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AN XUV/VUV FREE-ELECTRON LASER OSCILLATOR

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ABSTRACT

It is shown, from computations based on a detailed theoretical model, that modest improvements in electron beam and optical mirror technologies will enable a free-electron laser, driven by an rf linear accelerator, to operate in the 50-200-nm range of optical wavelengths.

INTRODUCTION

The problems and prospects of extending free-electron laser technology from the visible and near infrared, where such devices are currently operating, to the ultraviolet were extensively discussed at a recent conference.^{1,2} Although significant technical problems must be overcome before free-electron lasers (FELs) can be operated in the VUV (100-200 nm) and the XUV (50-100 nm), the present lack of other kinds of laser sources of coherent radiation at these wavelengths (see however Ref. 3 for a review of the generation of short wavelengths by harmonic conversion processes) together with the intrinsic properties of FELs—continuous tunability in wavelength and output in the form of a train of picosecond pulses—make the development of such devices potentially very rewarding.

The basic problem to be solved in order to operate a free-electron laser oscillator in the XUV/VUV is unchanged:^{1,2} because only low reflectivity mirrors ($\sim 50\%$) are available at the shorter wavelengths, a laser must have a large weak-signal gain to overcome the losses of the optical resonator. Hence, uniform wiggler magnets only are considered as other (tapered) magnet designs, which yield high efficiency operation at large optical power levels, suffer from proportionately low small-signal gains. Although the optical klystron design has been successfully used to enhance the small-signal gain of an FEL,⁴ this device is useful only if the quality of the driving electron beam is very good,⁵ as in an electron storage ring.

The necessity of having a large small-signal gain forces one to consider long wiggler magnets which in turn put increasingly severe constraints on allowable electron beam quality. The emittance and energy spread of the electron beam reduce the available small-signal gain from a given wiggler from that which a perfectly monoenergetic electron beam would produce. Although the importance of these effects was recognized and treated in an earlier study of a linac-driven XUV free-electron laser,² the present work introduces a more refined theoretical model which treats the effects of an imperfect electron beam in a more realistic way.

In the following sections, the theoretical model is presented and is followed by a tabulation of XUV/VUV system parameters which are derived from the small-signal gain theory and realistic estimates of achievable mirror reflectivities and electron beam quality from rf linacs. Gain saturation curves at five different wavelengths are calculated for the given system parameters and are used to obtain laser output (peak) powers versus mirror reflectivities at optimized output coupling values. The designs developed in this work are refinements of the basic system design given in Ref. 2.

ELECTRON DYNAMICS

Consider a beam of relativistic electrons propagating along the $\hat{z}$ -axis of a plane-polarized uniform wiggler magnet. The wiggler is assumed to be coaxial with an optical resonator in which only the lowest-order Gaussian mode is excited. The vector potential for the combined static magnetic field and the optical mode is given by $\vec{A} = \hat{x} A_x = \hat{x} [A_w(x,y) + (c/\omega)E(x,y,z) \sin(k_z z - \omega t + \phi(x,y,z))]$ where

$$A_w = (B_w/k_w) \cosh(k_w y) \sin(k_w z) \quad (1)$$

and $E(x,y,z)$ and $\phi(x,y,z)$ are the electric field amplitude and phase of the lowest-order Gaussian mode.⁶ The vector potential of Eq. (1) gives rise to a static magnetic field primarily polarized in the y -direction with a wavelength $\lambda_w = 2\pi/k_w$ and a peak amplitude B_w :

$$\vec{B} = \hat{y} B_w \cosh(k_w y) \cos(k_w z) - \hat{z} B_w \sinh(k_w y) \sin(k_w z) \quad (2)$$

The basic variables of the theory are the relativistic factor γ_o of an electron, whose total energy is $\gamma_o mc^2$, and its phase $\psi = k_w z + KZ - \omega t + \phi(x,y,z)$. By differentiating ψ with respect to z (denoted by ψ'), one can easily show that for a strongly relativistic electron ($\gamma_o \gg 1$) moving primarily along the $\hat{z}$ -axis ($\beta_x, \beta_y \ll 1$), ψ obeys the following equation of motion:

$$\psi' = k_w + \phi' - (k/2\gamma_o^2) [1 + \gamma_o^2 (\beta_x^2 + \beta_y^2)] \quad (3)$$

The dependence of the transverse velocities ($\beta_x = v_x/c$, $\beta_y = v_y/c$) on the electron's z -coordinate will be derived next.

The motion of an electron in the $\hat{x}$ -direction is determined from conservation of this component of canonical momentum:

$$\gamma_o m v_x = \frac{|e|\hbar}{c} A_x + v_{xi} \quad (4)$$

Note that A_x does depend on the coordinate x through E and ϕ , but that, for the magnetic field strengths and optical intensities of interest in this work, the contribution of the optical field E to A_x is some six orders of magnitude smaller than that of the static

magnetic field A_w . Hence, Eq. (4) is a good approximation, and leads to

$$\beta_x^2 \cong (a_w^2/2\gamma_0^2)[1 + (k_w y)^2] + \beta_{xi}^2 \quad (5)$$

where $a_w = |e|B_w \lambda_w / (2\pi mc^2)$ is the dimensionless vector potential of the static magnetic field ($a_w \sim 1$ usually), we have averaged β_x^2 over a wiggler wavelength, and we have assumed that the electron's y-coordinate is small enough to take only the first order term in y of Eq. (1). Also, β_{xi} is the electron's initial x-velocity upon entering the wiggler.

In the $\hat{y}$ -direction, electrons execute betatron oscillations described by the following trajectory:

$$y(z) = y_i \cos(k_\beta z) + (\beta_{yi}/k_\beta) \sin(k_\beta z) \quad (6)$$

Here y_i (β_{yi}) is the electron's initial y-position (velocity) on entering the wiggler, and the betatron wavelength λ_β is given by

$$\lambda_\beta = 2\pi/k_\beta = \sqrt{2}\gamma_0 \lambda_w / a_w \quad (7)$$

Note that, for this simple harmonic motion, conservation of energy leads to the following relation:

$$\beta_y^2(z) + (k_\beta y(z))^2 = \beta_{yi}^2 + (k_\beta y_i)^2 \quad (8)$$

Hence, from Eqs. (5), (6), and (8) we have

$$\begin{aligned} 1 + \gamma_0^2(\beta_x^2 + \beta_y^2) &\cong 1 + \frac{a_w^2}{2} [1 + (k_w y)^2 + \gamma_0^2 \beta_{xi}^2 + \gamma_0^2 \beta_{yi}^2] \\ &= 1 + \frac{a_w^2}{2} + \gamma_0^2 [\beta_{xi}^2 + \beta_{yi}^2 + (k_\beta y_i)^2] \end{aligned} \quad (9)$$

We can now use (9) in (3): note that the terms that depend on the electron's initial position and velocity are small, so that we write the phase equation as

$$\psi' = k_w + \phi^1 - (k/2\gamma^2)(1 + \frac{1}{2} a_w^2) \quad (10)$$

where γ is the effective value that is shifted slightly from the initial value γ_0 by $\delta\gamma$:

$$\gamma = \gamma_0 - \delta\gamma \quad (11)$$

$$\delta\gamma = \frac{\gamma_0^{3/2}}{1 + \frac{1}{2}a_w^2} \left[\beta_{xi}^2 + \beta_{yi}^2 + (K_p \gamma_i)^2 \right] \quad (12)$$

Eqs. (11) and (12) are the central results of this theory: in the FEL, the motion of an electron which enters the wiggler with initial transverse velocities (β_{xi}, β_{yi}) and positions (x_i, y_i) can be calculated as if its energy is slightly reduced by $\delta\gamma_i$ from its initial value γ_0 . Accompanying Eqs. (10) - (12) is the usual equation of motion for γ in a plane-polarized wiggler.⁷ These equations, coupled to Maxwell's equations for the electromagnetic field, are solved numerically⁷ to calculate the gain of the FEL.

The initial positions and velocities of electrons entering the wiggler are determined from the transverse phase space distribution of the beam. We shall take this distribution to be the product of uncorrelated Gaussians:

$$\begin{aligned} \rho(x_i, \beta_{xi}; y_i, \beta_{yi}) dx_i d\beta_{xi} dy_i d\beta_{yi} \\ = (\epsilon_x \epsilon_y)^{-1} \prod_{j=1}^4 d\alpha_j \exp [-(\alpha_j / \bar{\alpha}_j)^2] \end{aligned} \quad (13)$$

where $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\} = \{x_i, \beta_{xi}, y_i, \beta_{yi}\}$ and the barred quantities are the constant parameters of the distributions. The transverse emittances ϵ_x and ϵ_y are given by the corresponding phase space areas

$$\epsilon_x = \pi \bar{x}_i \bar{\beta}_{xi}; \quad \epsilon_y = \pi \bar{y}_i \bar{\beta}_{yi} \quad (14)$$

and we will assume that they are both equal to the emittance ϵ :

$$\epsilon_x = \epsilon_y = \epsilon \quad (15)$$

According to Eqs. (11) and (12), the distribution of initial values corresponds to a distribution of γ values. Although we have not derived an analytical expression for this distribution starting from Eq. (13), we have numerically generated this distribution for a given emittance ϵ . Such a distribution is shown in Fig. 1 with parameters chosen for the 82 nm laser example. Note that the effective energy is always less than the nominal energy $\gamma_0 mc^2$ which, in this case, equals 204.4 MeV. This is because γ reflects the reduction in the electron's $\hat{z}$ -velocity as it propagates away from the wiggler's axis. If the electron has a finite transverse initial

velocity, clearly v_z will be reduced from what it would have been if no initial transverse velocity were present. Also, if the electron has an initial $\hat{y}$ -displacement, it moves in a region of higher magnetic field which produces a larger wiggler velocity (v_x) and a consequent reduction of v_z . The theory is solved⁷ with a spread of electron energies generated numerically for each case and always qualitatively resembling the shape in Fig. 1.

A distribution of electron γ 's corresponds, through the basic FEL resonance condition $\lambda = \lambda_w (1 + \frac{1}{2} a_w^2) / (2\gamma^2)$, to a distribution of optical wavelengths λ . An important point to note here is that the distribution of optical wavelengths corresponding to Fig. 1 is about ten times wider than the gain bandwidth (homogeneous width) due to a monoenergetic beam of electrons⁸ which is determined solely by the length and wavelength of the wiggler magnet. Hence, the FEL in the present study is strongly inhomogeneously broadened. Since the monoenergetic beam gain curve is antisymmetric about the resonance wavelength,⁸ the net gain due to the energy spread distribution of Fig. 1 is approximately given by the derivative of that distribution. Hence, one qualitatively expects to see a gain vs wavelength that has a sharp absorption feature (negative gain) at short wavelengths followed by a low, broad region of positive gain to longer wavelengths. This shape is indeed seen in the numerical calculations. In what follows below, the reported values of gain will refer to the maximum gain calculated for any optical wavelength.

One thus sees that the effect of the emittance of the beam is to severely inhomogeneously broaden the gain and thus to lower the peak gain. If one crudely models the shape of Fig. 1 as a Gaussian, one finds that the peak small-signal gain varies strongly with ϵ as ϵ^{-3} : one power of ϵ comes from the electron beam density (ϵ determines the beam area), while ϵ^{-2} comes from a Lorentzian broadening factor. Hence, one notes that it is very important to minimize the emittance of the electron beam in order to maximize the gain.

Another source of energy spread is the finite temporal duration of the electron pulse which implies that some electrons arrive either earlier or later than the moment of peak rf field in the accelerating gaps of the linac. The shape of Fig. 1 has included in it this effect for a Gaussian electron pulse of 9.1 ps full width at half maximum and an rf frequency of 1.3 GHz.

TWO-DIMENSIONAL EFFECTS

Although the theoretical model presented above is essentially one-dimensional, there are two important two-dimensional effects that cannot be neglected: ϵ determines the transverse area of the electron beam, and the gain for a Gaussian resonator mode differs significantly from that of a plane optical wave.

In the $\hat{y}$ -direction, the betatron motion keeps the electron beam confined. If the beam size is "matched" to the wiggler at its entrance, that is the $\hat{y}$ -radius y_i is chosen according to

Fig. 1. Distribution of effective energy due to transverse emittance.

$$\bar{y}_i^2 = \varepsilon \lambda_\beta / (2\pi^2) \quad (16)$$

then the motion of individual particles will keep this envelope radius constant as the beam propagates down the wiggler. In the $\hat{x}$ -direction, there is no constraining field, so we take the beam to be externally focused such that at the middle of the wiggler the beam is circular (L_w is the length of the wiggler):

$$\bar{x}^2(z) = \bar{y}_i^2 [1 + (z - 0.5 L_w) / RR_e]^2 \quad (17)$$

$$RR_e = \pi \bar{y}_i^2 / \varepsilon \quad (18)$$

In general, the beam cross-section is elliptical. We further average $\bar{x}(z)$ over the length of the wiggler to finally obtain an average electron beam area A_{eb} :

$$A_{eb} = 0.5 \pi \bar{y}_i^2 \{ \sqrt{1 + \mu^2} + \mu^{-1} \ln [\mu + \sqrt{1 + \mu^2}] \} \quad (19)$$

$$\mu = 0.5 L_w / RR_e \quad (20)$$

We are currently investigating the possibility of using a chain of quadrupoles external to the wiggler to keep the beam area small. The results of this study will be published elsewhere.⁹

We use the results of an earlier study¹⁰ to relate the one-dimensional plane wave gain, G_{pl} , to the gain for a Gaussian mode, G_g :

$$G_g = G_{pl} \cdot f_g \cdot f_c \quad (21)$$

where G_{pl} would be the Colson formula⁸ for a monoenergetic electron beam but here is obtained numerically according to the theory above, and the filling factor f_g is given by

$$f_g = 1/(1 + A_{lb}/A_{eb}) \quad (22)$$

A_{lb} is the area of the light beam at the focus and is equal to the Rayleigh range of the resonator RR_1 multiplied by one-half the optical wavelength. f_c is a correction factor which depends on $\alpha = \sqrt{\lambda L_w/A_{eb}}$ and $q = L_w/RR_1$ and which is tabulated in Ref. 10. There is an optimum value of q for each α , but values of α greater than 5 do not raise further the maximum gain.

SYSTEM RESULTS

We have used the above theory with the previously specified wiggler magnet² to determine those electron beam properties that lead to acceptable small signal gains at XUV and VUV wavelengths. The wiggler has $B_w = 7500$ g, $\lambda_w = 1.6$ cm, and $a_w = 1.12$. We find that a twelve meter wiggler is needed for the shorter wavelengths, but that a six-meter wiggler is sufficient for the longer wavelengths. The system properties are specified below in Table I.

Table I System parameters

λ , nm	50	82	100	150	200
γ_0	510.33	400.0	360.86	294.59	255.12
$\bar{n}$, cm-rad	3.25×10^{-6}	4.88×10^{-6}	5.40×10^{-6}	6.62×10^{-6}	7.64×10^{-6}
$\frac{\epsilon_n}{\bar{n}}$, cm-rad	1.66×10^{-3}	1.95×10^{-3}	1.95×10^{-3}	1.95×10^{-3}	1.95×10^{-3}
I, Amp	100	100	100	100	100
L_w , cm	1200	1200	1200/600	600	600

Can low emittance values be achieved with high currents? The present state of the art is summarized on Fig. 2 where one sees recent data plotted with the Lawson-Penner curves² which represent the empirically observed performance of somewhat older linacs. The

Fig. 2. Emittance vs charge per bunch for linacs.

normalized emittance $\epsilon_n = \gamma \epsilon$ is usually a constant for a given linac. The charge per bunch times the rf frequency is the average current in the linac which can be related to the peak current (listed in Table I) by assuming a pulse shape. To achieve FELs at wavelengths ≥ 82 nm we require a normalized emittance (shown on Fig. 2 as the starred dot to which the arrow points) about a factor of 100 below the LP curve. However, as can be seen from Fig. 2, several recent results already fall below the LP curve, so that we are here asking for slightly more improvement over recent results. To achieve oscillation at 50 nm requires a 15% better ϵ_n than for the longer wavelengths. These improvements appear not to be excessive, and approach currently under study² is to use a photo cathode electron gun that would replace the buncher sections of a linac in which most of the emittance growth is believed to occur.

Using the theory above with the parameters of Table I, we compute the gain saturation curves shown in Fig. 3 for three XUV wavelengths. Using the same resonator design as before,² we show in Fig. 4 the expected peak output power as a function of mirror reflectivity for optimized output coupling at each reflectivity. One notes that peak powers of hundreds of kilowatts are possible if sufficient reflectivity is available; on the other hand, the present design will not oscillate at 50 nm unless a reflectivity of 60% is achieved.

Fig. 3. XUV gain saturation curves.

Fig. 4. XUV laser peak output power vs mirror reflectivity.

Fig. 5 shows calculated gain saturation curves for the VUV wavelengths using a six-meter wiggler. The peak output powers, for optimized output-coupling fractions, are shown for these cases in Fig. 6 where one sees that peak powers of a megawatt are achievable at longer wavelengths with good mirrors. In both wavelength ranges, thermal distortion of the grazing angle of incidence reflectors² limit the average power output to about 200 W (for $\lambda/20$ distortions).

Fig. 5. VUV gain saturation curves.

Fig. 6. VUV laser peak output power vs mirror reflectivity.

CONCLUSIONS

A realistic theory of the effects of an electron beam's emittance and energy spread on the gain of a free-electron laser has been used to predict required system parameters for an FEL operating in the 50-200 nm range of optical wavelengths. Modest improvements in rf linac and mirror technologies are required to yield peak powers of ~ 100 kW in the XUV and ~ 1 MW in the VUV. The wavelength tunability and short pulse output of such a laser, at these power levels, make this device attractive for a wide range of scientific studies.

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