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**THERMAL CONVECTION
WITH LARGE VISCOSITY VARIATION
IN AN ENCLOSURE WITH LOCALIZED HEATING ¹**

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ABSTRACT

The present study is undertaken in order to gain an understanding of convective transport in a magma chamber. We have chosen to represent the chamber by an enclosure with localized heating from below. Results of both laboratory experiments and computer modeling are reported. The experimental apparatus consists of a transparent enclosure with a square planform. An electrically heated strip, with a width equal to one-fourth of the length of a side of the enclosure, is centered on the lower inside surface of the enclosure. For the experiments reported here, the top of the fluid layer is maintained at a constant temperature and the depth of the layer is equal to the width of the heated strip. The large viscosity variation characteristic of magma convection is simulated by using corn syrup as the working fluid. Measured velocity and temperature distributions as well as overall heat transfer rates are presented. The experiment is numerically simulated through use of a finite element computer program. Numerically predicted streamlines, isotherms, and velocity distributions are presented for the transverse vertical midplane of the enclosure. Good agreement is demonstrated between predictions and measurements.

INTRODUCTION

The present study is undertaken in order to gain an understanding of convective transport in a magma chamber and is being pursued in support of the Magma Energy Extraction Program at Sandia National Laboratories (Ortega, *et al.*, 1987). The approach taken in our studies is to first characterize the convection in the magma chamber and then to examine the convective heat transfer to an energy extraction device inserted into the magma chamber. Typically, a magma chamber is periodically recharged at a discrete location (Clark, *et al.*, 1987). Hence, we elect to represent the magma chamber as an enclosure with lo-

calized heating from below. Specifically, the enclosure chosen for study has a square planform with a heated strip centered on the lower inside surface of the enclosure. The large viscosity variation characteristic of magma convection (Clark, *et al.*, 1987) is simulated by using corn syrup as a working fluid. Results from laboratory experiments and computer modeling are presented.

Thermal convection is of continuing interest (Globe and Dropkin, 1959, Chu and Goldstein, 1973, Adrian, *et al.*, 1986) because of its application in the modeling of flow and energy transfer in geophysics, meteorology, and astrophysics as well as in numerous applications to engineering systems. The numerical modeling of Torrance and Turcotte (1971), which treats thermal convection between horizontal free boundaries, appears to be the first such study to consider large viscosity variations. For a Rayleigh number of 3600, the viscosity variation was found to exert a significant influence on the flow field. Richter, *et al.* (1983) and Booker (1976) experimentally investigated thermal convection with large viscosity variations in a horizontal fluid layer using a variety of working fluids including a golden syrup. In addition, Richter, *et al.* (1983) also measured horizontally-averaged temperatures in the fluid layer. Their results indicate that the Nusselt number Nu for the variable viscosity cases can be approximated by the constant viscosity results if the correlation is cast in a form where the Rayleigh number Ra is normalized with the Rayleigh number at the onset of convection, i.e., $Nu = C(Ra/Ra_c)^n$, where C is a constant, the c -subscript indicates a critical value, and the viscosity is evaluated at the average of the boundary temperatures. Furthermore, temperature measurements showed the existence of a stagnant conduction zone above the actively convecting part of the layer.

Previous studies of enclosure convection with localized heating from below have been concerned mainly with cases involving a circular heater on the lower surface of an enclosure (Torrance, *et al.*, 1969, Kamotuni, *et al.*, 1983). Boehm (1977) carried out numerical studies of natural convection in air due to strip-wise

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heating, from below, on a horizontal surface. For a Grashof number of 10^5 , a plume-like structure was found to exist above the heated strip.

It appears from the literature review that the combination of large viscosity variation and localized heating in an enclosure has never been studied before. In addition to the fact that localized heating is appropriate for modeling magma chamber flow, this particular geometry produces a convective flow field which is relatively straightforward to model numerically.

EXPERIMENTAL PROGRAM

Apparatus

A schematic of the test section is shown in Fig. 1. The enclosure is an open top box with a square planform measuring 55.9 cm on a side and 60 cm high and is constructed from 13 mm thick Lexan (polycarbonate) sheets. A heater assembly consisting of a centered heated strip measuring 13.6 cm by 55.9 cm is attached to the lower inside surface with silicone adhesive. The heater assembly is a three layer structure. The top layer is made of a sheet of 13 mm thick Lexan with a centered recess machined into the sheet to accept the heated strip. This top layer is glued to two 10 mm thick Lexan glazing sheets. The glazing sheets are extruded sheets with longitudinal cells, providing structural strength and effective insulation. The heated strip consists of a 3 mm thick copper plate with a thin, flexible, etched foil, heater glued on the underside. The resistance of the foil heater is nominally 8 ohms. The temperature of the heated strip is monitored by 12 thermocouples embedded in the copper plate. In addition, there are 7 thermocouples in the unheated portion of the bottom plate. The side wall temperature is similarly monitored with 4 thermocouples. The thermocouples are made of 0.25 mm copper-constantan wires mounted flush with the surface. Thermocouple locations are shown in the schematic.

The fluid layer is bounded from above by a constant temperature plate made of 25.4 mm thick brass. The temperature of the plate is maintained constant by the circulation of cooling water, from a temperature controlled bath, in twenty 13 mm by 19 mm parallel channels machined in the back of the plate. The back of the plate is sealed by a 6 mm thick brass plate. The plate temperature is monitored by thirteen type-E (chromel-constantan) thermocouples. During the experiment, the plate temperature was found to be uniform within 0.05°C for plate temperatures ranging from 15 to 50°C and within 0.1°C for plate temperatures below 6°C . The constant temperature plate is supported from an overhead frame with three threaded adjusting rods, which allow the depth of the fluid layer to be varied. The fluid layer depth for the present series of experiments was fixed at 13.6 cm, the same as the heater width. The horizontal dimension of the layer is thus essentially four times that of the layer depth.

Experimental Method

Three types of data are obtained from the experiments: (1) overall heat transfer rates, (2) velocity fields, and (3) temperature profiles. For overall heat transfer measurements, the entire test section is insulated with 13 mm of packing foam and 3.8 cm of urethane insulation. The packing foam forms a conforming seal between the urethane sheet and the Lexan wall to prevent infiltration. For each run, the top surface temperature and the heater power were set to the desired values and a data point was

Fig. 1. Schematic of the test section showing thermocouple locations.

obtained when the heated strip temperature reached a steady state. The average temperature of the central half of the strip is used as the characteristic temperature of the heated strip. The heat transfer coefficient is defined in terms of the temperature difference between the heated strip and upper surface and the area of the heated strip. Typically, the temperature difference varies less than 2% over the central region of the heated strip.

The velocity field was made visible by taking time lapse photographs of scattered light from seeded particles in the working fluid. The particles used were glass balloons with typical sizes ranging from 20 to 100 μm . Because of the large viscosity of the fluid and the small size of the particles, the particles can remain in suspension essentially indefinitely (several months). Approximately 0.1 cm^3 of glass balloons were added to each 20 l bucket of syrup. The resulting particle density was estimated to be in the range of 30 - 100 particles per cm^3 of fluid. The light source used was a 15 mW He-Ne laser. A 10 mm diameter quartz rod acting as a cylindrical lens was used to create a vertical sheet of laser light cutting through the central plane of the test section. Three minute time exposures were used to obtain streamline patterns.

For velocity determinations, the laser beam is interrupted by a shutter to create a multiple exposure of instantaneous positions of particles at fixed intervals apart. One second exposures at 15 to 30 s apart were found to be satisfactory. A typical example of

multiple-exposure particle tracking is shown in Fig. 2. The photograph corresponds to case A (see later section for details) in the present study. Our method is a variation on the time exposure method used by Carey, *et al.* (1981) and is less demanding photographically. It also allows three-dimensional flow effects normal to the illuminated plane detecting region (laser sheet) to be readily detected, which otherwise may be misinterpreted from a time exposure as lower velocities. Shadowgraphs were also used to visualize the temperature field. A 6 mm aperture illuminated with a slide projector was used as the light source. A parallel beam of light was then produced by a 25 cm diameter f/6 spherical mirror and subsequently directed through the test section onto a screen placed in contact with the test section wall. A double pane window of Lexan was used to minimize the heat loss during photography.

A vertical temperature profile was obtained using a specially made L-shaped probe. The probe body was made of 1.8 mm O.D. stainless steel tubing with 0.13 mm thermocouple wires strung through the center. The vertical leg of the probe passes through a penetration in the upper, constant temperature, plate. The horizontal leg of the probe consists of a stainless steel section approximately 3.5 cm long and a double-bore alumina tube glued inside of the stainless steel tube and extending 3 mm beyond it. The pair of thermocouple wires (copper-constantan) emerging from the alumina tube are welded to form the measuring junction. Because of the large viscosity of the working fluid, the temperature measurement must be accomplished rapidly before the entire flow field adjusts substantially to the presence of the probe. A typical temperature profile measurement requires approximately one hour.

Fig. 2. Example of multiple-exposure particle tracking. Case A (see text for details). Exposure interval: 15 s.

Working Fluid

The working fluid used is a commercial corn sweetener commonly known in the food industry as 42/43 corn syrup (*Critical Data Book*, 1975). It has a solid content of 80.3 %, by weight. Detailed specifications for the material have been published (*Critical Data Book*, 1975). Although this syrup has been used in a number of studies (Olson, 1984), the temperature dependence of some of the thermophysical properties has not previously been available. As a consequence, the temperature dependence of thermal conductivity and viscosity was measured as part of the experimental program. Equations used for the determination of fluid properties are summarized in the appendix.

Since only provisional values of thermal conductivity are available (*Critical Data Book*, 1975), a line heat source probe was used to measure the thermal conductivity of corn syrup as a function of temperature in the range 9 – 60°C. Details of the experimental method are similar to those described by Hickox, *et al.*, (1986). A total of 27 runs covering five temperatures were made. The maximum span of data within one temperature was less than 5%. Thermal conductivity k was found to be a linear function of temperature with a maximum deviation of the data from the fit (given in the appendix) of 2% (Littel, *et al.*, 1988).

Since the variation of viscosity with temperature is of prime importance in the present studies, the viscosity of the syrup was measured in a series of falling ball experiments. The experiments were carried out in a 17.3 cm diameter cylinder at five temperatures ranging from 9 – 51°C. Wall effects were corrected according to the procedure described by Happle and Brenner (1973). The data were consistent within 0.5% in repeated runs and were in excellent agreement with graphical data (*Critical Data Book*, 1975). The experimental data and high temperature data (*Critical Data Book*, 1975) were combined to fit the viscosity temperature dependence with a super-exponential form (Richter, *et al.*, 1983) with a maximum deviation of the fit from the data in the range of interest of no more than 5%.

Density and specific heat were obtained from published data (*Detailed Tables ...*, 1984, *Critical Data Book*, 1975), and the thermal expansion coefficient was calculated by differentiation of the curve of density versus temperature. Typical values, at 25 °C, for density, specific heat, thermal expansion coefficient, thermal conductivity, and viscosity are, respectively, 1.423 gm/cm³, 2.30 J/gmK, 3.96×10^{-4} K⁻¹, 0.380 W/mK, and 748 poise.

COMPUTATIONAL APPROACH

For computational purposes, a steady, planar, flow field was assumed for the vertical midplane of the enclosure. The nondimensional continuity, incompressible Navier-Stokes, and energy equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$\sqrt{\frac{Ra_o}{Pr_o}} \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial x} \left[f(\theta) \frac{\partial u}{\partial x} \right] + \frac{\partial}{\partial y} \left[f(\theta) \frac{\partial u}{\partial y} \right], \quad (2)$$

$$\sqrt{\frac{Ra_o}{Pr_o}} \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial P}{\partial y} + \sqrt{\frac{Ra_o}{Pr_o}} \theta + \frac{\partial}{\partial x} \left[f(\theta) \frac{\partial v}{\partial x} \right] + \frac{\partial}{\partial y} \left[f(\theta) \frac{\partial v}{\partial y} \right], \quad (3)$$

and,

$$h(\theta) \sqrt{Ra_o Pr_o} \left(u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} \right) = \frac{\partial}{\partial x} \left[g(\theta) \frac{\partial \theta}{\partial x} \right] + \frac{\partial}{\partial y} \left[g(\theta) \frac{\partial \theta}{\partial y} \right], \quad (4)$$

where the Boussinesq approximation has been invoked, the hydrostatic pressure has been absorbed into the pressure term, and all physical properties are treated as temperature dependent. The horizontal and vertical coordinates are denoted by x and y , and the corresponding velocity components by u and v . Gravity g acts in the negative y -direction. The nondimensional pressure is P and the nondimensional temperature is $\theta = (T - T_o)/\Delta T$, where T is the temperature, T_o is the temperature of the upper surface of the enclosure, and ΔT is the overall temperature difference between the heated strip and the upper surface. The Rayleigh number is $Ra_o = g\beta\Delta TW^3/\nu_o\alpha_o$, and $Pr_o = \nu_o/\alpha_o$ is the Prandtl number, where the o -subscript indicates properties evaluated at T_o . Also, $\nu_o = \mu_o/\rho_o$ and $\alpha_o = k_o/\rho_o c_o$ where μ_o , ρ_o , k_o , and c_o are, respectively, the reference viscosity, density, thermal conductivity, and specific heat. For the nondimensionalization, the reference length is the heated strip width W , the reference velocity is $\sqrt{Ra_o Pr_o \alpha_o}/W$, and the reference pressure is $\mu_o \sqrt{Ra_o Pr_o \alpha_o}/W^2$. The variations of physical properties with temperature are given by $\mu/\mu_o = f(\theta)$, $k/k_o = g(\theta)$, and $c/c_o = h(\theta)$, which can be obtained from the expressions presented in the appendix. For computational purposes, the variation of density with temperature is represented in the usual manner by $\rho/\rho_o = 1 - \beta\Delta T\theta$, where β is the thermal expansion coefficient, and is assumed constant.

Numerical solutions to equations (1)–(4) are sought subject to the conditions of no slip on all solid boundaries, $\theta = 1$ on the heated strip, $\theta = 0$ on the upper surface, and zero heat flux on the remaining boundaries. A finite element computer program (*FIDAP User's Manual*, 1988), based on the Galerkin formulation of the finite element method, was utilized for the numerical simulations. Initial simulations of the entire flow field produced flows that were symmetric about the vertical midplane and in essential agreement with experimental observations. Hence, the numerical simulations described here were performed for a symmetric half of the flow field. The computational domain was discretized using a nonuniform mesh of 600, 9-node, quadrilateral elements, as shown in Fig. 3. A discontinuous pressure discretization was used and a consistent penalty function approach was adopted to enforce the incompressibility constraint. Steady-state solutions were obtained by first using approximately 10 successive substitution iterates with a relaxation factor of 0.5 followed by 10 to 20 Newton-Raphson iterates, with the same relaxation factor. Depending on the particular numerical values of the parameters, it was sometimes necessary to increment the parameters of a converged solution to obtain converged solutions for larger Ra_o , Pr_o , or viscosity variations. Convergence was assumed whenever $\|u_i - u_{i-1}\|/\|u_i\| \leq 0.01$ and $\|R(u_i)\|/\|R(u_o)\| \leq 0.01$ where $\|\cdot\|$ denotes the Euclidian norm,

u_i is the solution vector, at iteration i , and $R(u_i)$ is the residual solution vector. Grid refinement studies were performed prior to the selection of the grid geometry used in the simulations in order to assure that the grid spacing was appropriate for the resolution of the flow field to be studied.

Fig. 3. Computational grid. Nondimensional width and depth are 2.06 and 0.98, and the heated strip half-width is 0.5.

RESULTS

Heat Transfer

A total of thirty heat transfer runs were made with average heat input ranging from 0.067 to 0.136 W/cm², top surface temperature from 4.1 to 50.1°C and heated strip temperature from 42.7 to 74.6°C. The largest top-to-bottom viscosity ratio across the layer was 1397; the smallest ratio was 3.12. Using the mean of the heater and upper surface temperature as the reference temperature and the layer depth (same as the heater width in the present case) as the characteristic dimension, the Rayleigh number was calculated to vary from 5×10^4 to 2×10^6 , and the Prandtl number from 9.2×10^3 to 3.3×10^5 . As shown in Fig. 4, the Nusselt number for the heated strip was found to be well correlated with the power law formulation

$$Nu_m = 0.54(Ra_m)^{0.25} \left(\frac{\mu_o}{\mu_h} \right)^{-0.035}, \quad (5)$$

where the subscripts m , o , and h indicate that properties are evaluated at the mean, upper surface, or heated strip temperatures. It is interesting to note that the largest effect of property variation is accommodated by evaluating properties at the mean temperature. The additional power law correction based on the viscosity contrast is relatively small. For a viscosity contrast of 1000, the correction is only 21%. In this respect, the present result is in agreement with the observations made by Richter, *et al.*, (1983) and Carrigan (1987). The *rms* deviation of the data from the fit is 2.46%, and the maximum deviation is 7.7%. A quite remarkable result, considering the large viscosity variation involved.

Lloyd and Moran (1974) extended an observation by Goldstein, *et al.* (1973) and proposed a universal correlation for natural convection from arbitrary planforms in an extended medium as

$$Nu = 0.54 Ra_L^{0.25}, \quad (6)$$

Fig. 4. Overall heat transfer correlation.

where the characteristic length L^* is defined as the ratio of surface area to surface perimeter. In the present case, if the perimeter is taken to mean the perimeter available for entrainment, excluding the ends of the heated strip, there is then an exact correspondence between the present result and the Lloyd and Moran correlation. However, this exact correspondence may be somewhat fortuitous since the entrainment mechanism is different for the two cases.

Velocity and Temperature Fields

Comparisons between experimental and computational results were investigated for three cases. Pertinent parameters for the cases studied are summarized in Table 1. The primary distinguishing feature of each of the three cases is the viscosity contrast μ_o/μ_h which ranges from 10 to 1000. The three cases have essentially the same mean Rayleigh number Ra_m . All simulations were performed for a fluid layer of nondimensional half-width 2.06 and depth 0.98.

In Fig. 5, computed isotherms and streamlines are shown along with photographs of tracer particle path lines, rendered visible by illumination with a vertical sheet of laser light, for cases A and C. The flow field is laminar, steady, and symmetrical with respect to the vertical midplane and consists of two counter-rotating cells driven by a plume rising from the heated strip. There is good qualitative agreement between the observed and computed flow structure. For the higher viscosity contrast (case C), it is apparent that the flow is more strongly inhibited in the vicinity of the upper surface when compared with case A, which has a much smaller overall viscosity variation.

Measured and computed vertical velocity distributions on two horizontal planes are compared in Fig. 6 for all three cases. For each case, a vertical velocity profile was determined along

a horizontal reference plane passing through the vortex centers of the two cells. The velocity is normalized with respect to the maximum centerline velocity on the reference plane. A second horizontal plane was established for each case at a distance of $W/4$ above the initial reference plane. On this upper plane, the velocities are also normalized by the maximum centerline velocity determined for the original reference plane. Experimental data for these planes are plotted as symbols and the results of the numerical simulations are plotted as continuous curves. The maximum centerline velocities for each case and each horizontal plane are tabulated along with the elevations of the planes in Ta-

Fig. 5a. Computed isotherms, streamlines, and photographs of particle paths for case A.

Case	T_o °C	ΔT °C	Ra_o $\times 10^{-4}$	Ra_m $\times 10^{-4}$	Pr_o $\times 10^{-5}$	Pr_m $\times 10^{-5}$	μ_o/μ_h	k_o/k_h	c_o/c_h	ρ_o/ρ_h
A	29.2	20.5	6.05	21.7	2.53	0.73	9.9	0.98	0.97	1.01
B	15.6	37.0	1.38	20.1	19.78	1.37	101.3	0.97	0.95	1.02
C	5.3	51.9	0.28	19.4	134.39	1.96	1026.0	0.96	0.92	1.02

Table 1. Parameters for the three cases simulated.

Fig. 5b. Computed isotherms, streamlines, and photographs of particle paths for case C.

Table 2. The quantitative agreement between measurements and numerically predicted results is good. The magnitudes of the reference velocities differ by as much as 14% when experimental data are compared with numerical predictions, as is apparent from an inspection of the information in Table 2. However, when scaled with the reference velocity for each case, the distributions have nearly the same shapes and the experimentally determined distributions are in excellent agreement with predictions.

Case	Plane 1 Elevation (y/W)	V_1 Centerline (cm/min)	Plane 2 Elevation (y/W)	V_2 Centerline (cm/min)
A (meas.)	0.44	1.25	0.69	0.73
A (calc.)	0.42	1.38	0.67	0.79
B (meas.)	0.41	1.38	0.66	0.73
B (calc.)	0.39	1.49	0.64	0.84
C (meas.)	0.40	1.68	0.65	0.88
C (calc.)	0.39	1.92	0.64	1.05

Table 2. Comparison of measured and computed maximum centerline velocities.

Fig. 6. Measured and computed vertical velocity distributions on the horizontal planes identified in Table 2.

The nondimensional vertical temperature distribution along the center of the enclosure is plotted in Fig. 7. Experimental data are indicated by symbols and the continuous curves are the computed distributions. Reasonably good agreement is obtained between experimental and computed values. In all cases, a conduction layer is observed to exist adjacent to the upper surface. The layer occupies approximately 15% of the depth of the fluid layer. With increasing viscosity contrast, the conduction layer grows progressively thicker and a correspondingly larger fraction of the temperature drop occurs in this layer. As a result, the underlying convective layer grows thinner. This is most likely the reason that the elevation of the vortex centers of the convective cells decreases with an increase in viscosity contrast, as evident from a consideration of the data in Table 2.

Fig. 7. Measured and computed temperature distributions on the vertical center plane of the enclosure.

It is interesting to note from the isotherm plots in Fig. 5 that the heat flux is a maximum at the center on the top surface where the rising plume impinges on the surface. Correspondingly, the heat flux is a minimum at the center of the heated strip where the plume leaves the surface. The variation of heat flux on the heated strip is demonstrated qualitatively in Fig. 8 by a slit-deflection shadowgraph (Jacob, 1967). In this method,

a slit of parallel light directed along the heated surface, is deflected away from the surface as a result of the temperature induced gradient in the refractive index of the fluid next to the surface. To first order, the amount of deflection is proportional to the temperature gradient, and is thus an indication of the local heat flux. Shown in Fig. 8 is the flux distribution for case C. The heat flux is relatively constant over the surface except for the substantial dip at the base of the rising central plume and the transition at the edges of the heated strip.

Fig. 8. Heat flux distribution on the heated strip for case C by the slit-deflection shadowgraph method.

CONCLUDING REMARKS

Measurements made in the present series of experiments show that, even with very large viscosity variations, overall heat transfer rates can still be correlated in terms of conventional, constant viscosity, formulations provided that properties are evaluated at the mean temperature. Residual variable viscosity effects can then be represented by a multiplicative power law correction factor involving the viscosity contrast.

Computer modeling and flow visualization showed that the flow field for the present configuration consists of two counter-rotating cells driven by a central plume rising from the heated strip. Computed velocity and temperature fields are in good agreement with detailed measurements. It appears that the bulk of the viscosity variation is confined to an essentially stagnant conduction layer next to the cold surface.

The results obtained here are for a specific configuration. However, the observed trends for large viscosity variations are likely to hold for other enclosure configurations involving laminar thermal convection. While by no means complete, the present investigation does provide important insights to the understanding and modeling of magmatic convection. In addition, it has produced useful engineering guidelines for the calculation of energy extraction from magma.

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APPENDIX. Physical properties of 42/43 corn syrup.

Viscosity :

$$\mu = a_0 \exp[a_1 \exp(-T/a_2)], \quad (^\circ\text{C}, \text{poise})$$

$$a_0 = 0.2412, \quad a_1 = 12.5867, \quad a_2 = 55.7805$$

Thermal conductivity :

$$k = b_0 + b_1 T, \quad (^\circ\text{C}, \text{W/mK})$$

$$b_0 = 0.3724, \quad b_1 = 3.034 \times 10^{-4}$$

Specific heat :

$$c = c_0 + c_1 T + c_2 T^2, \quad (^\circ\text{C}, \text{J/gmK})$$

$$c_0 = 2.2005, \quad c_1 = 3.9532 \times 10^{-3}, \quad c_2 = -6.7883 \times 10^{-6}$$

Density (as used in numerical simulations) :

$$\rho = \rho_0 [1 - \beta(T - T_0)], \quad (^\circ\text{C}, \text{gm/cm}^3)$$

$$\rho_0 = 1.4314, \quad \beta = 4.1218 \times 10^{-4}$$

Density and thermal expansion coefficient (best estimate) :

$$\rho = \rho_0 / (1 + \Delta), \quad (^\circ\text{C}, \text{gm/cm}^3)$$

$$\rho_0 = 1.4255, \quad \Delta = (d_0 + d_1 T + d_2 T^2) / 10000$$

$$d_0 = -74.5333, \quad d_1 = 3.5691, \quad d_2 = 7.8788 \times 10^{-3}$$

$$\beta \equiv \frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)$$

NOMENCLATURE

a_0, a_1, a_2	Constants in expression for viscosity
b_0, b_1	Constants in expression for thermal conductivity
C	Constant
c	Specific heat
c_0, c_1, c_2	Constants in expression for specific heat
d_0, d_1, d_2	Constants in expression for density
$f(\theta)$	Nondimensional viscosity
$g(\theta)$	Nondimensional thermal conductivity
g	Gravity
$h(\theta)$	Nondimensional specific heat
k	Thermal conductivity
Nu	Nusselt number
P	Nondimensional pressure
Pr	Prandtl number
Ra	Rayleigh number
$R(u_i)$	Residual solution vector
T	Temperature
u, v	Nondimensional velocity components
u_i	Solution vector
x, y	Nondimensional spatial coordinates
W	Width of heated strip
α	Thermal diffusivity
β	Coefficient of volumetric thermal expansion
ΔT	Temperature difference
θ	Nondimensional temperature
μ	Viscosity
ν	Kinematic viscosity
ρ	Density
subscripts	
c	Critical value
h	Heated strip
i	Iteration number
L^*	Characteristic length
m	Mean value
o	Reference value