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SMALL THERMAL OSCILLATION ANALYSIS OF THE MOTA

Hanford Engineering Development Laboratory

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Hanford Engineering Development Laboratory

George L. Guthrie

September 1978

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SMALL THERMAL OSCILLATION ANALYSIS OF THE MOTA

George L. Guthrie

ABSTRACT

This report analyzes the MOTA (Materials Open Test Assembly) as a thermal regulator. Its purpose is to make the methods available to others, not to give a tabulation of specific numerical results.

The approach used is to set up differential equations assuming small sinusoidal variations in the temperatures and flow rates. The equations are subjected to both closed-form and digital-finite-difference analysis to discover trends, stabilities and instabilities, and allowable degrees of regulation.

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SMALL THERMAL OSCILLATION ANALYSIS OF THE MOTA (FF028)

I. INTRODUCTION

The MOTA (Materials Open Test Assembly) was designed to achieve a degree of thermal regulation compatible with the generation of useful materials property data obtained by irradiation of candidate reactor structural materials in the FFTF. Attaining a high degree of regulation is limited by the necessity to avoid undamped thermal oscillations. This report documents some of the analyses used to select usable configurations and determine effects of parameter choices, and investigates limitations on allowable gains of the sensor-control-valve assembly. The main purpose of the document is to make the methods available to others, rather than to give a tabulation of specific numerical results.

A rough view of the MOTA is shown in Figure 1. For purposes of analysis the MOTA is divided into four regions. They are, starting at the bottom, (1) the heaters, (2) the specimens, (3) the free flow space (which may contain specimens), and (4) the valve and valve sensor. In its operation, the MOTA has an insulated outer wall and has sodium flowing in an upward direction driven by the reactor coolant circulation pumps. The gamma heating of the heater rods raises the temperature of the flowing Na in the MOTA above the temperature of the Na coolant in the main part of the core. As the Na coolant in the MOTA, which contains totally enclosed stagnant NaK, flows past the valve bellows, the temperature of the flowing MOTA coolant determines the temperature of the NaK in the bellows.

The NaK in the bellows, via its volume versus temperature characteristics, controls the valve, which regulates the upward directed coolant flow. The MOTA Na flow rate controls the temperature of the coolant stream in the MOTA, and this provides temperature regulation. The degree of regulation depends on several variables including the expansion characteristics of the bellows and the area and slope of the valve seat.

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FIGURE 1. Schematic of MOTA Test Capsule Layout.

In analyzing the MOTA as a regulator, several approaches are possible. One is to subdivide the MOTA into separate cells, each with a heat generation rate, heat capacity, and thermal conduction coefficient to all neighboring cells, and then to do a finite-element time-step analysis. This type of approach has been used previously but will not be considered in the present report. A second type of analysis involves setting up the differential equations and assuming small sinusoidal variations in the temperatures and flow rates. The sinusoidal amplitudes are assumed to be small and second order infinitesimals are discarded. The equations are then subjected to both closed-form and digital-finite-difference analysis to discover trends, stabilities and instabilities, and allowable degrees of regulation. The present report treats this second type of analysis.

II. THEORY

A. GENERAL

A comparison of the heat carried vertically by the flowing Na to that being transported in either the upward or downward directions by thermal-diffusion conduction processes shows that non-convected heat transfer can be neglected. As a further simplification, it is assumed that turbulence in the flowing Na will, on a time average basis, reduce the significance of the radial temperature gradient in the flowing sodium. The thermal conductivity of Na is about three times that of stainless steel at the temperatures of interest, and the radial thickness of the flow streams is expected to be small in most cases. Consequently, the neglect of the radial temperature gradient in the flowing Na coolant will not have serious consequences.

Using these assumptions, the problem can be regarded as being essentially one dimensional in physical space, with two coupled differential equations. One equation relates to the $T_1(x,t)$ relationship of the coolant temperature, T_1 , as a function of vertical position X , and time t . A second equation deals with the $T_2(x,t)$ relationship of the temperature, T_2 of the "isolated" region at each level where the "isolated" region might consist of the heaters, samples, sensor-bellows, or whatever is in contact with the sodium stream at a particular elevation. The two equations are coupled due to the fact that at each vertical level the coolant stream exchanges heat with the "isolated" items of the MOTA.

Thus,

$$T_1 = T_1 (x, t, T_2[x, t]) \quad [1]$$

and

$$T_2 = T_2 (x, t, T_1 [x, t]) \quad [2]$$

For purposes of small signal analysis, the equations can be uncoupled. The equation in T_1 contains a term which depends on the heat input contributed from the "isolated" region at each level, and this contribution depends on T_2 . However, this contribution can be broken down into two parts, a steady component and a time varying output. The explicit dependence on T_2 can be circumvented for each of the two parts. The steady output can be calculated and substituted directly into the equation for T_1 , and the time varying part can be expressed in terms of T_1 alone, rather than T_2 , so that T_2 is completely eliminated from the T_1 equation. Once this simplification is made, the analysis of the MOTA stack becomes much easier. When the MOTA stack analysis is simplified, it is possible to look at the MOTA control loop as being composed of three elements linked head to tail and feeding signals in a circular loop. The three elements are: (1) the MOTA stack (the heater region, specimen region, free flow space, and the valve and sensor region up to the outside surface of the temperature sensor), (2) the sensor itself, and (3) the valve. The "stack" is regarded as a black box which has a sinusoidally varying fluid flow as input and gives a sinusoidal temperature as output, namely the temperature immediately outside the sensor or bellows. The bellows is regarded as a black box which takes as its input the sinusoidally varying temperature outside and converts this input to the sinusoidally varying average temperature of the bellows. The valve is regarded as a black box which takes the average value of the bellows temperature and converts it to a flow rate. In this way of thinking, the "valve" includes the expansion characteristics of the sensor. At a given frequency, for a sine wave excitation analysis, each of the black boxes can be regarded as an operator which multiplies the amplitude and shifts the phase angle of the sine wave, and all of this can be represented using complex numbers familiar to electrical engineers and servo-analysts. The product of the three operators is the negative of the usual transfer function familiar in servo-analysis.⁽¹⁾

The problem then is to: (1) analyze the stack, (2) analyze the bellows as a temperature-to-temperature converter, and (3) decide what the

implications of (1) and (2) are with regard to possible values of valve gain (d flow /d temperature) and the consequent effects on regulation and stability.

B. STACK ANALYSIS

1. Derivation of Principal Equations

The equation of heat balance (enthalpy) for the flowing sodium in an infinitesimal differential length of the MOTA stack is:

$$\left\{ -Q \frac{\partial T}{\partial x} + G(x) + \text{IFFIR}(x) = C_{Na} \frac{\partial T}{\partial t} \right\} dx \quad [3]$$

In Equation 3 , Q is the product of the flow and the heat capacity per unit volume for the coolant. (Q will be referred to as "flow" and will require care in unit conversions.) G(x) is the total steady-state heat generation per unit length, caused by gamma heating both in the flowing sodium and in the "isolated" parts at the same vertical level. IFFIR is the sinusoidal heat flow per unit length from the isolated region coming into the flowing sodium (In-Flow from Isolated Region) and C_{Na} is the heat capacity per unit length of the flowing sodium coolant at a particular level in the MOTA. The philosophy behind the equation is that the flowing sodium at a height x, over a differential length dx is watched, and, remembering that the enthalpy and energy balances are not affected by the change in identity of atoms in the flow stream, the temperature and energy balance are considered. The $-Q \frac{\partial T}{\partial x}$ term is due to the fact that the colder sodium entering a region of differential length dx brings in less enthalpy than that removed by the hotter sodium exiting at the top of the same region of differential length dx.

We assume that the valve has set up a sinusoidal oscillation in Q and investigate the "black box" properties of the stack. When

$$Q = Q_0 + \epsilon \cos(\omega t) \cdot K \quad [4]$$

Q_0 is the steady state value of the flow, and K is a constant having units of watt $^{\circ}\text{C}^{-1}$. If $K = 1.0 \text{ watt}/^{\circ}\text{C}$, K can be dropped from future equations if we stay in CGS units. Also,

$$T = T_0 + \epsilon (A \cos[\omega t] + B \sin[\omega t]) \quad [5]$$

where $A(x)$ and $B(x)$ are functions of the vertical position, x . The problem then reduces to the one of finding A and B as functions of x . T_0 is the steady state value of T . Then, $(A^2 + B^2)^{1/2}$ will give the amplitude of T (sinusoidal) at any level for a unit oscillatory component of Q , and $\arctan (-B/A)$ will give the phase angle. Under our assumptions all the heat is removed by the flowing sodium so that:

$$Q_0 \frac{\partial T_0}{\partial x} = G(x) \quad [6]$$

for steady state heat balance. Then, when Equations [4], [5], and [6] are substituted in Equation [3], we obtain:

$$\begin{aligned} & \frac{-G(x)}{Q_0} \cos(\omega t) - Q_0 \left(\cos(\omega t) \frac{\partial A}{\partial x} + \sin(\omega t) \frac{\partial B}{\partial x} \right) + \text{IFFIR}/\epsilon \\ & = C_{\text{Na}}(-\omega \sin(\omega t) + \omega \cos(\omega t)) \end{aligned} \quad [7]$$

In combining Equations [3], [4], [5], and [6] to get Equation [7], terms in ϵ^2 have been ignored. Equation [7] can be integrated directly for various configurations of material which contribute to IFFIR. A given type of "isolated region" is "driven" by the sinusoidal temperature of the coolant in contact with its surface and supplies to the coolant an in-phase and out-of-phase contribution of heat content. (In-phase and 90° out-of-phase with the harmonic variation of T at that level.) It is also possible to calculate phase angles between the heat flow (IFFIR) and the coolant flow. Mathematically, we can express this in the form:

$$\text{IFFIR} = F_1(A,B) \sin(\omega t) + F_2(A,B) \cos(\omega t) \quad [8]$$

where F_1 and F_2 functions can be determined. When those forms are known, the $\sin(\omega t)$ coefficients and the $\cos(\omega t)$ coefficients in Equation [7] can be collected separately to form two coupled equations in A and B as functions of x. Then, using the boundary conditions:

$$A(0) = 0 \quad [9a]$$

$$B(0) = 0 \quad [9b]$$

we can use a digital computer to integrate up the stack to get A(x) and B(x) at the outer surface of the valve.

The details of the preceding arguments follow. V(x) is the in-phase oscillatory heat flow per unit oscillatory temperature drive per unit length, flowing out of the flowing sodium and into the isolated region, flowing in-phase with the temperature in the sodium stream. W(x) is similar but is the out-of-phase component of the oscillatory heat flow, considered positive when the flow out of the sodium and into the isolated region leads the applied driving temperature.

With these definitions, Equation [7] becomes:

$$\begin{aligned} & -\frac{G(x)}{Q_0} \cos(\omega t) - Q_0 \left(\frac{\partial A}{\partial x} \cos(\omega t) + \frac{\partial B}{\partial x} \sin(\omega t) \right) - V(x)(A \cos(\omega t) \\ & + B \sin(\omega t)) - W(x)(-A \sin(\omega t) + B \cos(\omega t)) = C_{Na}(-\omega \sin(\omega t) \\ & + \omega \cos(\omega t)) \end{aligned} \quad [10]$$

Collecting the coefficients of the $\sin(\omega t)$ and $\cos(\omega t)$ terms separately to form two equations, we find:

$$\frac{\partial A}{\partial x} = -\frac{B(x)}{Q_0} (C_{Na} \omega + W(x)) - \frac{G}{Q_0^2} - \frac{A(x)}{Q_0} \cdot V(x) \quad [11]$$

and

$$\frac{\partial B}{\partial x} = \frac{A(x)}{Q_0} (C_{Na} \omega + W(x)) - \frac{B(x)}{Q_0} \cdot V(x) \quad [12]$$

Equations [11] and [12] can easily be integrated on a digital computer. It is only necessary to start with $A = B = 0$ at $x = 0$ and go upwards in finite steps, letting:

$$A(x + dx) = A(x) + \frac{\partial A}{\partial x} \cdot dx \quad [13]$$

$$B(x + dx) = B(x) + \frac{\partial B}{\partial x} \cdot dx \quad [14]$$

In the notation used in the programming, the terminology involved the quantities $Y(x)$ and $Z(x)$ defined by:

$$Y(x) = \frac{1}{Q_0} [C_{Na} \omega + W(x)] \quad [15]$$

and

$$Z(x) = \frac{1}{Q_0} \cdot V(x) \quad [16]$$

Using $Y(x)$ and $Z(x)$ rather than $V(x)$ and $W(x)$, we find that Equations [11] and [12] become:

$$\frac{\partial A}{\partial x} = -B(x) Y(x) - G(x)/Q_0^2 - A(x) Z(x) \quad [17]$$

$$\frac{\partial B}{\partial x} = A(x) Y(x) - B(x) Z(x) \quad [18]$$

The following procedure is used in analyzing the stack. We assume a specific configuration at each level x and calculate $Y(x)$ and $Z(x)$ at each level. We assume a steady temperature at $x = 0$ so that $A = B = 0$ at $x = 0$ and use Equations [13] and [14] together with Equations [17] and [18] to find $A(x)$ and $B(x)$ on the outside of the sensor (bellows), using a digital computer to integrate our way up the stack. Then the amplitude of the stack "gain" is $(A^2 + B^2)^{1/2}$ evaluated at the sensor elevation, and leading phase angle is $\arctan(-B/A)$, also evaluated at the sensor elevation, and where the phase angle is referred to the fluid flow as a reference.

The actual development of formulas for $Y(x)$ and $Z(x)$ depend on the configuration chosen. The details have been worked out for three cases. They are: (1) The sodium stream is partially insulated from an "isolated region" in contact with the stream. The internal conductivity of the "isolated region" is high compared to the conductivity between the "isolated region" and the sodium coolant. The "isolated region" has a fixed heat capacity. Therefore, the "isolated region" is all at one uniform temperature internally but has a temperature difference between itself and the coolant stream; (2) The "isolated region" is in intimate surface contact with the coolant stream but has a finite internal thermal conductivity and finite heat capacity per unit volume. The "isolated region" is a solid cylinder; and (3) same as (2) above, but the cylinder is hollow, insulated on the outer radius, and has the sodium flowing up through the center.

2. Specific Examples of $Y(x)$ and $Z(x)$

The derivation of $Y(x)$ and $Z(x)$ for the three cases above is available in HEDL Notebooks N-2019, N-1978, and N-1958. In the present document, we will merely state the results. They are as follows:

a. AC-RC Case, Isothermal Semi-Isolated Region. For case (1), the AC-RC case, so called because it is analogous to an RC combination in electrical engineering AC theory:

$$Y(x) = \frac{1}{Q_0} \left\{ C_{Na} \omega + \frac{\omega C_{ISO}}{\omega^2 \tau^2 + 1} \right\} \quad [19]$$

$$Z(x) = \frac{\omega^2 \tau^2 C_{ISO}}{Q_0 \{\omega^2 \tau^2 + 1\}} \quad [20]$$

In Equations [19] and [20], C_{ISO} is the heat capacity per unit length of the isolated region and τ is the RC time constant for thermal relaxation of the isolated region to an infinite capacity isothermal sink. C_{Na} is the heat capacity per unit length of the flowing sodium coolant, and ω is the frequency of the oscillation. If we define a term R , where $(1/R)$ is the thermal conduction per unit length between the isolated region and the sodium coolant, then $R \cdot C_{ISO} = \tau$ and Equations [14] and [20] can be written:

$$Y(x) = \frac{1}{Q_0} \left(C_N \omega + \left[\frac{1}{R} \right] \frac{\omega \tau}{\omega^2 \tau^2 + 1} \right) \quad [21]$$

$$Z(x) = \frac{1}{Q_0} \left(\left[\frac{1}{R} \right] \frac{\omega^2 \tau^2}{\omega^2 \tau^2 + 1} \right) \quad [22]$$

This form is convenient for comparison to the next two cases.

b. Solid Cylinder in Contact with Na Coolant Stream. Case 2, the solid cylinder,

$$Y(x) = \frac{1}{Q_0} \left\{ C_{Na} \omega + 4\pi K \sum_{j=1}^{\infty} \frac{\omega \left(\frac{HR_0^2}{KX_j^2} \right)}{\omega^2 \left[\left(\frac{HR_0^2}{KX_j^2} \right)^2 + 1 \right]} \right\} \quad [23]$$

$$Z(x) = \frac{1}{Q_0} 4\pi K \sum_{j=1}^{\infty} \frac{\omega^2 \left[\left(\frac{HR_0}{KX_j} \right)^2 \right]^2}{\omega^2 \left[\left(\frac{HR_0}{KX_j} \right)^2 \right]^2 + 1} \quad [24]$$

In Equations [23] and [24], R_0 is the radius of the cylinder, H is the heat capacity per unit volume, K is the thermal conductivity of the material in the cylinder, and X_j are the arguments for which the zero order Bessel function of the first kind has

$$J_0(X_j) = 0 \quad [25]$$

The similarity to Case 1 is obvious if we relate $4\pi K$ to $(1/R)$ and only use the first term in the summations, letting $HR_0^2/(KX_j^2) = \tau$, where the $HR_0^2/(KX_j^2)$ is the relaxation time of a cylinder to a surrounding bath, when the initial temperature distribution is the fundamental mode of the cylinder.

c. Hollow Cylinder with Flow Up Center. Here we find:

$$Y(x) = \frac{1}{Q_0} C_{Na} \omega + 4\pi K \sum_{N=1}^{\infty} \frac{\omega \frac{K}{C} \lambda_N^2}{\left[\left(\frac{b}{a} \frac{U_{0N}(b)}{U_{1N}(a)} \right)^2 - 1 \right] [\omega^2 + \left(\frac{K}{C} \lambda_N^2 \right)^2]} \quad [26]$$

$$Z(x) = \frac{1}{Q_0} 4\pi K \sum_{N=1}^{\infty} \frac{\omega^2}{\left[\left(\frac{b}{a} \frac{U_{0N}(b)}{U_{1N}(a)} \right)^2 - 1 \right] [\omega^2 + \left(\frac{K}{C} \lambda_N^2 \right)^2]} \quad [27]$$

In Equations [25] and [26], U_{ON} is a linear combination of $J_0(\lambda_N r)$ and $Y_0(\lambda_N r)$ where:

$$U_{ON}(\lambda_N r) = C_1 J_0(\lambda_N r) + C_2 Y_0(\lambda_N r) \quad [28]$$

with λ_N , and C_1/C_2 chosen so that $U_{ON}(\lambda_N r)$ has zero value at $r = a$ and zero slope at $r = b$. Then we define $U_{1N}(r)$ to be:

$$U_{1N}(r) = C_1 J_1(\lambda_N r) + C_2 Y_1(\lambda_N r) \quad [29]$$

We are working with eigenfunctions in the annular ring region, and the eigenfunctions are chosen to have zero value on the inner surface and zero slope on the outer surface. K is the conductivity of the hollow cylinder and C is the heat capacity per unit volume. The pair of Equations [26] and [27] can be made to resemble the pair of Equations [23] and [24] if we call $X_j/R_0 = \lambda_j$ in Equations [23] and [24] and then multiply the top and bottom of Equation [23] by $\lambda_j/(KH)$ and similarly in Equation [24].

However, there is no expression in the Equations [23] and [24] pair to resemble the expression involving b , a , $U_{ON}(b)$ and $U_{1N}(a)$ that appears in Equation [26] and [27].

3. Stack Information Inferred from Closed Form Analysis

Several things about the stack can be inferred from closed form manipulation without resort to use of a computer.

a. Stack Gain - Ultra Safe Valve Gain. Consider the quantity $(A^2(x) + B^2(x))^{1/2}$ and call it

$$\text{Amp}(x) = \sqrt{A^2(x) + B^2(x)} \quad [30]$$

This quantity gives us the temperature oscillation amplitude at elevation x for a unit amplitude "flow" oscillation (where flow includes the heat capacity per unit volume for sodium, by our choice of definition of Q). For a high degree of regulation and a high gain valve, $Amp(x)$ should be small at the sensor to keep the total gain around the loop < 1 to avoid oscillations in a very simplified viewpoint. The differentiation of Equation [29] gives:

$$\frac{\partial Amp}{\partial x} = \frac{A \frac{\partial A}{\partial x} + B \frac{\partial B}{\partial x}}{\sqrt{A^2 + B^2}} \quad [31]$$

or, substituting from Equations [17] and [18]:

$$\frac{\partial Amp}{\partial x} = -A \frac{G}{2} \frac{1}{Q_0 \sqrt{A^2 + B^2}} - Z \sqrt{A^2 + B^2} \quad [32]$$

$Amp(x)$ should be kept small at the sensor. In Equations [22], [24] and [27], $Z(x)$ is always positive. Also, the $(A^2 + B^2)^{1/2}$ term in Equation [32] is always a damping term. The worst, or least damped case, is therefore found where $Z = 0$. Similarly, examining the first term of Equation [32], the worst case is for $A > 0$ and $B = 0$, for then $\partial Amp / \partial x$ has its most positive value, namely G/Q_0^2 . Therefore, taking $Z = 0$ and $A < 0$ in Equation [31],

$$MAX \ AMP = \int_0 [\frac{\partial Amp}{\partial x}]_{MAX} dx = \frac{1}{Q_0^2} \int_0 G(x) dx \quad [33]$$

is an upper bound on the possible amplitude. But $\int_0 G(x) dx / Q_0$ is ΔT , the end-to-end temperature difference of the stack. Also, if we note that the "gain" of the bellows is always less than unity, we can conclude that a valve gain of

$$\frac{Q_0}{\Delta T} = \text{safe valve gain} \quad [34]$$

cannot possibly sustain oscillations. Returning to Equation [32], we also note that the maximum decrement of the amplitude occurs when $Z(x)$ is large.

b. Phase Information. Another relationship gained from Equations [17] and [18] is the expression giving the rate of rotation of the phase angle with travel distance up the stack.

If we let θ be the angle by which the temperature leads the flow, then

$$\theta = \arctan (-B(x)/A(x)) \quad [35]$$

$$\frac{d\theta}{dx} = \frac{B \frac{\partial A}{\partial x} - A \frac{\partial B}{\partial x}}{A^2 + B^2} \quad [36]$$

and, substituting from Equations [17] and [18], we find that:

$$\frac{d\theta}{dx} = Y(x) - \frac{G(x)}{Q_0^2} \cdot \frac{B}{A^2 + B^2} \quad [37]$$

Therefore, in a region where $G(x) = 0$, the phase angle will change most rapidly with distance for a high value of $Y(x)$.

c. Closed Form Stack Solution with No Isolation. Considerable information can be obtained from the closed form solution of a simple case where the heater and specimens and other items which make up the isolated region are in intimate contact with the coolant stream, and where $G(x)$ is a constant, independent of x_0 .

The differential equation is then:

$$-Q \frac{\partial T}{\partial x} + G = C \frac{\partial T}{\partial x}$$

where C is now the total heat capacity per unit length, and the other terms are as previously used.

If we assume that $Q = Q_0 + \epsilon \cos(\omega t) \cdot K$, as we did before, we are able to find a closed form solution for T_{osc} , the oscillatory part of the temperature at any position x in the stack, for a unit oscillatory component of Q . K is defined and treated as before. The solution is:

$$T_{osc}(x,t) = \frac{G}{Q_0 C \omega} \sin(\omega t - \frac{x \omega C}{Q_0}) - \frac{G}{Q_0 C \omega} \sin(\omega t) \quad [38]$$

The oscillatory part of the temperature at the top of the stack is then given by:

$$T_{osc}(L,t) = \frac{G}{Q_0 C \omega} \sin(\omega t - \frac{L \omega C}{Q_0}) - \frac{G}{Q_0 C \omega} \sin(\omega t) \quad [39]$$

The identity $\sin(A-B) = \sin A \cos B - \cos A \sin B$ can be used to rewrite Equation [39]. When this is done, we obtain:

$$T_{osc}(L,t) = \frac{G}{Q_0 C \omega} \left\{ \left[\cos\left(\frac{L \omega C}{Q_0}\right) - 1 \right] \sin(\omega t) - \sin\left(\frac{L \omega C}{Q_0}\right) \cos(\omega t) \right\} \quad [40]$$

This should be regarded as being of the form:

$$T_{osc}(L,t) = A \{ B \sin(\omega t) + C \cos(\omega t) \} \quad [41]$$

Looking at Equation [40] from this point of view, we see that although the amplitudes and phase angles shift with frequency, the general trend is toward lower amplitudes with higher frequencies, due to the presence of the $(1/\omega)$ factor multiplying all the terms of Equation [40].

At very low frequencies, if we take the limits as $\omega \rightarrow 0$, we find that the coefficients of $\sin(\omega t)$ and $\cos(\omega t)$ become:

$$\lim_{\omega \rightarrow 0} \frac{G}{Q_0 C \omega} \left[\cos\left(\frac{L \omega C}{Q_0}\right) - 1 \right] = 0 \quad [42]$$

for the coefficient of the $\sin(\omega t)$ term and $-GL/Q_0^2$ for the $\cos(\omega t)$ term. Therefore, at very low frequencies, Equation 40 becomes:

$$T_{osc}(L,t) = -\frac{LG}{Q_0^2} \cos(\omega t) \quad [43]$$

If we use the fact that $LG/Q_0 = \Delta T$, the end-to-end temperature rise in the stack, then at very low frequencies, Equation 44 becomes:

$$T_{osc}(L,t) = \frac{-\Delta T}{Q_0} \cos(\omega t) \quad [44]$$

and the oscillatory temperature is 180° out of the phase with the applied fluid oscillation and has an amplitude ratio of $(\Delta T/Q_0)/1.0$. This holds only in the limit as $\omega \rightarrow 0$.

If we make a vector diagram to represent Equation 38, some interesting facts become obvious. In this diagram, shown in Figure 2, ω is fixed and the vectors for various values of x , the distance up the stack, are plotted. We represent the oscillatory items as projections of rotating sectors where the vectors rotate in a counterclockwise direction. A vector in the direction of the $+R$ axis is chosen to represent the oscillatory part of Q , namely $\cos(\omega t)$. Then since $\sin(\omega t)$ lags behind $\cos(\omega t)$ by 90° , the $\sin(\omega t)$ terms are plotted in the $-j$ direction. Consequently, the $-G/Q_0 C \sin(\omega t)$ term is plotted in the $+j$ direction, and the $G/Q_0 C \omega \sin(\omega t - L\omega C/Q_0)$ term can be added vectorially to the tip of the $-b/Q_0 C \omega \cos(\omega t)$ vector. Since this additional term is of constant length and varying phase angle, it generates a circle; since it is of equal length with the previous $+j$ vector, the circle is tangent to the R axis at the origin, as shown in Figure 2.

From Figure 2, we see that for a simple case of a uniform G field and any frequency ω , the sinusoidal temperature at all points in the stack always leads the applied oscillatory fluid flow by 0° to 180° . This relationship only holds for positions in the stack and does not apply in

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FIGURE 2. Stack Temperature Vs. Position. This is a vectorial representation of the oscillatory temperature at various vertical positions in the stack for a fixed value of ω , with a constant G field.

a free flow space above the stack. We also see that if we find a position for which the phase shift is 0^0 , then the amplitude is zero also, and this holds for an arbitrary choice of ω .

In the case where the sensor is at the top of the stack, the oscillatory stack temperature also leads the oscillatory fluid flow by 0^0 to 180^0 , and 0^0 phase shift is accompanied by zero gain. The "black box" which represents the conversion between the average bellows temperature and the flow through the valve has 0^0 phase shift since a positive increase in temperature results in more flow. The "black box" which represents the conversion between the outside of the bellows and the average temperature of the bellows always has a phase shift of 0^0 to 90^0 lagging, as can be seen by viewing it via the analogous RC electrical combination. The sum of the phase shifts of the 3 black boxes should add to zero for stable oscillation, and this can only occur for those frequencies for which the tip of the sector of Figure 2 lies in the first quadrant, for a position at the top of the stack. From this, we arrive at the following points: (1) If the sensor can be kept at the top of a uniform stack and if the phase shift in the sensor can be kept near zero, then oscillation can occur only for situations represented by the vector in Figure 2 being at the lower part of the first quadrant, where the stack gain is zero. As a result, if this set of circumstances can be maintained, the critical gain of the valve is infinite; (2) The presence of an extended free flow space between the stack and the sensor at each frequency has the effect of shifting the phases of the points in Figure 2 toward lagging angles as viewed at the sensor. For small phase shifts, the overall effect is that the stack gain associated with a given phase shift (in the 0^0 to 90^0 leading range) becomes larger, and the general effect is a decrease in the allowed gain of the valve. Thus, free-flow space delay times which delay the transmission of stack temperatures to the sensor are in general deleterious unless the amplitude of the oscillatory wave can be decreased significantly in transit. This general trend has been confirmed by digital calculations.

Next, consider the case of a sensor with a variable time constant built into the sensor, and with the sensor located at the top of the uniform stack analyzed above. In the limit of zero time constant, the phase shift in the sensor is zero for all frequencies, and 0° net shift is only possible for the particular frequency for which the vector of Figure 2 is in the lower part of the first quadrant where the stack gain is zero and the critical gain of the valve is infinite. For very large time constants, the gain in the sensor tends towards zero, with increasing values of the thermal time constant. This argument holds because very low frequencies need not be considered. For extremely low frequencies, the stack shift is $\sim 180^\circ$, and these frequencies cannot be compensated to resonance by a sensor having a phase shift which resides in the 0° to 90° leading range. Therefore, for very large thermal time constants in the sensor, the critical gain of the valve becomes larger for longer thermal time constants of the sensor.

We have shown the following general trends: (1) short transit times from the top of the stack to the sensor are beneficial; (2) extremely short thermal time constants in the sensor are in general beneficial, especially if the sensor is near the top of a nearly uniform stack, (3) extremely long thermal time constants in the sensor are also beneficial, and (4) thermal resonances at very high frequencies are not important due to low stack gains caused by the $1/\omega$ factor in Equation [39].

C. SENSOR ANALYSIS

The sensor has been analyzed using two models. The first model details a situation analogous to an electrical RC circuit. The sensor has a thermal resistance between itself and the coolant stream, all at one temperature, and has a finite known heat capacity. The second model details a situation where the outer surface of the bellows is in intimate thermal contact with the coolant and has known cylindrical radius and a finite and known heat capacity per unit volume and thermal conductivity.

The bellows is assumed to be long compared to its width; and the temperature input at the surface all comes at one level, x , which is an admitted inconsistency at short wavelengths and higher frequencies. (However, in this discussion these are not of importance.)

1. AC-RC Sensor Analysis

The "gain" of the sensor for Case 1 is found to be:

$$\text{Sensor gain}_1 = (1 + \omega^2 \tau^2)^{-1/2} \quad [45]$$

while the phase angle is:

$$\theta = - (\pi/2 - \arctan (\frac{1}{\omega\tau})) \quad [46]$$

quoted as a leading angle, referred to the temperature of the coolant. In Equations [45] and [46], τ is the RC type time-constant of the electrical engineering analogy or is the product of the total heat capacity of the sensor and the thermal resistance linking it to the coolant.

2. Bessel Analysis of a Cylindrical Sensor

For the second model of the sensor, where radial heat flow is assumed in a cylindrical sensor, it is found that the gain is given by:

$$\text{Sensor gain}_2 = \sqrt{D^2 + E^2} \quad [47]$$

where D and E are given by:

$$D = 1 - 4 \sum_{j=1}^{\infty} \left\{ \frac{1}{1 + \left[\frac{Kx_j}{HR_0} \right]^2} \cdot \frac{1}{x_j^2} \right\} \quad [48a]$$

$$E = \frac{4K}{H\omega R_o^2} \sum_{j=1}^{\infty} \frac{1}{1 + \left[\frac{Kx_j^2}{HR_o^2 \omega} \right]^2} \quad [48b]$$

The phase angle, quoted as a lagging angle referred to the temperature applied on the surface of the sensor, is given by:

$$\theta = \arctan \left(\frac{E}{D} \right) \quad [49]$$

D. COMBINED STACK AND SENSOR ANALYSIS

When the gains of the three elements of the loop (stack, sensor, and valve) are expressed as complex numbers and a product is taken, the result is a complex number which is, in fact, the negative of the so-called transfer function.⁽¹⁾ If a valve gain of unity is assumed, a modified negative transfer function is obtained, which must be multiplied by the valve gain to get the true negative transfer function. This modified function can be plotted to get information about the system. A plot of this type is shown in Figure 3. The significance of such plots is discussed in textbooks on control theory and servo-mechanisms.⁽¹⁾

In a very simplified viewpoint, the system will not oscillate if the valve gain is kept low enough so that the plot of the negative transfer function does not enclose the point (1.0). One way of avoiding oscillation is to restrict the gain of the valve to numbers less than the reciprocal of the intercept of the modified negative transfer function on the x-axis.

The valve gain obtained by the method just described will have units of (heat capacity) x (volume flow rate) x (1/temperature) due to the way we defined flow in the stack. (Flow is the volume rate of flow times the heat capacity per unit volume.) When the method described above is used to calculate a value of valve gain low enough to prevent oscillation, the

FIGURE 3. A Modified (Assumed Unity Valve Gain) Negative Transfer Function Produced by Program CYLB.

calculated gain must then be divided by the heat capacity per unit volume for sodium if the results are to be interpreted as a valve gain in units of (volume/time)/temperature.

E. DEGREE OF REGULATION POSSIBLE

We now will calculate the degree of regulation possible with a given valve gain. The following definitions apply:

H.G. = total heat generated per unit time in the stack

T = end-to-end temperature rise in the stack

S = slope of valve equation in units of (flow/unit time)/(unit temperature change)

Q_F = flow in units of volume/time

C_{Na} = heat capacity per unit volume for sodium coolant

The heat balance requires that:

$$H.G. = T Q_F C_{Na} \quad [50]$$

With a fixed, non-variable valve, the flow would be constant, and changes in gamma heat would be related to changes in temperature by:

$$\left(\frac{d(H.G.)}{dT}\right)_{\text{FIXED VALVE}} = Q_F C_{Na} \quad [51]$$

With a variable valve we find that:

$$\left(\frac{d(H.G.)}{dT}\right)_{\text{VARIABLE}} - C_{Na} \left[Q_F + T \frac{dQ_F}{dT}\right] = C_{Na} [Q_F + TS] \quad [52]$$

The question is how many degrees change (end-to-end) occurs with the variable valve, in the case of a gamma heating change that would have caused a 1°C change with a fixed valve. The answer is given by:

$$\frac{\Delta T_{\text{VARIABLE}}}{\Delta T_{\text{FIXED}}} = \left(\frac{d(H.G.)}{dT} \right)_{\text{FIXED}} / \left(\frac{d(H.G.)}{dT} \right)_{\text{VARIABLE}} \quad [53]$$

$$\left(\frac{\Delta T_{\text{VARIABLE}}}{\Delta T_{\text{FIXED}}} \right) = \frac{1.}{1. + TS/Q_F} \quad [54]$$

As an example, end-to-end temperature differences of 300°C, together with a valve gain of 0.8 cm³/°C and a volume flow of 50 cm³/sec give:

$$\left(\frac{\Delta T_{\text{VARIABLE}}}{\Delta T_{\text{FIXED}}} \right) = 0.172 \quad [55]$$

III. COMPUTER PROGRAMS

A. PROGRAM CYLB (CYLINDER-BESSEL)

Program CYLB is a Fortran program that runs on the Richland Boeing CYBER. It could be run elsewhere if subprograms were made available to calculate values for Bessel functions of the first and second kind. A listing of the program appears in Appendix A and a flow chart appears in Figure 4. The purpose of the program is to provide the material needed to make a plot of the modified negative transfer function.

CYLB is a digital finite difference program which integrates Equations [17] and [18] to find $A(x)$ and $B(x)$ at the outer surface of the sensor, calculates the gain of the sensor, and multiplies the two separate gains to obtain the modified negative transfer function. It does this for a sequence of frequencies and prints the results. The user specifies certain loading and geometry parameters. Required input items are: (1) sodium cross-sectional flow area; (2) density of packing of iron; (3) density of packing of stagnant non-flowing sodium; (4) position of top of the specimen zone; (5) length of free flow zone; (6) radius, heat capacity, thermal conductivity, and number of solid cylindrical rods in Zones 1, 2, and 3; (7) presence or absence and details of geometry and thermal properties of a possible hollow cylinder in Zone 3; and (8) radius and thermal properties of the NaK-filled bellows in Zone 4.

As now programmed, CYLB has cells which are each 1.32 cm long. The heater zone is set to include the first 48 cells. The sensor-valve is 8 cells long, and the temperature at the outer surface of the sensor is assumed to be the temperature at the third cell of the fourth zone.

In the calculations, it is found that errors arise at high frequencies and short wave lengths unless the cells are subdivided. In the subdivision, the values of $Y(x)$ and $Z(x)$ are assumed to be independent of the position inside the 1.32 cm long cell, but Equations [13] and [14] together with

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FIGURE 4. Flow Diagram for Program CYLB.

Equations [17] and [18] are integrated in small substeps. The number of substeps is set by an operator input value of parameter NJ in line 690 of the program, which must be set before the program is run.

Before running the program, the operator must set the dimensional and material parameter choices in lines 220 through 280, lines 360 and 370, and lines 760 and 720. The choice of whether a hollow cylinder exists or not is set in line 620. The number of substeps in a cell is set in line 690, and the number of Bessel functions to be used in each part of the calculation is set in lines 730, 740, and 750. The significance of the choices and meanings of the symbols are explained in comment statements interspersed in the program area from line 380 to line 710. The XJ data starting in line 290 is a listing of the arguments for which $J_0(x) = 0$, and should not be disturbed. The setting of the number of Bessel functions in lines 730 to 750 should be done remembering that the results appear to be almost independent of the choice of values between 5 and 16, while 40 is an upper limit and larger numbers require longer running time. After the program parameters are suitably set, the program is run while the subprograms RBESY and NBESJ are attached and available. On the Richland CYBER this can be accomplished by the commands:

```
Edit CYLB, S
Attach (NBESJ, MATHSCIENCES)
RUN, F, N
CON OUTPUT
XEQ, LDSET=LIB=NBESJ, LOAD=LGO, EXECUTE
```

In the foregoing there are no spaces except one after "edit" and one after "con". When the program runs, it absorbs the operator input choices up to line 870, then swings over to line 1400 (Program Statement No. 3), and proceeds to calculate and assign at each cell level all groups of factors which are independent of ω and can be stored and reused during generation of the transfer function. The free flow area of

sodium, linear density of sodium, and linear density of iron in each cell area are assigned in preparation for the calculation of the gamma heating in each cell. In line 1820 the total linear mass to be used in calculating gamma heating is assigned to each cell and is called, temporarily incorrectly, G (for gamma heating). Then in lines 1830 through 1940 the gamma heating at each level is correctly calculated and stored with the aid of a subroutine GHEAT, and the total energy input of the entire stack is calculated and stored. At this time, the program also calculates and stores the coolant flow required to maintain the desired steady-state end-to-end temperature difference (300°C).

If the end-to-end temperature difference of the stack is desired as an adjustable input variable, it can be entered by changing line 1900 = $\text{RK2} = \text{SUMG}/300$ to 1900 = $\text{RK2} = \text{SUMG}/\text{ENTEM}$ and setting ENTEN in a new line inserted near the start of the program. After the gamma heating of each cell is calculated and stored, the required flow is calculated and stored, and nearly all reusable parameter groups independent of ω are calculated and stored for each cell, the question is asked in line 1980 whether a hollow cylinder exists in region 3. If so, the program swings over the line 2690 and calculated reusable parameter groups independent of ω which will arise in calculations of $Y(x)$ and $Z(x)$ for the hollow cylinder in region 3 of the stack. These items are stored and the program returns to line 1990. Process flow shortly thereafter swings to Program Statement No. 4 in line 880.

At this point, all reusable parameter groups independent of ω have been calculated and stored for each cell. The program then proceeds in a loop from line 880 (Program Statement No. 4) to line 1320 (Program Statement No. 47) and back, choosing progressively larger values of ω , and, for each choice of ω , integrating equivalently Equations [17] and [18] or Equations [13] and [14], and calculating the complex loop gain in each case, for an assumed valve gain of unity. The results are printed out for each value of ω , using the print command and format of lines 1300 and 1310.

The "gain" of the sensor is calculated following Equation [42] through [44] of this report, and the calculation is done by subroutine SENANG, called in line 1240. Then the negative modified transfer function is calculated in lines 1250 and 1260, using the equivalent of complex number multiplication.

In the loop from Program Statement 4 to Program Statement 47, immediately before the integration of Equations [17] and [18] (as accomplished in lines 1080 through 1200), the question is asked in line 1070 whether there is a hollow cylinder in region 3, and if so, the subroutine CYL is used to modify $Y(x)$ and $Z(x)$ for cells in that region.

The "WRITE" statement in line 1300 of the program produces items called W2, GNMAX, ANGLE, I, G7, AR(NT), BR(N7), SP, and YP. This makes a total of 9 columns of output. The more familiar terms or definitions associated with these quantities are as follows:

1. W2 = omega, is the angular frequency.
2. GNMAX is the maximum allowed gain in $\text{cm}^3/\text{O}^\circ\text{C}$ if this value of omega happens to be the one which causes the negative modified transfer function to cross the X-axis.
3. ANGLE is the total phase shift angle of the negative modified transfer function. It is sometimes garbled due to the manner in which the computer calculates an arctan and should be checked against columns 8 and 9 for offset errors which are multiples of $\pi/2$.
4. I is the loop number of the loop between Program Statements 4 and 47.
5. G7 is the gain of the sensor.
6. AR(NT) in Column 6, and

7. BR(NT) in Column 7 are the $A(x)$ and $B(x)$ coefficients of Equations [17] and [18] evaluated at the sensor. These can be used to calculate the stack gain and stack angle.
8. XP in Column 8, and
9. YP in Column 9 are the vector components of the modified negative transfer coefficient, or the negative of the transfer coefficient as it would be for a valve gain of unity.

The normal way of using the program is to set the adjustable input parameters and then set the "DO LOOP" parameters of lines 930 and 950 to do a coarse search. The values shown are suitable. Then Columns 1, 8, and 9 of the printout are observed to determine approximately the values of ω for which Columns 8 and 9 indicate a plot of XP and YP will give crossings of the positive XP axis. The "DO LOOP" parameters of lines 930 and 950 are then set to search in these regions with a finer search and GNMAX of Column 2 is observed for those values of ω for which the plot of YP versus XP crosses the positive XP axis.

B. PROGRAM BITT (AR-RC ANALYSIS)

Program BITT was a predecessor to Program CYLB. The logic and the flow are similar to CYLB, but there are noticeable differences. Program BITT uses the same general small signal analysis approach, and the main purpose is to integrate Equations [17] and [18] and determine the maximum valve gain that can be used without encountering sustained oscillations. Program BITT is listed in Appendix B and a flow chart appears in Figure 5.

The major differences between CYLB and BITT are the following: (1) BITT uses the AC-RC approach, i.e., BITT uses Equations [21] and [22] to calculate the $Y(x)$ and $Z(x)$ factors needed to integrate Equations [17]

FIGURE 5. Flow Diagram for Program BITT.

and [18]. That is, the semi-isolated regions at each level are assumed to be internally isothermal and are connected to the flowing sodium by a simple thermal resistance. CYLB used a continuous medium at each level and employed an analysis of time dependent radial flow in a cylinder. This involved Equations [23], [24], [25], [26], and [27]; (2) As a result of Item (1), the input to CYLB is different; (3) The output of BITT is different from that of CYLB. As it now stands, the BITT program does not give the negative transfer function. As now programmed, it can be easily used to give a safe value of valve gain which will avoid sustained oscillations.

BITT takes each frequency and calculates (a) the valve sensor time constant which would give a 0° phase angle for the negative transfer function, and (b) the valve gain in $\text{cm}^3/^\circ\text{C}$ which would result in an overall loop gain of 1.0 for operation at that frequency with the stated sensor time constant. Printing is suppressed for frequencies for which it is impossible to find a sensor-time constant that will give a net 0° phase shift for the 3 black-box loop.

In running the program, the operator enters general input data items in lines 200 through 230. These are: (1) GNAS1, the linear density in gms/cm of stagnant, non-flowing sodium in each of the four stack regions; (2) GFE1, the linear density of iron in gms/cm in each of the 4 stack regions; (3) A4, the cross-sectional flow area of flowing sodium in each of the four stack regions; and (4) TAUL1, the time constants of the isolated or semi-isolated parts of the cell in each of the four stack regions. These time constants are to be entered as time constants for leakage of heat from the isolated part of the cell, through the existing thermal resistance, calculated on the assumption that the heat is flowing into an infinite sink.

In BITT, as now written, the stack has heaters in cells 1-48, and has specimens in cells 49-69. The length of region 3 is adjustable and is NC

cells long. This adjustment can be made in line 310. The thermal sensor is in the third cell out of eight cells in the sensor-valve region, Region No. 4. In BITT, as in CYLB, the cells are each 1.32 cm long. However, just as in CYLB, the cells are subdivided for integration purposes and the degree of subdivision is adjustable in line 290.

In the processing of running, the program absorbs and stores the input data constants while executing the lines from 200 to 470. In lines 480 and 490 it calculates the heat capacity per unit length of all stagnant items in each of the four regions. Program flow then swings to the program area from lines 1020 to 1260 where the input data is used to assign values of mass densities and flow areas to each individual cell. Then in lines 1280 to 1320 the mass density and area information is used to calculate a heat generation rate for each cell, calculated with the aid of subroutine GHEAT. In lines 1330 to 1430 this information is used together with the end-to-end temperature rise of 300°C to calculate the steady state flow of sodium required to maintain the required 300°C temperature difference. If the end-to-end temperature difference is desired as an input variable, it can be set in line 1360.

In line 1440, program flow swings back to line 510 and a loop is begun in line 560. In the loop, a value of omega is set, the $Y(x)$ and $Z(x)$ factors are calculated, and the stack gain at the sensor level is calculated after A and B at the sensor are obtained by integrating Equations 17 and 18. The phase angle of the temperature in the flowing sodium, outside the sensor, is calculated, and then a calculation is made of the sensor relaxation time required to give an offsetting phase shift. This tells the sensor relaxation time required to get a net 0° phase shift around the 3 black-box loop. Then, assuming such a sensor relaxation time, the program calculates the valve gain $(\text{cm}^3/\text{sec})/^{\circ}\text{C}$ required to give a total loop gain of 1.0. For cases where it is impossible to compensate to 0° phase shift, printing is suppressed. This suppression occurs in lines 800 and 810. When compensation is possible which will produce 0° net phase shift, the printout gives:

In Column 1, ω

In Column 3, the sensor relaxation time required to obtain a net 0° phase shift.

In Column 2, the valve gain in $(\text{cm}^3/\text{sec})/^\circ\text{C}$ which gives a total loop gain of 1.0 with a sensor relaxation time as shown in Column 3.

In Column 4, the frequency $f = \omega/2\pi$.

In Column 5, the loop number, so that the operator can see which loops were printed and which were not.

In actual operation, one would ordinarily set the 4 time constants in the 4 regions, set the other adjustable parameters, and enter the command RUN, F. Then the output table would be searched to find all values of ω for which the 0° net phase shift occurred in conjunction with a time constant which was in fact the same as the time constant entered for Region 4. The maximum valve gain listed for each such value of ω would then be noted and recorded.

Additional printing can be obtained by BITT by setting IJUMP = 0 in line 280. This causes a complete printout of A, B, Y, Z, and $(A^2 + B^2)^{1/2}$ for each cell for each value of ω .

IV. REFERENCES

1. Principles of Servo-Mechanisms, Gordon S. Brown and Donald P. Campbell, John Wiley and Sons, 1948, Chapters 5 and 6.

APPENDIX A

LISTING OF PROGRAM CYLB

```

100=      PROGRAM CYLB(OUTPUT,TAPE6=OUTPUT)
110=C      VARIABLE NUM OF BESSEL FUNCTS, VARIABLE LENGTH FREE FLOW
120=C      LENGTH,OUTPUT GIVES TRANSFER FUNCTION AND CRIT GAIN.
130=C      PROGRAM HAS PROVISION FOR HOLLOW CYLINDER IN REGION 3.
140=      DIMENSION A(120),GFE(120),GNAS(120),G(120),T(120)
150=      DIMENSION GFE1(4),GNAS1(4)
160=      DIMENSION AR(120),BR(120),A4(4),RKK(120)
170=      DIMENSION RK(4),H(4),RZ(4),XJ(40),RN(4),XA(120),XB(120),RNN
(120)
180=      DIMENSION CYLRAD(2),CYLKC(2)
190=      COMMON/B/Y(120),Z(120)
200=      COMMON/A/RNORM(40),RLAMB(40)
210=      COMMON/C/L2,NL,NU,RK2,W2,CYLCON,CYLCAP
220=      DATA H/4.5028,1.7452,2.25,1.61/
230=      DATA CYLRAD/1.,2./
240=      DATA CYLKC/1.,2./
250=      DATA RZ/.3175,.4487,1.52,.7747/
260=      DATA RK/.2302,.4623,.40,.6076/
270=      DATA RN/19.,7.,4.,1./
280=      DATA A4/1.7391,3.085,.32,.7161/
290=      DATA XJ/2.4048256,5.5200781,8.6537279,11.7915344,14.9309177
,18.071
300=      10640,21.2116366,24.3524715,27.4934791,30.6346065,33.7758202
,36.917
310=      10984,40.0584258,43.1997917,46.3411884,49.4826099,52.6240518
,55.765
320=      15108,58.9069839,62.0484692,65.1899648,68.3314693,71.4729816
,74.614
330=      15006,77.7560256,80.8975559,84.0390908,87.1806298,90.3221726
,93.463
340=      17188,96.6052680,99.7468199,102.8883743,106.0299309,109.1714
896,112
350=      1.3130503,115.4546127,118.5961766,121.7377421,124.8793089/
360=      DATA GFE1/58.6894,16.7241,22.3,2.31/
370=      DATA GNAS1/0.,.995,3.3,1.485/
380=C      RKK() IS CONDUCTIVITY IN EACH CELL BUT RK() IS SINGLE VALUE OF
390=C      CONDUCTIVITY OF RODS IN EACH OF FOUR REGIONS. RNN() IS NM
BER OF
400=C      RODS IN EACH CELL, RN() IS SINGLE VALUE IN EACH OF 4 REGION
S.
410=C      CYLKC() IS 1. CONDUCTIVITY OF HOLLOW ROD IN REGION 3, 2. HE
AT CAP/
420=C      UN. VOL. OF SAME. CYLRD() IS THE INNER AND OUTER RADIIIS OF
THE HOL
430=C      CYL IN REGION 3. ICYL IS AN INDICATOR TO TELL IF THE HOLLO
W

```

440=C CYLINDER IN REG. 3 EXISTS OR NOT. 1=YES, 0=NO. L1 IS TOP C
 ELL
 450=C OF SPECIMEN REGION. L2 IS BOTTOM CELL OF REGION 3. NB1,2,
 3, ARE
 460=C NUMBER OF BESSEL FUNCTIONS IN 1 RODS, 2,BELLOWS TYPE SENSOR
 ,
 470=C AN 3 THE HOLLOW CYLINDER OF REGION 3. TOP OF HEATER REGIO
 N
 480=C IS SET AT THE 48TH CELL. NU IS TOP CELL OF MOTA, NT IS SENSOR L
 OC
 490=C ATION. NL IS TOP OF REGION 3, NM IS BOTTOM OF REGION 4. ALL TH
 ESE
 500=C DERIVED BY PROGRAM FROMCELL 48 (FIXED) ,L1, AND NC. IF ICYL IS
 SET
 510=C TO BE 1, THEN INSERT DUMMY NUMBERS FOR RK AND H IN REGION 4
 TO
 520=C PREVENT OVERFLOW ERRORS, BUT SET RN(4)=0. "A" IS CROSS SEC
 TIONAL
 530=C FLOW AREA OF FLOWING SODIUM, GFE IS GRAMS PER CM OF FE, GNAS IS
 GRAMS
 540=C PER CM OF STAGNANT NA NOT ACCOUNTED FOR BY "A". Y,Z,AR,BR, ARE
 DISCUSS
 550=C ED ON PAGE 127 OF N-1958.
 560=C RK() IS AVERAGE CONDUCTIVITY IN A CYLINDER. H() IS THE PUV
 570=C AVERAGE HEAT CAPACITY IN A CYLINDER. RZ() IS THE RADIUS B
 580=C A CYLINER. XJ() ARE THE ZEROES OF JZERO. RN() IS THE NUM
 590=C BER OF ISOLATED CYLINDERS IN EACH REGION. XA() IS KAPPA/
 600=C (H*RZ**2)
 610=C LET NC BE TH NUMBER OF CELLS IN THE THIRD REGION. NORM=4
 620= ICYL=1
 630= CYLCON=CYLKC(1)
 640= CYLCAP=CYLKC(2)
 650= RIN=CYLRAD(1)
 660= ROUT=CYLRAD(2)
 670= L1=69
 680= L2=L1+1
 690= NJ=5
 700= RNJ=NJ
 710=C NJ IS NUMBER OF SUBDIVISIONS OF A CELL,MUST BE 2 OR MOR
 720= NC=23
 730= NB1=16
 740= NB2=16
 750= NB3=16
 760= W1=.00628318*2.*.025
 770= AR(1)=0.
 780= BR(1)=0.
 790=C RK2 IS THE HEAT CAP PER UNIT VOL OF SODIUM TIMES FLOW RATE.
 800=C RK2S IS RK2**2 OR IT IS Q.ZERO**2
 810=C RK2DS IS 2.*Q.ZERO**2
 820= TALE=-1.
 830= NL=L1+NC
 840= NM=L2+NC
 850= NT=NM+3
 860= NU=NL+8

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870=      GO TO 3
880=      4 CONTINUE
890=C      1.1295 IS A ROUGH VALUE FOR THE HET CAP PER UN VOL OF
900=C      NA.
910=C      1.255 IS NOMINAL HEAT CA P OF NA PER UNIT MASS
920=C      1.32 IS CELL LENGTH. .58 IS NOM HEAT CAP FE PER UN MASS
930=      DO 47 I=1,50
940=      RI=I
950=      W2=RI*W1*60.
960=      DO 55 IJ=1,NU
970=      Y(IJ)=0.
980=      DO 37 ID=1,NB1
990=      37 Y(IJ)=Y(IJ)+XJ(ID)**2*XB(IJ)/(W2**2+(XJ(ID)**2*XA(IJ))**2)
1000=      Y(IJ)=(RNN(IJ)*Y(IJ)+A(IJ)*1.1295)*W2*1.32/RK2
1010=      Z(IJ)=0.
1020=      DO 38 ID=1,NB1
1030=      38 Z(IJ)=Z(IJ)+1./(W2**2+(XA(IJ)*XJ(ID)**2)**2)
1040=      Z(IJ)=4.*3.14159*RKK(IJ)*(W2**2)*Z(IJ)*RNN(IJ)
1050=      Z(IJ)=Z(IJ)*1.32/RK2
1060=      55 CONTINUE
1070=      IF(ICYL.EQ.1) CALL CYL(NB3)
1080=      DO 41 J=2,NU
1090=      AR1=AR(J-1)
1100=      BR1=BR(J-1)
1110=      Y1=Y(J-1)/RNJ
1120=      Z1=Z(J-1)/RNJ
1130=      G1=G(J-1)/RNJ
1140=      DO 88 IZ=1,NJ
1150=      AR2=AR1-BR1*Y1-G1/RK2S-AR1*Z1
1160=      BR2=BR1+AR1*Y1-BR1*Z1
1170=      AR1=AR2
1180=      88 BR1=BR2
1190=      AR(J)=AR2
1200=      BR(J)=BR2
1210=      41 CONTINUE
1220=      GANSTK=SQRT(AR(NT)**2+BR(NT)**2)
1230=      THETA=ATAN(-BR(NT)/AR(NT))
1240=      CALL SENANG(RK(4),XJ,H(4),RZ(4),W2,G7,T9,NB2)
1250=      XP=G7*(AR(NT)*COS(T9)-BR(NT)*SIN(T9))*1.1295
1260=      YP=-G7*(BR(NT)*COS(T9)+AR(NT)*SIN(T9))*1.1295
1270=      GNMAX=1./(1.1295*G7*GANSTK)
1280=      ANGLE=THETA-T9
1290=      FREK=W2/(2.*3.14159)
1300=      WRITE (6,48) W2,GNMAX,ANGLE,I,G7,AR(NT),BR(NT),XP,YP
1310=      48 FORMAT(1H ,F5.2,E8.2,E8.2,I3,5(1X,E8.2))
1320=      47 CONTINUE
1330=      43 STOP
1340=C      PROGRAM PARTA(INPUT,TAE5=INPUT, ETC)
1350=C      THIS PROGRAM SETS THE INITIAL CONDITIONS FO THE MOTA
1360=C      SEE HEDL N 1958 (T24) P27
1370=C      GFE IS THE GRAMS OF STEEL PER CIM, GNAS IS THE GRAMS OF
1380=C      STAGNANT SODIUM PER CM. A IS THE CROSS-SECTIONAL
1390=C      FLOW AREA OF THE FLOWING SODIUM.
1400=      3 CONTINUE

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1410=      XY=RK(1)/(H(1)*RZ(1)**2)
1420=      XY2=XY*4.*3.14159*RK(1)
1430=      DO 21 IK=1,48
1440=      XA(IK)=XY
1450=      XB(IK)=XY2
1460=      RNN(IK)=RN(1)
1470=      GFE(IK)=GFE1(1)
1480=      GNAS(IK)=GNAS1(1)
1490=      RKK(IK)=RK(1)
1500=  21  A(IK)=A4(1)
1510=      XY=RK(2)/(H(2)*RZ(2)**2)
1520=      XY2=XY*4.*3.14159*RK(2)
1530=      DO 22 IK=49,L1
1540=      XA(IK)=XY
1550=      XB(IK)=XY2
1560=      RNN(IK)=RN(2)
1570=      GFE(IK)=GFE1(2)
1580=      GNAS(IK)=GNAS1(2)
1590=      RKK(IK)=RK(2)
1600=  22  A(IK)=A4(2)
1610=      XY=RK(3)/(H(3)*RZ(3)**2)
1620=      XY2=XY*4.*3.14159*RK(3)
1630=      DO 23 IK=L2,NL
1640=      XA(IK)=XY
1650=      XB(IK)=XY2
1660=      RNN(IK)=RN(3)
1670=      GFE(IK)=GFE1(3)
1680=      GNAS(IK)=GNAS1(3)
1690=      RKK(IK)=RK(3)
1700=  23  A(IK)=A4(3)
1710=      XY=RK(4)/(H(4)*RZ(4)**2)
1720=      XY2=XY*4.*3.14159*RK(4)
1730=      DO 24 IK=NM,NU
1740=      XA(IK)=XY
1750=      XB(IK)=XY2
1760=      RNN(IK)=RN(4)
1770=      GFE(IK)=GFE1(4)
1780=      GNAS(IK)=GNAS1(4)
1790=      RKK(IK)=RK(4)
1800=  24  A(IK)=A4(4)
1810=      DO 25 IK=1,NU
1820=      G(IK)=1.32*((1./1.3)*GFE(IK)+GNAS(IK)+.9*A(IK))
1830=      RIK=IK
1840=      RL1=(RIK-.5)*1.32-20.*2.54
1850=      CALL GHEAT(GH,RL1)
1860=  25  G(IK)=G(IK)*GH
1870=      SUMG=0.
1880=      DO 32 IE=1,NU
1890=  32  SUMG=SUMG+G(IE)
1900=      RK2=SUMG/300.
1910=C      RK2 IS FLOW RATE TIMES HEAT CAP PER UN VOLUME.
1920=      RK2S=RK2**2
1930=      RK2DS=2.*RK2S
1940=      RRR=RK2/1.1295

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1950=C      RRR IS FLOW IN CUB CM PER SEC.
1960=      WRITE(6,33) RRR
1970=      33 FORMAT(1H ,23HFLOW IN CM3 PER SEC IS ,E10.4)
1980=      IF(ICYL.EQ.1) CALL CYLSET(RIN,ROUT,NB3)
1990=      IF(ICYL.EQ.1) WRITE(6,75) RIN,ROUT,NB3
2000=      75 FORMAT(1H ,18HANNULAR RING. RIN=,F6.2,1X,SHROUT= ,F6.2,1X,9
HNUM BE
2010=      ISS=,I3)
2020=      GO TO 4
2030=      END
2040=      SUBROUTINE SENANG(RK1,X,H,R,OMEG,G,T9,NB2)
2050=      DIMENSION X(40),D(40),XSQ(40)
2060=C      RDI IS CONDUCTIVITY, X IS A SET OF ZEROES OF JZERO, R IS
2070=C      THE CYLINDER RADIUS, G IS THE GAIN AND T9 IS THE PHASE
2080=C      SHIFT, LAGGING BEHIND THE DRIVING TEMPERATURE ON THE OUT
2090=C      ER SURFACE, H IS THE HEAT CAPACITY PER UNIT VOLUME.
2100=C      SENANG CALCULATES THE GAIN (LESS THAN 1.) AND PHASE
2110=C      SHIFT (LAGGING NGLE POSITIVE) OF TH CYLINDRICAL SENSOR
2120=C      DRIVEN ON THE OUTER SURFACE BY A SINUSOIDAL TEMPERATURE.
2130=C      SEE HEDL-N-1978 PAGES 45 AND 46.
2140=      DO 6 I=1,NB2
2150=      6 XSQ(I)=X(I)**2
2160=      RSQ=R**2
2170=      DO 7 I=1,NB2
2180=      7 D(I)=((RK1*XSQ(I))/(H*RSQ*OMEG))**2
2190=      E1=0.
2200=      E2=0.
2210=      DO 8 I=1,NB2
2220=      E3=1./(1.+D(I))
2230=      E1=E1+E3/XSQ(I)
2240=      8 E2=E2+E3
2250=      E1=1.-4.*E1
2260=      E2=4.*E2*(OMEG**(-1))*RK1/(H*RSQ)
2270=      G=SQRT(E1**2+E2**2)
2280=      T9=ATAN(E2/E1)
2290=      RETURN
2300=      END
2310=      SUBROUTINE GHEAT(GH,X)
2320=C      THES SUBROUTIE CALCULATES GAMMA HEAT IN WATTS PER GRAM
2330=C      FOR SODIUM WHERE THE INPUT IS IN CM VERTICAL DISTANCE FROM
2340=C      THE REACTOR MIDPLANE IN FTR. GOOD FOR SUBASSEMBLIES 1202
2350=C      2202 AND 3202. SEE HEDL N 1958 P20-21.
2360=      IF(X.LT.-42.) GO TO 70
2370=      IF(X.GE.-42..AND.X.LE.45.) GO TO 71
2380=      IF(X.GT.45.) GO TO 72
2390=      70 GH=63.7575+4.81749*X+.133554*X**2+1.60624E-3*X**3
2400=      GH=GH+7.14177E-06*X**4
2410=      GH=1./GH
2420=      GO TO 73
2430=      71 GH=6.59365-1.72503E-02*X-1.66901E-03*X**2+4.28092E-06*X**3
2440=      GH=GH+8.26642E-08*X**4
2450=      GO TO 73
2460=      72 GH=226.471-1.60379E+01*X+.417065*X**2-4.71612E-03*X**3
2470=      GH=GH+1.96912E-05*X**4

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2480=      GH=1./GH
2490= 73  RETURN
2500=      END
2510=      SUBROUTINE CYL(NB3)
2520=      COMMON/A/RNORM(40),RLAMB(40)
2530=      COMMON/B/Y(120),Z(120)
2540=      COMMON/C/L2,NL,NU,RK2,W2,CYLC ON,CYLCAP
2550=      ZAD=0.
2560=      YAD=0.
2570=      DO 12 IJ=1,NB3
2580=      MAG3=RNORM(IJ)*(1./(W2**2+((CYLC ON/CYLCAP)*RLAMB(IJ)**2)**2
))
2590=      MAG4=MAG3*(CYLC ON/CYLCAP)*RLAMB(IJ)**2
2600=      ZAD=ZAD+MAG3
2610= 12  YAD=YAD+MAG4
2620=      YAD=(YAD*4.*3.14159*CYLC ON*W2)*(1.32/RK2)
2630=      ZAD=(ZAD*4.*3.14159*CYLC ON*W2**2)*(1.32/RK2)
2640=      DO 14 IJ=L2,NL
2650=      Y(IJ)=Y(IJ)+YAD
2660= 14  Z(IJ)=Z(IJ)+ZAD
2670=      RETURN
2680=      END
2690=      SUBROUTINE CYLSET(A,B,N)
2700=      DIMENSION ZROB(40)
2710=      DIMENSION DJ(2),EJ(2),FY(2),GY(2),XROB(40)
2720=      COMMON/A/RNORM(40),RLAMB(40)
2730=      RLIM=.000001
2740=      CALL BESZER(N,A,B,ZROB,RLIM)
2750=      DO 26 IJ=1,N
2760=      CALL NBESJ(ZROB(IJ),1,DJ)
2770=      CALL RBESY(ZROB(IJ),1,FY,IER)
2780=      XROB(IJ)=(B/A)*ZROB(IJ)
2790=      CALL RBESY(XROB(IJ),1,GY,IER)
2800=      CALL NBESJ(XROB(IJ),1,EJ)
2810=      UONB=DJ(1)*GY(1)-FY(1)*EJ(1)
2820=      UINA=DJ(1)*FY(2)-FY(1)*DJ(2)
2830=      RLAMB(IJ)=ZROB(IJ)/A
2840=      UMAG=((B/A)*(UONB/UINA))**2-1.
2850= 26  RNORM(IJ)=1./UMAG
2860=      END
2870=      SUBROUTINE BESZE(N,A,B,ZROB,RLIM)
2880=      DIMENSION ZROB(40)
2890=C      N IS NUMBER OF ZEROES DESIRED, A AND B ARE INNER AND OUTER
2900=C      RADIUS, ZROB IS A COLUMN VECTOR OF ARGUMENTS FOR WHICH
2910=C       $Y1/J1((B/A)*ZROB)=Y0/J0(ZROB)$ . RLIM IS FINE-NESS OF ZERO
2920=C      DETERMINATION.
2930=      XT=.09
2940=      DXT=.05
2950=      DO 27 I=1,N
2960=      CALL ZER(A,B,XT,DXT,RTXT,RLIM)
2970=      ZROB(I)=RTXT
2980= 27  XT=RTXT+20.*RLIM
2990=      RETURN
3000=      END

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3010=      SUBROUTINE ZER(A,B,ZT,DXT,RTXT,RLIM)
3020=C      A AND B ARE INNER AND OUTER RADIUS, ZT IS STARTING POINT, D
XT IS      STEP LENGTH AND RTXT IS LPACE OF ZERO VALUE OF EIGEN FUNCTI
3030=C      ON,
3040=C      RLIM IS SMALL STEP TEST TO SEE IF THE FINE-NESS OF STEP IS
SUFF-      ICIENT TO STOP THE HUNT PROCESS.
3050=C      DYT=DXT
3060=      RBA=B/A
3070=      XT=ZT
3080=      CALL VALTS(XT,DIF,RBA)
3090=      GDIF=DIF
3100=      23 XT=XT+DYT
3110=      CALL VALTS(XT,DIF,RBA)
3120=      RTT=DIF/GDIF
3130=      IF(RTT.LT.0.) GO TO 22
3140=      GO TO 23
3150=      22 GDIF=DIF
3160=      DYT=-DYT/3.
3170=      ABDYT=ABS(DYT)
3180=      IF(ABDYT.GT.RLIM) GO TO 23
3190=      RTXT=XT
3200=      RETURN
3210=      END
3220=      SUBROUTINE VALTS(XTS,DIF,RBA)
3230=      DIMENSION BJ(2),BY(2),CJ(2),CY(2)
3240=      CALL NBESJ(XTS,1,CJ)
3250=      CALL RBESY(XTS,1,CY,IER)
3260=      XTB=RBA*XTS
3270=      CALL NBESJ(XTB,1,BJ)
3280=      CALL RBESY(XTB,1,BY,IER)
3290=      AJ=CJ(1)
3300=      AY=CY(1)
3310=      DIF=BY(2)*AJ-AY*BJ(2)
3320=      RETURN
3330=      END
3340=

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APPENDIX B

LISTING OF PROGRAM BITT

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100=      PROGRAM BITT(OUTPUT,TAPE6=OUTPUT)
110=C      THIS IS AN AC-RC TYPE PROGRAM FOR DOING A SMALL THERMAL OSC
ILL       ATION ANALYSIS OF THE MOTA.  THE SEMI ISOLATED REGIONS AT E
120=C      LEVEL IN THE STACK ARE ASSUMED TO BE INTERNALLY ISOTHERMAL
ACH       ARE CONNECTED TO THE FLOWING SODIUM AT THAT LEVEL BY A RESI
AND       STANCE
140=C      DIMENSION A(120),GFE(120),GNAS(120),G(120),T(120)
150=      DIMENSION RT(120),CTS(120)
160=      DIMENSION TAUL(120),AR(120),BR(120),Y(120),Z(120)
170=      DIMENSION GFE1(4),GNAS1(4)
180=      DIMENSION CTS1(4),A4(4),TAUL1(4)
190=      DATA GNAS1/0.,.995,3.3,1.485/
200=      DATA GFE1/58.6894,16.7241,22.3,2.31/
210=      DATA A4/1.7391,3.085,.32,.7161/
220=      DATA TAUL1/.509,.31,2.25,.275/
230=      CTS() IS TOTAL HEAT CAPACITY OF STAGNANT ITEMS
240=C      TAUL() IS TAU THE RELAXATION TIME AT LEVEL, ASSUMING INFIN
ITE       HEAT CAPACITY OF THE FLOWING PART.
260=C      LET NC BE THE NUMBER OF CELLS IN THE THIRD REGION NORM=4.
270=C      IJUMP=1
280=      NJ=5
290=      RNJ=NJ
300=      NC=23
310=      W1=.00628318*2.*.025
320=      AR(1)=0.
330=      BR(1)=0.
340=      RK2 IS THE HEAT CAP PER UNIT VOL OF SODIUM TIMES FLOW RATE.
350=C      RK2S IS RK2**2 OR IT IS Q.ZERO**2
360=C      RK2DS IS 2.*Q.ZERO**2
370=C      TALE=-1.
380=      NL=69+NC
390=      NM=70+NC
400=      NT=73+NC
410=      NU=77+NC
420=      THE TAUL 1-4 ARE THE TAU'S FOR EQUILIBRATION OF THE STATIC
430=C      MATERIAL TO THE FLOW STREAM IF THE FLOW IS AN INFINITE
440=C      SINK
450=C      CTS1-4 IS HEAT CAPACITY TOTAL,(STAGNANT ONLY) AND INCLUDES
460=C      FE+NONFLO SODIUM AND IS ON A PER CM BASIS.
470=C      DO 69 I=1,4
480=      69 CTS1(I)=.58*GFE1(I)+1.255*GNAS1(I)
490=      GO TO 3
500=      4 CONTINUE
510=      1.1295 IS A ROUGH VALUE FOR THE HEAT CAP PER UN VOL OF
520=C      NA.
530=C

```

```

540=C      1.255 IS NOMINAL HEAT CAP OF NA PER UNIT MASS
550=C      1.32 IS CELL LENGTH. .58 IS NOM HEAT CAP FE PER UN MASS
560=      DO 47 I=1,50
570=      RI=I
580=      W2=RI*W1*60.
590=      DO 55 IJ=1,NU
600=      Y(IJ)=(1.32*W2/RK2)*(A(IJ)*1.1295+(GFE(IJ)*.58+GNAS(IJ)*1.2
55)/((W
610=      12**2)*TAUL(IJ)**2+1.))
620=      55 Z(IJ)=(1.32*CTS(IJ)/RK2)*(TAUL(IJ)*W2**2)/((W2*TAUL(IJ))**2
+1.)
630=      DO 41 J=2,NU
640=      AR1=AR(J-1)
650=      BR1=BR(J-1)
660=      Y1=Y(J-1)/RNJ
670=      Z1=Z(J-1)/RNJ
680=      G1=G(J-1)/RNJ
690=      DO 88 IZ=1,NJ
700=      AR2=AR1-BR1*Y1-G1/RK2S-AR1*Z1
710=      BR2=BR1+AR1*Y1-BR1*Z1
720=      AR1=AR2
730=      88 BR1=BR2
740=      AR(J)=AR2
750=      BR(J)=BR2
760=      RT(J)=SQRT(AR(J)**2+BR(J)**2)
770=      41 CONTINUE
780=      GANSTK=SQRT(AR(NT)**2+BR(NT)**2)
790=      THETA=ATAN(-BR(NT)/AR(NT))
800=      IF(THETA.GT.0..AND.AR(NT).GT.0.) TALE=+1.
810=      IF(THETA.LT.0..OR.AR(NT).LT.0.) GO TO 47
820=      TAUVAL=-BR(NT)/(AR(NT)*W2)
830=      G2=1./SQRT(1.+(W2*TAUVAL)**2)
840=      GNMAX=1./(1.1295*G2*GANSTK)
850=      FREK=W2/(2.*3.14159)
860=      WRITE(6,48) W2,GNMAX,TAUVAL,FREK,I
870=      IF(IJUMP.EQ.1) GO TO 47
880=      WRITE(6,99) AR
890=      WRITE(6,99) BR
900=      WRITE(6,99) Y
910=      WRITE(6,99) Z
920=      WRITE(6,99) RT
930=      99 FORMAT(14(1H ,9(E8.2),/))
940=      48 FORMAT(1H ,F7.4,1X,F8.3,1X,F8.2,1X,F7.4,1X,14)
950=      47 CONTINUE
960=      43 STOP
970=C      THIS PART SETS THE INITIAL CONDITIONS FOR THE MOTA
980=C      SEE HEDL N 1958 (T24) P27
990=C      GE IS THE GRAMS OF STEEL PER CM, GNAS IS THE GRAMS OF
1000=C     STAGNANT SODIUM PER CM. A IS THE CROSS-SECTIONAL
1010=C     FLOW AREA OF THE FLOWING SODIUM.
1020=      3 CONTINUE
1030=      DO 21 IK=1,48
1040=      GFE(IK)=GFE1(1)
1050=      GNAS(IK)=GNAS1(1)
1060=      TAUL(IK)=TAUL1(1)

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1070=      CTS(IK)=CTS1(1)
1080=    21 A(IK)=A4(1)
1090=      DO 22 IK=49,69
1100=      GFE(IK)=GFE1(2)
1110=      GNAS(IK)=GNAS1(2)
1120=      TAUL(IK)=TAUL1(2)
1130=      CTS(IK)=CTS1(2)
1140=    22 A(IK)=A4(2)
1150=      DO 23 IK=70,NL
1160=      GFE(IK)=GFE1(3)
1170=      GNAS(IK)=GNAS1(3)
1180=      TAUL(IK)=TAUL1(3)
1190=      CTS(IK)=CTS1(3)
1200=    23 A(IK)=A4(3)
1210=      DO 24 IK=NM,NU
1220=      GFE(IK)=GFE1(4)
1230=      GNAS(IK)=GNAS1(4)
1240=      TAUL(IK)=TAUL1(4)
1250=      CTS(IK)=CTS1(4)
1260=    24 A(IK)=A4(4)
1270=      DO 25 IK=1,NU
1280=      G(IK)=1.32*((1./1.3)*GFE(IK)+GNAS(IK)+.9*A(IK))
1290=      RIK=IK
1300=      RL1=(RIK-.5)*1.32-20.*2.54
1310=      CALL GHEAT(GH,RL1)
1320=    25 G(IK)=G(IK)*GH
1330=      SUMG=0.
1340=      DO 32 IE=1,NU
1350=    32 SUMG=SUMG+G(IE)
1360=      RK2=SUMG/300.
1370=C      RK2 IS FLOW RATE TIMES HEAT CAP PER UN VOLUME.
1380=      RK2S=RK2**2
1390=      RK2DS=2.*RK2S
1400=      RRR=RK2/1.1295
1410=      WRITE(6,33) RRR
1420=    33 FORMAT(1H ,23HFLOW IN CM3 PER SC IS ,E10.4)
1430=C      RRR IS FLOW IN CM3 PER SEC
1440=      GO TO 4
1450=      END
1460=      SUBROUTINE GHEAT(GH,X)
1470=C      THIS SUBROUTINE CALCULATES GAMMA HEAT IN WATTS PER GRAM
1480=C      FOR SODIUM WHERE THE INPUT IS IN CM VERTICAL DISTANCE FROM
1490=C      THE REACTOR MIDPLANE IN FTR. GOOD FOR SUBASSEMBLIES 1202
1500=C      2202 AND 3202. SEE HEDL N 1958 P20-21.
1510=      IF(X.LT.-42.) GO TO 70
1520=      IF(X.GE.-42..AND.X.LE.45.) GO TO 71
1530=      IF(X.GT.45.) GO TO 72
1540=    70 GH=63.7575+4.81749*X+.133554*X**2+1.60624E-3*X**3
1550=      GH=GH+7.14177E-06*X**4
1560=      GH=1./GH
1570=      GO TO 73
1580=    71 GH=6.59365-1.72503E-02*X-1.66901E-03*X**2+4.28092E-06*X**3
1590=      GH=GH+8.26642E-08*X**4
1600=      GO TO 73

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```
1610= 72 GH=226.471-1.60379E+01*X+.417065*X**2-4.71612E-03*X**3
1620=    GH=GH+1.96912E-05*X**4
1630=    GH=1./GH
1640= 73 RETURN
1650=    END
..
```