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EXCITED BY ENERGETIC-TRAPPED PARTICLES IN LARGE-SIZE TOKAMAKS

By

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THEORY OF RESISTIVE MAGNETOHYDRODYNAMIC INSTABILITIES
EXCITED BY ENERGETIC-TRAPPED PARTICLES IN LARGE-SIZE TOKAMAKS

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ABSTRACT

It is shown that, in present-day large-size tokamaks, finite resistivity modifies qualitatively the stability properties of magnetohydrodynamic instabilities resonantly excited by the unfavorable precessional drift of energetic-trapped particles, i.e., the so-called "fishbone"-type instabilities. Specifically, it is found that (i) the $n = 1$ energetic-trapped particle-induced internal kink ("fishbone") instability is strongly stabilized by resistive dissipation and (ii) finite resistivity lowers considerably the threshold conditions for resonant excitations of high- n ballooning/interchange modes. The possibility of exciting fishbones by alpha particles in ignition experiments is also considered.

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MASTER

I. INTRODUCTION

In recent years, observations of magnetohydrodynamic (MHD) instabilities and associated particle loss in tokamak experiments with high-power near-perpendicular neutral beam injection^{1,2} have stimulated considerable theoretical interest in MHD modes resonantly excited by the precessional drift of energetic particles trapped in the region with unfavorable curvature, i.e., the outside of a tokamak. These so-called "fishbone"-type MHD instabilities, thus, have, generically, mode frequencies, ω , comparable to the trapped-particle precessional frequencies, $\bar{\omega}_{dh}$.³ Theoretical models based on this resonance excitation mechanism have been rather successful in explaining many experimental features observed in both the $n = 1$ fishbone⁴ and high- n "precursor"^{5,6} events in tokamaks the size of PDX. Here n is the toroidal mode number. To extend these theoretical results to current high-field large-size tokamaks such as TFTR, JT-60, and JET we must examine the validity of the core-plasma ideal MHD approximation employed in the theoretical models. Note that $\bar{\omega}_{dh} = E/BR$ with $E = v^2/2$ and R the major radius. Hence, in large devices, $\bar{\omega}_{dh}$ may become comparable to or even smaller than the characteristic resistive frequency $\omega_R = S_M^{-1/3} \omega_A$ and finite-resistivity effects may be significant. Here, S_M is the magnetic Reynold number, $\omega_A = (1 + 2 q_s^2)^{1/2} V_A/q_s R$ is the toroidal Alfvén frequency, q_s is the safety factor q at the singular surface $r = r_s$, and V_A is the toroidal Alfvén velocity.

In the next section, we first consider the effects of finite resistivity on the $n = 1$ fishbone internal kink instability. The high- n fishbone-type resistive ballooning/interchange instabilities are then considered in Sec. III. Finally, in Sec. IV, noting that fusion alpha particles will carry significant β ($\equiv$ plasma pressure/magnetic pressure) values in future ignition experiments, we also consider the possible excitations of α -particle-induced fishbone instabilities.

II. $n=1$ RESISTIVE FISHBONE INSTABILITIES⁷

As in the ideal fishbone theory,⁴ we consider a large-aspect-ratio ($\epsilon \equiv a/R \ll 1$) circular tokamak plasma consisting of core (c) and hot (h) components and perturbations of predominantly the $m=1$, $n=1$ internal kink mode. Here, m is the poloidal mode number. We adopt the following formal orderings: $\beta_{pc} \sim 1$, $\beta_{ph} \sim \epsilon$, $T_c/T_h \sim \epsilon^2$, and $|\omega/\omega_A| \sim |\bar{\omega}_{dh}/\omega_A| \sim \epsilon^2$. In addition, we order $\omega \sim \omega_R$, i.e., $S_M^{-1/3} \sim \epsilon^2$, which represents an optimal ordering for the resistive effects. Consistent with these orderings, the hot component is described by the gyrokinetic equation, but the core plasma must be treated as a resistive MHD fluid obeying the Ohm's law

$$\delta \underline{B} = \underline{\nabla} \times (\underline{\xi} \times \underline{B}) - (\eta c^2 / 4\pi i \omega) \nabla^2 \delta \underline{B}, \quad (1)$$

where $\underline{\xi}$ is the (core-plasma) fluid displacement and η is the resistivity.

By separating the regions outside and inside the singular layer located at $q(r_s) = 1$, we can derive the dispersion relation variationally. Since resistivity is ignorable in the outside region, the only modification to the ideal fishbone theory is the replacement of the inertial-layer Lagrangian by the following resistive-layer Lagrangian

$$D_R[\underline{\xi}] = -2\pi R r_s^3 \int_{-\infty}^{\infty} dx \{ 3\omega^2 \rho_m |d\underline{\xi}_r/dx|^2 + i B k_{||}' \times \underline{\xi}_r^* d^2 \delta B_r / dx^2 \}, \quad (2)$$

where $x = r - r_s$, $k_{||}' = q'(r_s)/R$, and δB_r from Eq. (1) acts as the constraint.

Following the analyses of Coppi *et al.*,⁸ it is straightforward to solve the resistive-layer Euler-Lagrange equation and match the solutions in the two regions. The final dispersion relation is then given by

$$\delta\hat{W}_{fc} + \delta\hat{W}_k + 8 S_M^{-1/3} \Omega^{-5/4} \frac{\Gamma[(\Omega^{3/2} + 5)/4]}{\Gamma[(\Omega^{3/2} - 1)/4]} = 0, \quad (3)$$

where $\Omega = -i\omega/\omega_R$, $\delta\hat{W}_{fc}$, defined in Ref. 4, is the normalized core-plasma contribution to the ideal MHD δW ,

$$\delta\hat{W}_k = \frac{2^{3/2} M_h \pi^2}{B_0^2} < \int_{1-R/R}^{1+R/R} d(\alpha B) \int_0^\infty dE E^{5/2} \frac{K_2^2}{K_b^2} \frac{\omega}{\bar{\omega}_{dh} - \omega} \left(\frac{\partial}{\partial E} + \frac{\omega_{*h}}{\bar{\omega}_{dh}} \right) F_{oh} >, \quad (4)$$

$\langle y \rangle \equiv (2/r_s^2) \int_0^{r_s} y \, r dr$, $\alpha = v_\perp^2/v^2$, $\omega_{*h} = -(i/\omega_c)(\hat{e}_\parallel \times \nabla \ln F_{oh}) \cdot \nabla$, $K_b = \oint (d\theta/2\pi)(1 - \alpha B)^{-1/2}$, $K_2 = \oint (d\theta/2\pi) \cos\theta (1 - \alpha B)^{-1/2}$, and only $m = 1$, $n = 1$ perturbations are kept in Eq. (4).

In the $|\Omega| \gg 1$ ideal limit, we recover from Eq. (3) the ideal fishbone dispersion relation⁴

$$\delta\hat{W}_{fc} + \delta\hat{W}_k - i\omega/\omega_A = 0, \quad (5)$$

and the corresponding threshold condition, using a model slowing-down distribution function as an example, is given by

$$\langle \varepsilon_{ph,t} \rangle_{cr}^I \sim \bar{\omega}_{dm}/\omega_A, \quad (6)$$

where the subscript t denotes trapped particles and $\bar{\omega}_{dm} = \bar{\omega}_{dh}(E=E_{inj})$. Note that below the threshold, i.e., for $\varepsilon_{h,t} < \langle \varepsilon_{h,t} \rangle_{cr}^I$, the singularity associated with damped ideal fishbone modes is regularized by finite resistivity with a characteristic width of $S_M^{-1/3}$.

In the $|\Omega| \ll 1$ regime of interest here, i.e., the resistive limit, Eq. (3) reduces to

$$\delta \hat{W}_{fc} + \delta \hat{W}_k = A_s e^{i5\pi/8} S_M^{-1/3} (\tilde{\Omega})^{-5/4}, \quad (7)$$

where $\tilde{\Omega} \equiv \omega/\bar{\omega}_{dm}$ and $A_s = -8 (\bar{\omega}_{dm}/\omega_R)^{-5/4} \Gamma(5/4)/\Gamma(-1/4) > 0$. The instability threshold condition then becomes, again, for a model slowing-down distribution function,

$$2^{9/4} \geq \langle \epsilon \beta_{ph,t} \rangle_{cr}^R S_M^{1/3} / A_s \geq 1 \quad (8)$$

and $1/2 < \tilde{\Omega}_{cr} < 1$, that is,

$$\langle \epsilon \beta_{ph,t} \rangle_{cr}^R \sim A_s S_M^{-1/3} \sim \left(\frac{\omega_R}{\bar{\omega}_{dm}} \right)^{9/4} \frac{\bar{\omega}_{dm}}{\omega_A} \sim \left(\frac{\omega_R}{\bar{\omega}_{dm}} \right)^{9/4} \langle \epsilon \beta_{ph,t} \rangle_{cr}^I. \quad (9)$$

Thus, in the resistive limit, the threshold β value of the energetic-trapped particles is raised dramatically over the ideal-limit value. Since physically the threshold condition can be understood in terms of balancing negative dissipation due to the trapped-particle precession mode with positive dissipation of Alfvén wave resonance absorption at the singular layer, this stabilization then corresponds to enhancement of positive dissipation due to finite resistivity.

IV. HIGH- n RESISTIVE FISHBONE-TYPE BALLOONING/INTERCHANGE MODES⁹

This study is in part motivated by observations of high- n ($n \sim 5$) precursors to fishbones in PDX experiments, which have been theoretically interpreted as high- n ideal MHD ballooning modes resonantly excited by trapped

beam-ion precessional drift, i.e., similar to the $n = 1$ fishbone instability. In addition to the above-mentioned issue concerning validity in large-size devices, the applicability of the ideal-MHD theoretical models appears to be also severely restricted to either singular-type energetic trapped particle distribution functions or regimes near ideal-MHD ballooning-mode marginal stability. Specifically, note that the dispersion relation of high- n ideal fishbone-type ballooning modes is of the same form as that of $n = 1$ ideal fishbone, Eq. (5), with appropriate replacements of $\delta\hat{W}_{fc}$ and $\delta\hat{W}_k$ in terms of surface quantities. For $\epsilon\beta_{pc} \sim \epsilon \ll (\epsilon\beta_{p,MHD})_{cr}^I \sim 1$, i.e., away from ideal-MHD marginal stability, $\delta\hat{W}_{fc} \sim 1$ due to field-line bending and, hence, balancing the real part of Eq. (5) dictates $\text{Re}(\delta\hat{W}_k) \sim O(\epsilon\beta_{ph,t}) \sim 1$ except for nonanalytic F_{oh} such as the model slowing-down distribution function which yields logarithmically singular $\delta\hat{W}_k$. Since field-line bending and, hence, $\delta\hat{W}_{fc}$ can be minimized by finite resistivity, in this case high- n resistive fishbone-type instabilities could be excited with regular F_{oh} and $\epsilon\beta_{ph,t} \sim \epsilon^2$, more consistent with experimental conditions.

Employing two small parameters ϵ and $\lambda \equiv (n^2/S_M)^{1/3}$, the orderings adopted here are $\beta_{pc} \sim 1$, $\beta_{ph} \sim \epsilon$, $\omega_R = \lambda\omega_A$, $\bar{\omega}_{dh} \sim \epsilon^{4/3}\omega_R$, and, in a maximal scheme, $\lambda \sim \epsilon^3$. The frequency ordering $|\bar{\omega}_{dh}/\omega_R| \sim \epsilon^{4/3} < 1$, corresponds to the resistive limit of interest in large-size tokamaks and also implies eigenmode properties similar to those of the electrostatic resistive interchange mode. The dispersion relation can then be straightforwardly derived and is given, in the incompressible limit and for the lowest (most unstable) eigenstate, by

$$\delta\hat{W}_k - D_{RC} + (-i\omega/\omega_R)^{3/2} = 0, \quad (10)$$

where $D_{RC} = E + F \sim 0$ ($\epsilon^2 s_{pc}^2$) < 0 represents favorable average curvature.¹⁰

We remark that inclusion of compressibility does not alter our final conclusions in any substantial way. Stability properties predicted by Eq. (10) have been analyzed using the Nyquist technique for both model slowing-down and Maxwellian F_{oh} , and are qualitatively similar. In particular, the instability threshold condition is given by

$$\langle \epsilon s_{ph,t} \rangle_{cr}^R \sim \max [|D_{RC}|, (\bar{\omega}_{dh}/\omega_R)^{3/2}] \sim 0 (\epsilon^2). \quad (11)$$

Here, $\langle \rangle$ denotes surface average. Equation (11) indicates that, with the field-line bending stabilizing term minimized by finite resistivity, high-n resistive fishbone-type ballooning/interchange instabilities can be excited for $s_{pc} \sim 1$ and $s_{ph,t} \geq \epsilon$, which is much lower than the ideal prediction [$\epsilon(s_{pc} + s_{ph,t}) \sim 0$ (1) for a Maxwellian F_{oh}] and, hence, appears to agree better with the PDX experiments. Also, we remark that since the eigenmodes are highly localized around the mode-rational surfaces, finite banana width may assert a stabilizing influence. In this case, it can be shown that the threshold condition is increased by a factor $(\rho_{bh}/\rho_R) \ln(\rho_{bh}/\rho_R)$ where ρ_{bh} is the typical energetic-trapped particle banana width and $\rho_R \sim r_s (nS_M)^{-1/3} (\bar{\omega}_{dh}/\omega_R)^{1/4}$ is the typical mode radial width. Finally, we note that, by performing a three-region asymptotic matching, the usual high-n resistive ballooning/tearing dispersion relation^{11,12} can be generalized to include effects due to energetic-trapped particles such that both ideal and resistive fishbone-type modes are derivable in the appropriate limits.

IV. ALPHA-PARTICLE-INDUCED FISHBONE INSTABILITY¹³

In proposed tokamak ignition experiments, fusion α particles generally contribute a significant fraction of the total β values and, hence, the possibility of exciting fishbone-type instabilities and an associated anomalous loss of α particles needs to be seriously considered. Assuming typical reference parameters $R = 135$ cm, $a = 42$ cm, $B = 10^2$ kG, and $n_0 = 10^{15}/\text{cm}^3$, the precessional frequency of the 3.5 MeV α particles is then $\bar{\omega}_{d\alpha} \sim 10^5 \text{ n}$ and $\omega_A \sim 2 \times 10^7$. That is $\bar{\omega}_{d\alpha}/\omega_A \sim n \cdot 5 \times 10^{-3} \geq (n^2/S_M)^{1/3}$ for, typically, $S_M \geq 10^7$. We, therefore, need only to consider ideal fishbone-type modes. Furthermore, since β values are usually sufficiently below $(\beta_{\text{MHD}})_{\text{cr}}^I$, high- n modes are stable and only the $n = 1$ fishbone instability may be excited. From Eq. (6), we find $\langle \beta_{\alpha,t} \rangle_{\text{cr}}^I \sim 1.7 \times 10^{-3}$ or, noting $n_{\alpha,t}/n_\alpha \sim 1/3$ for isotropic α -particle distributions, $\langle \beta_\alpha \rangle_{\text{cr}}^I \sim 0.5\%$. Taking $\beta_\alpha/\beta_{\text{total}} \sim 0.1$,¹⁴ we then derive $(\beta_{\text{total}})_{\text{cr}}^I \sim 5\%$ as the corresponding instability threshold condition, which may be exceeded in some designs. Finally, it is instructive to note the scaling $\beta_{\alpha,t}/\langle \beta_{\alpha,t} \rangle_{\text{cr}}^I \sim n_\alpha/n_0^{1/2}$ and, hence, high-density ignition experiments are more favorable with regard to the stability of the fishbone.

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