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**A THERMAL ANALYSIS OF PSEUDO-STEADY
CIRCULATING WELLBORES**

Douglas D. Cline

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A THERMAL ANALYSIS OF PSEUDO-STEADY CIRCULATING WELLBORES

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ABSTRACT

A mathematical model for the fluid temperature distribution inside a wellbore undergoing circulation is developed. The formulation assumes that the heat transfer processes between the formation and the circulating fluid in the wellbore occur in a pseudo-steady manner. A closed form solution to the counterflow energy equations is developed in a manner which allows for arbitrary temperature distributions in the formation. Results are presented for both linear as well as bilinear formation temperatures.

As a result of this investigation it is concluded that the outlet fluid temperature is a nonunique function of the geothermal gradient. In addition, for a specified wellbore configuration, an optimum set of circulation conditions may be identified for which the outlet fluid temperature is a maximum for a given inlet fluid temperature.

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NOMENCLATURE

c_p	Specific heat capacitance of fluid
h	Flow enthalpy
h_{fp}	Primary fluid heat transfer coefficient
h_{fa}	Annulus fluid heat transfer coefficient
k	Thermal conductivity
k_p	Primary wellbore parameter, Equation (15c)
k_a	Annulus wellbore parameter, Equation (15d)
$\tilde{k}$	Product of k_a , k_p
L	Total well depth
$\dot{m}$	Mass flow rate of circulation
q	Heat transfer rate per unit depth
r	Radial direction
t	Time
T	Temperature
$\tilde{T}$	Geothermal temperature at slope discontinuity
U_{pa}	Overall heat transfer coefficient between primary and annulus fluids
U_{a5}	Overall heat transfer coefficient between annulus fluid and outermost casing
U_{eff}	Effective heat transfer coefficient due to soil
z	Axial coordinate
z^*	Normalized well depth, z/L
$\tilde{z}$	Depth of bilinear slope discontinuity
z_0	Normalized depth of slope discontinuity, $\tilde{z}/L$
α	Heat transfer parameter, Equation (23)
α_∞	Soil thermal diffusivity
θ	Normalized temperature, T/T_{in}

$\Delta\theta_o$	Normalized outlet temperature, Equation (27a,c)
$\Delta\theta_{bh}$	Normalized bottom-hole temperature, Equation (27b,d)
θ_∞	Geothermal temperature distribution, equation (14)
κ	Biot number, Equation (3c)
λ	Fourier number, Equation (3b)

Subscripts

a	Annulus
bf	Formation at bottom-hole depth
bh	Bottom-hole
g	Geothermal slope
in	Inlet
o	Outlet
p	Primary
s	Formation at surface
1	Near-surface gradient
2	Secondary gradient
∞	Far-field soil

1. INTRODUCTION

The estimation of fluid temperatures during drilling and circulation requires an understanding of the thermal processes which occur inside the wellbore. This knowledge may be used to predict the behavior of drilling fluids and drilling equipment in the extreme temperatures of the down-hole environment. In geothermal drilling, this problem is of particular importance since underground reservoirs of geothermal steam and hot water are likely to produce temperature gradients in the formation which are more severe than those encountered in petroleum drilling. To aid in the determination of down-hole temperatures during drilling and circulation, mathematical models have been developed which simulate the various heat transfer processes between the formation and the wellbore. As with most fluid-heat transfer problems, the formulation of these models is based upon the governing conservation equations of mass, momentum, and energy. The results of these efforts have led either to simplified mathematical models,^{1,2} or to the development of wellbore thermal simulator codes^{3,4} which are capable of detailed thermal analysis.

In spite of these advancements, many questions regarding heat transfer in circulating wellbores have yet to be answered. In particular, questions regarding the sensitivity, uniqueness, and the general trends produced by the mathematical solutions must be resolved in order to apply these models towards solving practical thermal problems.

To gain some insight into these problem areas, a pseudo-steady-state thermal wellbore model has been developed which incorporates the salient thermal phenomena which occur between the formation and the drilling fluids circulating in the wellbore. The present model is based upon the well known counterflow heat exchanger equations which may be found in many standard heat transfer texts.⁵ A closed form solution to the governing equations is developed which allows for arbitrary temperature distributions in the formation. Thus problems associated with both linear and bilinear geothermal gradients may be addressed.

Previous Work

Edwardson⁶ et al. have calculated temperature disturbances in the formation using an assumed temperature distribution for the circulating fluid in the annulus region. Later, Tragesser⁷ et al. used the simplified temperature profile method of Edwardson to calculate transient temperature profiles during cementing operations. The calculated temperatures were required to satisfy energy balance equations at discrete intervals of depth inside the wellbore. The theoretical analysis of Raymond¹ solved the governing energy conservation equations assuming a linear temperature profile in the formation. Transient temperature distributions for the primary and annular fluids were obtained by numerical integration of the governing equations, while pseudo-steady state solutions for the temperature profiles were obtained by a similar integration of the

heat diffusion equation in the formation. The work of Holmes and Swift² neglected initial transients during the onset of circulation and used a steady state approximation in the energy balance equations for the heat transferred between the formation and the annular fluid. However, as shown by Raymond, this approach is unrealistic since no true steady state is ever achieved due to thermal diffusion in the formation.

The analyses described above have been primarily concerned with the predictive capabilities of wellbore thermal models. The present study departs from this trend in that the governing equations have been parameterized so as to accentuate non-unique relationships between calculated temperatures and a particular characteristic of the circulating system, e.g., flow rate, geothermal gradient. It will be shown that under certain conditions, the assumption of uniqueness may lead to significant errors when estimating down-hole temperatures from known circulation data.

2. THEORETICAL MODEL

Presented in Fig. 1 is a wellbore configuration which receives heat from a bilinear temperature distribution in the formation. During the drilling process, the drilling fluid travels down the primary bore and returns to the surface through the annular region between the drill pipe and the casing. The heat transfer between the soil and the circulating fluid decreases with time as the thermal gradients in the soil adjacent to the wellbore become small. The temperature difference between the inlet fluid and the outlet fluid must, in some manner, reflect the net heat gained by the fluid from the formation.

Simple analytical models for the transient heat transfer in the formation have been developed by many investigators prior to the widespread use of digital computers.^{8,9} Many of the simplifying assumptions presented in these earlier works have been implemented into the present formulation. It will therefore be assumed that the flow of heat from the formation to the wellbore will occur primarily in the radial direction. Furthermore, it will be assumed that the thermal resistance in the wellbore is small compared to the soil thermal resistance. Consequently, heat transmission inside the wellbore occurs rapidly compared to that in the soil. Under these assumptions, the flow of heat between the formation and the wellbore shall be considered to be pseudo-steady. The analysis of Raymond showed that these conditions are typically achieved within approximately four hours of circulation or after one or two circulation cycles. The principal

FIGURE 1. SCHEMATIC OF WELLBORE CONFIGURATION

difficulty to overcome in this type of formulation is the modeling of the transient heat transfer in the formation. The approach utilized in the present study is to approximate this process with an effective overall heat transfer coefficient which incorporates the time dependent function of Ramey.⁸ The formulation of this coefficient is similar to the approach developed by Willhite.⁹

2.1 Heat Transfer

The heat transfer to the circulating fluids is illustrated in Fig. 2, where the wellbore is shown to consist of a simple tube-shell arrangement. The drilling fluid travels down the primary bore and returns up the annulus. Heat is supplied to the wellbore by an arbitrary geothermal temperature distribution which is assumed to be at least a piecewise continuous function of depth.

For a cylindrical coordinate geometry, the heat transfer rate per unit depth between the annulus fluid and that in the primary bore is given by,

$$q_p = 2\pi U_{pa} (T_a - T_p) \quad (1a)$$

where the subscripts p and a denote the primary and annular regions respectively. The overall heat transfer coefficient between the annulus fluid and the primary fluid is given by

$$U_{pa} = \left[\frac{1}{r_1 h_{fp}} + \frac{\ln(r_2/r_1)}{k_{12}} + \frac{1}{r_2 h_{fa}} \right]^{-1} \quad (1b)$$

FIGURE 2. PREDOMINANT MODES OF HEAT TRANSFER
IN THE CIRCULATING SYSTEM

Similarly, the heat transfer rate per unit depth between the well-formation interface and the annulus fluid is given by

$$q_a = 2\pi U_{a5} (T_5 - T_a) \quad (2a)$$

The overall heat transfer coefficient between the outermost casing and the annulus fluid is given by

$$U_{a5} = \left[\frac{1}{r_3 h_{fa}} + \frac{\ln(r_4/r_3)}{k_{34}} + \frac{\ln(r_5/r_4)}{k_{45}} \right]^{-1} \quad (2b)$$

Note that additional levels of casing string and cement may be accounted for in the above relation by simply including the appropriate thermal resistances.

In order to model the pseudo-steady heat transfer from the formation, the solution of Carslaw and Jaeger¹⁰ for the heat flux across an infinite, homogeneous medium at temperature T_∞ to an internally bounded cylinder at temperature T_5 is utilized. This relationship is given by

$$q = \frac{8}{\pi} k_\infty (T_\infty - T_5) \int_0^\infty \frac{e^{-\lambda \psi^2}}{\left[\left(\frac{\psi}{\kappa} \right) J_1(\psi) + J_0(\psi) \right]^2 + \left[\left(\frac{\psi}{\kappa} \right) Y_1(\psi) + Y_0(\psi) \right]^2} \frac{d\psi}{\psi} \quad (3a)$$

where

$$\lambda = \frac{\alpha_\infty t}{r_5^2} \quad (3b)$$

$$\kappa = \frac{U_{a5} r_5}{k_\infty} \quad (3c)$$

The thermal conductivity of the infinite medium is given by k_{∞} while α_{∞} represents its thermal diffusivity. The functions J_0 , J_1 , Y_0 and Y_1 represent Bessel functions of order zero and one, respectively. Equation (3a) may be written in a more concise form as follows

$$q = \frac{2\pi k_{\infty} (T_{\infty} - T_5)}{F(\lambda, \kappa)} \quad (4)$$

where $\frac{\pi^2}{4F(\lambda, \kappa)}$ represents the integral term appearing in Equation (3a). The above equation may be compared to the formulation derived by Willhite.⁹ In the nomenclature of Willhite, (as well as that of Ramey⁸) it can be seen that

$$f(t) = F(\lambda, \kappa) \quad (5)$$

where values of $f(t)$ have been tabulated by Willhite. For long circulation times, i.e., greater than 7 days, approximate values for the function $f(t)$ may be obtained from the following relationship.⁸

$$f(t) = \ln[2\sqrt{\lambda}] - 0.29 \quad (6)$$

Using Equation (2a) to eliminate T_5 in Equation (4) gives the heat transfer rate per unit length between the formation and the annulus fluid as

$$q = \frac{2\pi k_{\infty} U_{a5}}{k_{\infty} + U_{a5} F(\lambda, \kappa)} (T_{\infty} - T_a) \quad (7)$$

By comparison with Equation (2a), an effective overall heat transfer coefficient between the formation and the annulus may be defined as

$$U_{\text{eff}}(\lambda, \kappa) = \frac{k_{\infty} U_{a5}}{k_{\infty} + U_{a5} F(\lambda, \kappa)} \quad (8)$$

and thus the heat transfer to the annulus is given by

$$q_a = 2\pi U_{\text{eff}} (T_{\infty} - T_a) \quad (9)$$

Notice that as λ approaches zero ($t \rightarrow 0$), Equation (8) reduces to $U_{\text{eff}} = U_{a5}$, while as λ becomes infinite ($t \rightarrow \infty$), U_{eff} approaches zero.

In the formulation of Equation (9) two points are worthy of consideration. First, as noted by Hickox¹¹ the temperature T_5 appearing in Equation (3) is time dependent and thus exact solutions should account for this dependency through a superposition of solutions to account for the "thermal history" of the wellbore. However, it is evident from previous analyses of other investigators that the variation in the heat transfer with time is sufficiently small that significant errors are not introduced by neglecting the superposed solutions. A second assumption which may not be apparent in the above formulation concerns the variation of U_{eff} with depth. In general, the thermal resistance between the formation and the wellbore will be a function of depth owing to variations in casing layers as well as local fluid film coefficients. For the purposes of estimating an effective heat transfer coefficient, it will be

assumed that the effect of these nonuniformities are small. Therefore, the effective overall heat transfer coefficient in Equation (9) is to be treated as being only time dependent and independent of the axial coordinate.

2.2 Energy Equation

The governing equations for the fluid energy balance in the primary and annular regions may be determined from fundamental thermodynamics. For a steady flow process in the primary bore, the axial variation of the flow enthalpy is given by:

$$\frac{dh_p}{dz} = \frac{q_p}{\dot{m}} + g \quad (10a)$$

The corresponding conservation equation for the fluid in the annular region is given by

$$-\frac{dh_a}{dz} = \frac{-q_p + q_a}{\dot{m}} - g \quad (10b)$$

In the development of Equations (10a) and (10b), the independent coordinate z is taken to be positive downward. Thus, the formulation of Equation (10b) expresses the variation of h_a with respect to z , while the flow is directed up the annulus as shown in Fig. 2.

Assuming the drilling fluid to be incompressible and considering the effects of potential energy changes to be negligible allows Equations (10a,b) to be written as

$$c_p \frac{dT_p}{dz} = \frac{q_p}{\dot{m}} \quad (11a)$$

$$- c_p \frac{dT_a}{dz} = \frac{-q_p + q_a}{\dot{m}} \quad (11b)$$

Equations (1a) and (9) may be substituted into the above relations to yield the following coupled set of linear ordinary differential equations,

$$\frac{dT_p}{dz} = \frac{2\pi U_{pa}}{\dot{m} c_p} (T_a - T_p) \quad (12a)$$

and

$$\frac{dT_a}{dz} = \frac{2\pi U_{pa}}{\dot{m} c_p} (T_a - T_p) - \frac{2\pi U_{eff}}{\dot{m} c_p} (T_\infty(z) - T_a) \quad (12b)$$

The dependent variables and independent coordinate are to be normalized by the following dimensionless variables,

$$\theta_p = \frac{T_p}{T_{in}} \quad (13a)$$

$$\theta_a = \frac{T_a}{T_{in}} \quad (13b)$$

$$z^* = \frac{z}{L} \quad (13c)$$

where T_{in} corresponds to the inlet temperature of the circulating fluid and L represents the total well depth. The bilinear geothermal gradient is to be normalized in accordance with the labeling in Fig. 1. The appropriate relationships over each subinterval are defined below.

$$\theta_{\infty 1} = \theta_{g1} z^* + \theta_{s1} \quad \text{on} \quad 0 \leq z^* \leq z_0 \quad (14a)$$

$$\theta_{\infty 2} = \theta_{g2} z^* + \theta_{s2} \quad \text{on} \quad z_0 \leq z^* \leq 1 \quad (14b)$$

where

$$\theta_{g1} = \frac{\tilde{T} - T_s}{T_{in} z_0}, \quad \theta_{s1} = \frac{T_s}{T_{in}} \quad (14c)$$

$$\theta_{g2} = \frac{T_{bf} - \tilde{T}}{T_{in}(1-z_0)}, \quad \theta_{s2} = \frac{\tilde{T}}{T_{in}} - \frac{T_{bf} - \tilde{T}}{T_{in}} \left(\frac{z_0}{1-z_0} \right) \quad (14d)$$

and

$$z_0 = \frac{\tilde{z}}{L}$$

Incorporating the newly defined variables into Equations (12a,b) results in the following dimensionless equations.

$$\frac{d\theta_p}{dz^*} = k_p (\theta_a - \theta_p) \quad (15a)$$

$$\frac{d\theta_a}{dz^*} = k_p (\theta_a - \theta_p) - k_a (\theta_{\infty}(z^*) - \theta_a) \quad (15b)$$

where

$$k_p = \frac{2\pi U_{pa} L}{\dot{m} c_p} \quad (15c)$$

$$k_a = \frac{2\pi U_{eff} L}{\dot{m} c_p} \quad (15d)$$

The dimensionless groups k_p and k_a indicate the relative contributions of radial conduction and film coefficient heat transfer to the forced convection inside the wellbore.

One would therefore expect that large values for the wellbore parameters would correspond to conditions in which the heat transfer processes inside the wellbore are conduction dominated. Similarly, small values for the wellbore parameters would correspond to conditions in which the wellbore heat transfer is convection dominated. Notice that the annulus parameter, k_a is an implicit function of circulation time as required by Equation (8). Also, the normalized geothermal gradient θ_∞ appearing in Equation (15b) has been left as an unspecified thermal potential.

The boundary conditions to be applied to Equations (15a,b) are as follows,

$$z^* = 0 \quad , \quad \theta_p = 1 \quad (16a)$$

$$z^* = 1 \quad , \quad \frac{d\theta_p}{dz^*} = 0 \quad (16b)$$

The first condition requires that the inlet fluid temperature be specified, while the latter imposes the condition that the primary and annular fluid temperatures match at the bottom of the wellbore.

2.3 Solution Procedure

The governing dimensionless equations and their corresponding boundary conditions constitute a coupled system of equations for which a closed form solution is sought. Expressing Equations (15a,b) in operator form, and eliminating θ_a from Equation (15b) yields a linear second order ordinary differential equation for θ_p .

$$\frac{d^2 \theta_p}{dz^{*2}} - k_a \frac{d\theta_p}{dz} - \tilde{k} \theta_p = -\tilde{k} \theta_\infty(z^*) \quad (17)$$

where $k = k_a k_p$.

The method of Laplace Transforms is used to solve the above equation. The convolution integral theorem¹² is utilized in the solution procedure in order to develop a general solution in terms of the unspecified geothermal temperature distribution, $\theta_\infty(z^*)$. The solution for θ_p is then used in conjunction with Equation (15a) to determine the temperature distribution for the annulus fluid. This procedure yields the following general results:

Primary Fluid:

$$\begin{aligned} \theta_p(z^*) = & \frac{1}{(A-B)} (Ae^{Az^*} - Be^{Bz^*}) + \frac{(C-k_a)}{(A-B)} (e^{Az^*} - e^{Bz^*}) \\ & - \frac{\tilde{k}}{(A-B)} \int_0^{z^*} \theta_\infty(\tau) (e^{A(z^*-\tau)} - e^{B(z^*-\tau)}) d\tau \end{aligned} \quad (18)$$

Annulus Fluid:

$$\begin{aligned} \theta_a(z^*) = & \frac{1}{k_p(A-B)} (A^2 e^{Az^*} - B^2 e^{Bz^*}) + \frac{(C-k_a)}{k_p(A-B)} (Ae^{Az^*} - Be^{Bz^*}) \\ & - \frac{\tilde{k}}{k_p(A-B)} \frac{d}{dz^*} \left\{ \int_0^{z^*} \theta_\infty(\tau) (e^{A(z^*-\tau)} - e^{B(z^*-\tau)}) d\tau \right\} \\ & + \frac{1}{(A-B)} (Ae^{Az^*} - Be^{Bz^*}) + \frac{(C-k_a)}{(A-B)} (e^{Az^*} - e^{Bz^*}) \\ & - \frac{\tilde{k}}{(A-B)} \int_0^{z^*} \theta_\infty(\tau) (e^{A(z^*-\tau)} - e^{B(z^*-\tau)}) d\tau \end{aligned} \quad (19)$$

where

$$A = \frac{k_a}{2} + \frac{1}{2}\sqrt{k_a^2 + 4\tilde{k}} \quad (20a)$$

$$B = \frac{k_a}{2} - \frac{1}{2}\sqrt{k_a^2 + 4\tilde{k}} \quad (20b)$$

The constant C, (which represents $\frac{d\theta_p}{dz^*}$ evaluated at $z^* = 0$) must be determined from the requirement that the bottom-hole temperatures match, i.e., Equation (16b). The thermal contribution from the geothermal gradient in each of the above expressions appears in the form of a convolution integral with τ being a dummy variable of integration. The only requirement to be met by θ_∞ is that it be piecewise continuous and integrable over the closed interval $0 \leq z^* \leq 1$. For situations in which θ_∞ is a piecewise function (as in the bilinear case) the integral must be evaluated over each subinterval. Similarly, the derivative of the integral (appearing in Equation (19)) must be evaluated by applying Leibnitz's rule to each piecewise integral.

The bilinear relationship for the geothermal gradient (Equation (14)) may be substituted into each of the equations presented above. Performing the necessary mathematical operations in a piecewise manner yields the final results for the temperature distribution in the primary and annular fluids:

Primary Fluid:

$$\theta_p(z^*) = \{A+\alpha\} \frac{e^{Az^*}}{(A-B)} - \{B+\alpha\} \frac{e^{Bz^*}}{(A-B)} - \frac{\tilde{k}}{(A-B)} I \quad (21)$$

Annular Fluid:

$$\theta_a(z^*) = \left\{ \frac{A(A+\alpha)}{k_p} \right\} \frac{e^{Az^*}}{(A-B)} - \left\{ \frac{B(B+\alpha)}{k_p} \right\} \frac{e^{Bz^*}}{(A-B)} \quad (22)$$

$$- \frac{k_a}{(A-B)} \frac{dI}{dz^*} + \theta_p(z^*)$$

where α is a function of only k_a , k_p and the geothermal temperature distribution. The expression for α is given by

$$\alpha = \frac{k_a k_p}{(Ae^A - Be^B)} \left\{ \theta_{s1} \left[\left(e^A - e^{A(1-z_o)} \right) - \left(e^B - e^{B(1-z_o)} \right) \right] \right.$$

$$+ \theta_{g1} \left[\left(\frac{e^A}{A} - z_o e^{A(1-z_o)} - \frac{e^{A(1-z_o)}}{A} \right) \right.$$

$$\left. - \left(\frac{e^B}{B} - z_o e^{B(1-z_o)} - \frac{e^{B(1-z_o)}}{B} \right) \right] + \theta_{s2} \left[e^{A(1-z_o)} - e^{B(1-z_o)} \right] \quad (23)$$

$$+ \theta_{g2} \left[\left(\frac{e^{A(1-z_o)}}{A} (z_o^{A+1}) - \frac{1}{A} \right) - \left(\frac{e^{B(1-z_o)}}{B} (z_o^{B+1}) - \frac{1}{B} \right) \right] \left. \right\}$$

$$- \frac{A^2 e^A - B^2 e^B}{Ae^A - Be^B}$$

The values for the integrals and their derivatives evaluated over each subinterval are given as follows,

On the subinterval $0 \leq z^* \leq z_o$:

$$I = \left(\frac{\theta_{s1} A + \theta_{g1}}{A^2} \right) (e^{Az^*} - 1) - \left(\frac{\theta_{s1} B + \theta_{g1}}{B^2} \right) (e^{Bz^*} - 1) + \frac{\theta_{g1} (A-B)}{AB} z^* \quad (24a)$$

$$\frac{dI}{dz^*} = \theta_{s1} \left(e^{Az^*} - e^{Bz^*} \right) + \theta_{g1} \left(\frac{e^{Az^*} - 1}{A} - \frac{e^{Bz^*} - 1}{B} \right) \quad (24b)$$

On the subinterval $z_0 \leq z^* \leq 1$:

$$\begin{aligned} I = & \theta_{s1} \left(\frac{e^{Az^*} - e^{A(z^*-z_0)}}{A} - \frac{e^{Bz^*} - e^{B(z^*-z_0)}}{B} \right) \\ & - \theta_{g1} \left[\left(z_0 \frac{e^{A(z^*-z_0)}}{A} - \frac{e^{Az^*} - e^{A(z^*-z_0)}}{A^2} \right) \right. \\ & \left. - \left(z_0 \frac{e^{B(z^*-z_0)}}{B} - \frac{e^{Bz^*} - e^{B(z^*-z_0)}}{B^2} \right) \right] \\ & + \theta_{s2} \left(\frac{e^{A(z^*-z_0)}}{A} - \frac{e^{B(z^*-z_0)}}{B} + \frac{A-B}{AB} \right) \\ & + \theta_{g2} \left[\left(\frac{z_0 e^{A(z^*-z_0)} - z^*}{A} + \frac{e^{A(z^*-z_0)} - 1}{A^2} \right) \right. \\ & \left. - \left(\frac{z_0 e^{B(z^*-z_0)} - z^*}{B} + \frac{e^{B(z^*-z_0)} - 1}{B^2} \right) \right] \end{aligned} \quad (25a)$$

$$\begin{aligned}
\frac{dI}{dz^*} = & \theta_{s1} \left[\left(e^{Az^*} - e^{A(z^*-z_o)} \right) - \left(e^{Bz^*} - e^{B(z^*-z_o)} \right) \right] \\
& + \theta_{g1} \left[\left(\frac{e^{Az^*} - e^{A(z^*-z_o)}}{A} - z_o e^{A(z^*-z_o)} \right) \right. \\
& \left. - \left(\frac{e^{Bz^*} - e^{B(z^*-z_o)}}{B} - z_o e^{B(z^*-z_o)} \right) \right] \\
& + \theta_{s2} \left[e^{A(z^*-z_o)} - e^{B(z^*-z_o)} \right] \\
& + \theta_{g2} \left[\left(\frac{e^{A(z^*-z_o)}}{A(z_o^{A+1})} - \frac{1}{A} \right) - \left(\frac{e^{B(z^*-z_o)}}{B(z_o^{B+1})} - \frac{1}{B} \right) \right]
\end{aligned} \tag{25b}$$

It is interesting to note that the relationships for the case of a linear geothermal gradient are inclusive in the above set of equations. By setting $z_o = 1$ in the relationship for α and considering the subinterval in Equation (24) to be defined by $0 \leq z^* \leq z_o = 1$, the appropriate relationships for the linear gradient may be determined.

A result of particular importance for the purposes of this analysis is an expression for the outlet temperature. Evaluating Equation (22) at $z^* = 0$ gives the simple expression

$$\theta_o - \theta_{in} = \theta_o - 1 = \frac{k_a + \alpha}{k_p} \tag{26}$$

This relationship for the temperature difference between the outlet and inlet flows must reflect the net heat exchange between the formation and the counterflowing fluids. Since the effect of the geothermal gradient is present only in the α parameter, it is this term in Equation (26) which determines the influence of the geothermal gradient on the outlet fluid temperature. Therefore α represents the theoretical mechanism which transmits thermal information about the particular form of the geothermal gradient to the outlet temperature.

3. DISCUSSION OF RESULTS

The equations developed in the previous section have been solved for the nondimensional temperature distribution in the primary and the annular flows as a function of the dimensionless groups k_p , k_a and the geothermal gradient which may be either linear or bilinear in form. An explicit equation for the outlet fluid temperature has been derived from these results. These equations may be evaluated using the simple program supplied in Appendix A.

In order to demonstrate how this solution may be applied towards the thermal analysis of a circulating wellbore, a sample test case has been chosen based on the numerical examples of Wooley.⁴ The circulation times were chosen to be 10 days and 60 days. Fluid circulation rates of 10 gal/min ($6.3 \times 10^{-4} \text{ m}^3/\text{s}$) and 100 gal/min ($6.3 \times 10^{-3} \text{ m}^3/\text{s}$) were assumed to exist for the duration of each of the two circulation times. By choosing these rates, trends for both conduction dominated circulation as well as convection dominated circulation may be established. Details of the wellbore configuration and the circulating fluid properties are given in Appendix B. The evaluation of the overall heat transfer coefficients needed to calculate k_p and k_a for each of the flowing conditions described above is also presented in Appendix B.

3.1 Evaluation of the Wellbore Parameters

For each circulation condition, two uniform overall heat transfer coefficients were estimated for the sample wellbore,

one for the heat transfer to the primary fluid and another for the heat transfer to the annular fluid (see Appendix B). These values were required to compute the wellbore parameters k_p and k_a .

By Equation (15c) it is obvious that the primary parameter, k_p , may be calculated directly from the known wellbore geometry for a given mass flow rate. The annulus parameter, k_a , may be calculated once the effective heat transfer coefficient given by Equation (8) has been determined. This calculation requires the specification of a circulation time in order to estimate $F(\lambda, \kappa)$ using the tabulated values given by Willhite (or the long term approximation given by Equation (6)). Thermal properties for the formation have been assumed to be 2.0 Btu/hrft^{°F} (3.46 W/m^{°C}) and 0.0476 ft²/hr (1.23x10⁻⁶ m²/s) for the thermal conductivity and the thermal diffusivity, respectively. Performing the calculations outlined above for each mass flow rate at times $t = 10$ days and $t = 60$ days yields the following tabulated values.

TABLE 1
Calculated Values of the Wellbore Parameters

Time (days)	$\frac{\text{gal}}{\text{min}} \left(\frac{\text{m}^3}{\text{s}} \right)$	Primary Parameter k_p	Annulus Parameter k_a
10	10 (6.3x10 ⁻⁴)	18.83	3.391
	100 (6.3x10 ⁻³)	13.14	0.361
60	10 (6.3x10 ⁻⁴)	18.83	2.691
	100 (6.3x10 ⁻³)	13.14	0.282

Before proceeding further with the discussion of results, several important points should be made regarding the above parameters. Obviously, both k_p and k_a are functions of the particular circulation conditions under consideration. As the flow rate is increased the overall heat transfer coefficients U_{pa} and U_{a6} increase due to the increased values for the fluid film coefficients (see Appendix B). As the flow rate is increased, the increasing denominator in Equation (15c) (for k_p) is offset by a corresponding increase in the numerator. The net effect is to cause a comparatively small reduction in the primary parameter k_p . In contrast, the influence of flow rate on k_a is such that its denominator increases while its numerator decreases. Once the thermal resistance of the soil is accounted for in Equation (8), the effective heat transfer coefficient will show a net reduction, i.e., $U_{eff} \leq U_{a6}$. The reduction in the effective heat transfer coefficient combined with the increased mass flow rate produces a significant reduction in k_a as the flow rate is increased. Also, the effect of increasing well depth, L , would result in a corresponding increase in both wellbore parameters. Finally, it should be reiterated that the annular parameter k_a decreases in an exponential manner with increasing circulation time, while the primary parameter, k_p is independent of the circulation time.

3.2 Comparisons For a Linear Geothermal Temperature

The wellbore parameters k_p and k_a have been determined from the known wellbore geometry and circulation conditions which

include fluid and soil thermal properties as well as a specified circulation time. It is important that the simplifying assumptions which were made in the formulation of these parameters be verified. For this purpose, the present theory shall be compared with numerical solutions generated by the GEOTEMP wellbore thermal simulator code.¹³

The geothermal temperature is taken to be linear with a surface temperature of 70°F (21.1°C) and a bottom-hole temperature of 145°F (62.7°C) at 5000 ft (1524 m). For an inlet temperature of 70°F (21.1°C), Equation (14a) gives $\theta_s = 1.0$ and $\theta_g = 1.0714$ (with $Z_o = 1$) for the normalized formation temperature. Evaluating Equations (21) and (22) for the flow rate of 10 gal/min ($6.3 \times 10^{-4} \text{ m}^3/\text{s}$) produces the results presented in Figs. 3A and B for circulation times of 10 and 60 days respectively. The comparison between the parameterized theory and the GEOTEMP solutions is considered to be very good at both circulation times. The closed form solution accurately predicts both the bottom-hole temperature and the resulting outlet temperature. Note that the temperature in the returning fluid closely follows the formation temperature at the low flow rate. As the wellbore circulates for the 50-day time interval, the bottom-hole fluid temperature decreases slightly with time while the outlet fluid temperature increases. The parameterized theory predicts these trends quite well. In Fig. 4A and B, wellbore temperature distributions are presented for the same two circulation times at a flow rate of 100 gal/min ($6.3 \times 10^{-3} \text{ m}^3/\text{s}$). At the higher flow

$T_{in} = 70^{\circ}\text{F}$ (21.1°C)

$K_p = 18.830$

$K_a = 3.391$

$\dot{m} = 10 \text{ gal/min}$ ($6.3 \times 10^{-4} \text{ m}^3/\text{s}$)

$T = 10 \text{ DAYS}$

● NUMERICAL SOLUTION
 — PRESENT THEORY
 - - GEOTHERMAL GRADIENT

FIGURE 3A. COMPARISON OF THEORY TO NUMERICAL SOLUTION AT 10 DAYS CIRCULATION TIME -- LOW FLOW RATE

$T_{in} = 70^{\circ}\text{F} (21.1^{\circ}\text{C})$

$K_p = 18.830$

$K_a = 2.691$

$\dot{m} = 10 \text{ gal/min} (6.3 \times 10^{-4} \text{ m}^3/\text{s})$

$T = 60 \text{ DAYS}$

FIGURE 3B. COMPARISON OF THEORY TO NUMERICAL SOLUTION AT 60 DAYS CIRCULATION TIME -- LOW FLOW RATE

$T_{in} = 70^{\circ}\text{F} (21.1^{\circ}\text{C})$

$K_p = 13.140$

$K_a = 0.3606$

$\dot{m} = 100 \text{ gal/min} (6.3 \times 10^{-3} \text{ m}^3/\text{s})$

$T = 10 \text{ DAYS}$

● NUMERICAL SOLUTION

— PRESENT THEORY

--- GEOTHERMAL GRADIENT

FIGURE 4A, COMPARISON OF THEORY TO NUMERICAL SOLUTION AT 10 DAYS CIRCULATION TIME -- HIGH FLOW RATE

$T_{in} = 70^{\circ}\text{F} (21.1^{\circ}\text{C})$

$K_p = 13.140$

$K_a = 0.282$

$\dot{m} = 100 \text{ gal/min} (6.3 \times 10^{-3} \text{ m}^3/\text{s})$

$T = 60 \text{ DAYS}$

● NUMERICAL SOLUTION

— PRESENT THEORY

- - - GEOTHERMAL GRADIENT

FIGURE 4B. COMPARISON OF THEORY TO NUMERICAL SOLUTION AT 60 DAYS CIRCULATION TIME -- HIGH FLOW RATE

rate, it can be seen that the solution no longer follows the geothermal gradient. The bottom-hole temperature has been substantially reduced for the higher circulation rate, at both circulation times. Again the agreement between the present theory and the numerical solutions is very good. From these results it may be concluded that the assumption of a uniform heat transfer coefficient has a negligible effect on the circulating temperature distributions for long circulation times.

3.3 Effect of Circulation Time

To present the trends predicted in the two test cases described above in a concise form, normalized outlet temperatures and their corresponding bottom-hole temperatures were computed for constant values of $k_p = 18.83$ and $k_p = 13.14$, i.e., for the two values of constant mass flow rate. The time-dependent parameter, k_a , was considered to be an independent variable during the evaluation of the appropriate equations. The results of these calculations are presented in Figs. 5A and B for the outlet fluid temperature and bottom-hole fluid temperatures, respectively. These quantities have been normalized by the inlet temperature of the drilling fluid using the following definitions:

$$\Delta\theta_o \equiv \frac{T_o - T_{in}}{T_{in}} = \theta_o - 1 \quad (27a)$$

$$\Delta\theta_{bh} \equiv \frac{T_{bh} - T_{in}}{T_{in}} = \theta_o - 1 \quad (27b)$$

--- $\dot{m} = 10 \text{ gal/min } (6.3 \times 10^{-4} \text{ m}^3/\text{s}) \quad K_p = 18.83$

— $\dot{m} = 100 \text{ gal/min } (6.3 \times 10^{-3} \text{ m}^3/\text{s}) \quad K_p = 13.14$

$\theta_s = 1.0$

$\theta_g = 1.071$

FIGURE 5A. EFFECT OF CIRCULATION TIME ON
OUTLET FLUID TEMPERATURE

$$K_a = \frac{2 \pi U_{\text{eff}} L}{\dot{m} c_p}$$

--- $\dot{m} = 10$ gal/min (6.3×10^{-4} m³/s) $K_p = 18.83$

— $\dot{m} = 100$ gal/min (6.3×10^{-3} m³/s) $K_p = 13.14$

$\theta_s = 1.0$

$\theta_g = 1.071$

FIGURE 5B. EFFECT OF CIRCULATION TIME ON BOTTOM-HOLE FLUID TEMPERATURE

The right-hand scale shows the corresponding actual temperatures in °F. For interpreting the normalized results in °C, the following definitions must be applied:

$$\Delta\theta_o \equiv \frac{T_o - T_{in}}{T_{in} + 17.77^\circ\text{C}} \quad (27c)$$

$$\Delta\theta_{bh} \equiv \frac{T_{bh} - T_{in}}{T_{in} + 17.77^\circ\text{C}} \quad (27d)$$

where -17.77°C represents the appropriate reference temperature for 0°F . The circular and triangular symbols denote the theoretical solutions for the two test cases at 10 and 60 days circulation time. The square symbol denotes the onset of pseudo-steady conditions as defined by the time required to produce three circulation cycles. For the flow rates of 10 gal/min ($6.3 \times 10^{-4} \text{ m}^3/\text{s}$) and 100 gal/min ($6.3 \times 10^{-3} \text{ m}^3/\text{s}$), the required times were computed to be approximately 72 hrs and 7.2 hrs respectively. The trends predicted in these results are quite revealing. For the low mass flow rate case ($k_p = 18.83$), it can be seen that the outlet fluid temperature increases with time (which corresponds to decreasing values of k_a) very slightly over the 50-day time interval. For the higher mass flow rate ($k_p = 13.14$), the theory predicts the outlet fluid temperature to decrease during the 50-day circulation time interval. The temperature changes referred to here are quite small. With a 70°F (21.1°C) inlet fluid temperature, the theory predicts an outlet fluid temperature

increase of 0.12°F (0.07°C) for the 10 gal/min ($6.3 \times 10^{-4} \text{ m}^3/\text{s}$) flow rate. The decrease in outlet temperature at 100 gal/min ($6.3 \times 10^{-3} \text{ m}^3/\text{s}$) is predicted to be 0.23°F (0.13°C). It should also be pointed out that the parameterized theory predicts that an optimum flow rate must exist for a given circulation time such that the outlet fluid temperature is a maximum. Conversely, it is also true that for a given mass flow rate, an optimum circulation time must exist for which the outlet fluid temperature is a maximum. For example, if the wellbore flowing at 10 gal/min ($6.3 \times 10^{-4} \text{ m}^3/\text{s}$) were allowed to circulate sufficiently long in time, the outlet temperature would be observed to increase along the line of constant $k_p = 18.83$ shown in Fig. 5A until a maximum value was reached, at $k_a = 0.75$, after which the outlet temperature would begin to decrease. Since k_a is a much stronger function of flow rate, than of circulation time (as shown previously), it is evident that the maximum outlet temperature can more easily be achieved by varying the flow rate. This procedure would be slightly complicated by the fact that both k_p and k_a are functions of the mass flow rate. In general, for a specified wellbore configuration and a known circulation rate, the outlet temperature is a double valued function with respect to time, i.e., the same outlet fluid temperature may be seen at two different circulation times. Because of the small magnitude in the changing temperature this effect would be immeasurable during normal drilling and circulation operations.

The corresponding normalized bottom-hole fluid temperatures shown in Fig. 5B predict that the bottom-hole temperatures decrease with time (decreasing k_a), for a given circulation rate, regardless of whether the outlet temperature is increasing or decreasing. The particular value of k_a merely determines the rate at which the temperature is decreasing. For the 50-day circulation interval with an inlet fluid temperature of 70°F (21.1°C), the decrease in bottom-hole fluid temperature is approximately 1.1°F (0.62°C) with the flow rate of 10 gal/min ($6.3 \times 10^{-4} \text{ m}^3/\text{s}$) and approximately 3.5°F (1.95°C) for the flow rate of 100 gal/min ($6.3 \times 10^{-3} \text{ m}^3/\text{s}$).

3.4 Generalized Thermal Trends For Linear Geothermal Temperatures

The results presented in Figs. 5A and B demonstrate how significant thermal trends for the normalized outlet and bottom-hole fluid temperatures may be represented in terms of the annular parameter, k_a . These results may be further generalized by allowing the primary parameter k_p to be varied as well. The solution to the governing equations may then be evaluated for a range of k_a , k_p values. The resulting temperature trends are then a function of only the particular form of the geothermal temperature distribution, i.e., θ_s and θ_g . Results for the normalized outlet fluid temperature are presented in Fig. 6A for values of $\theta_s = 1.0$ and $\theta_g = 1.07$. For a given value of k_a , it can be seen that the outlet temperature increases as k_p is reduced. Physically, this trend may be interpreted as a reduction in the overall heat transfer coefficient between the

$$K_p = \frac{2\pi U_{pa} L}{\dot{m} c_p}$$

$$K_a = \frac{2\pi U_{eff} L}{\dot{m} c_p}$$

$$\theta_s = 1.0$$

$$\theta_g = 1.071$$

FIGURE 6A. EFFECT OF WELLBORE PARAMETERS ON OUTLET FLUID TEMPERATURE -- LINEAR GRADIENT

primary fluid and the annulus fluid. The increased outlet temperature thus reflects the reduced heat transfer from the annulus region to the primary bore. Also in the figure, it is apparent that as k_a approaches zero, $\Delta\theta_o$ approaches zero (corresponding to $T_{in} = T_o$). As k_a is reduced, the degree of influence of the formation temperature is reduced. The various terms appearing in Equation (15d) suggest that this condition will occur in wellbores circulating at high mass flow rates and/or with drilling fluids of large heat capacitance. Small values of k_a will also be realized in wellbores of shallow depth or with low thermal conductance (low U_{eff}) between the formation and the annular fluid. It should be noted that low values of U_{eff} correspond to conditions in which the materials used in the wellbore construction are of low thermal conductance and/or the wellbore has circulated for a sufficiently long time such that the thermal gradients in the soil adjacent to the wellbore are small. From these results, it may be inferred that the outlet fluid temperature is a nonunique function with respect to mass flow rate. That is, for a given wellbore configuration and a specified formation temperature, the same outlet temperature may be obtained at two different times, for two different flow rates. This observation causes little difficulty in wellbore thermal analysis since the flow rate is typically a known circulation characteristic.

The corresponding trends for the bottom-hole temperature are presented in Fig. 6B. The trends predicted in Fig. 5B have

$$K_p = \frac{2\pi U_{pa}L}{\dot{m}c_p}$$

$$K_a = \frac{2\pi U_{eff}L}{\dot{m}c_p}$$

$$\theta_s = 1.0$$

$$\theta_g = 1.071$$

FIGURE 6B. EFFECT OF WELLBORE PARAMETERS ON BOTTOM-HOLE FLUID TEMPERATURE -- LINEAR GRADIENT

now been generalized to include a range of values for k_p . Note that for large values of both k_p and k_a the bottom-hole fluid temperature approaches the bottom-hole temperature of the formation, i.e., $\Delta\theta_{bh} \approx 1$. Physically, this condition will occur in very deep wells and/or with low circulation rates and/or at early circulation times. As the annular parameter becomes small, the normalized bottom-hole temperature approaches zero, a result which is in agreement with the trends presented in Fig. 6A. Finally, it should be pointed out that the bottom-hole fluid temperature is not a double-valued function of k_a for a given value of k_p as was the case for the outlet temperature results.

3.5 Uniqueness With Respect To Formation Temperature

In any wellbore undergoing circulation, the geothermal temperature distribution represents a major unknown which may only be determined using expensive well logging measurements. To investigate the uniqueness of the parameterized solutions with respect to the geothermal gradient, values for normalized outlet fluid temperatures and normalized bottom-hole fluid temperatures were computed with various θ_s , θ_g combinations for fixed values of the wellbore parameters k_p and k_a . Physically this corresponds to an examination of a wellbore circulating at a constant flow rate for a specified circulation time. The values for the wellbore parameters chosen for this analysis correspond to the 10-day circulation time. In Figs. 7A, B results for the 10 gal/min (6.3×10^{-4} m³/s) circulation are

$$\theta_s = \frac{T_s}{T_{in}}$$

$$\theta_g = \frac{T_{bf} - T_s}{T_{in}}$$

$T_{in} = 70^\circ\text{F} (21.1^\circ\text{C})$

$K_p = 18.830$

$K_a = 3.391$

$\dot{m} = 10 \text{ gal/min} (6.3 \times 10^{-4} \text{ m}^3/\text{s})$

$T = 10 \text{ DAYS}$

FIGURE 7A. INFLUENCE OF LINEAR GEOTHERMAL GRADIENT ON OUTLET FLUID TEMPERATURE -- LOW FLOW RATE

$$\theta_s = \frac{T_s}{T_{in}}$$

$$\theta_g = \frac{T_{bf} - T_s}{T_{in}}$$

FIGURE 7B. INFLUENCE OF LINEAR GEOTHERMAL GRADIENT ON
BOTTOM-HOLE FLUID TEMPERATURE -- LOW FLOW RATE

presented. The negative values of $\Delta\theta_o$ and $\Delta\theta_{bh}$ correspond to conditions for which the circulating temperatures would be less than the inlet temperature. The horizontal dashed lines represent the theoretical normalized temperatures obtained from the previous analysis where $\theta_s = 1.0$, $\theta_g = 1.07$. It is apparent that the outlet fluid temperature and the bottom-hole fluid temperature vary linearly with increasing geothermal gradient, for a given surface temperature. With an inlet fluid temperature of 70°F (21.1°C), the variation of θ_g from zero, to a maximum value of two, results in an increase in the outlet temperature of only 5°F (2.7°C). Over the same θ_g interval, Fig. 7B indicates that the bottom-hole fluid temperature would be expected to increase approximately 120°F (66.6°C). It can therefore be inferred, based on the relative slopes of the two figures, that the bottom-hole temperature is considerably more sensitive to the geothermal slope, as would be expected. Also, by comparison of the relative spacing of the lines in the two figures, it can be inferred that the bottom-hole fluid temperature is also more sensitive to changes in θ_s . This result is also to be expected since the temperature of the formation at the bottom-hole depth has been defined as $\theta_\infty(z^*=1) = \theta_{s1} + \theta_{g1}$ (by setting $z_o = 1$ in Equation (14a)). With regards to uniqueness, it can be seen that the solutions are unique for fixed values of either θ_s or θ_g . For example, given the known outlet fluid temperature and the formation surface temperature, the geothermal slope could be determined. It should be emphasized that to perform such an

analysis, the accuracy required in the outlet fluid temperature would not be obtainable using present measurement techniques. In Fig. 8A and B similar plots are given for the normalized outlet and bottom-hole fluid temperatures which correspond to the 100 gal/min ($6.3 \times 10^{-3} \text{ m}^3/\text{s}$) circulation case at 10 days circulation time. Again the parameterized theory predicts a linear variation of the two temperatures with increasing geothermal slope. By comparison of Fig. 8A with the results in Fig. 7A, it can be deduced that the outlet temperature for the 100 gal/min ($6.3 \times 10^{-3} \text{ m}^3/\text{s}$) flow rate is more sensitive to the geothermal slope, while it is slightly less sensitive to the local formation surface temperature. By a similar comparison, it can be seen that the bottom-hole temperature is less sensitive to both θ_s and θ_g at the higher flow rate. These results would be expected in view of the trends presented in Figs. 6A and B.

3.6 Graphical Determination Of The Geothermal Gradient

The trends predicted by the parameterized solution in Fig. 7 and 8 indicate that, in general, a given outlet fluid temperature may be calculated at a known flowing condition, from an infinite number of θ_s , θ_g combinations. Therefore, for a given wellbore flowing at a specified circulation rate, it would appear to be impossible to infer the correct geothermal temperature distribution based solely on the known outlet temperature without some additional information.

The information required to solve for the unique θ_s , θ_g combination may be obtained by an investigation of the circulating

$$\theta_s = \frac{T_s}{T_{in}}$$

$$\theta_g = \frac{T_{bf} - T_s}{T_{in}}$$

$T_{in} = 70^\circ\text{F} (21.1^\circ\text{C})$

$K_p = 13.140$

$K_a = 0.3606$

$\dot{m} = 100 \text{ gal/min} (6.3 \times 10^{-3} \text{ m}^3/\text{s})$

$T = 10 \text{ DAYS}$

FIGURE 8A. INFLUENCE OF LINEAR GEOTHERMAL GRADIENT ON
OUTLET FLUID TEMPERATURE -- HIGH FLOW RATE

$$\theta_s = \frac{T_s}{T_{in}}$$

$$\theta_g = \frac{T_{bf} - T_s}{T_{in}}$$

$T_{in} = 70^\circ\text{F} (21.1^\circ\text{C})$

$K_p = 13.140$

$K_a = 0.3606$

$\dot{m} = 100 \text{ gal/min} (6.3 \times 10^{-3} \text{ m}^3/\text{s})$

$T = 10 \text{ DAYS}$

FIGURE 8B. INFLUENCE OF LINEAR GEOTHERMAL GRADIENT ON
BOTTOM-HOLE FLUID TEMPERATURE -- HIGH FLOW RATE

outlet temperatures at two different flow rates. Such an analysis requires that the outlet temperature trends presented in Fig. 7A and Fig. 8A be examined in a conjunctive manner. It is apparent that both Fig. 7A and Fig. 8A reveal that an infinite number of combinations of θ_s and θ_g may produce the two known outlet temperatures. Given this observation, it may then be deduced that at least one θ_s , θ_g combination must exist which will produce the two known outlet temperatures at each of the two circulation conditions. In Fig. 9, the results of Fig. 7A have been superimposed onto the results presented in Fig. 8A (which now appear as dashed lines). The horizontal lines denote the normalized values for the outlet fluid temperature obtained from the parameterized solution at each of the two flow conditions. The vertical lines specify the value of the normalized geothermal slope necessary to produce the known outlet temperature at each surface temperature examined. Notice that at $\theta_s = 0.95$, different values of θ_g are required to produce the two known outlet fluid temperatures. This is also true of the solutions for which $\theta_s = 1.05$. However, the two lines of constant $\theta_s = 1.0$ intercept the two known outlet temperatures at the same value of the geothermal slope, i.e., $\theta_g = 1.07$. It may be concluded from this graphical analysis that $\theta_s = 1.0$, $\theta_g = 1.07$ are the unique combination of values which specify the local geothermal temperature distribution. Using only knowledge of the outlet fluid temperature and the known circulation conditions, at two different flow rates, the unique formation temperature

$$\theta_s = \frac{T_s}{T_{in}}$$

$$\theta_g = \frac{T_{bf} - T_s}{T_{in}}$$

FIGURE 9. DETERMINATION OF LINEAR GEOTHERMAL TEMPERATURE DISTRIBUTION FROM KNOWN OUTLET FLUID TEMPERATURES

distribution has been determined. The accuracy required in the known outlet temperature would be beyond current measurement capabilities. However, the fact that this procedure can be performed in theory leaves open the possibility that it may someday be possible.

3.7 Comparisons For a Bilinear Geothermal Temperature

The previous sections have been concerned with circulating wellbores under the influence of a linear temperature distribution in the formation. For geothermal applications, the formation temperature is altered due to the presence of underground geothermal sources. For these applications, the geothermal temperature may be approximated by two linear segments such that the resulting distribution appears bilinear in form. To demonstrate the versatility of the present formulation, the following discussion shall consider the effects of a bilinear geothermal temperature distribution. The bilinear temperature distribution to be used in the following analysis was taken from measured data of the Republic Geothermal Well #1.¹⁴ The surface temperature of the undisturbed formation was taken to be 70°F (21.1°C), while the bilinear temperature slopes were determined from temperatures of 280°F (137.7°C) at 2000 ft (609.6 m) and 310°F (154.4°C) at 5000 ft (1524 m). For an inlet temperature of 70°F (21.1°C), the normalized formation parameters are given as $\theta_{s1} = 1.0$, $\theta_{g1} = 7.5$, $\theta_{s2} = 3.714$, and $\theta_{g2} = 0.714$ with $z_o = 0.40$. The wellbore configuration and the circulation conditions are to be those used in the previous analyses. Since k_p and k_a are functions of

only the wellbore configuration and the circulation conditions, the values of the wellbore parameters computed in the previous section may be used for the present analysis.

With the wellbore circulating at 10 gal/min ($6.3 \times 10^{-4} \text{ m}^3/\text{s}$) for a duration of 10 days ($k_p = 18.83$, $k_a = 3.39$), circulating temperature distribution were compared with numerical solutions given by the GEOTEMP code. These results are shown in Fig. 10A. As was the case for the linear formation temperature, the temperature in the annulus region attempts to follow the geothermal temperature over each linear segment. However, since the fluid temperature must behave in a continuous manner, Equation (22) accurately predicts that the annulus fluid temperature cannot follow the geothermal temperature at the point of the discontinuity in the geothermal slope. It is also worth pointing out that the maximum temperature does not occur at the bottom hole as one might expect, but actually occurs in the annulus region at approximately 4500 ft (1372 m). This same result was observed by both Tragesser, et.al., and Raymond in their theoretical analysis of wellbore circulation. Again the parameterized solution can be seen to agree extremely well with the GEOTEMP results.

Temperature distributions are presented in Fig. 10B for the wellbore flowing at 100 gal/min ($6.3 \times 10^{-3} \text{ m}^3/\text{s}$) which is also at 10 days circulation time. At this higher flow rate, the circulating temperatures no longer follow the geothermal temperature. As with the other temperature distributions presented previously,

$T_{in} = 70^{\circ}\text{F}$ (21.1°C)

$K_p = 18.830$

$K_a = 3.391$

$\dot{m} = 10 \text{ gal/min}$ ($6.3 \times 10^{-4} \text{ m}^3/\text{s}$)

$T = 10 \text{ DAYS}$

$\theta_{s1} = 1.0$, $\theta_{g1} = 7.5$

$\theta_{s2} = 3.714$, $\theta_{g2} = 0.714$

$Z_0 = 0.4$

● NUMERICAL SOLUTION
 — PRESENT THEORY
 - - GEOTHERMAL GRADIENT

FIGURE 10A. COMPARISON OF THEORY TO NUMERICAL SOLUTION AT 10 DAYS CIRCULATION TIME -- LOW FLOW RATE

$T_{in} = 70^{\circ}\text{F} (21.1^{\circ}\text{C})$

$K_p = 13.140$

$K_a = 0.3606$

$\dot{m} = 100 \text{ gal/min} (6.3 \times 10^{-3} \text{ m}^3/\text{s})$

$T = 10 \text{ DAYS}$

$\theta_{s1} = 1.0, \theta_{g1} = 7.5$

$\theta_{s2} = 3.714, \theta_{g2} = 0.714$

$Z_o = 0.4$

● NUMERICAL SOLUTION

— PRESENT THEORY

-- GEOTHERMAL GRADIENT

FIGURE 10B. COMPARISON OF THEORY TO NUMERICAL SOLUTION AT 10 DAYS CIRCULATION TIME -- HIGH FLOW RATE

the comparison between the GEOTEMP code and the parameterized theory is very good.

3.8 Generalized Thermal Trends For Bilinear Geothermal Temperatures

The trends predicted in the previous comparisons may be generalized in order to obtain plots of normalized outlet and bottom-hole fluid temperatures as a function of the wellbore parameters k_p and k_a . The calculated values for the outlet temperature are presented in Fig. 11A. Qualitatively, these trends are similar to those shown in Fig. 6A; however, the outlet fluid temperatures predicted for the bilinear case are substantially higher than those predicted for the linear case. As an example of this difference, consider an arbitrary wellbore which is circulating under pseudo-steady conditions such that $k_p = 10$ and $k_a = 1.25$. For an inlet temperature of 70°F (21.1°C), Fig. 6A may be used to determine that the resulting outlet temperature would be approximately 75.1°F (23.94°C). Under the same circulation conditions the bilinear results predict an outlet temperature of approximately 100.1°F (37.83°C). Obviously this increase in outlet fluid temperature is a result of the hotter bilinear geothermal temperature.

The corresponding bottom-hole fluid temperature trends are presented in Fig. 11B. For large values of the wellbore parameters, the bottom-hole temperature approaches the geothermal temperature at the bottom hole depth, i.e., $\Delta\theta_{bh} = 3.4$. Comparison of these results with those of Fig. 6B reveal that, in general, the bottom-hole temperature for the bilinear formation will be

$$K_p = \frac{2\pi U_{pa} L}{\dot{m} c_p}$$

$$K_a = \frac{2\pi U_{eff} L}{\dot{m} c_p}$$

FIGURE 11A. EFFECT OF WELLBORE PARAMETERS ON OUTLET FLUID TEMPERATURE -- BILINEAR GRADIENT

$$K_p = \frac{2\pi U_{pa} L}{\dot{m} c_p}$$

$$K_a = \frac{2\pi U_{eff} L}{\dot{m} c_p}$$

$$\theta_{s1} = 1.0 \quad \theta_{g1} = 7.5$$

$$\theta_{s2} = 3.714 \quad \theta_{g2} = 0.714$$

FIGURE 11b. EFFECT OF WELLBORE PARAMETERS ON BOTTOM-HOLE FLUID TEMPERATURE -- BILINEAR GRADIENT

substantially higher than those for the linear formation case, as expected.

3.9 Effect of Bilinearities on Circulating Temperatures

As wells are drilled into geothermal reservoirs it is frequently desirable to estimate down-hole circulation temperatures. Based on the results from the previous analyses, it is clear that these estimates are virtually impossible without some knowledge of the particular form of the geothermal gradient. In order to assess the errors introduced in estimating the location of the bilinear slope discontinuity, the following analysis will examine variations in the baseline bilinear gradient used in the previous analysis. The variations assumed in the present study are shown together with the baseline gradient (slope discontinuity at 2000 ft (609.6m)) in Fig. 12A. The first approximation assumes the geothermal temperature varies linearly with depth, while the latter two assume the slope discontinuity occurs at 1000 ft (304.8 m) and at 1500 ft (457.2 m). It has been assumed that the formation temperature at the bottom-hole depth is the same for all three cases. This assumption represents idealized conditions from the standpoint of estimating the underground temperatures. The value for the primary parameter was chosen from an approximate average of the 10 gal/min ($6.3 \times 10^{-4} \text{ m}^3/\text{s}$) and the 100 gal/min ($6.3 \times 10^{-3} \text{ m}^3/\text{s}$) circulation conditions. Thus $k_p = 15$ was used in conjunction with a range of k_a values for determining outlet fluid temperatures and the corresponding bottom-hole temperatures. The reasoning behind this approach is based on the observation

————— BASELINE, $\tilde{Z}=2000$ ft (609.6 m)
 - - - - - LINEAR
 - . - . - $\tilde{Z}=1000$ ft (304.8 m)
 - . . . - $\tilde{Z}=1500$ ft (457.2 m)

FIGURE 12A. APPROXIMATIONS TO THE BASELINE
BILINEAR TEMPERATURE DISTRIBUTION

$$K_p = \frac{2\pi U_{pa} L}{\dot{m} c_p} = 15$$

$$K_a = \frac{2\pi U_{eff} L}{\dot{m} c_p}$$

$$T_{in} = 70^\circ F (21.1^\circ C)$$

FIGURE 12B. EFFECT OF VARIATIONS IN THE GEOTHERMAL GRADIENT ON OUTLET FLUID TEMPERATURE

that k_a is a much stronger function of flow rate, than k_p . By considering a particular average value of k_p , the effects of changing flow rate may be approximated by considering changes in the k_a parameter alone.

The predicted outlet fluid temperature results are presented in Fig. 12B. It is immediately apparent in these results that the linear approximation does a poor job in predicting the outlet temperatures for the baseline gradient. However, note that for $k_a < 0.25$, all four formation yield approximately the same outlet temperatures. For small values of k_a the heat transfer in the wellbore is convection dominated and thus heat conduction from the geothermal gradient is diminished. Consequently, the fluid outlet temperature does not reflect the particular form of the geothermal gradient. It can also be seen that for large values of k_a , the solutions from the two deviated geothermal slopes coalesce into the solution obtained for the baseline gradient. For large values of k_a , the wellbore heat transfer is conduction dominated and the degree of influence of the geothermal gradient is enhanced. Therefore, for the range of large k_a values, the first geothermal slope, i.e., θ_{s1} , θ_{g1} , exhibits a maximum influence on the resulting outlet temperature. Since the near-surface geothermal slope is the same for all three cases, the outlet fluid temperatures are identical. Only for the range of k_a defined by the interval $1 < k_a < 10$ is the influence of the secondary geothermal slope apparent. The discussion associated with Equation (26) revealed that the α parameter was the

theoretical mechanism for transmitting down-hole thermal information to the surface. Hence the influence of the secondary slope on the α term in Equation (26) is maximized for the k_a interval specified above. Because of this influence, changes in the down-hole secondary slope produce small, yet theoretically distinguishable changes in the outlet temperature. As the position of the approximate geothermal discontinuity approaches the baseline geothermal discontinuity, the outlet fluid temperatures for the approximations tend to converge towards the baseline outlet fluid temperatures.

The bottom-hole fluid temperatures calculated using the geothermal temperatures in Fig. 12A are presented in Fig. 12C. Here it is shown that all three geothermal temperature approximations closely follow the baseline values for the bottom-hole fluid temperature. Again for small values of k_a ($k_a < 0.25$) the bottom-hole temperatures are identical for all four gradients owing to the diminished influence of the formation temperature in this range of k_a . It may be concluded from these results that the bottom-hole fluid temperature is not greatly influenced by the particular form of the geothermal temperature distribution so long as the geothermal temperatures at the bottom-hole depth are identical.

The table below summarizes the maximum fluid temperature differences at the outlet and bottom-hole locations between the baseline geothermal gradient and the three approximations.

TABLE 2

Summary of Differences Between Baseline
And Approximated Gradients

Geothermal Gradient	Max ΔT_o °F(°C) From Baseline	Max ΔT_{bh} °F(°C) From Baseline
Linear	11.55 (6.42)	36.00 (20.0)
$\tilde{Z}=1000$ ft (304.8m)	3.325 (1.85)	22.75 (12.64)
$\tilde{Z}=1500$ ft (457.2m)	1.225 (0.68)	12.25 (6.81)

$$K_a = \frac{2\pi U_{eff} L}{\dot{m} c_p}$$

$$K_p = \frac{2\pi U_{pa} L}{\dot{m} c_p} = 15$$

$$T_{in} = 70^\circ F (21.1^\circ C)$$

FIGURE 12c. EFFECT OF VARIATIONS IN THE GEOTHERMAL GRADIENT ON BOTTOM-HOLE FLUID TEMPERATURE

4. SUMMARY AND CONCLUDING REMARKS

To gain a better understanding of the thermal processes which occur in a circulating wellbore, a pseudo-steady thermal model has been formulated in terms of two nondimensional groups which indicate the relative contributions of radial conduction and forced convection. The solution to the governing energy equations has been determined in a manner which allows for discontinuous geothermal gradients. As a result of this solution procedure, a nondimensional parameter has been identified which determines the influence of the geothermal gradient on the annulus outlet fluid temperature.

In the preceding discussion of results it has been shown that the parameterized theory correlates very well with numerically generated solutions for both linear as well as bilinear geothermal gradients. Although numerous questions regarding sensitivity and uniqueness have been addressed, it is obvious that all possible situations could not be discussed within the context of the present study. Nevertheless, based on the analyses conducted herein, the following general conclusions regarding pseudo-steady circulation can be made.

1. For a given wellbore configuration, it is possible to define a set of circulation conditions such that the outlet fluid temperature is a maximum for a given inlet fluid temperature.
2. For circulation rates which are typical of those used in present drilling operations, the pseudo-steady heat transfer in the wellbore is dominated by forced convection. However,

as wells are drilled to greater depths, radial conduction becomes increasingly important in the heat transfer processes.

3. In circulating wellbores dominated by forced convection both the outlet fluid temperature and the bottom-hole fluid temperature decrease with increasing circulation time. For wellbores circulating such that radial conduction dominates, the outlet fluid temperature increases slightly with time, while the bottom-hole fluid temperature remains essentially unchanged.
4. In general, for a given circulation condition, it is possible to define an infinite number of linear geothermal gradients which will produce a given outlet fluid temperature. By examination of the outlet temperature at two different flow rates, it is theoretically possible to ascertain the unique geothermal temperature distribution for the circulating wellbore.
5. For bilinear geothermal gradients, significant changes in the secondary geothermal slope produce only small changes in the resulting outlet fluid temperature. Unless the wellbore is circulated near the optimum flow conditions defined in this analysis, the changes in the outlet temperature are imperceptible, and thus incapable of being measured.

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APPENDIX A
Program For Equation Evaluation

APPENDIX A. PROGRAM FOR EQUATION EVALUATION

PROGRAM WELL(INPUT,OUTPUT)

PROGRAM WELL COMPUTES WELLBORE TEMPERATURES FOR LINEAR AND BILINEAR THERMAL SOURCES.

IMPLICIT DOUBLE (A-H,K,O-Z)
COMMON /BLOK1/ AMB,KP,KA,A,B,ZST
COMMON /BLOK2/ ZO,THTS1,THG1,THTS2,THG2

PRINT 2
2 FORMAT(31X"PROGRAM WELL")
PRINT 4
4 FORMAT("HOW MANY SOLUTIONS DO YOU WANT .LE. 10")
READ *, ILAST
PRINT 6
6 FORMAT("/INPUT 1=COMPLETE, 0=LIMITED SOLUTION TYPE")
READ *, ITYPE

PRINT 8
8 FORMAT("/INPUT INITIAL VALUES FOR KP, KA")
READ *, KPINT, KAINC
PRINT 10
10 FORMAT("/READ FORMATION TEMPERATURE TERMS, ZO, THTS1, THG1,
+THTS2, THG2")
READ *, ZO, THTS1, THG1, THTS2, THG2

INCREMENT ON KA PARAMETER

KAINC=.25
KA=KAINC
KP=KPINT

DO 100 I=1,ILAST
IF(I.GT.1)KA=KA+KAINC

DEFINITION OF IMPORTANT TERMS

A=KA/2.+0.5*DSQRT(KA*KA+4.*KA*KP)
B=KA/2.-0.5*DSQRT(KA*KA+4.*KA*KP)
KTILD=KA*KP
AMB=A-B
EXPA=DEXP(A)
EXPB=DEXP(B)
EXPAZ=DEXP(A*(1.-ZO))
EXPBZ=DEXP(B*(1.-ZO))

```

      PRINT 14,I
14  FORMAT(/15("+" )=SOLUTION "I3,15("+" ))
      PRINT 15, KP, KA
15  FORMAT("PRIMARY COEFF.= "F15.4,/"ANNULAR COEFF.= "F15.4)

```

DEFINITION OF PARAMETER ALFA

```

      ALF1=THTS1*(EXPA-EXPAZ-EXPB+EXPBZ)
      ALF2=THG1*(EXPA/A-ZO*EXPAZ-EXPAZ/A-EXPB/B+ZO*EXPBZ+EXPBZ/B)
      ALF3=THTS2*(EXPAZ-EXPBZ)
      ALF4=THG2*(EXPAZ*(ZO*A+1.)/A-1./A-EXPBZ*(ZO*B+1.)/B+1./B)
      ALF5=KTILD*(ALF1+ALF2+ALF3+ALF4)/(A*EXPA-B*EXPB)
      ALF=ALF5-((A*A*EXPA-B*B*EXPB)/(A*EXPA-B*EXPB))

```

```

      PRINT 16, ALF
16  FORMAT(/"ALPHA= "E15.5)
      PRINT 18
18  FORMAT(/"      ZSTAR          THETAP          THETAA          THETAW")

```

```

      IEND=26
      IF(ITYPE.EQ.0) IEND=2
      DO 100 IZ=1,IEND
      ZST=FLOAT(IZ-1)/25.
      IF(ITYPE.EQ.0.AND.IZ.EQ.2)ZST=1.0

```

```

      CALL INTGRT(EVAL,DEVAL)

```

```

      THP=(A+ALF)*DEXP(A*ZST)/AMB-(B+ALF)*DEXP(B*ZST)/AMB-
      •KTILD*EVAL/AMB

```

```

      THA=(A*(A+ALF)/KP)*DEXP(A*ZST)/AMB-(B*(B+ALF)/KP)*DEXP(B*ZST)/AMB
      + -KA*DEVAL/AMB+THP

```

```

      THG=THTS1+THG1*ZST
      IF(ZST.GT.ZO) THG=THTS2+THG2*ZST
      PRINT 20, ZST, THP, THA, THG

```

```

20  FORMAT(2X,4E12.5)

```

```

100 CONTINUE

```

```

      STOP
      END

```

```

      SUBROUTINE INTGRT(EVAL,DEVAL)

```

```

      IMPLICIT DOUBLE(A-H,K,O-Z)

```

```

      COMMON /BLOK1/ AMB,KP,KA,A,B,ZST

```

```

      COMMON /BLOK2/ ZO,THTS1,THG1,THTS2,THG2

```

```

      IF(ZST.GT.ZO) GOTO 1000

```

```

      TERM1=(THTS1*A+THG1)*DEXP(A*ZST)/(A*A)-(THTS1*B+THG1)*DEXP(
      +B*ZST)/(B*B)

```

```

      TERM2=THG1*AMB*ZST/(A*B)

```

```

      TERM3=(THTS1*A+THG1)/(A*A)-(THTS1*B+THG1)/(B*B)

```

```

      EVAL=TERM1+TERM2-TERM3

```

```

DEVAL=THTS1*(DEXP(A*ZST)+DEXP(B*ZST))
+*THG1*((DEXP(A*ZST)-1.)/A-(DEXP(B*ZST)-1.)/B)
RETURN

```

1000 CONTINUE

```

EAZ=DEXP(A*ZST)
EBZ=DEXP(B*ZST)
EAO=DEXP(A*(ZST-ZO))
EBZO=DEXP(B*(ZST-ZO))

```

```

TERM1=THTS1*((EAO-EAZ)/A-(EBZO-EBZ)/B)
TERM21=ZO*EAO/A-(EAO-EAZ)/(A*A)
TERM22=ZO*EBZO/B-(EBZO-EBZ)/(B*B)
TERM2=THG1*(TERM21-TERM22)
TERM3=THTS2*(EAO/A-EBZO/B+AMB/(A*B))
TERM41=(ZO*EAO-ZST)/A+(EAO-1.)/(A*A)
TERM42=(ZO*EBZO-ZST)/B+(EBZO-1.)/(B*B)
TERM4=(TERM41-TERM42)*THG2
EVAL=TERM1-TERM2+TERM3+TERM4

```

```

TERM1=THTS1*(EAO-EAZ-EBZ+EBZO)
TERM21=(EAO-EAZ)/A-ZO*EAO
TERM22=(EBZ-EBZO)/B-ZO*EBZO
TERM2=THG1*(TERM21-TERM22)
TERM3=THTS2*(EAO-EBZO)
TERM41=EAO*(ZO*A+1.)/A-1./A
TERM42=EBZO*(ZO*B+1.)/B-1./B
TERM4=THG2*(TERM41-TERM42)
DEVAL=TERM1+TERM2+TERM3+TERM4
RETURN
END

```

APPENDIX B

Example Wellbore Configuration

APPENDIX B. EXAMPLE WELLBORE CONFIGURATION

During the course of the development of the present theory, several simplifying assumptions were introduced. To investigate the ability of the present solution to simulate circulating temperature distributions in a realistic situation, a sample wellbore configuration was selected from the numerical examples of Wooley.⁴ The casing schedule to be used in the sample calculations is given as follows:

Casing	I.D., in(cm)	O.D., in(cm)	Depth, ft(m)	Cemented Length ft(m)
1	3.958(10.05)	4.500(11.43)	5000(1524.0)	5000(1524.0)
2	8.681(22.05)	9.625(24.44)	5000(1524.0)	2200 (670.6)
3	12.347(31.36)	13.375(33.97)	3000 (914.4)	2200 (670.6)
4	19.124(48.57)	20.000(50.80)	1000 (304.8)	1000 (304.8)
5	29.000(73.66)	30.000(76.20)	100 (30.5)	100 (30.5)

In the casing schedule specified above, it is obvious that the overall heat transfer coefficient between the formation and the annular fluid will vary with depth due to the multiple levels of casing and cement. The present theory assumes the overall heat transfer coefficient is independent of depth. Nevertheless, an estimate for a uniform coefficient can be made by neglecting casings 4 and 5, and assuming the remaining three casings to extend to the bottom-hole depth of 5000 ft (1524m). The wellbore/formation interface was assumed to exist at the outermost diameter of the third casing.

The drilling fluid to be circulated in the wellbore is considered to have non-newtonian characteristics with the following properties.

Density: 10 lbm/gal (1198.3 kg/m³)

Plastic Viscosity: 15 centipoise (0.015 Pa-s)

Yield Point: 5 lb_f/100 ft² (2.394 Pa)

Specific Heat: 0.89 Btu/lb_m^oR (3.72x10³ J/kg^oC)

Assuming the drilling fluid to be water based, values for the fluid thermal properties may be computed from a power law viscosity model using the above information.¹² From these results, fluid film coefficients may be determined from standard Nusselt number correlations using known values for the Reynolds number and for the Prandtl number of the flow. It has been assumed that the flow in the casing is fully developed and thus axial variations in the Nusselt number are considered to be small. For flow rates of 10 gal/min (6.3x10⁻⁴ m³/s) and 100 gal/min (6.3x10⁻³ m³/s) the following film coefficients were calculated.

TABLE B-1

Calculated Film Coefficients For
Wellbore Thermal Study

<u>Mass Flow</u>	<u>Primary Fluid Coefficient</u>	<u>Annulus Fluid Coefficient</u>
$\frac{\text{gal}}{\text{min}} \left(\frac{\text{m}^3}{\text{s}} \right)$	$\frac{\text{Btu}}{\text{hrft}^{2\text{o}}\text{F}} \left(\frac{\text{W}}{\text{m}^{2\text{o}}\text{C}} \right)$	$\frac{\text{Btu}}{\text{hrft}^{2\text{o}}\text{F}} \left(\frac{\text{W}}{\text{m}^{2\text{o}}\text{C}} \right)$
10 (6.3x10 ⁻⁴)	72.10 (409.4)	24.11 (136.9)
100 (6.3x10 ⁻³)	556.30(3158.8)	186.20(1057.3)

Before the overall heat transfer coefficients can be computed, thermal properties of the casing materials must be specified. For the present study, the thermal conductivity of steel was taken to be 26.0 Btu/hrft^{°F} (45.0 W/m^{°C}) while the thermal conductivity of the cement was considered to be 0.50 Btu/hrft^{°F} (0.87 W/m^{°C}). Substituting these values along with values for the film coefficients into Equations (1b) and (2b) yields the estimates for the overall heat transfer coefficients in the circulating wellbore. The values used in the present analysis are presented in Table B-2 below.

TABLE B-2

Overall Heat Transfer Coefficients
For Each Circulation Condition

<u>Mass Flow</u>	<u>U_{pa} (Primary/ Annulus) Coefficient</u>	<u>U_{a6} (Annulus/ Outer-casing) Coefficient</u>
$\frac{\text{gal}}{\text{min}} \left(\frac{\text{m}^3}{\text{s}} \right)$	$\frac{\text{Btu}}{\text{hrft}^{\circ\text{F}}} \left(\frac{\text{W}}{\text{m}^{\circ\text{C}}} \right)$	$\frac{\text{Btu}}{\text{hrft}^{\circ\text{F}}} \left(\frac{\text{W}}{\text{m}^{\circ\text{C}}} \right)$
10 (6.3x10 ⁻⁴)	3.22 (5.57)	1.61 (2.78)
100 (6.3x10 ⁻³)	22.50 (38.94)	1.92 (3.32)

The symbol U_{a6} has been used here to denote the overall heat transfer coefficient between the annular fluid and the wellbore-formation interface. This coefficient must be modified in Equation (8) in order to include the effects of transient temperature gradients in the formation.

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4750 V. L. Dugan
4751 J. R. Tillerson
4756 H. M. Dodd
5500 O. E. Jones
5510 D. B. Hayes
5512 D. F. McVey
5512 A. Ortega
5512 F. G. Blottner
5512 L. A. Mondy
5513 D. W. Larson
5513 D. D. Cline (10)
5520 T. B. Lane
5530 W. Herrmann
5540 O. E. Jones (Acting)
3141 Tech Library (5)
3151 W. L. Garner (3)
3154-3 DOE/TIC (R. P. Campbell)
6011 J. B. Szymanski
8266 E. A. Aas