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**DYNAMICALLY GENERATED FERMIONIC MASSES\***

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**ABSTRACT**

The fermion mass hierarchy problem is presented in the context of supersymmetric vector-like theories as well as within effective gauge theories with Yukawa interactions between Higgs-like scalars and chiral fermions. We present the results for the latter theories from the point of view of Higgs mechanism as well as from that of dynamical symmetry breaking through formation of two-body condensates.

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**I. INTRODUCTION**

The fermion mass hierarchy problem is very complex. It can be subdivided into the following three questions: (i) the origin of the hierarchy between fermionic families  $e, \mu, \tau, \dots$ , i.e., the interfamily hierarchy  $m_e \gg m_\mu \gg m_\tau$ , (ii) the origin of the hierarchy within each family, i.e., the intrafamily hierarchy  $m_d > m_u, m_s > m_c, m_b > m_t$ , and (iii) origin of the Cabibbo mixing angles between fermionic families. Within realistic models of gauge theories serious attempts<sup>1-8</sup> have been made in order to get at least a partial answer to this problem. These attempts usually suffer from one or a few of the following deficiencies: enlarged gauge symmetry structure,<sup>6</sup> the unusual and often proliferated representations of the Higgs fields,<sup>4</sup> exotic fermions<sup>2,3</sup> and extra parameters which are put in the theory by hand. The origins of fermion mass hierarchy as it may arise from the superstring theories<sup>7</sup> is under investigation. However, the low energy phenomenology<sup>9</sup> based on superstring theories still suffers from the problems of too fast proton decay, inadequate neutrino masses, etc..

At present, gauge theories alone do not seem to provide an ultimate answer to this problem. It was argued<sup>9</sup> a long time ago that the fermions and possibly other particles of the gauge theories are composites of the more fundamental entities, preons. Gauge theories are, then, only effective interactions of the underlying preonic dynamics. It is believed that the composite structure of fermions within effective gauge theories, together with the effects of the underlying preonic dynamics, possibly arising<sup>10</sup> from superstring theories should offer an explanation for the origin of the fermion masses.

Therefore, in order to understand the flavor problem, it is very important to study the origin of chiral and flavor symmetry breaking within theories based on different kinds of dynamics, like gauge theories and/or supersymmetric gauge theories.

In vector-like theories strong statements<sup>11,12</sup> have been made about the dynamical breakdown of the flavor symmetry. Vector-like theories are CP conserving gauge theories with no interaction between scalars and fermions. First Coleman and Witten<sup>11</sup> showed that in  $N_C \rightarrow \infty$  chromodynamics the chiral symmetry of  $N_f$  flavors  $U(N_f)_L \times U(N_f)_R$  must be broken down to the diagonal  $U(N_f)_{L+R}$ , i.e., if chiral symmetry is broken it is broken in such a way as to preserve the flavor symmetry. This claim has been generalized to any vector-like theory by Vafa and Witten.<sup>12</sup> They proved that in a vector-like theory dynamical breakdown of the flavor symmetry cannot take place.

The Vafa-Witten constraint is very restrictive because aesthetic arguments almost force us to assume that the flavor symmetries should be broken spon-

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taneously. Theories<sup>9</sup> based on underlying composite field dynamics in general therefore face a stumbling block of the Vafa-Witten constraint. Namely a vector-like theory as a primordial preonic force is ruled out.

However, this constraint need not apply to the case where there is an interaction between fermions and scalars. It is therefore important to study such theories, because they may shed a new light on the spontaneous (dynamical) breakdown of the flavor symmetry. An obvious aesthetically appealing extension of the vector-like theories,<sup>13</sup> is to the super-symmetric vector-like theories, because in such theories scalars emerge in a natural and compelling manner.

## II. SUPERSYMMETRIC VECTOR-LIKE THEORIES

One could conjecture that having any Yukawa type interaction in a vector-like theory is enough to claim that the Vafa-Witten constraint is not applicable. However, this may not be the case for the supersymmetric vector-like theory, because one may imagine that the larger symmetry of the theory, i.e., supersymmetry, could ensure that this constraint would still be applicable in this case.

It has been shown<sup>14</sup>, however, that this constraint does not apply to the supersymmetric version of such theories. This conclusion is due to the observation that in supersymmetric vector-like theories the contribution of the fermionic measure to the path integral is complex in general, unlike in vector-like theories where it is positive definite. This in turn implies that an upper bound on the fermionic propagator in any background gauge-field configuration cannot be proven to exist and so the constraint of flavor preservation does not apply.

The structure of the effective potential for all the two-body condensates in the supersymmetric gauge theories is under investigation. It may shed light on the nature of flavor breaking in such theories.

## III. GAUGE THEORIES WITH YUKAWA-TYPE INTERACTIONS - HIGGS MECHANISM

Another interesting theory to be studied is an effective gauge theory with Higgs-like scalar fields  $\Phi_{ab}$  ( $a, b = 1, 2, \dots, N_f$ ) and chiral fermions  $\psi_a^L, \psi_b^R$  ( $a = 1, 2, \dots, N_f$ ) which interact via the universal Yukawa interaction:

$$\mathcal{L}_Y = h \left( \sum_{a, b=1}^{N_f} \Phi_{ab} \bar{\psi}_a^L \psi_b^R + \text{h.c.} \right) \quad (1)$$

On the other hand the self-interaction of  $\Phi_{ab}$ 's is governed by the most general

renormalizable Higgs potential:

$$V = m_\Phi^2 \text{tr} \Phi^\dagger \Phi + \lambda_1 \text{tr} \Phi^\dagger \Phi \Phi^\dagger \Phi + \lambda_2 (\text{tr} \Phi^\dagger \Phi)^2 \quad (2)$$

which respects the global  $SU(N_f)_L \times SU(N_f)_R \times U(1)_F$  symmetry of the theory. Such a theory may emerge at the composite level as an effective theory based on the underlying supersymmetric primordial gauge interactions.

We shall first study the case with gauge interactions being weak, so that fermionic masses are not generated dynamically through formation of the two-body condensates, but rather through nonzero vacuum expectation value (VEV) of  $\Phi_{ab}$ . It has been shown<sup>15</sup> that there are two such minima of the potential(2). The first one preserves the flavor symmetry:

$$\langle \Phi \rangle = \begin{pmatrix} \kappa & & \\ & \kappa & \\ & & \ddots \\ & & & \kappa \end{pmatrix}, \quad \kappa = \sqrt{\frac{-m_\Phi^2}{2(N_f \lambda_2 + \lambda_1)}} \quad (3a)$$

with

$$\{-m_\Phi^2, N_f \lambda_2 + \lambda_1, \lambda_1\} > 0 \quad (3b)$$

Thus, the breaking pattern is  $SU(N_f)_L \times SU(N_f)_R \times U(1)_F \rightarrow SU(N_f)_L \times R \times U(1)_F$ .

The other solution breaks the flavor symmetry spontaneously:

$$\langle \Phi \rangle = \begin{pmatrix} \sigma & & \\ & \sigma & \\ & & \ddots \\ & & & \kappa \end{pmatrix}, \quad \kappa = \sqrt{\frac{-m_\Phi^2}{2(\lambda_1 + \lambda_2)}} \quad (4a)$$

with

$$\{-m_\Phi^2, \lambda_1 + \lambda_2, -\lambda_1\} > 0 \quad (4b)$$

Thus, the flavor symmetry  $SU(N_f)_L \times SU(N_f)_R \times U(1)_F$  can be spontaneously broken down to  $SU(N_f - 1)_L \times SU(N_f - 1)_R \times U(1)_F$  so that only a fermion of one flavor acquires a mass while others still remain massless. All the other possibilities turn out to be a saddle point.

This is an interesting observation which explicitly shows that the flavor symmetry can be broken spontaneously via the Higgs-type mechanism. However, one does not get a handle on the question of which vacuum is preferable. In principle, parameters of the Higgs potential are free; their value could possibly emerge from an underlying dynamics which is responsible for the formation of scalar fields.

Within a concrete model with  $N_f = 4$ , i.e.,  $e$  and  $\mu$  families, and the left-right symmetric gauge group  $SU(2)_L^{e+\mu} \times SU(2)_R^{e+\mu}$ , the pattern of fermionic masses was investigated<sup>15</sup> by adding to the Higgs potential *soft* flavor symmetry breaking terms, which of course still respect the above gauge symmetry. The desired fermion mass matrix can be obtained as a consequence of a subtle interplay of the soft terms respecting the gauge symmetry and the hard (dimension four) terms respecting the full flavor symmetry. The nontrivial result of this analysis is that if the Cabibbo mixing between the  $e - \mu$  family is determined to be  $\theta_C = O(\epsilon < 1)$  then the inter-family hierarchy is necessarily determined to be  $m_u/m_e = O(\epsilon^2 \ll 1)$ , which is in agreement with experimental observations.

#### IV. GAUGE THEORIES WITH YUKAWA-TYPE INTERACTIONS - DYNAMICAL SYMMETRY BREAKING

Another aspect to be studied is dynamically generated fermionic masses within the proposed theory. Due to strong interactions of the theory these masses arise through formation of the two-body condensates  $\langle \psi_L^i \psi_R^j \rangle \neq 0$  rather than through the nonzero VEV of  $\Phi_{ab}$ . We would like to see whether there is an indication that condensates  $\langle \psi_L^i \psi_R^j \rangle$  which break flavor symmetry dynamically can be formed.<sup>16</sup> We shall therefore study the effective potential<sup>17</sup> (free energy) for the two-body condensates in the presence of all the interactions of the theory. The form of this potential can indicate which vacuum for the two-body condensate is preferable.

One would like to evaluate the free energy for the non-local two-body operator of the fermionic fields:

$$\langle \psi(x)_a \bar{\psi}(y)_b \rangle_K = \int \frac{d^4 p}{(2\pi)^4} \exp[-ip(x-y)] S_{ab}(p) \quad (5)$$

with

$$S_{ab}(p) = \left[ \not{p} + \Sigma(p^2) \right]_{ab} \quad (6)$$

For the sake of simplified notation, the vector notations for the space-time indices is suppressed. Also the equations are written in Euclidean space. The non-local source  $K(x, y)$  is chosen so that the operator  $S_{ab}(p)$  corresponds to the physical

(nontrivial) propagator of the theory with  $\Sigma_{ab}(p^2)$  denoting the dynamically induced fermionic mass matrix element for flavors  $a, b$ . The free energy (effective potential)  $\Gamma$  as a function of  $S$  assumes the following functional form:

$$\Gamma = -tr \ln S^{-1} + tr \{ S^{-1} - \beta \} S - (\text{diagrams}) \quad (7)$$

Here the integration over momenta is implied and trace implies the trace over the flavor and space-time indices. The term (diagrams) includes all the two-particle irreducible vacuum diagrams presented on Fig. 1. We included only contributions from 1-gauge boson exchange and 1-scalar exchange, i.e., graphs are proportional to  $g^2$  and  $h^2$  only.<sup>11</sup>

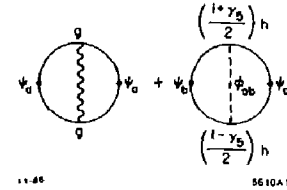


Figure 1 Diagrams contributing to the free energy  $\Gamma$  up to order  $g^2$  and  $h^2$ . The blobs on the solid lines denote the fermionic propagator with a dynamically induced mass. The wiggly and dashed lines denote the gauge boson and the Higgs-like scalar propagators, respectively.

The free energy  $\Gamma$  gives the proper equation of motion, i.e., by taking a stationary point of the functional derivative of  $\Gamma$  with respect to  $S$ , one recovers the gap-equation. The method takes into account corrections to the mass  $\Sigma(p^2)$  of the fermionic propagator to all orders while the corrections to the coupling constant renormalization are taken perturbatively order by order in the coupling constant expansion. Thus, any truncation in the coupling constant expansion is inconsistent. We are aware of this problem, however, even the results of such an inconsistent expansion can shed a light on the nature of dynamically generated masses for the fermions.

<sup>11</sup> As argued below, this truncation is not justified, however, it may give an indication which symmetry pattern is preferable. Also, the chosen truncation to the leading order in  $g^2$  and  $h^2$  allows for the possibility that both  $g^2$  and  $h^2$  are comparable in magnitude.

$\Gamma$  can be presented in the following form:<sup>12</sup>

$$\Gamma_0 = \frac{1}{4\pi^2} \int_0^\infty dp p^3 Tr_f \left\{ -\ln[p^2 + \Sigma^2(p^2)] + \frac{2\Sigma^2(p^2)}{[p^2 + \Sigma^2(p^2)]} \right\} \quad (8)$$

$$\Gamma_{1-GB} = -\frac{3(N_f^2 - 1)g^2}{64N_c\pi^4} \int_0^\infty \int_0^\infty dp dk \frac{p^3 k^3}{\text{Max}(p^2, k^2)} \quad (9)$$

$$Tr_f \left\{ \frac{\Sigma(F^3)\Sigma(k^2)}{[p^2 + \Sigma^2(p^2)][k^2 + \Sigma^2(k^2)]} \right\}$$

$$\Gamma_{1-SC} = -\frac{h^2}{32\pi^4} \int_0^\infty \int_0^\infty dp dk p^3 k^3 Tr_f \frac{1}{[p^2 + \Sigma^2(p^2)]} Tr_f \frac{1}{[k^2 + \Sigma^2(k^2)]} \quad (10)$$

$$\sum_{\ell=0}^{\infty} \frac{(2\ell+1)!!}{(2\ell+2)!!(\ell+2)} \left( \frac{2kp}{k^2 + p^2 + m_\phi^2} \right)^{2\ell+2}$$

with

$$\Gamma = \Gamma_0 + \Gamma_{1-GB} + \Gamma_{1-SC} \quad (11)$$

Here  $Tr_f$  denotes the trace over the flavor indices. The following comments, are in turn

- (i) From (8) one sees that  $\Gamma_0$  is not bound from below, however it does have a local minimum at  $\Sigma = 0$ .
- (ii) The value of  $\Sigma(p^2)$  should fall-off faster than  $1/p^3$  in order for  $\Gamma_{1-GB}$  and  $\Gamma_{1-SC}$  to be finite.
- (iii) One can also explicitly see that  $\Gamma_0$  and  $\Gamma_{1-GB}$  can be written as  $Tr_f F(\Sigma^2)$ , with  $F$  being a general function of  $\Sigma^2$ , thus allowing only for the flavor symmetric solution. Therefore,  $\Gamma_{1-GB}$  is necessary, for including new terms, which have a chance of breaking flavor symmetry.
- (iv) One can also observe that a term in  $\Gamma_{1-GB}$  proportional to  $\Sigma\Sigma$  is negative, thus indicating that the chirally symmetric vacuum with  $\Sigma \equiv 0$  can be unstable. On the other hand, the terms in  $\Gamma_0$  and  $\Gamma_{1-SC}$  proportional to

<sup>12</sup> We shall assume that  $\Sigma$  is a real matrix in the flavor space, i.e., we shall not study dynamical breakdown of CP.

$\Sigma\Sigma$  are positive. Thus, in order to allow for dynamical chiral symmetry breaking, gauge interactions should necessarily be present.

- (v)  $\Gamma_{1-SC}$  can be written as  $[Tr_f G(\Sigma^2)]^2$ . Here  $G$  is a function of  $\Sigma^2$ . This allows for the possibility that the flavor symmetric vacuum is unstable (see also Sect. III). However, the expansion in terms of  $\Sigma^2$  shows that the term proportional to  $(Tr_f \Sigma^2)^2$  has the negative sign. Also this term is damped by a factor  $1/N_f$  relative to the term proportional to  $Tr_f \Sigma^4$ . Therefore there is no tendency to destabilize the flavor symmetric vacuum.
- (vi)  $\Gamma_{1-SC}$  is also logarithmically divergent. In an effective theory with the cut-off parameter  $\Lambda$ , with  $\Lambda$  being the compositness scale of the fermions  $\psi_{L,R}^i$  and/or scalar fields  $\Phi_{\Delta 1}$ ,  $\Gamma_{1-SC}$  is finite. However the question of the underlying dynamics and what its contribution to the effective potential might be, remains unanswered.

Therefore, if one neglects the additional effects of the underlying dynamics, one can claim that in the theory with the cut-off parameter, there is an indication that the flavor-symmetric vacuum is stable.

However, our main goal is to study a renormalizable theory and see whether in such a theory there is an indication for the dynamical breakdown of flavor.

In order to obtain the proper-finite form of the free energy  $\Gamma$  the bare unrenormalized fermionic propagator  $S$  should be reexpressed in terms of the renormalized propagator  $S_R$ . One has to take into account the wave function renormalization effects, only. After doing so<sup>17</sup> the result for  $\Gamma_{REN}$  is finite, however it has a different functional form from the one of  $\Gamma$ . Namely, the term proportional to  $Tr_f \Sigma^2$  is positive, while the term proportional to  $(Tr_f \Sigma^2)^2$  is again negative and again damped by  $1/N_f$  compared to the term  $Tr_f \Sigma^4$ . Therefore, from the form of  $\Gamma_{REN}$  one has an indication that the flavor-symmetric vacuum remains stable in the renormalizable theory.

One can explicitly evaluate<sup>17</sup>  $\Gamma_{REN}$  with a particular Ansatz for  $\Sigma$  presented on Fig. 2 with the dotted line:

$$\Sigma(p^2) = \begin{cases} \sigma \Lambda_{HC}, & p \leq \Lambda_{HC} \\ 0, & p > \Lambda_{HC} \end{cases} \quad (12)$$

Here  $\Lambda_{HC}$  is intuitively a scale at which  $g$ , the gauge coupling constant and correspondingly the Yukawa coupling  $h$  become large,<sup>13</sup> i.e., of order 1. We

<sup>13</sup> It can be shown<sup>18</sup> that for this theory  $h$  evolves proportionally to the gauge coupling  $g$  at low momenta, i.e.,  $h$  reaches very soon the infrared fixed point proportional to  $g$ .

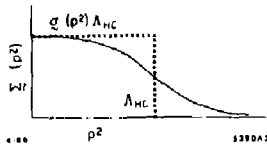


Figure 2 The momentum dependence of the dynamically induced mass (solid line). The scale  $\Lambda_{HC}$  denotes (intuitively) the scale at which the coupling constants become strong. The dotted line represents an approximate Ansatz for  $\Sigma$ , with  $\sigma$  being a variational parameter.

choose  $\sigma$ , the constant matrix in the flavor space, to be a variational parameter. This simplified Ansatz agrees with our intuition (see Fig. 2, solid line) for what the momentum dependence of  $\Sigma$  should be. On the other hand the essential symmetry structure in the flavor space is still encompassed in the variational parameter  $\sigma$ . The explicit form for  $\Gamma_{REN}$ ,<sup>17</sup> with the above Ansatz again substantiates all the claims made before.

Observation that the terms which have a chance of breaking the flavor symmetry, are suppressed by a factor  $1/N_f$  relatively to other terms of the form  $Tr_f F(\Sigma^2)$ , is crucial when studying a theory with  $N_f \rightarrow \infty$  and  $\hbar^2 N_f = \text{const}$ . In this case one sees that to *all orders* in  $\hbar$  the only contribution to  $\Gamma$  which survives is the one where the empty fermionic line denotes the free propagator of the massless fermion. This contribution is obviously of the form  $\hbar^2 N_f Tr_f F(\Sigma^2)$  and has *not* a chance of breaking flavor.

## V. GAUGE THEORIES WITH ASYMMETRIC YUKAWA COUPLINGS

Having observed that flavor symmetric Yukawa couplings cannot destabilize the flavor symmetric vacuum, one can approach<sup>19</sup> this problem from a new angle. In a class of preonic theories<sup>13</sup> rather strong arguments have been given based in part on cosmological considerations and in part from rare processes, that there must be two scales: the metacolor scale,  $\Lambda_{MC} \sim 10^{14}$  GeV, associated with a binding primordial force in addition to the hypercolor (or technicolor) scale,  $\Lambda_{HC} \sim 10^5$  GeV, associated with the dynamically generated masses of composite fermions. Imagine that the underlying dynamics at  $\Lambda_{MC}$  permitted a breaking of flavor symmetry (owing perhaps to its supersymmetric nature) through formation of condensates analogous to that of the Higgs mechanism in

Grand Unified Theories. The questions are (i) can such a breaking of flavor be transmitted in just the right way to the effective Yukawa couplings of fermions and Higgs-like scalars, and (ii) can asymmetric Yukawa coupling together with flavor-symmetric gauge interactions provide a hierarchical pattern of fermionic condensates at  $\Lambda_{HC}$ .

The answer to these questions is yes.<sup>19</sup> One can show, that the formation of condensates of the type  $\sigma_L \sim (15, 1)$ ,  $\sigma_R \sim (1, 15)$ ,  $\omega \sim (6, 6)$  with transformation properties under  $SU(4)_L \times SU(4)_R$  as well as  $\Delta_R \sim (1, 3, 1D)$  with transformation properties under  $SU(2)_L^{c+\mu} \times SU(2)_R^{c+\mu} \times SU(4)_C$ , induces unequal Yukawa couplings and unequal masses of the Higgs-like scalars  $\Phi_{ab}$  in such a way that the desired condensate matrix for the chiral fermions emerges in a nontrivial way.<sup>19</sup> Of course the numerical values of the entries in the condensate matrix depend on the unknown couplings of  $\Phi_{ab}$  to  $\{\sigma, \omega, \Delta_R\}$  and the condensate pattern of  $\{\sigma, \omega, \Delta_R\}$ . However, there are unique predictions for the *structure* of the condensate matrix. In particular, the ratio  $m_u/m_c$  is inevitably related to the Cabibbo angle,  $\theta_C \equiv \theta_{uc} - \theta_{dc}$ . One obtains  $m_u/m_c = O(\frac{\Lambda^2}{\Lambda_{HC}^2} \sin \theta_{ud} \cos \theta_{ud})$  and  $\theta_{uc} \neq \theta_{dc}$  in general. Also  $\omega$  condensates induce nonzero  $m_b$  as well as nonzero  $m_d$  once  $m_c \neq 0$ . These results are encouraging and they await to be embedded in a realistic composite model.

We have presented different aspects of the flavor problem, as studied within gauge supersymmetric theories as well as gauge theories with Yukawa-type interactions between Higgs-like scalars and fermions. Certainly the ultimate answer to this problem is not given yet. However, the investigation of the above theories may provide at least a partial answer to this fundamental problem.

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