MUONIC MOLECULAR FORMATION UNDER LASER IRRADIATION
AND IN THE CLUSTERED ION MOLECULE
(The Effect of Protonium Additive on the Muon Catalyzed
Fusion cycle)

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INTRODUCTION

The number of fusions catalyzed by one muon depends on the rate of formation of the muonic molecule. The fast resonance formation rate for the $dt\mu$ molecule due to the shallow binding energy of ($J=1$, $v=1$) state produces more than 140 fusions per one muon for a liquid hydrogen density DT target.^{1,2,3}

The formation rate of the $dt\mu$ molecule is very sensitive to the differences in the vibrational and rotational level between D_2 and $[(dt\mu)-d-2e]$ molecules.^{4,5,6,7} The density effect of the normalized reaction rate has been studied by the resonance broadening due to collisional quenching.^{8,9,10} The surrounding molecules of the molecule forming $dt\mu$ act as the third body which takes out the excess energy forming $dt\mu$ from $t\mu$, and the formation reaction occurs with the excitation of the vibrational state just below the threshold energy. By using the laser as the third body, the rate of resonance formation can be increased. In my last paper,¹¹ I calculated the formation rate under high-intensity laser irradiation, using Vinitzky's model assuming that the laser interacts directly with the deuteron and modulates the interaction between $t\mu$ and d_2 nuclei. However, the laser interacts more strongly with the electrons, because the interaction energy of the laser and the charged particle is proportional to the velocity of the particle's motion, and the velocity of the electron is a few thousand times greater than the velocity of the nuclei. This interaction with electron was neglected in my last paper. In the present paper, the enhancement of the $dt\mu$ formation rate by the strong laser irradiation is studied, taking into account the laser electron interaction; I showed that the enhancement can be achieved by an intensity lower than the one described previously.

Instead of using the surrounding molecule as the third body enhancing or changing the rate of $dt\mu$ formation, the clustered ion can¹² also change the $dt\mu$ formation rate drastically because it has many more vibrational modes than the diatomic molecule.

Many clustered hydrogen ions exist as stable ions, and they are created at very high temperatures. Recently Jandel et al. discussed the $dt\mu$ fusion reaction in the high temperature environment created by the heat spike,¹³ where there is the possibility that D_2, T_2 molecules are transformed into the (D_3^+, e) (T_3^+, e) states. Thus it is worthwhile to study the rate of $dt\mu$ formation from these clustered ions. In this paper, the most simple clustered ion (the D_3^+ ion) is studied,^{14,15} as a first step. Due to the many modes which can be excited in the clustered ion, the temperature

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dependence of dt μ molecular formation is smoother than when the dt μ molecules are formed from diatomic molecules.

The addition of protonium would be beneficial, giving savings in fuel cost and also reducing the neutron wall loading: both factors have been criticized as regards their practical use in this fusion approach to energy production. Also, it will be beneficial to use this method as an energy source for future space propulsion.

Three effects of the addition of protonium to the D.T. mixture target can be considered. The first is an increase of the dt μ fusion cycle in a very dilute D.T. mixture. By adding the protonium to the low density D.T. gas mixture, the effective density becomes high, and dt μ molecular formation rate can be increased as the density increases by the third body effect.^{8,9,10} The second effect is the population change of the d μ ground state¹⁶ which may become a bottle neck in the fusion cycle. The third effect is a change in dt μ molecular formation when the protonium is in the form of a D-H molecule instead of a D₂ and H₂ molecule. These subjects will be discussed.

1. dt μ Formation under Laser Irradiation

When the laser irradiates the t μ -D₂ system, the kinetic part of the Hamiltonian $P_i^2/2M_i$ ($i = t, \mu, d_1, d_2, e_1, e_2$) is replaced by the $(\vec{P} + ie\vec{A}/C)^2/2M_i$ where $\vec{A}$ is the vector potential corresponding to the laser field, which is treated as the classical field due to the strong coherency. The term, $(e_i^2 A_i^2/2M_i C^2)$, expresses the interaction energy due to the ponderomotive force and changes only the phase of wave function. This term becomes large only when the laser intensity is much higher than the laser intensity considered here, therefore, this term can be neglected.

The linear term in A is $e\vec{P}_i \cdot \vec{A}/M_i C = e\vec{A}_i \cdot \vec{v}_i/C$ and it is proportional to the velocity v_i of the particle i . The velocities of the electron and muon are, respectively, of the order of $C/137$ and they are about a few thousand times the velocity of the nuclei motion, so that the interaction of the laser with electron cannot be neglected.

The interaction of the laser with the muon is comparable to the electron, but the muon is 200 times more strongly bound than the electron. Thus muon wave function is hardly changed by the laser field intensity which we are considering here. Therefore, we can neglect this muonic term.

The initial wave function of the t μ -D₂ molecule system and the final wave function of (dt μ -d-2e) molecular system under the laser with single frequency ω irradiation can be expressed as

$$\psi_i(r_k, t) = \sum_f \phi_{i m A}(r_k) e^{i(E_{i f} A + m \omega) t}, \quad (2.1)$$

where k stands for the muon (μ), triton (t), deuteron (d_1 and d_2) and electrons (e_1 and e_2). The wave functions are composed of many modes whose energies are separated with frequency ω .

The amplitude functions of $\phi_{imA}(r_k)$, $\phi_{fm'A}(r_k)$ and initial and final energies E_{iA} and E_{fA} are functions of the laser intensity A , and when the laser intensity increases, the amplitude of the high $|m|$ mode behaves like the function of $J_m(\frac{ePA}{\hbar c \omega})$ as discussed in the previous paper.¹¹ The rate of dtu molecular formation rate is expressed as

$$\lambda = \sigma v N_0 [\text{sec}^{-1}] , \quad (2.2)$$

where v is the relative velocity of the colliding beam, N_0 is the number density of the deuteron in the D,T mixture,

$$v d\sigma = 2\pi \hbar^{-1} \sum_{m,m'} |T_{fmim'}|^2 \delta(E_{fA} - E_{iA} + (m-m')\omega) \gamma(\epsilon, \epsilon_T) d\epsilon. \quad (2.3)$$

$\gamma(\epsilon, \epsilon_T)$ is the Maxwellian distribution function.

$$T_{fm,im'} = \int d r_k (\phi_{fmA}(r_k) H_{int} \phi_{im'A}(r_k)) \quad (2.4)$$

H_{int} is the interaction Hamiltonian of the tu atom and D_2 molecule and is expressed as

$$H_{int} = \sum_{k',k''} \frac{e_{k'} e_{k''}}{|r_{k'} - r_{k''}|} \quad (2.5)$$

where $k' = t, u$; $k'' = d_1, d_2, e_1, e_2$.

This interaction energy becomes large only when the tu atom comes close to the deuteron, because of the neutral charge on the tu atom. Thus, this can be approximated by the interaction energy between the dtu dipole moment and the deuteron (d_2) and the two electrons (e_1, e_2) as shown in Eq. (6):

$$H_{int} = d [\vec{p}/\rho^3 - (\vec{r}_{1e}/r_{1e}^3 + \vec{r}_{2e}/r_{2e}^3)] \quad (2.6)$$

In the paper by Vinitzky et al.⁴ and my previous paper,¹¹ only the first term of the interaction with the deuteron (d_2) is considered, and the second term of the interaction with the electron¹⁷ was neglected. When a strong laser field is applied to the D_2, T_2 (DT) mixture, the many modes with energies of $E_{Ai} + m'\omega$ and $E_{fA} + m\omega$ are excited, and the dtu molecule is formed by satisfying the energy conservation requirement through energy transfer between these modes. When the laser field is weak, the amplitudes of the model with high m, m' value are small: the transition matrix becomes small and it results in a small rate of formation. The amplitude change of the electronic wave function is calculated as a function of the strength of the laser field.

Under the single-mode laser field which is treated as the classical field, the schrödinger equation can be written as:

$$[H + \sum_i e \vec{P}_i \vec{A}(t)/c M_i] \psi(r_k, t) = -i\hbar \partial \psi(r_k, t) / \partial t \quad (2.7)$$

where H is the original Hamiltonian, and $\vec{A}(t)$ is the vector potential expressing the laser field which can be written as

$$\vec{A}(t) = \vec{A} \sin \omega t = \vec{A} [e^{i\omega t} - e^{-i\omega t}] / 2i \quad (2.8)$$

The wave function $\psi(r_k, t)$ is expanded as:

$$\psi(r_k, t) = \sum_{m=-\infty}^{\infty} \phi_m(r_k, t) e^{i(E+m\omega)t}, \quad (2.9)$$

and by substituting Eqs. (8) and (9) to Eq. (7) and calculating it we get

$$[H + m\omega] \phi_m(r_k) + \sum_i (e_i \vec{P}_i \vec{A}_i / 2i c M_i) [\phi_{m+1}(r_k) - \phi_{m-1}(r_k)] = E_A \phi_m(r_k) \quad (2.10)$$

This Schrödinger equation can be solved by expanding the wave function in term of some function, but the dimension of the matrix to be solved becomes very large for the large A value of a strong laser intensity.

To estimate the laser field strength required to enhance of $dt\mu$ molecular formation, we study the case when the laser field is linearly polarized in the direction of the hydrogen molecular axis. The study considers the case when the laser field affects only the electronic motion of the molecule; the study neglects the effect of nuclear motion resulting from perturbation of the electronic potential caused by the change in electron motion.

When the vector potential is in the direction of the diatomic nuclear axis, the electronic wave function can be expanded as the following polynomial functions:¹⁷

$$\phi_m(r_{e1}, r_{e2}) = \sum_{1,n,j,k,p} C_{mlnjkp} [1, n, j, k, p] \quad (2.11)$$

where $[1, n, j, k, p]$ stands for

$$(1/2\pi) \exp[-\delta(\lambda_1 + \lambda_2)] (\lambda_1^{1-\lambda_2} \lambda_2^{n\mu_1} j_{\mu_2} k_{\rho p} + \lambda_1^{n\lambda_2} \lambda_2^{1-\mu_1} k_{\mu_2} j_{\rho p}) \quad (2.12)$$

λ, μ are elliptic coordinate of electron and ρ is the interelectronic distance divided by the internuclear distances R of $(d-d)$ or $(dt\mu-d)$.

Due to the limitation of the machine memories of CRAY computer, the following ten terms are taken as the Hilbert space expansion, $[1, n, j, k, p] = [0, 0, 0, 0, 0], [0, 0, 0, 2, 0], [0, 0, 1, 1, 0], [1, 0, 0, 0, 0], [1, 0, 2, 0, 0], [0, 0, 1, 0, 0], [1, 0, 1, 0, 0], [0, 1, 1, 0, 0], [1, 1, 0, 0, 0]$ and $[2, 1, 0, 0, 0]$.

Figure 1 shows the rate of resonance $dt\mu$ formation as function of laser intensity. Curve 1 is a one photon ($m = 1$) exchange that balances the energy conservation for the formation process. The frequency ω corresponds to the energy difference of $[(dt\mu)-d-2e]$ and D_2 molecules ground state. The curves 2, 3, and 4 are the cases of two, three and four photons, respectively, which satisfy the energy conservation condition. In the low intensity region, the formation rates using the m photons increase as the m power of the intensity (I^m). As is the case for regular photon excitation and de-excitation, the cross-section of one photon process is independent of the photon intensity. The transition probability is proportional to:

$$|\langle \psi_f | \hat{A} \hat{V} | \psi_i \rangle|^2, \quad (2.13)$$

where ψ_i and ψ_f are, respectively, the wave function of the initial and the final state so that the reaction rate increases A^2 which is proportional to laser intensity I . In the case of m photons, transition probability in the perturbation calculation is expressed as:

$$\propto |\langle \psi_f | \hat{A} \hat{V} | \psi_k \rangle \langle \psi_k | \hat{A} \hat{V} | \psi_{k'} \rangle \dots \langle \psi_{k^{m-1}} | \hat{A} \hat{V} | \psi_k \rangle|^2 \quad (2.14)$$

so that the reaction rates become proportional to $A^{2m} I^m$.

Due to the limitation of the matrix dimension which can be handled by the computer in a reasonable time and of the computational method, only the case of up to ± 5 photons exchange is calculated here. When the laser intensity increases above that shown in Figure 1, more than 5 photons are excited and de-excited, and the accuracy of the calculation deteriorates.

In the one photon exchange, the high reaction rate requires higher laser intensity than an exchange of multiphotons, but for low rate reaction rate, the required intensity is lower than the multiphoton case. Therefore, the laser intensity and frequency that enhances the formation rate depends on the specified formation rate. In the case of $dt\mu$ formation rate, in which the largest formation is preferable, the enhancement using many multiphoton with low frequency laser is more efficient than the laser with high frequency; however, as shown in the figure, there seems to be a threshold laser intensity with the low frequency laser below which there is no enhancement of the rate of formation. Although we studied cases of multiphoton exchange up to $m = \pm 5$, the figure suggests that the intensity required from a laser with low frequency to achieve a $dt\mu$ formation rate of $\lambda_{dt\mu} = 10^{10} \text{ sec}^{-1}$ is about 10^8 watt/cm^2 . This value is 1/30 times smaller than that given in our previous study.¹¹

When the laser irradiates the D-T mixture, not only will the rate of $dt\mu$ formation be enhanced or decreased, but other side processes such as muon transfer, the excitation of the molecule's rotational motion associated with the muon catalyzed fusion, will

be changed. These reaction rates depend on the laser frequency and the intensity in the same way as they do for dt_μ formation. Figure 1 suggests that the competing phenomena for muon catalyzed fusion might be controlled by the proper choice of the laser's characteristics.

In this paper, we have treated the laser field as a classical field, so that the laser is completely coherent, and has no noise or other statistical nature. The formulation obtained for the single frequency mode laser can be easily extended to the case of P multi-frequency mode. The statistical nature of the laser can be treated as the ensemble average of the above transition probability.

2. dt_μ Molecular Formation in the Clustered Ion

When the t_μ atom collides with the clustered deuteron ion, the additional deuterons ions of the clustered ion play the role of third body as well as the deuteron participating for dt_μ formation. The dt_μ molecular formation rate is strongly affected by this spectator-participator deuteron nuclei.

Figure 2 shows the configurations of some clustered ions. In this paper the dt_μ formation rate in simplest clustered ion, D_3^+ ,^{14,15} ions, is calculated. This ion has been extensively studied and the calculated vibrational modes show the reasonable agreement with the experimental data.

The dt_μ molecular formation rate in clustered D_n^+ ions can be expressed by adding the $(n-1)$ deuteron and $(n-2)$ electrons to the formula for the D_2 molecule.

The interaction term can be expressed by the interaction energy between dt_μ dipole and the other charged particle. In this study, the electronic contribution is neglected and only the interactions with deuteron nuclei are taken into account.

The wave functions of the d_n^+ ion and the $[(dt_\mu) d_{n-1}]^+$ ion are calculated by using the electronic potential¹⁸ obtained by the Hartree Fock method.

The kinetic energy part of the Hamiltonian can be expressed with the normal coordinate S_1 , t_1 and r , which are defined in (3.1), but the parts associated with the t_1 and r are related to the translational motion and the rotational motion, they are neglected in this study.

$$\begin{pmatrix} \Delta x_1 \\ \Delta y_1 \\ \Delta x_2 \\ \Delta y_2 \\ \Delta x_3 \\ \Delta y_3 \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 0 & 0 & -b & a & -a & -b \\ 0 & 1 & -a & -b & -b & a \\ 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & -1 \\ 1 & 0 & b & a & -a & b \\ 0 & 1 & -a & b & b & a \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \\ r \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} \quad (3.1)$$

where $a = 1/2$, $b = \sqrt{3}$.

The interaction terms of the $dt\mu$ dipole and the other deuteron nuclei

$$-ed \cdot [\vec{R}_{31}/R_{31}^3 + \vec{R}_{21}/R_{21}^3] \quad (3.2)$$

are also expressed by this normal coordinate, where d is the dipole moment of $dt\mu$ molecule and R_{31} and R_{21} are, respectively, the distances of deuteron 3-1, and 2-1. The wave functions of the deuteron nuclei and $dt\mu$ nuclei are expanded by the polynomial of S_1 with gaussian weighting function as

$$\phi(S_1, S_2, S_3) = \sum_{\ell, m, n} a_{\ell, m, n} \phi_{\ell}(S_1) \phi_m(S_2) \phi_n(S_3) \quad (3.3)$$

$$\text{where } \phi_k(S_i) = S_i^k e^{-\alpha_i S_i^2}, \quad (k = 1, m, n \text{ for } i = 1, 2, 3) \quad (3.4)$$

The $dt\mu$ molecule is formed by the interaction energy of the $dt\mu$ dipole and the other deuteron nuclei, so that the two deuterons will assist in molecular formation in the case of the D_3^+ ion, instead of the one deuteron that assists in the D_2 molecule. The nuclear interaction energy is proportional to the inverse square of the equilibrium distance and for the D_3^+ ion, the equilibrium internuclear distance R_{eq} is a little larger ($\approx 1.66 a_0$, where a_0 = electron Bohr radius) than the distance in the D_2 molecule where $R_{eq} = 1.4 a_0$. Thus, the magnitude of the interaction energy is roughly $2/(1.66/1.4)^2 = 1.42$ times larger than that of the D_2 molecule, and the cross-section will be increased about $(1.42)^2 \approx 2$ times if they satisfy the energy conservation requirement.

The calculation was performed using the 15-dimensional Hilbert space. Table II shows the eigen value of the D_3^+ ion and $(Dt\mu)D_2^+$ ion and the transfer matrix element of dipole interaction. The 3rd and 4th columns of the table are the dipole transfer integral in the cases when the dipole polarization is directed as shown in Figure 4. The fifth column shows the absolute value dipole integrals averaged over the polarization angle. Due to this dipole integral value, the $dt\mu$ molecular formation rate from D_3^+ ion has a different temperature dependence from the D_2 molecule, as shown in Figure 3.

These vibrational frequency modes have strong isotopic dependence, thus the temperature dependence of $dt\mu$ formation rate can be changed by using a different isotopic composition. Also, it might be possible to produce a large increase in the formation rate, using a suitable clustered ion in the desired temperature range. Figure 5 shows the $dt\mu$ molecular formation rate for the D_3^+ ion which was calculated using the vibrational motion excitation. The formation rate is higher than for the diatomic case in the low temperature region.

The triple nuclei molecules like Li^7_3 and Li^6_3 molecule present interesting problems in physics. Recently, the Berry phase

Fig. 1. dtu formation under Irradiation

Fig. 2. Dn⁺ clusters configuration Fig. 3. dtu molecular formation from D₃⁺ clustered ion (E₁₁ is the dtu (J=1, V=1) energy level 0.63~0.66 eV) Table I The energy levels of D₃⁺ and dtu₂⁺ and the dipole element integral (Energy is unit of eV and D in unit of amu)

n	E _{D₃⁺}	E _{dtu₂⁺}	E _{dtu₂⁺} - E _{D₃⁺}	D ₀ ⁰	D ₂₀ ⁰	D ₄₀ ⁰
1	0.3887	0.3442	-0.0445	-5.34 (-2)	-3.282 (-2)	4.430 (-2)
2	0.6251	0.5380	0.1493	-3.496 (-2)	-1.479 (-1)	1.074 (-1)
3	0.6346	0.5558	0.1671	1.872 (-1)	-1.129 (-1)	1.545 (-1)
4	0.7160	0.6321	0.2434	-1.979 (-1)	-1.566 (-1)	1.784 (-1)
5	0.8731	0.7304	0.3417	-4.49 (-3)	-1.462 (-2)	1.075 (-2)
6	0.8750	0.7623	0.3736	-1.742 (-2)	-1.129 (-2)	1.468 (-2)
7	0.8892	0.7817	0.3930	-1.518 (-2)	-9.707 (-3)	1.274 (-2)
8	0.9652	0.8573	0.4686	4.339 (-3)	1.570 (-2)	1.151 (-2)
9	0.9941	0.8827	0.4940	5.086 (-3)	-1.033 (-2)	8.1425 (-3)
10	1.0329	0.9647	0.5760	1.446 (-2)	1.173 (-2)	1.316 (-2)
11	1.0769	0.9771	0.5884	-2.483 (-3)	2.631 (-2)	2.558 (-3)
12	1.1538	1.0235	0.6348	-9.093 (-3)	-7.29 (-3)	8.241 (-3)
13	1.2710	1.0951	0.7064	-1.275 (-2)	+6.479 (-3)	1.011 (-2)
14	1.2840	1.1462	0.7575	1.704 (-2)	2.418 (-3)	1.217 (-2)
15	1.3812	1.2476	0.8589	8.449 (-3)	2.683 (-3)	6.270 (-3)

factor¹⁹ was discussed in many quantum system. The molecular Aharonov-Bohm effect,²⁰ which is one of the Berry phase factors, produces a different eigen frequency from the one which does not take into account the Berry phase factor. It would be interesting to study whether or not the Berry phase factor has an effect on the $d\mu$ formation rate from the triple nuclei ion of D_3^+ .

4. The Effect of Protonium Additive on the Muon Catalyzed Fusion Cycle

4.1 Population of $d\mu$ Ground State

As discussed by Jandel et al.,²¹ the energy difference between excited $d\mu$ atom and $t\mu$ atom is only $48.17/n^2 \text{ eV}$ (where n is the principal quantum number of the excited state), and the binding energy of hydrogen molecule is about 4.6 eV . Therefore, the muon transfer process from $d\mu$ to $t\mu$ is suppressed because the excess energy of this transition is not sufficient to cause dissociation of the molecule, and this results in the bottleneck of the muon catalyzed fusion cycle.

The energy difference for the muon transfer from $p\mu$ to $t\mu$, or $d\mu$, is much larger than the one between $d\mu$ and $t\mu$. Therefore, the transfer is not suppressed below $n = 6$ for $t\mu$, and $n = 5$ for $d\mu$. If the muon transfer rate from $p\mu$ to $t\mu$ is much greater than that to $d\mu$, then the most of the muons captured by protonium are transferred to $t\mu$. To evaluate this, let us consider the ratio of these transfer rates. Roughly speaking, the transfer rate is expressed as the square of the transition matrix element times, the phase space, which is available for this transfer. The available phase space is approximately $3/2$ power of the energy difference of the above excited level minus the molecule's dissociation energy. If we assumed that the isotropic effect of the transition matrix element is very small, and the molecular dissociation energy is not taken into account, then the transfer rates of $p\mu \rightarrow t\mu$ and $p\mu \rightarrow d\mu$ are, respectively, 7.4 times and 4.6 times of the transfer $d\mu \rightarrow t\mu$. Therefore, the ratio of muon transfer rate $p\mu \rightarrow t\mu$ to $p\mu \rightarrow d\mu$ is 1.61 and only 62% of the muon capture by proton is transferred to the triton, even when there are large amounts of protonium are in the target mixture.

When we take into account the molecular dissociation energy, this ratio becomes large at high n states and a small percentage of muons captured by proton are transferred to the deuteron.

Table II shows the ratio of the muon transfer of $p\mu \rightarrow t\mu$ and $p\mu \rightarrow d\mu$ for $n = 2, 3, 4$ and 5 . For the $n = 5$ state, the ratio becomes 6.77, and at $n = 6$, only transfer to triton occurs. Therefore, if the population of the $n = 5$ and 6 of $p\mu$ is high, then the transfer to $t\mu$ is much greater than the one to $d\mu$, and thus can avoid the bottle-neck problem. But the Auger transition from $n = 5$ to $n = 4$ is $50\phi \cdot 10^{-11} \text{ sec}^{-1}$ and it is much larger than the transfer rate $p\mu \rightarrow t\mu$ ($17.4\phi \cdot 10^{11} \text{ sec}^{-1}$) (Fig. 4); thus the population of $p\mu$ $n = 5$ cannot be so large, and the effect of the addition of protonium on the fusion cycle is not very effective.

4.2 Resonance Muon Transfer

It must be pointed out that there is the possibility of resonance enhancement of muon transfer when $d\mu$ collides with T_2 or TD molecules. It was argued that the muon transfer from deuteron to triton in the collision of the highly excited state $d\mu$ to the molecules will be suppressed, due to insufficient energy to dissociate the molecule.²¹ However, even then, the excess energy is enough to excite the vibrational and rotational mode of the molecule.

Analogous to the $dt\mu$ resonance formation,^{22,4,5,6} the resonance muon transfer from $d\mu$ to $t\mu$ in the collision with tritium molecule can be expressed by the formula of:

$$\lambda_{d\mu \rightarrow t\mu} \propto \left| \langle \psi_{f,n}^+(t\mu) \psi_{f,v,k}^+(T,D) | H_{int} | \psi_{i,n}(d\mu) \psi_{i,o}(T_2) \rangle \right|^2 \delta[E_{d\mu,n} + E_{T_2,0} - (E_{t\mu,n} + E_{DT,v,k})] \quad (4.2.1)$$

In Eq (1), $\psi_{i,n}(d\mu)$ and $\psi_{i,o}(T_2)$ are the wave functions of the initial $d\mu$ atom in n^{th} excited state and the T_2 molecular state in the ground state: $\psi_{f,n}(t\mu)$ and $\psi_{f,v,k}(T,D)$ are wave functions of final $t\mu$ n^{th} excited state and the DT molecule in the vibrational v and rotational k state. The H_{int} is the interaction energy between the two states.

4.3 $dt\mu$ Molecular Formation by the D-H and D-T Molecules

When the proton is added to the DT mixture in the form of a D-H molecule, $dt\mu$ molecular formation rate shows very different characteristics than when the proton is added as a H_2 molecule.

D-H and $[(dt\mu)-p-2e]$ molecules have the smaller reduced nuclear masses than D-D and $[(dt\mu)-d-2e]$ molecules, so that the energy difference of excited $[(dt\mu)-p-2e]$ is wider than that of the $[(dt\mu)-d-2e]$ molecule. Table II shows the energy level differences between the $[(dt\mu)-p-2e]$ vibrational excited states and (D-H) ground state and those between the $[(dt\mu)-d-2e]$ and D_2 , which are calculated using the Morse potential approximation for the hydrogen electronic internuclear potential. The formula of energy level E_v for the v^{th} excited states in the Morse potential can be expressed in:

$$E_v = \frac{h}{a_0} \left(\frac{2D}{\mu} \right)^{1/2} \left(v + \frac{1}{2} \right) - \frac{1}{a_0^2 2\mu} \left(v + \frac{1}{2} \right)^2 - D, \quad (4.3.1)$$

where D is the depth of the Morse potential, μ is the reduced mass of participating nuclei including the attached muon and a_0 is the Bohr radius. In Figure 5, the resonance production rate of mesic molecular $dt\mu$ from D-H molecule, calculated by using the simple Vinitzky's formula of Eq. (4), which neglects the rotational motion

n	Energy Difference (Unit of eV)			
	$\frac{p\mu+t\mu}{p\mu+d\mu}$	$\frac{p\mu+t\mu}{p\mu+d\mu}$	$\frac{p\mu+d\mu}{d\mu+t\mu}$	$\frac{d\mu+t\mu}{p\mu+d\mu}$
2	1.70	45.8	33.5	12.0
3	1.89	20.3	14.9	5.35
4	2.43	11.4	8.38	3.01
5	6.78	7.32	5.36	1.93
6	=	5.08	3.72	1.33

Table II Energy differences of $[(d\mu)-p-2e]$ $[(d\mu)-d-2e]$, from the ground state of vibrational excitation $[(d\mu)-t-2e]$ molecules D-H, D-D and D-T

Table III Ratio of Muon Transfer Difference of $p\mu+t\mu$, $p\mu+t\mu$ to $p\mu+d\mu$ and Energy $p\mu+d\mu$ and $d\mu+t\mu$.

ν	$\frac{[(d\mu)-p-2e]}{-(D-H)}$	$\frac{[(d\mu)-d-2e]}{-(D-D)}$	$\frac{[(d\mu)-d-2e]}{-(D-T)}$ Vinitsky's	$\frac{[(d\mu)-t-2e]}{-(D-T)}$
0	-0.0249	-0.0317	-0.026	-0.0355
1	0.302	0.282	0.286	0.239
2	0.772	0.585	0.588	0.506
3	1.143	0.878	0.88	0.765

Fig. 4. Muon transfer between protonium, deuterium and tritium, and Auger transition of $n = 5 \rightarrow 4$ and $4 \rightarrow 3$. ϕ is density factor to liquid hydrogen density, X is the suppression factor discussed in reference 21.

Fig. 5. $dt\mu$ molecular formation rate from D-H, D_2 and D-T molecules

(Energy level $dt\mu$ ($\nu=1, j=1$))

of the molecule and the finite distance of the muon from the dt nuclei, are shown as the function of the target temperature:

Because of the smaller reduced nuclear mass of the $[(dt\mu)-p-2e]$ molecule than that of $[(dt\mu)-d-2e]$ molecule, the energy level interval of the former is wider than the latter, and when the $dt\mu$ ($J=1$, $v=1$) state energy level is $0.63 \sim 0.66$ eV, the energy conservation condition is satisfied by the excitation of the $v = 2$ level instead of the $v = 3$ level, as in the case of the D-H molecule. The transfer matrix element from ground state to excited v state decreases as the value of v increases. The value of transfer matrix for $v = 2$ state is about 4 times of that for $v = 3$ state.

Leon⁶ showed that the rotational motion excitation plays a very important role for $dt\mu$ formation when the vibrational $v = 2$ levels are just below the threshold energy. Due to the excitation of rotational motion, the matrix element from $v = 2$ can contribute, and this gives a large formation rate. This contribution should be studied, and a precise determination made of the vibrational and rotational excitation energy and the dipole interaction matrix element, including the electron screening effect.

In the case of D-T molecule,²³⁻²⁴ its small nuclear reduced masses compared with the D_2 molecule produces a smaller excitation level interval, as discussed by Leon.⁶ The energy differences of excited vibrational level of $(dt\mu-t-2e)$ molecule from the ground level of D-T molecule also are shown in Table III. The energy level of $v = 2$ is too low compared with the binding energy E_{00} of $dt\mu$ ($J = 1$, $v = 1$) state, and so the contribution due to rotational motion excitation becomes small. Thus only the $v = 3$ state contributes to the $dt\mu$ formation. Compared to the D_2 case, the $v = 3$ level is closer to the $dt\mu$ shallow binding energy E_{11} , which means that $dt\mu$ formation has its maximum at lower temperature. Figure 5 shows the $dt\mu$ molecular formation rate as a function of the temperature without taking into account the changes in rotational and other levels due to fine structure splitting, etc.^{9,25,26,27}

CONCLUSION

The interaction between laser and electron, which was neglected in the last paper, is strong and the laser intensity which is required to enhance the formation of $dt\mu$ can be reduced by taking into account this interaction. It is estimated from the limited number of photon exchange that by using multi photon exchange with the low frequency laser, the $dt\mu$ formation rate of 10^9 sec^{-1} can be achieved with an intensity of 10^8 watt/cm^2 . To verify this by numerical calculation requires solving the matrix equation with the very large number of dimensions.

The study suggests that the multiphoton mechanism of laser irradiation might be used for controlling the fusion cycle.²⁸⁻²⁹ The effect of laser irradiation in changing the motion of the nuclei due to the internuclear electron potential is neglected

here; a positive or a negative contribution depends upon the phase difference between nuclear and electronic motion. This fact should be studied in a future paper.

The study of $dt\mu$ molecular formation using the clustered ion shows that the temperature dependence of $dt\mu$ formation is quite different from that of the diatomic molecule, due to the many modes which exist in the clustered ion compared with the diatomic molecule, also there is the possibility of coherent enhancement.

The effects of addition of protonium to the deuterium tritium mixture target in the $dt\mu$ fusion cycle are studied in the context of a possible reduction in the population of $d\mu$ ground state and a change in the formation rate of $dt\mu$ by the D-H molecule from the one by the D_2 molecule. The effect of the protonium additive in reducing the $d\mu$ ground state population is not very marked, because of the limited muon transfer rate ratio of $p\mu + t\mu$ to $p\mu + d\mu$; only a small percentage of reduction can be achieved even when there are large quantities of protonium present in the target. Relating to the bottle-neck problem of high population of the $d\mu$ ground state, the muon can be more effectively transferred as resonance process in the same way as resonance $dt\mu$ molecular formation in the molecular state target. This rate is very large especially in the energy range where the suppression of muon transfer is due to insufficient energy to cause the molecule to dissociate. Thus, the bottle-neck problem relating to muon suppression of transfer of $d\mu$ to $t\mu$ might not exist. By adding protonium to the DT target in the form of the D-H molecule, the $dt\mu$ molecular formation rate will be enhanced in the high temperature mixture above 1000°C .

In a similar way the characteristics of $dt\mu$ formation rate in the D-T molecule also change.²³⁻²⁴ Simple calculation using Vinitzky's formula indicates that this also enhance the formation. But the formation rate is very sensitive to the data used in the calculation, and a more detailed calculation is required before a final conclusion can be drawn.

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