COMMENTS ON "ADIABATIC MODIFICATIONS TO PLASMA TURBULENCE THEORY"

BY

J. A. KROMMES

PLASMA PHYSICS LABORATORY

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY
Catto introduced in Ref. 1 an interesting and plausible modification of the usual "resonance-broadening" prescription for obtaining the nonlinear dielectric function. He argued reasonably that one should employ that prescription only for the nonadiabatic response, and that one should treat the adiabatic response essentially exactly. However, Misguich, in a recent Comment on Catto's work, found an apparent divergence in a form for the renormalized dielectric which he argued was equivalent to Catto's. Misguich was thus led to conclude that, at least for stationary turbulence, Catto's form was suspect, and that a more intricate renormalization might have to be used to obtain a sensible, convergent result.

I wish to argue that this conclusion is incorrect, at least for the reasons Misguich gives. My goal is not to criticize Misguich, whose work is detailed and instructive, but to exemplify some subtleties of renormalization. It is adequate to discuss the electrostatic, stationary, Gaussian-Markov approximation with constant diffusion coefficient $D$ and Maxwellian background distribution $f$. In the resonance-broadening approximation, the dielectric $\varepsilon$ then takes the form

$$\varepsilon(k,\omega) = 1 + \sum_{s} (k\lambda_{D})^{-2} \left[ \int d\nu dvk \right] U_{k,\omega}(\nu,\bar{\nu}) f(\bar{\nu}) ,$$

where

$$U_{k,\omega}(\nu,\bar{\nu}) \equiv \int_{0}^{\infty} \exp[\mathrm{i} \nu (\omega-k\bar{\nu}) - k^{2} D\tau^{3}/3] P_{\nu}(\nu - \bar{\nu} - i k D\tau^{2}, \tau) ,$$
\[
 P_v(z, \tau) \equiv (4\pi D\tau)^{-\frac{1}{2}} \exp\left(-\frac{z^2}{4D\tau}\right),
\]

and \( \lambda_D^2 \equiv T/4\pi m e_s^2 \). To simplify (1), it is convenient to change variables from \( (v, \overline{v}) \) to \( (u \equiv v-ikD\tau^2, \overline{v}) \) so that

\[
 I = \int_{-\infty}^{\infty} dv \int_{-\infty}^{\infty} d\overline{v} \exp(-ikvT)P_v(v-ikD\tau^2)ik\overline{v}\langle f(\overline{v}) \rangle
\]

\[
 = \exp(k^2D\tau^3) \int_{-\infty}^{\infty} dv \exp(-ikvT)ik\overline{v}\langle f(\overline{v}) \rangle \int_{-\infty}^{\infty} du \exp(-ikuT)P_v(u). \quad (4)
\]

A standard application of Cauchy's theorem enables one to shift the \( u \) contour upward onto the real axis, giving rise to

\[
 \int_{-\infty}^{\infty} du \exp(-ikuT)P_v(u) = \exp(-k^2D\tau^3). \quad (5)
\]

This factor cancels with the first term in (4), whereupon

\[
 I = -\frac{\partial}{\partial \tau} \int dv \exp(-iKVT)\langle f(v) \rangle. \quad (6)
\]

Then, upon inserting (6) into (1) and integrating by parts in \( \tau \), one finds

\[
 \epsilon(k, \omega) = 1 + \overline{\int (k\lambda_D)^{-2}} + \overline{\int (k\lambda_D)^{-2}} \int dv \langle f(v) \rangle
\]

\[
 \times \int_{0}^{\infty} dt (i\omega-k^2D^2) \exp[i(\omega-k\tau)\tau-k^2D\tau^3/3]. \quad (7)
\]

Catto makes subsidiary approximation which lead him to neglect the \( k^2D\tau^2 \) term. The result,
is the one which Misguich discusses.

Misguich claims to derive the convergent form (8) from a form [his Eq. (10)] which is apparently divergent. There are two facets to the resolution of this paradox: Misguich's derivation of (8) from his Eq. (10) is not consistent, and his divergent form itself does not follow from Eq. (1), but from an expression [Misguich's Eq. (5)] which does not seem to be consistent with the definition of the dielectric.\textsuperscript{5,6} In an attempt to include non-Markovian corrections, Misguich and Balescu\textsuperscript{7} argue that (1) should be replaced by

\[\varepsilon(k,\omega t) = 1 - \int_{k}^{2} \frac{d\tau}{k^2} e^{i\omega \tau} \frac{d\nu}{d\nu} \frac{U_k(\nu, \tau; \nu')}{\nu} \frac{\partial e(\nu, \tau; \nu)}{\partial \nu}. \quad (9a)\]

They write

\[\langle e(\nu, t; \tau) \rangle = \int d\nu' U_{k=0}(\nu, \tau; \nu') \langle e(\nu', t) \rangle \quad (9b)\]

and then take \(\langle e(\nu', t) \rangle\) to be stationary. However, the Eulerian function \(\langle e(\nu, t; \tau) \rangle\) is no less stationary than \(\langle e(\nu, t) \rangle\). The dielectric describes the result of probing the system after the turbulent state is set up. Since both Misguich and Balescu as well as I assume stationarity, \(\langle e \rangle\) is unchanging before the probe is applied and (9) is incorrect.

Nevertheless, Misguich proceeds from (9). One has

\[U_{k=0}(\nu, \tau; \nu) = P_{\nu}(\nu; \nu, -\tau). \quad (10)\]
Misguich, in a separate, also inconsistent approximation, now parses (9b) with (10) through the 3/3v operator, after which it partially cancels with \( P \langle v-v-ikD_t^2 \rangle \) in such a way that \( P \langle v-v-ikD_t^2 \rangle \) is effectively replaced in (2) by \( \Delta(v-v-ikD_t^2) \), where \( \Delta(z) \) is the Dirac delta function analytically continued from real to complex values of \( z \), e.g.,

\[
\Delta(z) = \lim_{\delta \to 0} (2\pi)^{-1/2} \exp\left(-z^2/4\delta^2\right). \tag{11}
\]

The manipulations leading up to (5) still hold; however, because of the delta function approximation to \( P \), (5) is replaced by unity, the term \( \exp(k^2D_t^3) \) in (4) is not cancelled, and in (2) a net factor of \( \exp(2k^2D_t^3/3) \) remains. The resulting time integral is divergent, as Misguich notes.

In an attempt to circumvent the divergence and obtain Catto's result, Misguich (effectively) returns to the form (1) (with \( P \) still replaced by \( \Delta \)) and integrates over \( v' \):

\[
\mathcal{E} = 1 - \frac{(k^3_0)}{2} \int d\nu' d\nu \int_0^\infty dt \exp\left[i(\omega-\nu')(t-k^2D_t^3/3)\right]
\]

\[
x(\nu-ikD_t^2) \langle f(v-ikD_t^2) \rangle. \tag{12}
\]

This result can be justified by Cauchy's theorem. It is obviously equivalent to the divergent result discussed above, as a change of variables to \( v' = v-ikD_t^2 \) reveals. However, Misguich now neglects the factor of \(-ikD_t^2\) inside (but not outside) \( \langle f \rangle \), arguing inconsistently that the action of the propagator on \( \langle f \rangle \) results in "higher order contributions" in D. The result,
\[ \varepsilon = 1 + \sum \lambda D \frac{-2}{f[v]} dv \langle f[v] \rangle \]

\[ \times \int_0^\infty dt \exp \{i(\omega-kv)t-k^2D\tau^2/3\}(iv-k^2D\tau^2), \]  

(13)

is convergent. It is equivalent to Catto's result and, upon integration by parts in \( t \), to (8). Thus, since the divergence Misquich discusses is spurious, Catto's result remains reasonable.

Other aspects of the nonlinear dielectric are discussed in Ref. 9. Recent work has attempted to systematically justify \( \varepsilon_{\infty} \) approximations similar to Catto's; Refs. 5 and 6 contain many references.

ACKNOWLEDGEMENTS

This work was jointly supported by the United States Air Force Office of Scientific Research Contract no. F44620-75-C-0037 and the United States Department of Energy Contract no. DE-AC02-76-CH03073.
REFERENCES

EXTERNAL DISTRIBUTION IN ADDITION TO TIC UC-20

ALL CATEGORIES

R. Askew, Auburn University, Alabama
S. T. Wu, Univ. of Alabama
Geophysical Institute, Univ. of Alaska
G. L. Johnston, Sonoma State Univ, California
H. H. Kuehl, Univ. of S. California
Institute for Energy Studies, Stanford University
H. D. Campbell, University of Florida
N. L. Oleson, University of South Florida
W. M. Stacey, Georgia Institute of Technology
Benjamin Wa, Iowa State University
Magne Kristiansen, Texas Tech. University
W. L. Wise, Natl Bureau of Standards, Wash., D.C.
Australian National University, Canberra
M. A. Austin-Munro, Univ. of Sydney, Australia
F. Cap, Inst. for Theor. Physics, Austria
Dr. M. Heindler, Institute for Theoretical Physics
Technical University of Graz
Ecole Royale Militaire, Brussels, Belgium
P. H. Sakanaka, Institute de Fisica, Campinas, Brazil
M. P. Bagnacki, MPB Tech. Sre. Anne de Bellevue, Quebec, Canada
C. R. James, University of Alberta, Canada
T. W. Johnson, INRS-Energie, Varennes, Quebec
M. H. Shagdorj, Univ. of Saskatchewan, Canada
Inst. of Physics, Academia Sinica, Peking, People's Republic of China
Inst. of Plasma Physics, Hefei, Anhwei Province, People's Republic of China
Library, Tung Hua Univ. Peking, People's Republic of China
Zhengwu Li, Southwestern Inst. of Phys., Leshan, Sichuan Province, People's Republic of China
Librarian, Culham Laboratory, Abingdon, England (2)
A.M. Dupas Library, C.E.N-Grenoble, France
Central Res. Inst. for Physics, Hungary
S. R. Sharma, Univ. of Rajasthan, JAIPLUR-3, India
R. Shingal, Indian Inst. of Technology, Bombay, India
A.K. Sundaram, Phys. Res. Lab., India
Biblioteca, Firenze, Italy
Biblioteca, Milano, Italy
C. Rastagni, Univ. Di Padova, Padova, Italy
Preprint Library, Inst. de Fisica, Pisa, Italy
Library, Plasma Physics Lab., Gokasho, U.P., Japan
S. U. Y. Japan Atomic Energy Res. Inst., Tokai-Mura Research Information Center, Nagoya Univ., Japan
S. Shoda, Tokyo Inst. of Tech., Japan
Inst. of Space & Aero. Sci., Univ. of Tokyo
T. Uchida, Univ. of Tokyo, Japan
H. Yanato, Toshiba R. & D. Center, Japan
M. Yoshikawa, JARI, Tokai Res. Est., Japan
Dr. T. Nakada, Toshiba Corporation, Kawasaki-Ku Kawasaki, 210 Japan
N. Yamada, Kansai Univ., Japan
R. England, Univ. National Autonoma de Mexico
B. S. Liley, Univ. of Waikato, New Zealand
S. Asama, Suuk Univ. Norge, Norway
J. L. Cabral, Univ. de Lisboa, Portugal
O. Petru, AIU, CUSA Univ., Romania
D. Villiers, Atomic Energy Bld., South Africa
A. Maurea, Comision De La Energy y Recursos Minerales, Spain
Library, Royal Institute of Technology, Sweden
Cen. de Res. En Phys. Des Plasmas, Switzerland
Librarian, Foss-Institut Voor Plasma-Physica, The Netherlands

Bibliothek, Stuttgart, West Germany
R.D. Buhrer, Univ. of Stuttgart, West Germany
Max-Planck-Institut fur Plasmaphysik, West Germany
Nucl. Res. Estab., Julich, West Germany
K. Schrader, Inst. For Theor. Physik, West Germany

EXPERIMENTAL

M. H. Brennan, Flinders Univ, Australia
H. Barman, Univ. of British Columbia, Canada
S. Screeenivasan, Univ. of Calgary, Canada
J. Radel, C.E.N.-B.P., Fontenay-aux-Roses, France
Prof. Schatzman, Observatoire de Nice, France
S. C. Sharma, Univ. of Cape Coast, Ghana
K. N. Aiyer, Laser Section, India
B. Buti, Physical Res. Lab., India
K. C. Chandra, S. Gujrat Univ., India
I.M. Las Dás, Bänaras Hindu Univ., India
S. Copeman, Tel Aviv Univ., Israel
E. Greenspan, Nuc. Res. Center, Israel
P. Rosenau, Israel Inst. of Tech., Israel
Inst. Center for Theor. Physics, Trieste, Italy
K. Kawakami, Nihon University, Japan
T. Nakayama, Ritsumekan Univ., Japan
S. Nagao, Tohoku Univ., Japan
J.L. Sakai, Toyama Univ., Japan
T. Tjotta, Univ. of Bergen, Norway
M.A. Heilberg, Univ. of Natal, South Africa
H. Wileman, Chalmers Univ. of Tech., Sweden
Astro. Inst., Sonnenborh Obs., The Netherlands
T. J. Boys, Univ. College of North Wales
K. Hubner, Univ. Heidelberg, A Germany
H. F. Kappeler, t.v. of Stuttgart, West Germany
K. H. Spatschek, Univ. Essen, West Germany

EXPERIMENTAL ENGINEERING

B. Gres, Univ. De Quebec, Canada
P. Lukac, Komenskeho Univ., Czechoslovakia
G. Horikoshi, Nal' Lab for High Energy Physics, Tsukuba-Gun, Japan

EXPERIMENTAL

J. J. Faust, Univ. of Adelaide, Australia
J. Fromkaeker, For Inst. for Atomic & Molec. Physics, The Netherlands

THEORETICAL

F. Verheest, Inst. Vor Theo. Mech., Belgium
J. Teichmann, Univ. of Montreal, Canada
T. Kanai, Univ. Paris V, France
R. K. Chhajlani, India
S. K. Trehan, Panjab Univ., India
T. Namikawa, Osaka City Univ., Japan
H. Narumi, Univ. of Hiroshima, Japan
Korea Atomic Energy Res. Inst., Korea
E. T. Kurlon, Upsala Univ., Sweden
L. Sterko, Univ. of UMEA, Sweden
S. R. Sarp, New Univ., United Kingdom

THEORETICAL

F. Verheest, Inst. Vor Theo. Mech., Belgium
J. Teichmann, Univ. of Montreal, Canada
T. Kanai, Univ. Paris V, France
R. K. Chhajlani, India
S. K. Trehan, Panjab Univ., India
T. Namikawa, Osaka City Univ., Japan
H. Narumi, Univ. of Hiroshima, Japan
Korea Atomic Energy Res. Inst., Korea
E. T. Kurlon, Upsala Univ., Sweden
L. Sterko, Univ. of UMEA, Sweden
S. R. Sarp, New Univ., United Kingdom