PROPAGATIONS OF DRIFT WAVES IN TOROIDAL PLASMA SYSTEMS

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Abstract

Drift wave patterns in toroidal plasmas are studied. The dispersion relation was simplified to retain both the shear and the toroidal coupling effects. Since the dispersion relation does not depend on the toroidal angle, ϕ , the dispersion is solved in the two-dimensional space made up with minor radius and poloidal angle. The dispersion relation can be reduced into second-order, partial differential equations of a hyperbolic type. The one-dimensional convective mode analysis, which was originated in the 1960's, was extended into the two-dimensional analysis. Depending on the strength of the magnetic shear, one can obtain either the convective or the localized solutions. The results show that the plasma is expected to be unstable for large azimuthal mode number and that the plasma instability tends to be more stabilized for large mass ions.

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I. Introduction

Drift waves in plasmas have been extensively studied theoretically, as the fluctuations caused by drift waves are considered to be most likely candidates for the cause of anomalous transports in toroidal plasmas. The analysis is, however, difficult, because, on one hand, the experimental data on fluctuations in modern-day toroidal plasmas are meager and, on the other, the number of possible modes for the drift waves are many, so that the non-linear estimate of fluctuations is exceedingly difficult.

However, there exist considerable amounts of data for warm toroidal plasmas where the Langmuir probe measurement could be performed. The results from both stellarators and multipole devices show that the spectra of density fluctuations widen in frequency, as (i) the magnetic shear is decreased and (ii) the ion mass is decreased (from Xe^+ to H^+).^{1,2}

It has been hitherto assumed that the behavior of the widening of the spectra be explained only with the fully-developed non-linear calculation of drift waves. This may not be true, if the present analysis is correct.

II. Idealized drift waves in a sheared, toroidal system.

The drift waves in modern-day tokamaks at about half the minor radius from the plasma edge, are probably caused either by the trapped electrons or by the strong ion temperature gradients. In the limits where we have, $|k_{\perp}| \rho_s < 1$, where ρ_s is the ion Larmor radius at the sum of the electron temperature, T_e , and ion temperature, T_i , and where $k_{\parallel}^2$ is small compared to ω_{pe}^2/c_s^2 , we may approximate the drift wave dispersion equation by

$$[L(s,j) + Q_1(s,j) - \epsilon_n T/\Omega] \hat{\phi}_j(s) = 0 \quad (1)$$

with

$$L = b_0 (\hat{s}^2 d^2/ds^2 - 1) \quad (2)$$

$$Q_1 = \frac{1}{\Omega} - 1 + \left[\frac{(s-j)}{\eta_s \Omega} \right]^2, \quad (3)$$

and

$$T\hat{\phi}_j(s) = \hat{\phi}_{j+1}(s) + \hat{\phi}_{j-1}(s) + \hat{s} \frac{\partial}{\partial s} [\hat{\phi}_{j+1}(s) - \hat{\phi}_{j-1}(s)]. \quad (4)$$

We followed the notation given in Eqs. (2) - (5) of Ref. 3, exactly.

Here $\hat{\phi}_j(s)$ is the Fourier transform of the potential fluctuations in the usual toroidal coordinates, (r, θ, ϕ) ,

$$\phi(r, \theta, \phi) = \sum_j \hat{\phi}_j(s) \exp\{i[m_0\theta + j\theta - n\phi - \omega t]\} \quad (4a)$$

where $|j| \ll m_0$, $s = (r - r_0)/\Delta r_s$, r_0 is the reference mode-rational surface $m_0 = nq(r_0)$, $\Delta r_s = 1/k_\theta \hat{s}$, $k_\theta = m_0/r_0$, and $\hat{s} = r_0/q$ at $r = r_0$. Also, $b_\theta = k_\theta^2 \rho_s^2$, $c_s = (T_e/m_i)^{1/2}$, $\Omega = \omega/\omega_{*e}$, $\eta_s = q b_\theta^{1/2}/\epsilon_n$, $\epsilon_n = r_n/R$, $r_n^{-1} = |d \ln n_0/dr|$, ω_{*e} = drift frequency = $b_\theta^{1/2} c_s/r_n$. The parameter, ϵ_n , is the ratio of the plasma equilibrium density scale length over the major radius and is assumed to be positive. The operator T arises because the magnetic field curvature as seen by the plasma is a function of poloidal angle, θ . The operator T is model-dependant. That is, the coefficients in front of $\hat{\phi}_{j\pm 1}$, and $\hat{s} \frac{\partial}{\partial s} \hat{\phi}_{j\pm 1}$ could be different from unity, if different toroidal equilibria were chosen. Also the magnetic well is completely neglected. However, this simplification is sufficient to represent drift wave behaviour in toroidal systems realistically.

Up to here the model is identical with that described in Ref. 3 [to the end of the paragraph following their Eq. (5)]. Now we take a different path.

In Ref. 3, to solve Eq. (1) in a tractable way, the authors let

$$Z \equiv s-j, \quad (5)$$

$$\hat{\phi}_j(s) \equiv \Phi(Z), \quad (6)$$

$$\hat{\phi}_{j\pm 1}(s) \equiv \Phi(Z \pm 1). \quad (7)$$

The assumption stems from the observation that, in a strong shear system, $|s| \sim O(1)$, the width of $\hat{\phi}_j(s)$ ($j=0$) in s is large enough so that at $s \sim \hat{\phi}_0(1)/\hat{\phi}_0(0) \sim O(1)$. If so, $|\hat{\phi}_j(s)|$ ($j \neq 0$) will be excited especially at $s \sim j$ where $k_{\parallel}$ for that mode is zero. Then the previous analysis of drift waves (e.g. Berk and Pearlstein)^{4,6} will not apply.

However these new assumptions, Eqs. (5) - (7), encounter the difficulty in that they allow the non-zero value even at $s(\approx j) \rightarrow \pm \infty$. Superficially all $m(=m_0+j)$ modes must be excited at once. The mode is termed the quasi-mode. We may visualize this quasi-mode, by expressing $\hat{\phi}_j(s)$ on the (s,j) plane by writing $\hat{\phi}_j(s) = \hat{\phi}(s,j)$. Then the $\hat{\phi} = \text{const.}$ contour lines can be shown as in Fig. 1. As is shown, the quasi-mode amplitude contour is not restricted in a finite domain. It is often argued that the factors, beyond the simplified model, such as the variation in the equilibrium density scale distance, are responsible for restricting $\hat{\phi}(s,j)$ in a finite domain.⁷

We shall analyze here Eq. (1) from a different point of view. We first note that $\hat{\phi}_j(s)(\equiv \hat{\phi}(s,j))$ is a function of a real space coordinate, s , and the mode number, j . If we were to solve the time-dependent problem, we would expect that $\hat{\phi}(s,j)$ is peaked near $\hat{\phi}(0,0)$ where the growth rate, or the imaginary part of ω , reaches the maximum. The analogy to the previous analysis of Coppi⁴ and Yoshikawa⁵ is obvious. Then the wave packet, as its amplitude grows, will spread out in both s and j directions. The behaviour of the spreading of the wavepacket is anisotropic in the (s,j) plane. Usually the spread speed is slowest on the $s=j$ axis.

Another way to view this problem is to try to solve Eq. (1) in the absence of the magnetic curvature (not equivalent to letting $\epsilon_n = 0$). Then there is no coupling between different j 's. The equation (1) is then satisfied with $\hat{\phi}_0(s) \neq 0$ and $\hat{\phi}_j(s) = 0$ ($j \neq 0$). The function $\hat{\phi}_0(s)$ is that analyzed previously.^{4,5} It is clear that assumptions (5) - (7) are incompatible with this solution.

III. Solution

We shall assume that toroidal coupling is sufficiently strong, that is, we assume that $\epsilon_n \sim b_\theta$. Then under conditions encountered experimentally, many m -modes can be simultaneously excited.

We may then approximate the operator by

$$T\hat{\phi}_j(s) = T\hat{\phi}(s,j) = 2\hat{\phi} + \frac{\partial^2 \hat{\phi}}{\partial j^2} + 2\hat{s} \frac{\partial^2}{\partial s \partial j} \hat{\phi} . \quad (8)$$

Thus Eq. (1) becomes

$$b_\theta \hat{s}^2 \frac{\partial^2 \hat{\phi}}{\partial s^2} - \frac{\epsilon_n}{\Omega} \left(\frac{\partial^2 \hat{\phi}}{\partial j^2} + 2\hat{s} \frac{\partial^2}{\partial j \partial s} \hat{\phi} \right) + \left[-b_\theta + \frac{1}{\Omega} - 1 - \frac{2\epsilon_n}{\Omega} \right] \hat{\phi} + \left[\frac{(s-j)}{\eta_s \Omega} \right]^2 \hat{\phi} = 0 . \quad (9)$$

The bracket of the third term can be considered an eigenvalue. Now this (hyperbolic) partial differential equation can be converted to a standard form by

$$Z = s - j ,$$

$$u = s + \alpha j ,$$

$$\alpha = \frac{\hat{s} + (b_\theta \Omega / \epsilon_n) \hat{s}^2}{\hat{s} - 1} ,$$

to

$$A \frac{\partial^2 \hat{\phi}}{\partial Z^2} + B \frac{\partial^2 \hat{\phi}}{\partial u^2} + \left(-b_\theta + \frac{1}{\Omega} - 1 - \frac{2\epsilon_n}{\Omega} \right) \hat{\phi} + \left(\frac{Z}{\eta_s \Omega} \right)^2 \hat{\phi} = 0 . \quad (10)$$

Two constants A and B (which can take either sign) are

$$A = b_\theta \hat{s}^2 - \frac{\epsilon_n}{\Omega} + \frac{\epsilon_n}{\Omega} 2\hat{s} , \quad (11)$$

$$B = b_\theta \hat{s}^2 - \frac{\epsilon_n}{\Omega} \alpha^2 - \frac{\epsilon_n}{\Omega} 2\hat{s}\alpha . \quad (12)$$

The characteristics of Eq. (10) may be determined by the sign of A and the u -axis. In Fig. 2, the domains of A and α are plotted vs. $(\hat{s}, \gamma)$ plane, where

$$\gamma \equiv \frac{b_0}{\varepsilon_n/\Omega} . \quad (13)$$

The three domains defined in Fig. 2 correspond to three domains of the orientation of the u' -axis (where u' axis is orthogonal to the u -axis) shown in Fig. 3. Here we assume that $\gamma > 0$ and $\hat{s} > 0$. In modern tokamaks, $\hat{s} \sim O(1)$ and $0 \leq \gamma \leq 1/\varepsilon_n \approx$ aspect ratio. We note that the assumptions leading to Eq. (1) require that $|\text{Im}(\Omega)| \ll \text{Re}(\Omega) \sim 1$ and $b_0 < 1$.

Now we consider that $\Omega = \Omega_0 + \Delta\Omega$ where

$$\Omega_0 \equiv \frac{1-2\varepsilon_n}{1+b_0} , \quad (14)$$

$$|\Delta\Omega| \ll |\Omega_0| . \quad (15)$$

Then the eigenvalue of Eq. (10) defined as

$$\lambda \equiv -(1+b_0) + \frac{1}{\Omega} (1-2\varepsilon_n) \quad (16)$$

may be expanded as

$$\begin{aligned} \lambda(\Omega) &= (\Omega - \Omega_0) \left(\frac{\partial \lambda}{\partial \Omega} \right)_{\Omega=\Omega_0} \\ &= -(\Omega - \Omega_0) \frac{1-2\varepsilon_n}{\Omega_0^2} \equiv (\Omega - \Omega_0) C_0 . \end{aligned} \quad (17)$$

To the lowest order, we can let $A(\Omega) = A(\Omega_0)$ etc., then Eq. (10) is simplified to

$$A_0 \frac{\partial^2 \hat{\phi}}{\partial Z^2} + B_0 \frac{\partial^2 \hat{\phi}}{\partial u^2} + \left(-i \frac{\partial \hat{\phi}}{\partial t} - \Omega_0 \hat{\phi} \right) C_0 + \frac{Z^2}{\eta_s^2 \Omega_0^2} \hat{\phi} = 0 . \quad (18)$$

Equation (18) is equivalent to a Schrodinger equation describing the dynamics of an electron with anisotropic mass in a two-dimensional space with potential extremum at $Z = 0$. Here we have converted $\Delta\Omega$ back to the time derivative. For convenience t is measured in the unit of $1/\omega_e$.

Now we let

$$\hat{\phi} = F(Z, t) G(u, t) . \quad (19)$$

Then we have the separation of variables, and from Eq. (18) we obtain

$$B_0 \frac{\partial^2 G}{\partial u^2} - i C_0 \frac{\partial G}{\partial t} - K^2 G = 0 , \quad (20)$$

$$A_0 \frac{\partial^2 F}{\partial Z^2} - i C_0 \frac{\partial F}{\partial t} + \left(K^2 - \Omega_0 C_0 + \frac{Z^2}{\eta_s^2 \Omega_0^2} \right) F = 0 , \quad (21)$$

where K^2 is an arbitrary constant. Both Eqs. (20) and (21) are standard forms solved by Lau and Briggs⁸ (and others).

We assume that the wavepacket is localized initially near $u = 0$ and $Z = 0$. That means that at $t = 0$, we have

$$G = \exp \left(- \frac{u^2}{\Delta u^2} \right) , \quad (22)$$

and

$$F = \exp \left(- \frac{Z^2}{\Delta Z^2} \right) . \quad (23)$$

The complete solutions of F and G (hence $\hat{\phi}$ and ϕ) can be solved from Eqs. (20) and (21) and are

given by

$$G(u,t) = \frac{\exp\left(i\left(\frac{K^2}{C_0}\right)t\right)}{\left(1 - \frac{4it}{C_0\Delta u^2/B_0}\right)^{1/2}} \exp\left[-\frac{u^2}{\Delta u^2\left(1 - \frac{4itB_0}{C_0\Delta u^2}\right)}\right], \quad (24)$$

$$|G(u,t)| = \frac{\Delta u}{\left|\Delta u^4 + \frac{16t^2}{C_0^2 B_0^2}\right|^{1/4}} \exp\left[-\frac{u^2}{\left(\Delta u^4 + \frac{16t^2 B_0^2}{C_0^2}\right)^{1/2}}\right], \quad (25)$$

and

$$F(z,t) = \frac{\exp(i\Omega_0\omega_{ce}t)}{H_1^{1/2}} \exp\left(-\frac{z^2/\Delta Z^2}{H_2}\right), \quad (26)$$

$$|F(z,t)| = \frac{1}{H_3^{1/4}} \exp\left(-\frac{z^2/\Delta Z^2}{H_3^{1/2}}\right). \quad (27)$$

Here we define

$$\alpha' \equiv C_e/A_0, \quad \beta \equiv \frac{1}{A_0\eta_s^2\Omega_0^2}, \quad \dot{\Omega}_0 \equiv \Omega_0 - \frac{K^2}{C_0}, \quad (28)$$

$$H_1 \equiv \cosh\left(\frac{2\sqrt{\beta}t}{\alpha'}\right) - \frac{2i}{\sqrt{\beta}\Delta Z^2} \sinh\left(\frac{2\sqrt{\beta}t}{\alpha'}\right), \quad (29)$$

$$H_2 \equiv \frac{\cosh\left(\frac{2\sqrt{\beta}t}{\alpha'}\right) - \frac{2i}{\sqrt{\beta}\Delta Z^2} \sinh\left(\frac{2\sqrt{\beta}t}{\alpha'}\right)}{\cosh\left(\frac{2\sqrt{\beta}t}{\alpha'}\right) + \frac{i\sqrt{\beta}\Delta Z^2}{2} \sinh\left(\frac{2\sqrt{\beta}t}{\alpha'}\right)}, \quad (30)$$

and

$$H_3 \equiv \left| \cosh \left(\frac{2\sqrt{\beta} t}{\alpha'} \right) \right|^2 + \frac{4}{\beta (\Delta Z)^4} \left| \sinh \left(\frac{2\sqrt{\beta} t}{\alpha'} \right) \right|^2 . \quad (31)$$

Note that K^2 comes into the solutions of F and G only as a phase factor. Equations (26) and (27) hold for both positive and negative A_0 .

Now we consider $A_0 > 0$ case. We note that $H_3 = 1$ at $t = 0$. Then after a time $t = \tau$ such that

$$H_3(\tau) = 4 , \quad (32)$$

the half width of the distribution of F in Eq. (27) will be doubled. As the wavepacket spreads out, the wavepacket will arrive in the domain where $k_{\parallel}^2 C_s^2 / \omega_e^2$ (which is proportional to Z^2) is no longer much less than 1. Then the ion Landau damping will kick in (we assumed that $T_i \approx T_e$). From the definition of η_s and noting $Z = s - j$, we get the upper limit of Z to be

$$Z_M^2 = c_1 \eta_s^2 \quad (33)$$

where, for our following example, c_1 was chosen to be 0.4. (For our analysis to be applicable, in view of the approximation of Eq. (8), Z_M should be greater than unity. Otherwise a different approach is needed.)

For our analysis here we choose ΔZ to be equal to Z_M . Then after a time τ defined in Eq. (32), the wavepacket is spread enough to be damped by ion Landau damping. The spread time τ is along the Z-axis. In addition, the spread time along the u-axis is also important.

Since the u-axis and the Z-axis cross with the angle, $\theta \left(\neq \frac{\pi}{2} \right)$, in general, the spread of the wavepacket along the u-axis also increases spread in Z towards $\pm Z_M$. The distance from the u' axis (defined as $s + \alpha j = 0$ in the (s, j) plane), $\Delta \sigma$, corresponding to Δu is (by a simple geometrical calculation)

$$\Delta\sigma^2 = \Delta u^2 \frac{1}{\alpha^2 + 1} . \quad (34)$$

If Δu equals $Z_M/\cos\theta$, the ion Landau damping takes place. Therefore, the life time of the wavepacket to survive the ion Landau damping, τ_L , must be

$$\omega_{*e}\tau_L < \min(\tau, \Delta\tau) , \quad (35)$$

where

$$\Delta\tau \equiv \frac{(1 + \alpha^2) Z_M^2}{4 |B| \cos^2 \theta} . \quad (36)$$

In general $\Delta\tau > \tau$, so we focus our attention to τ defined by Eq. (32).

If the growth rate due to destabilizing factors, such as the trapped electrons, in a linear system is to be such that $\gamma\tau_L \geq 10$, we expect the plasma to be convectively unstable. For example, a dissipative trapped electron mode may have

$$\gamma = c_\gamma \frac{\epsilon_n^{3/2} \eta_e \omega_{*e}^2}{\nu_{ei}} \left(\equiv \frac{1}{\tau_g} \right) , \quad (37)$$

where c_γ is a constant of the order of unity, η_e is $|d \ln T_e / d \ln n_e|$, and ν_{ei} is the electron-ion collision frequency. Also the values of $\omega_{*e}\tau_L$ are shown for different values of b_θ and $\hat{s}$ in Table 1 for the following tokamak parameters: $q = 2.5$, $\epsilon_n = 1/4.5$, $c_1 = 0.4$.

Just as an example, the growth time of the instability, τ_g , may be calculated for Eq. (25) with $c_\gamma = 1$. We assume that it takes $10 \tau_g$, before the instability grows to the non-linear threshold. Then for the fully established instability to exist, we must have $\tau_L > 10 \tau_g$. The values of $\omega_{*e} \tau_g$ are tabulated in Table 2 for the plasma parameters:

$$T_e = T_i = 2,000 \text{ eV}, \eta_e = 10^{20}/\text{m}^3, B = 5\text{T}, r_n = r_o = 0.5\text{m}, \eta_e = 1, c_\gamma = 1 \quad (38)$$

in addition to the parameters shown in Table 1. From Tables 1 and 2, the minimum shears, $\hat{s}$, needed to stabilize the dissipative trapped electron mode can be estimated. As expected, the shears needed are lower for the deuterium plasmas.

For $A_0 < 0$ (domain I in Fig. 2), Eq. (26) represents a harmonic oscillator of a quantum mechanics. Therefore, if the well is deep enough, the wave will be trapped in the Z direction. The borderline characterized by $A = 0$ is, therefore, the line where the characteristics of the drift wave change.

In the domain II and III where $A_0 > 0$, the effect is seen as the lengthening of the wave lifetime, τ_L , as $\hat{s}$ approaches to 0.5 in Table 1. In the limit of the $\epsilon_n \rightarrow 0$, the line between domains I and II approaches $\hat{s} \rightarrow 0$. It is not surprising, because $\hat{s} = 0$ (and $\epsilon_n = 0$) is the shearless case for a slab geometry. Thus, the wavepacket has the maximum lifetime. This result is consistent with the previous slab geometry analysis.⁴⁻⁵

IV. Physical picture.

1. Slab geometry

In a sheared, slab system without a toroidal coupling [$\epsilon_n = 0$ in Eq. (1)] we have $A_0 > 0$ and the problem of the convective mode of drift waves is understood in Refs. 4 and 5. Conceptionally, it is assumed that, at $t = 0$, a small amplitude perturbation localized at $r = r_0$ or equivalently $s = 0$, is born and increases in its amplitude. At the same time, the perturbation (wavepacket) spreads out radially. If the time to spread to the points where ion Landau damping becomes important, is smaller than $10/\gamma$ (say), the perturbation does not grow noticeably above the thermal background. The radial extent of the wavepacket is given in Eq. (22) in normalized unit. In physical distance,

$$\Delta r = \Delta r_s Z_M = \frac{q \rho_s}{\hat{s} \epsilon_n} = \frac{L_s}{r_n} \rho_s. \quad (39)$$

Here L_s is the shear length and L_s may be approximated to be $qR/\hat{s}$ in a tokamak.

2. Toroidal geometry

With a toroidal coupling, the problem is no longer one-dimensional. The wavepacket may be

created in a three-dimensional space. However the coefficients of Eq. (1) do not depend on the toroidal angle, ϕ . Thus the problem can be reduced to a two-dimensional problem. In the poloidal angle, θ , direction, for a given n , m_0 can be selected at a rational surface where $q(r_0) = m_0/n$. Then $m \ll m_0$ or $m \gg m_0$ modes will be damped quickly by the ion Landau damping at $r = r_0$, but if $m \sim m_0$, the perturbation may be excited by the toroidal coupling. The Fourier transform in θ , yields Eq. (1) and, if we assume that $m_0 \pm 1, m_0 \pm 2 \dots$ modes are to contribute (i.e., $Z_M \gg 1$), the problem is reduced to a partial differential equation with two independent variables, Eq. (9).

After that transformation, the problem is solved easily. If we assume that there exists a small perturbation localized at $(0,0)$ in the (s,j) plane, the perturbation spreads out. For $A_0 > 0$ ($\hat{s} > 1/2$) the problem in a toroidal geometry is then analogous to the problem in a slab geometry. The instability becomes important, if the growth rate is sufficiently large, so that the amplitude of the wave exceed $\exp(10)$ of the initial thermal level. In the process of spreading and magnifying, the higher $|j|$ modes are excited. Although not completely accurate, we may say that greater $|j|$ modes are excited via toroidal coupling, as the wavepacket spreads out. For $A_0 < 0$ ($\hat{s} \leq 1/2$) the wave is trapped near $Z = 0$ and corresponds to the previously obtained toroidicity-induced drift wave eigenmode.³

3. Comparisons with experiments

From the experimental results, it is known that the wave spectra become less turbulent (i.e., move towards monochromatic spectrum) as (i) the shear increases and (ii) the ion mass increases.^{1,2} Both effects are found to exist in the domain we have analyzed and are shown in Tables 1 and 2. According to this analysis subject to limitation mentioned in IV.4, the deuterium plasma will be stabilized against the dissipative trapped electron mode [with parameters specified by Eqs. (37) and (38)], if the shear parameter $\hat{s}$ is greater than 1.9. The lighter hydrogen plasma will require higher shear, $\hat{s} > 2.3$, for stability. This shear parameter is similar in magnitude to that achievable in a modern-day, circular-cross-section tokamak. Thus it is conceivable that a non-circular cross section tokamak may be constructed to have higher shear and is more stable against drift waves.

4. Limitation due to approximation

Although the results appear to qualitatively agree with the experimental observations (later to be discussed), we must be careful to point out the limitation of the present analysis. The equation we solved, Eq. (1), is based upon the approximations $b_0 \ll 1$ and $\epsilon_n \ll 1$, as well as $k_{\parallel}^2 c^2 / \omega_e^2 \ll 1$. Thus, the non-linear contribution such as b_0^2 , $k_{\parallel}^4$ will be expected to alter the nature of the solution if $b_0 \sim 1$ for example. While, on the other hand, minimizing or ignoring the importance of this analysis may not be wise because the result correctly shows the behavior of the instability in different limits, and experimental observations clearly support this analysis. The situation may be such that, in the absence of full non-linear study, the present analysis is the best we can achieve to understand the drift instabilities. With this preamble, we shall explore the physical significances of this analysis furthermore.

V. Summary

The Chen-Cheng equation [Eq. (1)] is solved by conversion of the difference operator T into a differential operator in the azimuthal wavenumber, j . The subsequent hyperbolic partial differential equation is reduced to one with separable variables Z and u . The axes of Z and u are, in general, not orthogonal. Depending upon the angle between the Z and the u axes, the parameter spaces (b_0 , ϵ_n , and $\hat{s}$) are divided into three domains, domain I, II, and III.

In domain I, the equation in Z has a trapped wave solution, or equivalently, the equation is a harmonic oscillator in quantum mechanics. Along the u -axis, however, the solution is that of an electron in vacuum and as such, the wavepacket expands. The solutions are given in Eqs. (24) and (26) but the detail of that behaviour is not discussed in this paper.

In domains II and III, the equation in Z takes the form of Weber's equation, or the equation of an electron at a potential hill. The wavepacket localized at $Z = 0$ will be spread away after τ along the Z axis. As the wavepacket spreads, the wave is damped by the ion Landau damping, since $k_{\parallel}^2$ is proportional to Z^2 . Therefore, we may assign the lifetime τ for the wavepacket.

If the imaginary part of ω , or growth rate, γ , is large enough, so that $10/\gamma$ becomes less than τ , then the wave increases its amplitude sufficiently above the thermal level. The non-linear wave coupling between other waves (including itself) will lead then to a turbulent plasma state.

The lifetimes of waves, or convection times are calculated and shown in Table 1. We note immediately that (i) large shear (increasing shear, $\hat{s}$) and (ii) small b_θ (longer azimuthal wavelength) have shorter convection times, hence, better stability. We also note that near $\hat{s} = 0.5$, where the drift wave transforms from convective mode to a trapped harmonic solution, the convection time increases.

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Figure Captions

Fig. 1. The contour lines of $\hat{\phi}(s,j)$ with the assumptions of Eqs. (5)–(7).

Fig. 2. Three domains in the $(\gamma, \hat{s})$ plane which determine the characteristics of the partial differential equation, Eq. (10). Here $\gamma = \Omega b_0/\epsilon_n$.

Fig. 3. The axes (Z, Z') and (u, u') in the (s, j) plane. The primed variables indicate the orthogonal coordinates. Regions I to III are the locations of the u' axis ($s + \alpha j = 0$) corresponding to Fig. 2.

Table 1

Convection Times for Different Values of $\hat{s}$ and b_0 .

$b_0 =$ $\hat{s}$	.2	.4	.6	.8
.5	17.72	20.69	23.66	26.63
.6	9.59	12.88	15.59	18.09
.7	7.24	10.02	12.3	14.39
.8	6	8.4	10.38	12.19
.9	5.19	7.33	9.08	10.68
1	4.61	6.54	8.12	9.57
1.1	4.17	5.93	7.37	8.69
1.2	3.82	5.44	6.77	7.99
1.3	3.53	5.04	6.27	7.4
1.4	3.28	4.7	5.85	6.9
1.5	3.07	4.4	5.48	6.47
1.6	2.89	4.14	5.16	6.1
1.7	2.72	3.91	4.88	5.76
1.8	2.58	3.71	4.63	5.47
1.9	2.45	3.53	4.4	5.2
2	2.33	3.36	4.19	4.96
2.1	2.22	3.21	4.01	4.74
2.2	2.13	3.07	3.84	4.53
2.3	2.04	2.94	3.68	4.35

Table 2

Ten times the dissipative trapped electron instability growth time and minimum shear, $\hat{s}$, required to stabilize the instability for different values of b_θ . Both hydrogen and deuterium cases were calculated.

	b_θ	.2	.4	.6	.8
H+	$10\tau_g$	7.55	5.34	4.36	3.77
	$\hat{s}$	0.6-0.7	1.2-1.3	1.9-2.0	> 2.3
D+	$10\tau_g$	10.67	7.55	6.16	5.34
	$\hat{s}$	0.5-0.6	0.8-0.9	1.3-1.4	1.8-1.9

Fig. 1

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Fig. 2