HEAVY QUARKS AND NEW PARTICLES*

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ABSTRACT

Some aspects of particles containing c, b, and heavier quarks are reviewed. It is shown how these particles can provide tests of the short-range nature of the strong interactions, and how elementary quantum mechanics can reveal the color and charge of the constituents and the number of states below flavor threshold. Some new results are presented on the inverse problem for confining potentials. Properties of flavored hadrons (masses and lifetimes), of gluonic bound states, and of particles containing very heavy objects are also discussed.

INTRODUCTION

New particles containing heavy quarks were first discovered just five years ago.¹ There now appears to be a rich variety of states containing the charmed quark ($m_c \approx 1\frac{1}{2} \text{ GeV}/c^2$),^{1,2} and particles containing the b quark ($m_b \approx 5 \text{ GeV}/c^2$) are beginning to appear.²⁻⁴ There is very likely at least one heavier quark and its corresponding family of hadrons.⁵⁻⁷ What can be learned from these new particles?

First, bound systems involving heavy quarks shed light on the strong interactions. The large masses of these quarks suggest simple but powerful nonrelativistic methods,⁸ and provide short-distance information not available from the lighter (u,d,s) quarks. Second, the masses, charges and decay properties of the quarks themselves help one understand the weak interactions and unification schemes.

In this review we shall discuss properties of states containing c, b, and heavier quarks from both standpoints: as a window on the strong interactions and as sources of information on the quarks.

In Sec. II, we shall be concerned with deeply bound states of a quark with the corresponding antiquark. The masses and leptonic widths of these quarkonium systems provide information on the short-distance quark-antiquark force, and their decays depend on coupling to (and properties of) the emitted quanta.

Certain properties of quarkonium states are not peculiar to a specific theory of strong interactions but follow only from known data, quantum mechanics, and assumptions about the constituents. This is the subject of Sec. III. There, the charge and color of the quarks in quarkonium states will be learned from observed leptonic widths of these states, and the number of bound states below threshold for decay of these states to flavored pairs will be predicted.

When they are far enough apart, the quark and antiquark in

*Invited talk at the 1979 Meeting of the Division of Particles and Fields of the American Physical Society, October 25-27, Montreal, Quebec.

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quarkonium each can be incorporated into a new (flavored) hadron, by the production of a light quark-antiquark pair. Remarks on this process and on properties of flavored hadrons are contained in Sec. IV. We include there a discussion of gluonic bound states ("glueballs") and particles containing very heavy objects (e.g., stable color sextet quarks).

Section V is devoted to a summary.

II. DEEP BINDING.

The quanta (gluons) of the strong interactions as described by quantum chromodynamics (QCD)^{9,10} can manifest themselves in several ways.²

Gluon exchange can give rise to a Coulomb-like potential between quarks and antiquarks.¹¹ It can lead to hyperfine splittings, for example between 3S_1 and 1S_0 quark-antiquark states.¹² Gluons can be emitted in the annihilation of a quark-antiquark pair.¹² For deeply bound states of heavy quarks, as first encountered in the J/ψ , this is a dominant process, since the quarks cannot separate from one another enough to dissociate into two flavored hadrons. Virtual gluon emission and absorption can significantly alter the structure of all processes involving heavy quarks.¹³

Our starting point for much of this discussion is the Schrödinger equation for a nonrelativistic system of reduced mass μ :

$$-\frac{\nabla^2}{2\mu} \Psi(\mathbf{r}) + V(\mathbf{r}) \Psi(\mathbf{r}) = E \Psi(\mathbf{r}) \quad (1)$$

The interaction between a quark and antiquark in a color singlet state can be described by the potential¹¹

$$V(r) = -\frac{4}{3} \frac{\alpha_s}{r} + ar \quad (2)$$

The first term describes single-gluon exchange, while the second is an approximation to the expected long-range quark-confining force. An effective strong coupling constant $\alpha_s \approx 0.4$ is stipulated by the charmonium and T spectra.^{11,14,15}

Alternatively, any trend toward short-distance Coulomb-like effects may be extracted directly from the behavior of masses and leptonic widths as functions of quark mass and quantum numbers. This may be done with the help of scaling properties of the Schrödinger equation (subsection A),^{8,16} or via inverse scattering methods (subsection B).¹⁷⁻¹⁹ It is possible to compare the value of α_s obtained in Eq. (1) with those obtained from other sources (subsection C).

A. Scaling.

A useful idealization of the quark-antiquark interaction is an effective-power-law potential

$$V(r) \sim r^V \quad (3)$$

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If the true interaction is (1), the effective power ν will approach 1 for large distances (light quarks, highly excited states) and -1 for short distances (heavy quarks, deeply bound states).

By scaling the Schrödinger equation, and applying semiclassical methods to determine the behavior of Ξ wave levels as a function of principal quantum number n , one obtains the dependences shown in Table 1.

Table 1. Scaling of the Schrödinger equation for a potential $V \sim r^\nu$.

Variable Quantity	Reduced mass μ	Principal quantum number n
Level spacing ΔE	$\mu^{-\nu/(2+\nu)}$	$(n - \frac{1}{4})^{2\nu/(2+\nu)}$ $\nu > 0$
		$[n - \frac{1+\nu}{2(2+\nu)}]^{2\nu/(2+\nu)}$ $\nu < 0$
Probability density at $r = 0$ $ \psi(0) ^2$	$\mu^{3/(2+\nu)}$	$(n - \frac{1}{4})^{2(\nu-1)/(2+\nu)}$ $\nu > 0$
		$[n - \frac{1+\nu}{2(2+\nu)}]^{(\nu-2)/(2+\nu)}$ $\nu < 0$

Examples:

	Coulomb ($\nu = -1$)	Linear ($\nu = 1$)
ΔE	μn^{-2}	$\mu^{-1/3} (n - \frac{1}{4})^{2/3}$
$ \psi(0) ^2$	$\mu^3 n^{-3}$	μ

Since ²⁰

$$|\psi(0)|^2 = \frac{\mu}{2\pi} \left\langle \frac{dV}{dr} \right\rangle, \quad (4)$$

the probability at the origin does not depend on n for the linear potential, for which $\langle dV/dr \rangle = \text{const.}$

A number of the observables quoted in Table 1 specify ν for the J/ψ and T families. Not all of these values need agree with one another, since they probe different parts of the potential.

ΔE vs. $m_Q = 2\mu$. The level spacings²¹⁻²³

$$\frac{T'(2S) - T(1S)}{\psi'(2S) - \psi(1S)} = \frac{558 \pm 10 \text{ MeV}}{589 \text{ MeV}} = 0.95 \pm 0.02 \quad (5)$$

yield, for $3 \leq m_b/m_c \leq 4$, the range

$$v = 0.09 \pm 0.05 \quad (6)$$

ΔE vs. quantum numbers. The position of the spir-averaged P wave charmonium levels, $\bar{\chi} = 3522 \text{ MeV}$, with respect to ψ and ψ' corresponds to²⁴

$$v = 0.15 \quad (7)$$

The ratio $(3S-2S)/(2S-1S)$, which is 0.59 ± 0.02 for $c\bar{c}$ and 0.70 ± 0.07 for $b\bar{b}$,²⁵ yields⁸

$$v(c\bar{c}) = 0.20 \pm 0.06 \quad (8)$$

$$v(b\bar{b}) = 0.45 \pm 0.25 \quad (9)$$

$|\Psi(0)|^2$ vs. m_Q . The leptonic widths of $Q\bar{Q}$ vector mesons V are related to squares of wave functions at the origin by²⁶

$$\Gamma(V \rightarrow e^+e^-) = \frac{16\pi \alpha^2 e_Q^2 N}{3 M_V^2} |\Psi(0)|^2 a \quad (10)$$

Here e_Q is the charge of Q in units of the proton's charge, and N is the number of colors (3 for ordinary quarks like c and b). The factor a accounts for strong radiative corrections²⁷; $a = 1$ will be taken for the present nonrelativistic discussion. For $c\bar{c}$ states,²⁸

$$1S: \Gamma(\psi \rightarrow e^+e^-) = 4.8 \pm 0.6 \text{ keV} \quad (11)$$

$$2S: \Gamma(\psi' \rightarrow e^+e^-) = 2.1 \pm 0.3 \text{ keV} \quad (12)$$

while for $b\bar{b}$ states²¹

$$1S: \Gamma(T \rightarrow e^+e^-) = 1.32 \pm 0.09 \text{ keV} \quad (13)$$

$$2S: \Gamma(T' \rightarrow e^+e^-) = 0.33 \pm 0.10 \text{ keV} \quad (14)$$

We then find, for $3 \leq m_b/m_c \leq 4$, that Eq. (10) and Table 1 imply²⁹

$$-0.66 \leq v \leq -0.09 \quad (1S) \quad (15)$$

$$\text{or} \quad -0.2 \leq v \leq 1.7 \quad (2S) \quad (16)$$

$|\Psi(0)|^2$ vs. n . The leptonic widths of higher S wave $c\bar{c}$ states can be incorporated into a fit for ν , yielding the range⁸

$$-0.09 \leq \nu \leq 0.14 \quad (17)$$

The leptonic widths (13) and (14) of T and T' lead to a lower value

$$\nu = -0.54 \pm 0.24 \quad (18)$$

The values of ν obtained above are compared with one another in Figure 1. There is not a clear-cut trend toward lower values as the quark mass increases, though one might have expected the heavier b

Fig. 1. Effective power ν from charmonium and upsilon data.

quarks in the T family to "see" shorter distances -- and, hence, more striking Coulomb-like effects -- than those in charmonium. This trend does appear to be visible in the ratios of 2S and 1S leptonic widths. This ratio should decrease further as the quark mass is raised beyond m_b if there is really a short-distance Coulomb-like force between quarks and antiquarks.

B. Inverse scattering results.

Quarkonium data can directly determine the potential between quarks. This "inverse method" enjoys a vast literature,^{30,31} which has been adapted and somewhat extended for the present purpose.^{17-19,32,33}

In one dimension, we may approximate an infinitely rising symmetric potential by a symmetric, reflectionless one having the same first N levels.¹⁷ The absence of reflection is a strong limitation on the form of the approximate potential, and is assumed only so that the problem reduces to linear algebra rather than inversion of integral equations. (See, however, Ref. 19 for a further discussion.)

One must also choose the lowest continuum energy E_0 of the approximate potential:

$$E_0 \equiv \lim_{|x| \rightarrow \infty} V(x), \quad (19)$$

to lie not far above the highest level specified. It has been shown¹⁹ that as long as

$$\lim_{N \rightarrow \infty} (E_0/E_N) = 1, \quad (20)$$

the Schrödinger wave functions and their derivatives for the N -level reflectionless approximation converge at $x = 0$ to the values corresponding to the potential being approximated. The choice¹⁷

$$E_0 = \frac{E_N + E_{N+1}}{2} \quad (21)$$

appears to provide good pointwise approximations to a number of simple confining potentials. An example is shown in Figure 2.

Some particularly simple results at $x = 0$ for N -level reflectionless, symmetric potentials and their normalized eigenfunctions have been derived.¹⁹ Examples are:

$$(K_n^2 \equiv E_0 - E_n):$$

$$V(0) = E_0 + 2 \sum_{i=1}^N (-1)^i K_i^2 \quad (\text{any } N) \quad (22)$$

$$= E_0 - 4 \sum_{i=1}^N |\psi'_{2i}(0)|^2 / K_i \quad (N \text{ even}) \quad (23)$$

$$|\psi_i(0)|^2 = \frac{K_i}{2} \frac{\prod_{j \text{ even}}^{N-1} |K_j^2 - K_i^2|}{\prod_{j \text{ odd} \neq i}^{N-1} |K_j^2 - K_i^2|} \quad \begin{matrix} (i \text{ odd,} \\ N \text{ odd}) \end{matrix} \quad (24)$$

$$|\psi'_i(0)|^2 = \frac{K_i}{2} \frac{\prod_{j \text{ odd}}^{N-1} |K_j^2 - K_i^2|}{\prod_{j \text{ even} \neq i}^{N-1} |K_j^2 - K_i^2|} \quad \begin{matrix} (i \text{ even,} \\ N \text{ even}) \end{matrix} \quad (25)$$

Figure 2. Eight-level approximation (dashed curve) to the harmonic oscillator potential $V(x) = x^2$ (solid curve). Levels (horizontal solid lines) are at $E_n = 2n = 1$ ($n = 1, \dots, 8$). Here $E_0 = 16$ (horizontal dashed line).

Equation 22, for example, dictates the choice (21) as the correct one for the harmonic oscillator.

The S wave levels of the three dimensional problem correspond to the odd-parity levels of the one-dimensional problem with a symmetric potential. Thus, quarkonium masses provide $E_2, E_4, \dots$, but $E_1, E_3, \dots$ are unspecified. Information on these quantities may be replaced by that on $|\psi_2'(0)|^2, |\psi_4'(0)|^2, \dots$, which comes from leptonic widths.³⁴ The lowest continuum energy E_0 and the quark mass $m_0 = 2\mu$ are constrained by requiring that observables besides E_2 and $|\psi_2'(0)|^2$ (which serve as input) be described in accord with experiment.^{17,18}

The charmonium masses and widths in Eqs. (11) and (12) give rise to the potential shown as the solid curve in Fig. 3. The choice $E_0 = 3.8$ GeV, $m_c = 1.1$ GeV/c² yields the correct values of the

Figure 3. Quarkonium potentials constructed by the inverse scattering method. Solid curve based on ψ, ψ' , with $E_0 = 3.8$ GeV, $m_c = 1.1$ GeV/c². Dashed curve based on T, T' , with $E_0 = 10.1$ GeV, $m_b = 4.5$ GeV/c². Horizontal lines denote 1S and 2S levels (solid) and 2P levels (dashed) for charmonium system (left) and upsilon system (right).

average P wave charmonium mass, the T - T' spacing, and the T, T' leptonic widths when the Schrödinger equation is solved for this potential. The 2S and 2P T levels lie closer to one another than the corresponding charmonium ones. The predicted ratio $\Gamma_{ee}(2S)/\Gamma_{ee}(1S)$ decreases as one passes from charmonium to upsilons.

The charmonium potential can be compared with one constructed directly from the T levels, shown as the dashed curve in Fig. 3.¹⁸ In the region of r probed by the T and T' levels, the two potentials agree well. Information on T'' and T''' levels will extend this comparison to larger interquark separations.

The agreement of the two potentials in Fig. 3 provides evidence for flavor independence of the quark-antiquark force. It also supports the conclusion that the quarks in the T family have charge $-1/3$ and are ordinary color triplets. The values of $|\Psi(0)|^2$ inferred from Eq. (10) would be cut in four if one replaced $e_Q = -1/3$ by $e_Q = 2/3$, and in half if $N = 3 \rightarrow N = 6$. This would have drastic effects on the shape of the potentials reconstructed from T and T',¹⁸ these potentials become much shallower, cease to be monotonic, and look nothing at all like Fig. 3. This is most easily seen from Eq. (23), which says that the depth of the potential is directly proportional to a weighted average of values of $|\Psi(0)|^2$.

A direct transcription from QED³⁵ to QCD²⁷ suggests that Eq. (10) receives important radiative corrections:

$$a = 1 - 16\alpha_s/3\pi < 1. \quad (26)$$

These, in fact, seem necessary if one is to describe the observed charmonium and upsilons leptonic widths with a potential of the form (2).^{11,14}

To explore this possibility we have constructed potentials from ψ and ψ' as described above,¹⁷ but with a common value of $a < 1$ in Eq. (10) for both charmonium and upsilons systems. The results are shown in Table 2.

As the importance of the radiative corrections increases, and $|\Psi(0)|^2$ is scaled upward from its nonrelativistic value [$a = 1$ in Eq. (10)], several trends are visible. The reconstructed potential becomes deeper at the origin. The lowest quarkonium systems become spatially more compact.³⁶ The ratios $\Gamma_{ee}(T')/\Gamma_{ee}(T)$ and $\Gamma_{ee}(T'')/\Gamma_{ee}(T)$ decrease. The T'(2S)- $\chi_b(2P)$ splitting tends to zero. All these trends entail an increased role for short-distance Coulomb effects, and confirm the ability of the inverse method to make good approximations to singular potentials.¹⁷⁻¹⁹

The T''-T' spacing in Table 2 increases as the potentials become deeper. The fourth T level lies at 10.62 ± 0.01 GeV, within 60 MeV of flavor threshold $E_{th}(bb)$ (estimated in Sec. III). The near constancy of E_{th} reflects the small range³⁷ in Table 2:

$$3.42 \leq m_b - m_c \leq 3.45 \text{ GeV}/c^2 \quad (27)$$

Table 2. Quarkonium potentials
fitting ψ , ψ' , χ , T' - T , $\Gamma_{ee}(T)$.

a^{-1}	1	2	3
E_0 (GeV)	3.8	3.775	3.75
m_c (GeV/c ²)	1.1	1.6	1.9
m_b (GeV/c ²)	4.554	5.025	5.334
$V(0)$ (GeV)	1.22	0.64	-0.31
$\Gamma_{ee}(T')$ (keV)	0.32	0.26	0.25
$\Gamma_{ee}(T'')$ (keV)	0.33	0.20	0.15
$T'' - T'$ (GeV)	0.34	0.40	0.44
$T' - \chi_b$ (GeV)	0.09	0.05	0.01
T''' (GeV)	10.63	10.62	10.61
E_{th} (b $\bar{b}$) (GeV)	10.69	10.62	10.64

C. Gluon emission and exchange.

There are several sources of information about the strong coupling constant α_s from charmonium. Many expressions involving gluon emission or exchange involve $P \equiv |\Psi_\psi(0)|^2/m_c^2$, which appears fortuitously to remain nearly constant in Table 2. The results of extracting α_s from charmonium observables are summarized in Table 3. Here we have assumed $|\Psi_\psi(0)|^2 \approx |\Psi_{\eta_c}(0)|^2$.

The results of Table 3 may be compared with the lowest-order renormalization-group formula^{10,45}

$$\alpha_s(Q^2) = \frac{12\pi}{(33-2n_f) \ln(Q^2/\Lambda^2)} \quad (28)$$

by estimating that α_s ([charmonium size]⁻²) $\approx \alpha_s$ ($[\frac{1}{2}\text{GeV}]^2$) ≈ 0.39 .⁴⁶
Then

$$\Lambda = 84 \text{ MeV}; \quad (29)$$

$$\alpha_s(m_c^2) = 0.24. \quad (30)$$

Only the observed decay width for $J/\psi \rightarrow \text{hadrons}$ appears to conflict with Eqs. (28)-(30) (in the absence of radiative corrections.³⁸) It would favor an even smaller value of Λ .⁴⁷ The radiative corrections to $\eta_c \rightarrow 2g$ appear to be large and positive,^{13,48} so that the

Table 3. Information on α_s from charmonium.

Source of information	Distance scale	Corrections to right-hand side	Data	α_s (uncorrected)
$V(r) = \frac{4}{3} \alpha_s + ar$	2 GeV ⁻¹ (Fig. 3)	Softening at short distances	Fit to charmonium levels (Ref. 11)	0.39
$\Gamma(J/\psi \rightarrow 3g) = \frac{40}{81} (\pi^2 - 9) \alpha_s^3 P$	m_c^{-1}	Unknown ³⁸	$\Gamma(J/\psi \rightarrow \text{hadrons}) = 45 \pm 12 \text{ keV}$ (Ref. 39)	0.15 ± 0.02
$\Gamma(\eta_c \rightarrow 2g) = \frac{8\pi}{3} \alpha_s^2 P$	m_c^{-1}	$1 + C(1/v) + 18.8 \alpha_s / \pi$ (Ref. 13)	$\Gamma(\eta_c \rightarrow \text{hadrons}) \approx 20 \pm 10 \text{ MeV}$ (Ref. 40)	0.28 ± 0.07
$\Gamma(J/\psi \rightarrow 2g + \gamma) = \frac{128}{81} (\pi^2 - 9) \alpha_s^2 \alpha_P$	m_c^{-1}	Unknown	Mark II (Ref. 41, 42)	Consistent with ≈ 0.2
$M(J/\psi) - M(\eta_c) = \frac{32\pi}{9} \alpha_s P$	m_c^{-1}	For some effects see Refs. 43-44	$\approx 115 \text{ MeV}$ (Ref. 40)	0.33 ± 0.05

$$P \equiv |\Psi(0)|^2 / m_c^2 = 0.031 \pm 0.004 \text{ GeV}^2 \text{ (Table 2)}$$

uncorrected α_s quoted for this process in Table 3 probably is an upper limit. The observed rate for $J/\psi \rightarrow 2g + \gamma$ is compatible with Eqs. (28)-(30), though the observed photon spectrum⁴² is distorted with respect to a free-gluon model.⁴⁵ Finally, the value of α_s extracted from the $J/\psi - \eta_c$ hyperfine splitting is consistent with (30), given theoretical uncertainties⁴³⁻⁴⁴ surrounding the prediction.

The gluon-jet events measured at PETRA⁵⁰ appear consistent with the prediction of Eqs. (28)-(29):

$$\alpha_s([\alpha_s E]^2) \approx 0.2 \text{ (E=15 GeV)}. \quad (31)$$

Several more predictions for $c\bar{c}$ radiative decays follow from the near-constancy of $P \equiv |\Psi(0)|^2 / m_c^2$. These are summarized in Table 4.

Table 4. Predictions for $c\bar{c}$ radiative decays.

Process	Corrections to right- hand side	Prediction (uncorrected)	Data
$\Gamma(\eta_c \rightarrow \gamma\gamma) =$ $\frac{64\pi\alpha^2}{27} P$	$1 + 0 (1/v)$ $-3.4 \alpha_s/\pi$ (Ref. 13)	12 keV	See Ref. 51
$\Gamma(J/\psi \rightarrow 3\gamma) =$ $\frac{2^{10}}{3^7} (\pi^2 - 9) \alpha^3 P$	$1 - 12.7 \alpha_s/\pi$ (Ref. 52)	5 eV	≤ 5.1 eV (Ref. 22,53) See Ref. 51
$\Gamma(J/\psi \rightarrow \gamma\eta_c) =$ $4\alpha e_c^2 k^3/m_c^2$	Many (Refs. 54,55)	For $m_c = 1.1, 1.9:$ 5, 1.7 keV	~ 0.7 keV (Ref. 56)

The first two differ from standard ones² only by kinematic factors which vanish in the zero-binding limit.⁵⁷

The value

$$\alpha_s(m_b^2) = 0.17 \quad (32)$$

obtained from Eqs. (28) and (29) implies $\Gamma(T+3g) \cong 50$ keV.⁵⁸ (For $0.15 \leq \alpha_s \leq 0.2$, this number ranges between 30 and 90 keV.) At present only $\Gamma(T \rightarrow \text{all}) > 25$ keV is known.²¹ Estimates like those in Table 3 predict

$$34 \text{ MeV} \leq T - \eta_b \leq J/\psi - \eta_c \approx 115 \text{ MeV}, \quad (33)$$

where the right-hand inequality comes from a scaling estimate of the largest possible value of $|\Psi(0)|_{b\bar{b}}^2$. As a consequence of the wide range in (33), we expect $2 \text{ eV} \leq \Gamma(T \rightarrow \gamma\eta_b) \leq 66 \text{ eV}$ for $m_b = 5 \text{ GeV}$. The smaller charge of the b quarks make the detection of $T \rightarrow \gamma\eta_b$ harder than $J/\psi \rightarrow \gamma\eta_c$. (See, however, Ref. 54).

III. BELOW FLAVOR THRESHOLD

Some aspects of quarkonium physics are not characteristic of short distances and, in fact, have little to do with specific form of the binding. Simple scaling and semiclassical arguments relate one family (e.g., charmonium) to another (upsilons, ...), revealing the charge of the constituents and the number of states below flavor

threshold.

A. Quark charge.

The ratio $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ has not always been helpful in determining quark charges. The step in R near charm threshold reflected the additional unit contributed by the charged lepton τ . The small step in R above the T levels⁵⁰ indeed rules out a charge $2/3$ quark, but has not yet been established at the expected ($e_b = -1/3$) value $\Delta R = 1/3$. A step in R of $4/3$ (above the highest present PETRA energy) could be the expected t quark ($e_t = 2/3$),^{5,6} a degenerate charge $-1/3$ quark "h" and a heavy lepton T' , or more exotic combinations.

One seeks independent information on quark charges, which comes from leptonic widths of quarkonium states. These also depend via Eq. (10) on $|\Psi(0)|^2$, on which bounds can be placed. The comparison of charmonium and upsilon levels in Fig. 3 shows that, while the lowest T level probes a short-range part of the potential not well specified by the lighter charmonium levels, the behavior of $\langle dV/dr \rangle$ (and hence, through Eq. (4), of $|\Psi(0)|^2$) for excited T levels is much more constrained by charmonium.^{17,59} It is thus the observed T' leptonic widths^{21,60} that have led most firmly to the conclusion that $e_b = -1/3$ for the quarks in the upsilon family.

If $V(r)$ is monotonic in r , one might expect on semiclassical grounds for heavier quarks to have more spatially compact wave functions.^{8,61} If dV/dr is monotonically decreasing in r (as in Eq. (1)), $\langle dV/dr \rangle$ will increase with quark mass. Then, for each nS $Q\bar{Q}$ state,

$$|\Psi_{nS}(0)|_{m_2}^2 \geq |\Psi_{nS}(0)|_{m_1}^2 \quad \text{for } m_2 \geq m_1 \quad (34)$$

Eq. (34) can be proved rigorously for ground states,^{59,62} and for all excited states in power-law potentials with $v < 1$ (See Table 1). It can be turned, by means of Eq. (10), into a powerful tool for analyzing the minimum signal due to a vector meson in e^+e^- annihilations.^{59,63}

B. Thresholds.

A simple semiclassical argument counts the number of quarkonium levels below flavor threshold.⁶⁴

The zero of energy may be taken at $2m_Q$ both for quarkonium levels and for flavored pairs. The energy of a pair of the two lowest flavored particles $\Gamma = Q\bar{Q}$ is then

$$\delta \equiv 2m_D - 2m_Q. \quad (35)$$

Semiclassically, this energy corresponds to the n^{th} quarkonium level, where n is given by⁶⁵

$$\int_0^{\delta} dr \sqrt{m_Q [\delta - V(r)]} = (n - \frac{1}{4})\pi. \quad (36)$$

Just as the mass of an atom is governed mainly by that of its constituents, one expects δ to vary slowly as a function of m_Q , approaching a fixed limit $\delta \rightarrow \delta_\infty$ as $m_Q \rightarrow \infty$.⁶⁶ Then, for large m_Q ,

$$n \text{ (flavor threshold)} \sim \sqrt{m_Q}. \quad (37)$$

Since flavor threshold lies just above the $\psi'(2S)$,

$$n = 2(m_Q/m_c)^{1/2}. \quad (38)$$

Since $m_b/m_c = 3$ to 4, one expects three (or, barely, four) narrow 3S_1 T levels. This agrees with many specific potential models,⁶⁶ and with Table 2. For $m_Q > 15 \text{ GeV}/c^2$, there should be at least six 3S_1 quarkonium levels below flavor threshold.

Eq. (36) and Fig. 3 imply that flavor threshold occurs at an interquark separation

$$r = V^{-1}(\delta) = 7-8 \text{ GeV}^{-1} \approx \frac{1}{2} \text{ fm}, \quad (39)$$

which should be just about the distance at which even very heavy quarks dissociate from one another into $Q\bar{Q} + Qq$.

Estimates of the actual excitation energy for flavor threshold, $E_{th} - E_1$, are more model-dependent. For $b\bar{b}$ production, we use the small variation (27) in $m_b - m_c$. For the lowest (0^-) states,

$$\begin{aligned} m_{D_b} &\approx m_D + m_b - m_c + (33 \text{ MeV})(1 - \frac{m_c}{m_b}) \\ &= 5.31 - 5.34 \text{ GeV}/c^2 \end{aligned} \quad (40)$$

The correction term on the right-hand side is an estimate of hyperfine and reduced-mass effects.^{64,66}

A state decaying to $\psi K\pi$ has been reported⁴ in the mass range (40). This decay mode has been suggested for the D_b .⁶⁷

For the illustrative heavy quark mass $m_Q = 16 \text{ GeV}$ one finds

$$\frac{1}{2} \text{ GeV} < E_{th} - E_1 < 2 \text{ GeV}. \quad (41)$$

The lower bound comes from scaling arguments and the assumption $v < 0$;⁶⁸ the upper bound is based on the potential of Ref. 11.^{69,70}

IV. ABOVE FLAVOR THRESHOLD.

Using what is learned from systems involving heavy quarks alone, one can discuss some properties of other particles.

A. Decays to flavored pairs.

One model for $Q\bar{Q} \rightarrow (Q\bar{q}) + (\bar{Q}q)$ (Q is heavy, q is light) envisions the light $q\bar{q}$ pair as created with no quantum numbers, i.e., in a 3P_0 state.⁷¹ This model implies that an S wave $Q\bar{Q}$ state should decay to $D\bar{D}$, $D\bar{D}^* + D^*\bar{D}$ in the ratios 1:4:7, aside from kinematic corrections.⁷² Furthermore, the model specifies that $D^*\bar{D}^*$ should be produced in a combination of states with total spin $S = 0$ and $S = 2$ in such a way that every allowed helicity amplitude for $Q\bar{Q} \rightarrow (Q\bar{q}) + (\bar{Q}q)$ occurs with equal magnitude.⁷³ This leads to an intensity ratio $I(S = 2)/I(S = 0) = 20$, and a definite angular distribution for the decay pions in

$$e^+e^- \rightarrow (Q\bar{Q})_{L=0} \rightarrow D^*\bar{D}^* \rightarrow \pi^0 D \quad (42)$$

namely:

$$W(\theta) = 1 - \frac{1}{5} \cos^2 \theta \quad (43)$$

Here θ is measured between the outgoing pion and the $e^\pm$ beam axis.⁷⁴

The 1:4:7 prediction is badly violated for $\psi(4.028)$.^{11,75} One suggested reason is the different nodes of D and D^* radial wave functions.¹¹ This would not alter the prediction (43), however. If (43) were found not to hold, one would have to conclude that the $\psi(4.028)$ is not a pure 3S_1 state⁷⁶, or simply that the quark pair creation model is wrong.⁷⁷

B. Masses of flavored hadrons.

Some control over hyperfine splittings is possible for flavored hadrons despite the relativistic nature of their light quarks. Crudely, we expect

$$\Delta E_{\text{hfs}} \sim \alpha_s |\psi(0)|^2 / m_Q m_q \quad (44)$$

for any meson composed of $Q\bar{q}$. Relations among baryon splittings also can be obtained. Here are some examples.

1. F*-F splitting. Since $m_s > m_u \approx m_d$, both quark masses (in the denominator of (44)) and reduced mass effects in $|\psi(0)|^2$ affect the

comparison of F^*-F and D^*-D splittings. Neglecting variations in α_s between F and D , and using a scaling law of Table 1 (for an effective power $\nu \approx 1$, characteristic of systems containing light quarks), we find

$$F^* - F \leq D^* - D \approx 143 \text{ MeV.} \quad (45)$$

2. Charmed baryons. Three baryons can be formed out of a charmed quark and two nonstrange quarks in an S wave.⁷⁸ They are the C_0 (cud), C_1 , and C_1^* (both cdd , cud , or cuu), also called Λ_c , Σ_c , and Σ_c^* .⁷⁹ The C_1-C_0 mass difference was predicted to be 160 MeV,⁷⁹ a figure supported by experiment.⁸⁰ It is worth reviewing^{81,82} the prediction, since the report⁸³ of a candidate for a doubly charged, weakly decaying object makes one ask whether $m(C_1^{**})-m(C_0^{**})$ could be less than m_π , rendering C_1^{**} stable with regard to strong decays.

If hyperfine splittings between quarks i and j are proportional to $-S_i \cdot S_j / m_i m_j$, the masses of the Λ - Σ - Σ^* and C_0 - C_1 - C_1^* systems may be parametrized by

$$m(\Sigma) - m(\Lambda) = z - 2x, \quad (46)$$

$$m(\Sigma^*) - m(\Lambda) = z + x, \quad (47)$$

$$m(C_1) - m(C_0) = z - 2y, \quad (48)$$

$$m(C_1^*) - m(C_0) = z + y, \quad (49)$$

The parameter $z \approx 206$ MeV is the mass difference between the $I = S = 0$ and $I = S = 1$ combinations of nonstrange quarks. The hyperfine interaction of the latter object with the strange or charmed quark is proportional to x (≈ 63 MeV) or y .

Previous estimates^{81,82} of $y/x = (D^*-D)/(K^*-K)$ entail

$$m(C_1) = m(C_0) + 160 \text{ MeV}/c^2 \approx 2430 \text{ MeV}/c^2 \quad (50)$$

$$m(C_1^*) = m(C_0) + 230 \text{ MeV}/c^2 \approx 2500 \text{ MeV}/c^2. \quad (51)$$

Reduced mass effects in $|\Psi(0)|^2$ (Eq. (44)) can be controlled with the help of Table 1. They increase y/x , but not enough to decrease $m(C_1)-m(C_0)$ below m_π .

The charmed-strange baryons csu , csd ($2490-2550 \text{ MeV}/c^2$) and css ($2750-2790 \text{ MeV}/c^2$)^{81,82} could contribute to inclusive proton production in e^+e^- annihilations^{41,84} at higher SPEAR energies. Their

avored nonleptonic decays lead to strangeness -2 (csu, csd) or -3 (css) final states. One way to produce them might be via beams of hyperons, which already have a strange quark.

3. $D_b^*-D_b$ splitting. The wave functions of the 0^- and 1^- $b\bar{q}$ states should resemble those of $c\bar{q}$, so that the major difference in their hyperfine interactions should arise from the heavy quark mass:

$$D_b^*-D_b \approx \frac{m_c}{m_b} [D^*-D] \approx 35-50 \text{ MeV.} \quad (52)$$

In Sec. III we estimated $2m_{D_b} = 10.65 \pm 0.04 \text{ GeV}/c^2$. By $E_{c.m.} = 10.8 \text{ GeV}$ there should then appear some "action" in the e^+e^- cross section, corresponding to the opening of all the thresholds for $D_b\bar{D}_b$, $D_b\bar{D}_b^* + c.c.$, and $D_b^*\bar{D}_b^*$. According to Table 2, the fourth S wave T level may be able to decay only to $D_b\bar{D}_b$, if it is above flavor threshold at all. The fifth S wave level may be a copious source of $D_b^*\bar{D}_b^*$.

C. Lifetimes.

It appears that $\tau(D^+) > \tau(D^0)$.^{41,83,85} The $\Delta S = \Delta C$ nonleptonic decays of D^+ lead to an exotic final state, while those of D^0 do not.⁷⁸ A specific mechanism which utilizes this difference to give selective enhancement of D^0 decays has been suggested.⁸⁶ It makes use of the weak transition $c\bar{u} \rightarrow s\bar{d}$ and invokes a special role for the non-exotic final state.⁸⁷ Gluon emission by the initial u quark is assumed to evade the helicity suppression that would normally affect an initial $0^-\bar{c}u$ state and light final quarks.⁸⁸

If nonleptonic decays of $F^+ = c\bar{s} \rightarrow u\bar{d}$ are enhanced by a similar mechanism, channels like $F^+ \rightarrow 2\pi^+\pi^-$ further dilute the already small signals⁸⁹ expected for F decays in any one mode.

Depending on the value of the pseudoscalar decay constant f_F ,⁹⁰ the decay $F^+ \rightarrow \tau^+ \nu$ ⁹¹ could become an important source of τ and ν_τ , the latter through beam-dump experiments.⁹² The lifetime of τ itself can serve as a calibration of short-track detectors and as a check of the sequential nature of τ and ν_τ , which entails the prediction⁹³

$$\tau(\tau) = 2.9 \times 10^{-13} \text{ sec.} \quad (53)$$

The b quark probably lives somewhat (but not much) less than 10^{-13} sec. ⁹⁴ The ratio $\Gamma(b \rightarrow u)/\Gamma(b \rightarrow c)$ can be measured by the spectrum, multiplicity, and charges of leptons in semileptonic b decays.⁹⁵

D. Exotic objects.

1. Glueballs. Several structures have been suggested for bound states of gluons, looking (for example) like bags, bagels, and bacteria.⁹⁶⁻¹⁰⁰ Are these objects heavy or light? In a lattice gauge theory,⁹⁸ a glueball is a closed flux loop (> 4 links), while the lightest $q\bar{q}$ states contain one link joining q and $\bar{q}$. Then

$$M(\text{glueball}) = 4 M(q\bar{q}) = 0 \quad (2 \text{ GeV}) \quad (54)$$

A toroidal glueball in the bag model (topologically, a bagel) weighs even more: around 8 GeV.⁹⁹ A "bacterial" (long, thin) glueball is predicted around 2 1/2 GeV in a string model based on QCD.¹⁰⁰ Other models also permit glueballs in this mass range.¹⁰¹

A large mass scale also is characteristic of QCD sum rules for gluons.¹⁰² Despite this, it has been suggested that η' contains a substantial amount of glue.¹⁰² The electromagnetic properties of these states are successfully described without this contribution, however. Moreover, the orthogonal states ought to exist if it really makes sense to talk of constituent gluons.

No evidence for narrow glueballs in the decays

$$J/\psi \rightarrow \gamma + (\text{glueball}) \quad (55)$$

occurs for photon energies above $0.6 E_{\text{max}}$, i.e., for glueball masses below 2 GeV.⁴² The reaction (55) still is of interest for glueballs between 2 and 3 GeV. The photon spectrum is distorted with respect to the free ($\gamma + 2$ gluon) one in such a way as to favor this high-mass region.

2. Heavy stable quarks. The lightest meson (π) is 800 MeV lighter than the lightest baryon (p), but this difference decreases rapidly when we go to charmed particles: $m(C_0^+) - m(D) = 400$ MeV. About 100 MeV of this, moreover, is due to the hyperfine depression of the D^0 (1S_0) mass; this depression will vanish when the charmed quark is replaced by a very heavy one. Thus as $m_Q \rightarrow \infty$ we expect the lowest baryon Qqq to lie only about 300 MeV above the lightest meson $Q\bar{q}$. If Q is absolutely stable, there are then two stable objects. The situation is even richer for color (anti) sextet quarks,¹⁰³ as summarized in Table 5. We have used elementary color-spin arguments,^{104,105} such as those which give $m(\Lambda) < m(\Sigma)$ or $m(C_0) < m(C_1)$, to tell which combination of light quarks is lightest.

Table 5 contains states separated by a unit of baryon number, < 1 unit of charge, and only $\approx m_p/3$ in mass. This should be a characteristic signal in mass-spectroscopic searches, which already have reached an impressive level for certain mass ranges.¹⁰⁶⁻¹⁰⁸

3. Charged scalar bosons can occur in theories¹⁰⁹ that seek to explain the origin of W and Z masses. The lightest bosons may be accessible to present e^+e^- experiments: $M = O(10 \text{ GeV})$. They will behave in these experiments as pointlike objects with a small asymptotic contribution to R :

Table 5. Lightest states involving heavy quarks Q.

Color of Q	State ^a	Mass	Charge
3	$Q\bar{u}$	M	$e_Q - 2/3$
	$Q[ud]$	$M + 300 \text{ MeV}/c^2$	$e_Q + 1/3$
6*	$Q\{ud\}$	M_1	$e_Q + 1/3$
	$Q[\bar{u}\bar{d}]$	$M_1 + 300 \text{ MeV}/c^2$	$e_Q + 1/3$
	$Q\{\bar{u}\bar{d}\}^2$	$M_1 + 600 \text{ MeV}/c^2$	$e_Q - 2/3$

a) Square brackets denote color antitriplet with $I = S = 0$; round brackets denote color sextet with $I = 0, S = 1$.

$$\Delta R (\text{spin } 0) = \frac{1}{4} \Delta R (\text{spin } \frac{1}{2}) \quad (56)$$

which sets in with a (slow) P-wave threshold factor. Colored scalar mesons could bind to form discrete states with small leptonic widths.

For all these reasons, charged scalar mesons are easy to miss in e^+e^- annihilations, and probably easier to miss elsewhere. No systematic exclusion of them has been performed in present data.¹¹⁰ Some of the techniques of Sec. II and III may apply to them if they are discovered.

V. CONCLUSIONS

The quarkonium families (J/ψ , T) have been of tremendous importance in illustrating how bound states of heavy quarks behave, and in shedding light on properties of the quarks themselves.

Quarkonium levels have provided a laboratory for testing quantum chromodynamics. The tests are necessarily crude for the J/ψ family. The $O(\alpha_s)$ corrections to gluon or photon emission are large for $Q^2 \approx m_c^2$, making the use of first-order perturbation theory unreliable. The effects of Coulomb interactions are not dominant in $c\bar{c}$ systems. Gluon jets are not visible in J/ψ decays. Nonetheless, some qualitative successes of QCD emerge, if one accepts a scale factor Λ below $\sim 0.1 \text{ GeV}$.

The chances of testing QCD in the T family are markedly better.¹¹¹ The decays of T contain hints of three gluon jets,¹¹² kindling some hope for the perturbation expansion. The small ratio of T' to T leptonic widths suggests an approach to short-distance Coulomb-like behavior.

The next family of heavy vector mesons might be a still better

QCD laboratory, especially if its quarks have charge $2/3$ (aiding production in e^+e^- reactions). This family will be heavy enough (if it exists at all) that its lowest states will display a number of features of any short-distance Coulomb interaction between quarks.¹¹³

Some of the best QCD tests in quarkonium await further work on radiative corrections. These must act to suppress $^3S_1 \rightarrow 3\pi$ decays (e.g., $J/\psi \rightarrow$ hadrons) if the picture of Sec. II.C. is to be self-consistent.

Some properties of quarkonium systems, distinct from tests of QCD, follow from ordinary quantum mechanics. Dependences on quark mass and quantum numbers often may be discerned by scaling, semiclassical, and inverse scattering methods. In passing, some new results have been obtained on the inverse problem for confining potentials.

The comparison of charmonium and upsilon families reveals the flavor independence of the quark-antiquark interaction. The conclusion $e_b = -1/3$ also was first obtained from this comparison, and the number of narrow upsilon levels was predicted to be three or (barely) four independent of details of the interaction.

The properties of quarkonium families involving heavier quarks are now well enough fixed by charmonium and upsilon data that these families can be told apart from more exotic objects, like composites of scalars or sextet quarks. When confronted with a step in R , one now knows where to look for narrow resonances in $e^+e^- \rightarrow$ hadrons, and how many should be found below flavor threshold, given that the observed ΔR comes from heavy color triplet quarks. The absence of narrow resonances in the predicted range would imply that ΔR is due to objects that do not form bound states (e.g. heavy leptons), or bind more shallowly. More deeply bound narrow resonances might be characteristic of the binding of highly colored (e.g., sextet) quarks.

Charged scalars (and $e_Q = -1/3$ quarks) are not excluded by present e^+e^- data in portions of the currently available energy range. The methods discussed here will not help to fill this lacuna but will be useful if such objects turn up in precise measurements of R .

Flavored objects containing the new quarks have been a source of information about both QCD and QFD (quantum flavor dynamics, the province of unified theories of the weak and electromagnetic interactions). The observed lifetimes of charmed particles entail an interplay of both, particularly in nonleptonic decays.

The successful prediction of charmed particle masses⁷⁹ has been a major triumph of simple QCD and quark model ideas. These now can be extrapolated with some confidence to heavier systems.

Specific final states in weak decays of heavier quarks ($b, \dots$) begin to shed light on areas where no detailed theoretical road map¹¹⁴ exists. Partial information is provided by effects like $D_b^0 - \bar{D}_b^0$ mixing, and by CP violation¹¹⁵ (which welcomes extra quarks⁵).

The elementary methods we have discussed should be able to help determine properties of new quarks from experimental data, and to help tell whether new effects are due to quarks at all.

ACKNOWLEDGMENTS

This work was supported in part by the U. S. Department of Energy under Contract No. EY-76-C-02-1764. I would like to acknowledge the hospitality of the Fermilab theory group during part of the preparation of this report. A particular debt is due to Chris Quigg for enjoyable collaborations on much of this material and to Waikwok Kwong, Jonathan Schonfeld, and Hank Thacker for joint work with the two of us on the inverse problem. I am also grateful to W. A. Bardeen, J. D. Bjorken, R. Cahn, T. P. Cheng, G. Feldman, P. Fishbane, S. Gasiorowicz, D. Geffen, B. Kayser, P. Langacker, G. P. Lepage, S. Meshkov, L. B. Okun', C. Peck, H. Sadrozinski, H. Suura, M. B. Voloshin, W. Wilson, and V. I. Zakharov, for many useful discussions.

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