT HEATING TO DESIGN OF 1/3 SCALE MODEL

Since μ_o is a constant, and if in both model and full scale 60 Hz electricity is used, and if equivalence between H_d and H_d' is achieved, the similarity criteria is reduced to:

$$\left[\frac{\left(\frac{i'_{rms}}{i_{rms}}\right)^2 \left(\frac{r_r'}{r_r}\right) \left(\frac{l'}{l}\right)}{\left(\frac{r_r'}{r_r}\right)^2 \left(\frac{\sigma_e'}{\sigma_e}\right)} \right] = 1 \quad (4.21)$$

Since the model power plug will be 1/3 scale, if $r_p' = r_p/3$, then $I' = I/3$. In addition, the current available for testing the model will be 1000 A. The similarity criteria is thus further simplified to:

$$\left[\frac{\left(\frac{1000}{3000}\right)^2 \left(\frac{r_p'}{r_p}\right) \left(\frac{1}{3}\right)}{\left(\frac{1}{3}\right)^2 \left(\frac{\sigma_e}{\sigma_e}\right)} \right] = 1 \quad (4.22a)$$

or

$$\left[\frac{\frac{1}{3} \left(\frac{r_p'}{r_p}\right)}{\left(\frac{\sigma_e}{\sigma_e}\right)} \right] = 1 \quad (4.22b)$$

If copper rebar is used in the scale model, similarity is easier to achieve. The electrical resistivity of copper is compared with carbon steel:

$$\begin{aligned} \sigma_e &= 1.6 \times 10^{-6} \text{ ohm-cm (copper)} \\ \sigma_e &= 10 \times 10^{-6} \text{ ohm-cm (carbon steel)} \end{aligned}$$

and the similarity criteria becomes:

$$\left[\frac{\frac{1}{3} \left(\frac{r_p'}{r_p}\right)}{\left(\frac{1.6}{10}\right)} \right] = 1 \quad (4.23a)$$

or

$$\left[2.0833 \left(\frac{r_p'}{r_p}\right) \right] = 1 \quad (4.23b)$$

or

$$r_r' = \frac{r_r}{2.0833} \quad (4.24)$$

In the full-scale cement wall, 1 in. diameter rebar will be used so that $r = 1/2$ in. In the model, copper rebar of 1/2 in. dia will be used so that Equation (4.24) is met. In this manner, $H_d = 1$ is achieved.

4.7 SUMMARY OF WALL REBAR SIMILARITY

Based on the analysis of induced-eddy-current similarity, a scale model could be built with the following geometric relationships;

$r_r = 1/2$ = ratio of radius of rebar

$r_p = 1/3$ = ratio of distance of rebar from power plug surface

$r = 1/3$ = ratio of radius of power plug.

Rebar in the model can be made of 1/2-in. dia solid copper bar stock (rather than carbon steel) while the wall cement will be of the same material as in actual cell walls. The copper rebar can be installed in the same geometric configuration as the carbon steel cell wall rebar. The recommended model rebar design is detailed in Figure 14 in the Appendix.

5.0 NATURAL-CONVECTION HEAT TRANSFER

There are two areas within the power plug affected by natural-convection heat transfer. Natural convection, sometimes called free convection, is a gravitational process wherein hot air near a heated surface rises through ambient air, due to its lower density (volume expansion), and in doing so conducts heat away from the surface. In the power plug, heat transfer through natural convection occurs from the exterior of the power plug shroud directly to the cell environment. The cell air acts as a heat sink where the temperature difference between the shroud surface and the ambient cell temperature is the heat transfer driving force. The magnitude of the natural convection depends upon this temperature difference. The second area of natural convection occurs in the outer annulus between the power plug cover sleeve and the wall liner. In this area, heat transfer will actually occur through a combination of forced and natural convection since a small amount of net air flow is anticipated. The direction of air flow will be through the annulus from the gallery to the cell. This flow is caused by a pressure drop from the operating gallery (at 1 atm) to the cell, which is maintained at a slight vacuum (2 to 5 in. H₂O). This pressure drop of 2 to 5 in. will be enough to cause air to flow through a high efficiency particulate air (HEPA) filter placed on the gallery side air inlet and through the annulus to the cell. (The HEPA filter is required as a standard precaution to contain airborne radioactive particles in the event of cell pressurization which would result in reverse air flow from the cell to the operating galley.)

For the geometrically similar model, the criterion for similarity in natural convective heat transfer is the equivalence of: the Nusselt modulus, N_{NU} ; the Grashof number, N_{GR} ; and the Prandtl number, N_{PR} , all of which are dimensionless groups derived from a differential equation describing natural convective heat transfer (Bird, Stewart, and Lightfoot 1960). The three similarity criteria are:

$$\frac{N_{NU}'}{N_{NU}} = N_{NU} = \frac{h}{kL\Delta T} = 1 \quad (5.1)$$

$$\frac{N_{GR}'}{N_{GR}} = N_{GR} = \frac{\beta g \Delta T L^3 \rho^2}{\mu} = 1 \quad (5.2)$$

$$\frac{N_{PR}'}{N_{PR}} = N_{PR} = \frac{C_p \mu}{k} = 1, \quad (5.3)$$

where the boldface symbols are model-to-full scale ratios.

The Nusselt modulus criterion can be met, as discussed in Section 3.0, if $H = 1/3$. The Prandtl number contains only fluid properties and is therefore automatically simulated when air is used, as discussed in Section 3.4.

The Grashof number contains fluid, gravitational, length and thermal properties. The fluid properties β , ρ , and μ will be equivalent in model and full scale since air at about the same temperature (and pressure) is the heat transfer medium in both cases. In addition, gravitational equivalence is also met automatically. The Grashof number can then be divided into two (dimensional) groups:

$$N_{GR} = \frac{\beta g \rho^2}{\mu} \cdot \Delta T L^3 = 1 \quad (5.4)$$

where, as discussed,

$$\frac{\beta g \rho^2}{\mu} = 1 \quad (5.5)$$

is automatically achieved, leaving as the similarity criterion:

$$\Delta T L^3 = \frac{\Delta T' L'^3}{\Delta T L^3} = 1 \quad (5.6)$$

For a 1/3 scale model, $L = 1/3$ so that $L^3 = 1/27$ and the similarity criterion becomes:

$$\frac{\Delta T}{27} = 1 \quad (5.7a)$$

or

$$\Delta T = 27 \text{ .} \quad (5.7b)$$

To achieve similarity in natural convection, Equation (5.7b) requires that the temperature driving force in the model be maintained 27 times higher than that in full scale.

Discussed below are the implications of this criterion given by Equation (5.7b) for achieving overall similarity between the 1/3 scale and full-scale power plugs. Treated first is natural convection from the shroud to the cell environment and then for natural convection in the outermost annular region within the wall plug.

5.1 NATURAL CONVECTION TO CELL ENVIRONMENT

Most of the heat loss from the power plug to the cell will be the forced convection of hot air out of the power plug shroud through vent holes directly to the cell. The air within the shroud is exhaust air from the two air-cooling circuits described in Section 3.4. Some radiant heat transfer will occur from the power plug surface to the shroud interior surface, causing a temperature rise in the shroud. Some of this radiant heat will be transferred out with the exhaust air leaving the shroud. The remainder of the heat will pass through the wall of the shroud and will be lost through natural convection to the cell. The extent of natural convection will depend on the exterior surface temperature of the shroud, which in turn depends on the forced convection cooling provided by the exhaust air that cools the interior shroud surface. Due to the forced-air cooling at the interior surface, it is anticipated that the

exterior shroud surface temperature will be maintained close to ambient cell temperature. In other words, it is anticipated that forced-convective cooling will overshadow cooling by natural convection. A reasonable intermediate case to imagine would be a shroud surface temperature of 51°C at an ambient cell temperature of 50°C , resulting in a temperature driving force (ΔT) of 1°C .

For similarity, Equation (5.7b) requires that the temperature driving force in the model must be 27°C . Since we are trying to achieve thermal similarity, the temperature of the model shroud must be the same as the full-scale shroud, or 51°C . The model "cell" environment must then be maintained at $51 - 27 = 24^{\circ}\text{C}$. Since the model will be operated in a room having an ambient temperature of about $25^{\circ}\text{C} \pm 1^{\circ}\text{C}$, similarity will be achieved for the conceptual case described. If however, forced convection within the shroud is less than anticipated, and natural convection is thus important, the exterior shroud temperature will be greater than 51°C . If the shroud temperature approached 55°C , for a ΔT of 5°C , the model ΔT would have to be maintained at $(27) \cdot (5) = 135^{\circ}\text{C}$ for similarity. This means that the model would need to be operated in an isothermal enclosure maintained at $50^{\circ}\text{C} - 135^{\circ}\text{C} = -85^{\circ}\text{C}$.

Since the actual shroud temperature is not known, it cannot be known beforehand at what temperature the model test environment should be maintained to achieve thermal similarity. For the test, it will be assumed for the reasons stated above that forced convection will effectively cool the shroud and that natural convection is not a significant heat transfer component. The validity of this assumption can be determined easily in the model by measuring the temperature of the shroud. A model shroud temperature between 50°C and 51°C will indicate a good fit while an exterior shroud temperature greater than 50°C will indicate a poor fit. A poor fit will mean that natural convection is more important than anticipated. At shroud temperatures greater than 51°C , the model will not be able to reject heat through natural convection as rapidly as in full scale, meaning that the model will represent the worst case natural-convection heat transfer.

Knowledge that the model represents a worst case situation is sufficient in any case to determine whether the overall cooling of the power plug is

satisfactory using the model data. On the other hand, best case situations in the model must be identified and avoided if possible.

5.2 MIXED-REGIME HEAT TRANSFER

As discussed, the power plug will be cooled through the combination of forced and natural convection and is thus governed by the kinematic and dynamic regimes respectively. As argued in the previous section, it is anticipated that power plug cooling will be primarily through forced convection and will thus be mainly in the kinematic regime, where similarity is easily achieved.

Heat transfer similarity in the outermost annulus region within the wall plug will be difficult to achieve since the relative contributions of forced and natural convection are difficult to estimate. It is probable that both modes of convection will be important and that heat transfer in that area will truly be in the mixed dynamic/kinematic regime.

The kinematic contribution in forced-convective heat transfer is determined by the Reynolds number, while the dynamic contribution in natural convection is determined by the Grashof number. The relative contributions of natural and forced convection in the dynamic/kinematic mixed regime can be assessed by comparing the Reynolds and Grashof numbers for a given heat transfer surface. A criterion has been developed (Kreith 1973) to determine when natural convection becomes important. This criterion is:

$$\frac{N_{Gr}}{N_{Re}^2} > 1, \quad (5.8)$$

which can be written:

$$\frac{N_{Gr}}{(N_{Re})^2} = \frac{\frac{\beta g \Delta T L^3 \rho}{\mu^2}}{\left(\frac{\rho L \vec{v}}{\mu}\right)} = \frac{\beta g \Delta T L}{\vec{v}^2} \quad (5.9)$$

This can be used as a mixed regime similarity parameter:

$$\frac{8g\Delta TL}{\bar{v}^2} = 1 \quad (5.10)$$

which reduces to:

$$\frac{L}{\bar{v}} = 1 \quad (5.11)$$

for cooling medium (air), thermal and gravitational similarity, and further to:

$$\bar{v} = \frac{\sqrt{3}}{3} = \frac{\bar{v}'}{v} \quad (5.12)$$

for a 1/3 scale geometrically similar model. Equation (5.12) expresses the cooling-air velocity ratio between model and full scale when heat transfer is in the dynamic/kinematic regime as defined by the criterion given by Equation (5.8). On the other hand, if Equation (5.8) indicates that heat transfer is in the kinematic regime, then Equation (3.8) for forced convection is the appropriate similarity criteria and $\bar{v} = 3$.

In operating the scale model, the flow of air through the outermost annulus will be varied from nonflow to high flow incrementally to determine the dependency of wall sleeve temperatures on flow rate. If it is found that an intermediate air flow rate is required to maintain acceptable wall sleeve temperatures, the Reynolds and Grashof numbers corresponding to the flow will be calculated from the measured data, allowing the use of Equation (5.8) to determine whether the heat transfer is controlled by mixed regime or kinematic regime. With this information, the required flow rate in the full-scale power plug can be predicted. Since the rate of natural convection in the model will always represent worst-case heat transfer, if it is found that the model wall

sleeve temperatures are acceptable with no axial air flow in the outer wall plug annulus, then it will have been determined that the wall sleeve temperatures in the full-scale power plug are acceptable.

6.0 OPERATION OF SCALE MODEL

In addition to maintaining geometric and materials similarity, heat fluxes must be carefully maintained so that thermal similarity is achieved. If thermal similarity is achieved, temperature profiles within the model will be identical to full-scale temperature profiles. With thermal similarity achieved, model operating parameters can be varied and their effect on temperature profiles within the model can be used to predict the operating characteristics of the full-scale power plug. Heat flux and operating parameters are discussed next.

6.1 CONTROL OF HEAT SOURCES

As derived in Section 2.2.5 the criterion for model heat input/loss similarity is given by Equation (2.11c) as

$$H = \frac{1}{3} . \quad (6.1)$$

The two heat sources that must be controlled in the model are the melting cavity heat source and joule heating within the power plug. As discussed in Section 3.0, the heat input from the melting cavity will be controlled by controlling the plate heater temperature located in the simulated model melting cavity. The temperature required to achieve $H = 1/3$ will be estimated from the full-scale heat input calculated using a computer-assisted mathematical model.

As derived in Section 3.6, $H = 1/3$ for joule heat within the power plug can be easily achieved by maintaining the model amperage at $1/3$ the required full-scale amperage (3000 A maximum) or from Equation (3.18):

$$i = \frac{1}{3} \quad (6.2)$$

6.2 VARIATION OF OPERATING PARAMETERS

With the heat sources controlled and maintained so that $H = 1/3$, the overall heat loss can be distributed in various ways to characterize the power-plug system operation. Each operating parameter is discussed in the following subsections.

6.2.1 Air Cooling Rates

There are four air-cooling circuits in the power plug that cover three different modes of heat transfer: forced convection, natural convection, and mixed-regime convection.

Forced convection occurs in electrode cooling circuit and in the inner annular region of the power plug within the cell wall. The air flow in both circuits is controllable and can be varied independently throughout the test to determine each of their contributions to overall power plug cooling. In addition, the effect of loss of air flow in each system can be simulated.

Natural convection occurs from the exterior surface of the shroud to the cell environment and from the operating gallery electrical connection to the gallery environment. Heat transfer by natural convection over both of these sections is not subject to direct control. Their effect on overall cooling, however, is expected to be negligible.

Mixed-regime convection occurs in the outermost wall sleeve annulus within the cell wall. The air flow in this region will be varied to determine the heat transfer requirements in the outer annulus.

6.2.2 Water Cooling Rate

A water jacket, located on the melter exterior, will tend to cool that part of the power plug passing through it. A water jacket appropriately located on the scale model will be used to determine the contribution of the melter water jacket to overall cooling of the power plug by varying the water flow rates.

6.2.3 Amperage

The effect of varying the power plug current on temperature profiles along the wall sleeve and wall rebar will be evaluated to determine when either joule

heating or eddy current induced rebar heating become significant. (It is anticipated that rebar heating will not be significant even at the maximum wall plug amperage.)

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7.0 SCALE-MODEL DATA INTERPRETATION

The similarity parameters derived in Sections 2.0 through 5.0 provide the information required to relate scale-model data to the full-scale power plug behavior.

7.1 VOLUMETRIC FLOW RATE

Volumetric flow rate, $\dot{V}$, is related to flow velocity, $\vec{v}$, and hydraulic diameter, D , by:

$$\dot{V} = \pi \frac{D^2}{4} \cdot \vec{v} \quad (7.1)$$

When Equation (7.1) is dimensionalized, it becomes:

$$\dot{V} = L^2 \vec{v} \quad (7.2)$$

and the resulting similarity criteria is:

$$\frac{\dot{V}}{L^2 \vec{v}} = 1. \quad (7.3)$$

In preceding sections it was found that in the kinematic regime $\vec{v} = 3$ and $L = 1/3$ so that Equation (7.3) becomes:

$$\dot{V} = \frac{1}{3} \quad (\text{kinematic regime}) \quad (7.4)$$

In the mixed regime, it was found that $\vec{v} = \frac{\sqrt{3}}{3}$, and Equation (7.3) becomes:

$$\dot{V} = \frac{\sqrt{3}}{27} \quad (\text{Mixed Regime}) \quad (7.5)$$

The full-scale volumetric flow rates corresponding to model flow rates can then be related by:

$$\dot{V} = 3\dot{V}' \quad (\text{kinematic regime}) \quad (7.6)$$

$$\dot{V} = 9\sqrt{3} \dot{V}' \quad (\text{mixed regime}) \quad (7.7)$$

where the primed quantities indicate model parameters.

7.2 PRESSURE DROP

The pressure drop, ΔP , resulting from viscous losses, incurred by a compressible fluid flowing through a duct is described by three dimensionless groups (Denn 1980):^(a) the Reynolds number, N_{RE} ; the friction factor, N_{FR} ; and the dimensionless pipe roughness, N_E , where

$$N_{FR} = \frac{\Delta p}{2\rho \dot{V}^2} \frac{D}{l} \quad (7.8)$$

$$N_E = \frac{k}{D} \quad (7.9)$$

and where l is the length of the duct and k is the pipe roughness in units of length. Table 1 lists values of k corresponding to various materials (Denn 1980).

The three dimensionless groups are not independent, and their dependency can be written:

$$N_{FR} = f(N_{RE}, N_E). \quad (7.10)$$

(a) The nomenclature, N_{FR} and N_E used in this report is not standard, but is done for succinctness.

TABLE 1. Surface Roughness for Various Materials

Material	k(mm)
Drawn tubing (brass, lead, glass, etc.)	1.5×10^{-3}
Commercial steel or wrought iron	0.05
Asphalted cast iron	0.12
Galvanized iron	0.15
Cast iron	0.46
Wood stave	0.2-0.9
Concrete	0.3-3
Riveted steel	0.9-9

Figure 9 illustrates the interdependency of N_{RE} , N_{FR} and N_E (Denn 1980). Note that at N_{RE} between 3000 and 30,000, N_{FR} is not a strong function of N_E for N_E less than about 0.001. As an example, consider a 2 in. steel tube. From Table 1, $k = 0.05$ mm and $D \sim 50$ mm for a 2-in. tube. N_E can be calculated:

$$N_E = \frac{0.05 \text{ mm}}{50 \text{ mm}} \quad (7.11a)$$

or

$$N_E = 0.001. \quad (7.11b)$$

Therefore if airflow through a steel tube is in the lower turbulent regime, the friction factor can be estimated from Figure 9 using the lower curve for smooth tubes to a good approximation. We will assume for simplicity that values of N_E in the model will be approximately the same as in full scale and that the effect of any difference is negligible. In addition, the airflow surfaces in the model will be made smoother either by using drawn tubing rather than pipe or by polishing the surfaces if pipe is used. In effect, a reduction in k in the model will accompany the factor-of-three reduction in hydraulic diameter so that $N_E = 1$ is approximated.

FIGURE 9. Friction Factor as a Function of N_{RE} and N_E

With $N_E = 1$ achieved to good approximation, Equation (7.10) can be redefined as a manageable similarity criteria for duct flow:

$$\frac{N_{FR}}{N_{RE}} = 1 \quad (7.12)$$

When the dimensionless groups are rewritten in terms of parameters, Equation (7.12) becomes:

$$\frac{N_{FR}}{N_{RE}} = \frac{\frac{\Delta p}{\rho v} \frac{L}{D}}{\frac{\rho L v}{\mu}} = 1 \quad , \quad (7.13)$$

which is reducible to:

$$\frac{\Delta p \mu}{\bar{v}^3 L} = 1 . \quad (7.14)$$

From previous derivations, $\mu = 1$, $\bar{v} = 3$ and $L = 1/3$ so that:

$$\frac{\Delta p \cdot 1}{27 \cdot \frac{1}{3}} = 1 \quad (7.15a)$$

or

$$\Delta p = 9. \quad (7.15b)$$

The full-scale pressure drop corresponding to model pressure drop can then be related by:

$$\Delta p = \frac{\Delta p'}{9} . \quad (7.16)$$

7.3 TEMPERATURE PROFILES

The scale model can be designed and operated to achieve thermal similarity. The first criteria for thermal similarity was derived as:

$$T = 1 \quad (7.17)$$

$$H = \frac{1}{3} . \quad (7.18)$$

Also, the expression for temperature gradients can be written:

$$\frac{T}{L} = \frac{1}{\frac{1}{3}} = 3 , \quad (7.19)$$

which states that temperature profiles will be three times as great in the scale model. The second criteria is the melting cavity boundary temperature, 1150°C, be maintained. With these three criteria achieved, temperatures measured at any point in the scale model will be identical with those that would be measured on a full scale power plug. The recommended thermocouple locations in the model are detailed in Figures 10, 13 and 14 in the Appendix.

7.4 WATER JACKET HEAT DUTY

The heat loss from the model power plug to the small water jacket located at the surface of the mock melting cavity can be calculated using measured values of cooling water flow rate and temperature rise. The rate of heat loss, H' , is then related to the full-scale heat loss, H , by the previously derived relation:

$$\frac{H'}{H} = H = \frac{1}{3} \quad (7.20)$$

or

$$H = 3H'. \quad (7.21)$$

7.5 AMPERAGE

As derived in Section 3.6, $i = 1/3$ or:

$$i = 3i'. \quad (7.22)$$

The value of i' can be measured in the model using a digital multimeter with a 5 to 1 current transformer accessory.

7.6 RESISTANCE

Electrical resistance, R , is an extensive property related to electrical resistivity, α_e , by:

$$R = \frac{\alpha_e \cdot L}{L^2} \cdot \quad (7.23)$$

With material similarity, $\sigma_e = 1$ (in the model power plug, not the wall rebar), and $L = 1/3$. Therefore,

$$R = \frac{1 \cdot \frac{1}{3}}{\frac{1}{9}} = 3 \quad (7.24)$$

and, finally,

$$R = \frac{R'}{3} \quad (7.25)$$

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NOMENCLATURE

A	ampere
h	hours
L	length
m	meter
s	seconds
T	tesla
V	volts
ω	angular frequency
W	watts
t	time, s or h
R	electrical resistance, ohms
A_c	cross-sectional surface of eddy current path in rebar
A_r	unit surface area of melting cavity refractory
A_s	surface area of rebar
r	cross-sectional radius of power plug, m or in.
r_r	cross-section radius of rebar, in.
r_p	distance away from surface of power plug, m or in.
l	length of rebar loop
H	steady-state heat input or heat loss, BTU/h
H_J	steady-state joule heat generation in power plug, watts
H_r	steady-state heat generation in rebar
H_d	surface heat flux in rebar, watts/ in. ²
$\vec{v}$	fluid velocity
$\dot{V}$	volumetric flow rate
$i(t)$	alternating current
i_m	maximum current value
i_{rms}	root mean square current
$B(t)$	magnetic field
ds	differential surface element
d l	differential length element
$d\phi_B/dt$	rate of change in magnetic flux with time
di/dt	rate of change in current with time

C_p	heat capacity
k	thermal conductivity

Greek Letters

β	volume expansivity
$\epsilon(f)$	induced-eddy voltage
$\epsilon_{\max}$	maximum-eddy voltage
ϵ_{rms}	root mean square eddy voltage
ρ	density
σ	Stefan-Boltzmann constant
σ_e	electrical resistivity
σ_s	surface tension
μ_0	permeability constant ($4\pi \times 10^{-7} \text{ T}\cdot\text{M/A}$)
ν	cycles per second (Hz)
$\Phi(t)$	magnetic flux
Δp	pressure drop
ΔT	temperature difference

Dimensionless Groups

N_E	Pipe roughness factor
$N_{FR}^{(a)}$	Friction factor
N_{GR}	Grashof number = $\beta g L^3 \Delta T / \nu^2$
N_{NU}	Nusselt modulus = $H / k L \Delta T$
N_{PE}	Peclet number = $N_{PR} \cdot N_{RE}$
N_{PR}	Prandtl number = $C_p \mu / k$
$N_R^{(a)}$	No accepted name = $H / \sigma_e L^2 (T_1^4 - T_2^4)$
N_{RE}	Reynolds number = $\rho L \bar{V} / \mu$
$N_{TH}^{(a)}$	Thring number = $\rho C_p \bar{V} / \sigma_e T^3$

(a) Nonstandard nomenclature.

APPENDIX

RECOMMENDED DESIGN OF MODEL POWER PLUG TEST APPARATUS

Figures 10 through 16 are detailed design prints of a recommended 1/3-scale model power plug test apparatus. The figures represent PNL print number SK-3-19882, sheets 1-7, prepared in conjunction with this report.

FIGURE 11. Scale Model Melter Power Plug Assembly (Sheet 2, Sk-3-19882)

FIGURE 14. Scale Model Melter Power Plug Details (Sheet 5, SK-3-19882)

FIGURE 15. Scale Model Melter Power Plug Electrical Details (Sheet 6, SK-3-19882)

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