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**A Method of Inferring the
Magnetic Field Profiles in the
Reversed-Field Pinch**

MASTER

University of California

LOS ALAMOS SCIENTIFIC LABORATORY

Post Office Box 1663 Los Alamos, New Mexico 87545

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A METHOD OF INFERRING THE MAGNETIC FIELD PROFILES
IN THE REVERSED-FIELD PINCH

by

Richard Gerwin

ABSTRACT

A diagnostic procedure is proposed that displays the toroidal and poloidal magnetic field profiles in the form of power series, through r^6 . In addition to the usual external measurements, only one internal point magnetic measurement is required, if cylindrical symmetry and pressure balance are assumed. (The plasma pressure profile is also assumed to be measured nonintrusively; alternatively, it might be ignored if beta is known to be sufficiently small.) The (zero-beta) method is tested here against full magnetic-profile shot-averaged data obtained from a multiple-probe device in ZT-40 with a ceramic liner. The agreement is quite good. The procedure seems simple enough for on-line use in RFP experiments. The availability of such profiles should enhance the effective application of theory to the RFP.

I. INTRODUCTION

It is important to be able to infer the toroidal and poloidal magnetic field profiles in the Reversed-Field Pinch (RFP) without using large internal multiple-probe instruments that degrade the plasma. In this report, a simple method of inferring the field profiles by means of external magnetic measurements, and a single-point internal magnetic measurement is described. The method is tested against profiles obtained with extensive, multi-point internal probe devices, and the agreement seems to be very good. Furthermore, the method appears to be simple enough for on-line use.

Knowledge of the field profiles in an undegraded RFP plasma has several important consequences for knowledge of:

- Poloidal Flux Loss
- Energy Confinement Time
- Transport Code Physics
- MHD Stability
- Microstability

We shall now briefly dwell on each of these topics.

1) From Faraday's law, we have a relation between the loop voltages on the axis and at the wall, in terms of the poloidal flux, namely

$$V_z(a) = V_z(0) + \frac{d}{dt} \left(l \int_0^a B_\theta(r) dr \right) \quad (1)$$

(Here, $l = 2\pi R$ is the major circumference.)

Since the gap voltage at $r = a$ is measured, and assuming magnetic fields and hence the poloidal flux can be somehow inferred to a satisfactory accuracy, one could then use Eq. (1) to calculate $V_z(0)$, the loop voltage on the magnetic axis. Since knowledge of the magnetic profiles implies knowledge of the current density $\vec{J}(r)$, one could then relate $V_z(0)$ to $J_z(0)$ by means of a parallel resistivity, $\eta_{||}$. Thus, one could have an indirect measurement of the parallel resistivity on the magnetic axis. This should then be compared with the resistivity obtained from a measurement of the electron temperature.

2) From power balance, one knows that the input power is partitioned so as to raise the internal magnetic and thermal energies, and also to supply a throughput of energy to the walls due to uncontrollable losses (thermal conduction, impurity radiation, and plasma radiation). Thus

$$I_z V_z + I_\theta V_\theta = \frac{d}{dt} \left(l \int_0^a \frac{B^2}{2\mu_0} 2\pi r dr \right) + \frac{d}{dt} \left(l \int_0^a \frac{P}{\gamma-1} 2\pi r dr \right) + (l \int_0^a \frac{P}{\gamma-1} 2\pi r dr) t_E^{-1} \quad (2)$$

where t_E is an overall effective thermal energy confinement time.

Now, the toroidal and poloidal currents and gap voltages are measured, and assuming that the field profiles can be somehow inferred, the magnetic energy could then be estimated. In addition, if the plasma pressure profile could be measured nonintrusively (laser interferometry for the density, for example), then Eq. (2) could provide an estimate of the effective energy confinement time t_E . (Macroscopic fluctuations could be explicitly modeled into Eq. (2). Otherwise, they will appear as an apparent drain on the plasma in the form of a reduced t_E).

3) Inferred magnetic field profiles (and measured plasma pressure profiles) could be used as initial conditions for, and serve as running comparisons with, cylindrical RFP transport codes. Such tandem modes of investigation could provide valuable clues as to the nature of the effective transport coefficients.

4) Such profiles could also be checked against ideal and resistive MHD stability criteria. (To do a complete job here, it is necessary to have measured the plasma pressure profile.) If the macroscopic fluctuations are sufficiently small, one might expect such mean-field (ensemble- or time-averaged) profiles to be near marginal stability.

5) Such profiles could also be examined for their susceptibility to micro-instabilities of various kinds. (Here it is desirable that the field profiles are known so accurately that drift velocities associated with the derived current densities are physically meaningful.)

II. GENERAL OUTLINE OF THE METHOD FOR INFERRING THE MAGNETIC FIELD PROFILES

We focus here on RFPs of large aspect ratio and therefore we ignore toroidal effects. (The ZT-40 device has $A = 5.7$.) Then, since the toroidal field, $B_z(r)$, is an even function with respect to the magnetic axis, an expansion in radial power series through r^6 involves the determination of four coefficients. (The poloidal field, $B_\theta(r)$, can then be determined from pressure balance if the plasma pressure profile is known to sufficient accuracy.)

The use of a power series expansion here is very convenient, as only trivial algebraic manipulations are required to calculate any quantity of interest. The method is also "unbiased". For instance, the leading term of the B_z expansion is not taken to be a Bessel function.

There are three external measurements that are routinely available, namely, the edge values of the toroidal and poloidal magnetic fields, and the internal toroidal flux. A single internal point measurement of the magnetic field is then required to determine the fourth coefficient of the power series, the coefficient r^6 .

Moreover, we find that this fourth coefficient generally has to be nonzero if the three external measurements are to be satisfied by the power series. Thus, one internal point measurement appears to be really essential to determine the fourth coefficient.

Such an internal single-point measurement is most straightforwardly performed by a single magnetic probe placed near the wall but it could also be performed nondestructively. (An example of the latter would be the observation of the Zeeman effect at a known point in the path of a narrow atomic beam aimed through the plasma along a minor chord.) Also, the internal point measurement could be replaced by a Faraday rotation measurement, thus sampling an entire internal chord, provided the density profile is independently measured.

III. EXPLICIT FORMULAS FOR THE DETERMINATION OF THE COEFFICIENTS

We shall represent the internal toroidal and poloidal magnetic fields as the respective power series

$$B_z = B_0 (1 + z_2 r^2 + z_4 r^4 + z_6 r^6), \quad (3)$$

$$B_\theta = B_0 (\gamma_1 r + \gamma_3 r^3 + \gamma_5 r^5), \quad (4)$$

where B_0 is the toroidal field on the magnetic axis.

Here, r is normalized to the discharge tube radius, a , so that $0 \leq r \leq 1$. The coefficients $z_2, z_4, z_6, \gamma_1, \gamma_3, \gamma_5$, and B_0 , are all to be determined from available measurements. Here, we have noted that, in a circularly symmetric, cylindrical configuration, the toroidal and poloidal magnetic fields are respectively even and odd functions across the magnetic axis. The externally measured quantities are B_z ,

$$B_z(1) = B_0 (1 + z_2 + z_4 + z_6) \quad (5)$$

and B_θ ,

$$B_\theta(1) = B_0(\theta_1 + \theta_3 + \theta_5) \quad , \quad (6)$$

and the toroidal flux in the discharge tube, Φ_z ,

$$\bar{B}_z \equiv \frac{\Phi_z}{\pi a^2} = \int_0^1 B_z 2r dr = \int_0^1 B_z dx \quad ,$$

with $x \equiv r^2$. Hence

$$\bar{B}_z = B_0 \left(1 + \frac{1}{2} z_2 + \frac{1}{3} z_4 + \frac{1}{4} z_6 \right) \quad . \quad (7)$$

In the following, we shall refer to the traditional F and Θ symbols used to describe the RFP behavior.

$$F \equiv B_z(1)/\bar{B}_z \quad (8a)$$

$$\Theta \equiv B_\theta(1)/\bar{B}_z \quad (8b)$$

Into the Eq. (8a), $B_z(1) = F\bar{B}_z$, we substitute the expressions, Eq. (5), and Eq. (7). The result can be used to solve for z_4 , as follows:

$$z_4 = - \frac{1-F + z_2(1-F/2) + z_6(1-F/4)}{1 - F/3} \quad . \quad (9)$$

Then, we turn back to Eq. (5) itself, substituting the above expression for z_4 , and then solving for the coefficient z_2 . In so doing, we write

$$\frac{B_z(1)}{B_0} = \frac{B_z(1)}{B_\theta(1)} \frac{B_\theta(1)}{B_0} = \frac{F}{\theta} \frac{1}{u} , \quad (10)$$

wherein we have introduced our basic unknown, u , as

$$u \equiv B_0/B_\theta(1) . \quad (11)$$

Thus, u is the central field normalized to the edge value of the poloidal field.

There results the following expression for z_2 .

$$z_2 = \frac{[(F/\theta u) - 1 + (1-F)/(1-F/3) - z_6\{1-(1-F/4)/(1-F/3)\}]}{[1 - (1-F/2)/(1-F/3)]} . \quad (12)$$

Notice that in Eqs. (9) and (12), z_6 is so far treated as a parameter. It will eventually be adjusted so that an internal point measurement can be satisfied.

Next, we turn to pressure balance in order to determine the θ coefficients. Pressure balance may be expressed as follows:

$$B_\theta^2 = \frac{1}{x} \int_0^x (B_z^2 + 2\mu_0 P) dx - (B_z^2 + 2\mu_0 P) , \quad (13)$$

where $x \equiv r^2$. We shall suppose that the plasma pressure profile can somehow be measured, and shall write it in the following form:

$$P = P_0(1 + P_2 r^2 + P_4 r^4 + P_6 r^6) . \quad (14)$$

From Eqs. (3) and (4), we note that both fields are taken to have the common amplitude factor, B_0 . We shall therefore divide Eq. (13) by B_0^2 and write

$$\frac{2\mu_0 P_0}{B_0^2} = \frac{2\mu_0 P_0}{B_0^2(1)} \frac{B_{\theta}^2(1)}{B_0^2} = \beta_P/u^2, \quad (15)$$

where we recall Eq. (11) for u , and where we have here defined β_P , a poloidal beta, with respect to the central plasma pressure, rather than the average plasma pressure.

Then, upon expanding each side of Eq. (13) through r^6 , and equating the coefficients of like powers of r , we find the following relations among the coefficients.

$$\theta_1 = \sqrt{-(1/2) (2z_2 + \beta_P P_2/u^2)} \quad (16a)$$

$$\theta_3 = -(z_2^2 + 2z_4 + \beta_P P_4/u^2)/(3\theta_1) \quad (16b)$$

$$\theta_5 = [- (3/4) (2z_2 z_4 + 2z_6 + \beta_P P_6/u^2) - \theta_3^2]/[2\theta_1] \quad (16c)$$

It makes no sense to try to use the pressure balance relation more accurately than indicated here (i.e., to higher order in r), because the higher coefficients that would then appear are inaccessible within the context of only one internal point measurement.

Ideally, one would like to have available a nondestructive measurement of the plasma pressure profile (β_P , P_2 , P_4 , P_6) to use in Eqs. (16). There are such methods for measuring density and electron temperature profiles, but the ion temperature profile measurement presents a difficult problem.

To summarize, z_2 is known in terms of u from Eq. (12), which also contains the external measurements, F and Θ . The coefficient z_4 is known in terms of z_2 from Eq. (9), which also contains F . The θ coefficients are known in terms of z_2 and z_4 from Eqs. (16), which also contain u explicitly through the plasma pressure profile coefficients. In all of the above, the coefficient z_6 is retained as an adjustable parameter. The reasons for this will become clear in the following section.

The final step in actually finding the coefficients of our power series is to substitute the expressions for the θ coefficients (Eq. (16)) into Eq. (6). This equation can obviously be written

$$u = 1/(\theta_1 + \theta_3 + \theta_5) , \quad (17)$$

and it amounts to a single nonlinear equation for u , with z_6 as a parameter.

Having obtained a solution, u , of Eq. (17), the coefficients $z_2, z_4, \theta_1, \theta_3, \theta_5$, may be explicitly calculated as discussed above, for each trial value of z_6 . They then yield magnetic field profiles which satisfy all of the external measurements. However, the coefficient z_6 is still free, and must be fixed by an internal point measurement of some sort.

IV. COMPARISON WITH EXPERIMENT

A code has been written to solve Eq. (17) and to display the resulting field profiles. In this section the procedure will be tested by comparing the results with full (minor radius) profiles obtained from multi-probe measurements on the Los Alamos Reversed-Field Pinch, ZT-40. Since the pressure profile has not yet been measured, and since beta is believed to be small (from the available temperature measurements and reasonable estimates of pumpout), we shall set the plasma pressure to zero when making this comparison. Note that the shots were obtained on the ceramic liner version of Zt-40. No quiescent periods were observed when the multi-probe device was inserted. Consequently, in order to obtain cylindrical magnetic profiles, it was necessary to average over fluctuations in some way.

In these nominally identical shots, the ten pairs of probe measurements, two centimeters apart, were made 335 μ s after the start of the toroidal current, and then were averaged over the shots. The shot numbers are 3856, 3857, 3858, 3861, 3862, and 3863. The measured values of F and θ were $F = -0.18$ and $\theta = 1.70$. This value of θ corresponds to an edge value of B_θ consistent with the total toroidal current, and constitutes a correction that had to be separately inserted due to the influence of the diagnostic port and toroidal effects on the B_θ probes near the edge. Jim Phillips also applied B_θ corrections that insure that the poloidal flux is properly balanced with

respect to the magnetic axis. These corrections (to remove the influence of the diagnostic port) are shown as the dashed curve in Fig. 1. By coincidence, they produce an edge value of B_θ consistent with the total toroidal current. (What this implies is that the decrease of the outer B_θ due to the toroidal R^{-1} effect must be approximately balanced by the increase in the outer B_θ due to the toroidal shift. One would not expect this coincidence to routinely occur.) In general, to use the methods presented here for cylindrical geometry, one should take the "measured" $B_\theta(1)$ value to be defined by the total toroidal current, I , as we do now. (Of course, I is now shot-averaged at 335 μ s.)

In Figs. 2 and 3 we now plot the two sides of Eq. (17), $u = F(u)$, with z_6 as a parameter, for the above experimental values of F and Θ , with plasma pressure ignored ($\beta_p = 0$). Notice that there are no solutions for $z_6 = 0$. In other words, a power series for B_z that is terminated at r^4 cannot satisfy the external constraints on F and Θ for vanishing (or small) plasma pressure. Thus, a fourth measurement becomes mandatory in order to fix the value of the coefficient z_6 . Note also, that for some values of this coefficient, there is more than one solution to Eq. (17); that is, distinct magnetic profiles can satisfy one set of external measurements.

In making further comparison with experiments, we shall pretend that we experimentally know B_z either at the probe position, $r = 16$ cm, or at $r = 18$ cm. (The tube radius is $r = 20$ cm.) Experimentally, one has

$$B_z(16)/B_\theta(20) = 0.289 \quad ,$$

(18)

$$B_z(18)/B_\theta(20) = 0.084 \quad .$$

From Figs. 2 and 3, we see that, for $z_6 = -0.3$ and -0.5 , there is only one root, whereas for $z_6 = -0.4$ there are three roots to consider. The results may be tabulated as follows:

Fig. 1. Comparison of ZT-40 shot-averaged multi-probe data (solid curves drawn through probe data at $r = 0, 2, 4, \dots, 20$ cm) with the power series inference (open circles). The shot-averaged data were obtained from J. Phillips, CTR-D0. The multi-probes were constructed by L. Burkhardt, CTR-2. The dashed curve is the corrected B_θ profile.

Fig. 2. A plot of Eq. (17) for various values of the coefficient z_6 .

Fig. 3.

A plot of Eq. (17), as in Fig. 2, but examined in detail at small values of u .

TABLE I

A Scan of Roots Corresponding to the Measured (F, θ) Values

z_6	u	$B_z(16)$	$B_z(18)$
-0.5	4.775	-0.686	-0.677
-0.4	3.095	-0.167	-0.276
	1.579	0.316	0.107
	1.141	0.456	0.218
-0.3	1.079	0.472	0.229

All of these roots satisfy the external measurements, F and θ .

From TABLE I, we see that the root that comes the closest to satisfying either of the internal measurements, Eq. (18), is the middle root of the case $z_6 = -0.4$. Therefore, we investigate this middle root more closely in TABLE II.

TABLE II

A Scan of the Middle Root

z_6	u	$B_z(16)$	$B_z(18)$
-0.41	1.524	0.334	0.122
-0.39	1.643	0.295	0.090
-0.387	1.665	0.288	0.085
-0.386	1.673	0.286	0.083
-0.385	1.680	0.283	0.0806
-0.380	1.722	0.269	0.070

From TABLE II, we see that the value, $z_6 = -0.387$, yields a root which not only satisfies the measured (F, θ) values, but also comes to within three decimal places of both $B_z(16)$ and $B_z(18)$. (This happy circumstance must be partly fortuitous, since the measurements are not all that accurate, and since an average over several shots has been performed.) The corresponding profiles have been plotted as the open circles in Fig. 1.

Except for a small toroidal shift, inaccessible to our model, the agreement is seen to be quite good. Moreover, the root finding and scanning procedure is simple enough to be used on-line.

V. CONCLUSION

In conclusion, we have seen that an RFP magnetic profile inference can be made in terms of power series through r^6 , provided that a single internal point magnetic measurement is used to supplement the external measurements. The best near-term approach would be to place a dogleg probe near the plasma edge, along the wall away from the diagnostic port, or a straight probe in conjunction with a metal flap that covers the port. In the long term, a nonintrusive Faraday rotation or cyclotron radiation measurement to infer B_z at $r = 0$ would be preferable, since the polynomial would then be "tied down" at both ends. Finally, the procedure described herein can only realize its full value in conjunction with a nonintrusive plasma pressure profile measurement.