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PARAMETRIC STUDY OF THE STABILITY MARGIN OF CABLE-IN-CONDUIT SUPERCONDUCTORS: THEORY*

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Summary

Recently, Lue, Miller, and Dresner¹ reported that the stability margin of cable-in-conduit superconductors is multivalued for certain combinations of transport current, ambient helium pressure, and externally imposed helium flow. There is a limiting transport current below which the stability margin is single-valued and equal to the upper stability margin. The theory of ref. 1 is used here to determine the scaling of the limiting transport current with critical temperature, ambient helium temperature, resistivity of copper, length of the heated zone, duration of the heat pulse, hydraulic diameter of the helium-filled part of the cable, volume fraction of copper in the metal, and volume fraction of metal in the cable. Combined with experimental data, the scaling relation provides a sound basis for design.

Introduction

The transit of helium through a coil wound with a cable-in-conduit conductor takes many minutes, whereas recovery from a thermal perturbation takes only tens of milliseconds. As far as recovery is concerned, then, the inventory of helium available for cooling is finite. This means that a large enough perturbation will quench the conductor. Accordingly, we judge the stability of cable-in-conduit conductors by the size of the largest sudden, uniform heat input from which the conductor can recover. When expressed per unit volume of metal, this largest heat input is called the stability margin, ΔH .

Implicit in this definition of the stability margin is the expectation that the conductor is stable against heat pulses smaller than ΔH and unstable against heat pulses larger than ΔH . But in some recent experiments on the stability of a single triplet in a tube cooled by supercritical helium, Lue, Miller, and Dresner¹ observed that sometimes the conductor was stable against the smallest heat pulses, unstable against larger heat pulses, again stable against even larger heat pulses, and finally unstable against the largest heat pulses. In other words, sometimes the stability margin is multivalued.

The experimental results of Lue et al. can be summarized in the two sketches in Fig. 1. In sketch a, the ordinate is ΔH , the energy density in the metal of the normalizing pulse, and the abscissa is I , the transport current (or p , the ambient pressure). The externally imposed helium flow velocity v is low (for example, near velocity v_0 in sketch b). After pulses in the hatched region the conductor quenches. After pulses in the open region it recovers. The Z-shaped boundary between the two regions determines the stability margin. The upper leg of the Z is called the upper stability margin; the lower leg, the lower stability margin. In experiments on a NbTi/Cu triplet, the upper and lower stability margins differed by roughly a factor of five, the upper margin being $\sim 200 \text{ mJ cm}^{-3}$ and the lower margin being $\sim 40 \text{ mJ cm}^{-3}$.

In sketch b, the ordinate is again ΔH but the abscissa is now v , the externally imposed flow velocity

of the helium. Fig. 1b corresponds to a transport current near I_0 in sketch a, for which there is multiple stability when $v = 0$.

Figs. 1a and 1b can be combined in a three-dimensional representation of the stability surface shown in Fig. 2. The surface has a fold near the points A, B, and F that makes the stability margin multivalued, for, if we slice the stability surface through the fold with a plane parallel to the ΔH - I plane, we get a Z-shaped curve, whereas if we slice the stability surface through the fold with a plane parallel to the ΔH - v plane, we get a smooth curve lying above a pair of intersecting segments.

We call the transport current corresponding to point B in Fig. 2 the limiting current, I_{lim} . When the transport current I is below the limiting current, the stability margin is single-valued and equal to the upper stability margin. Keeping $I < I_{lim}$ is a desirable goal in coil design, and the designer needs to know I_{lim} . In this paper, we use the schematic theory of ref. 1 to determine how I_{lim} scales with critical temperature, ambient helium temperature, copper resistivity, length of the heated zone, duration of the heat pulse, hydraulic diameter of the helium-filled part of the cable, volume fraction of copper in the metal, and volume fraction of metal in the cable. This scaling rule, together with the experimental data available in ref. 1, provides a basis for rational conductor design.

Theory of Multivalued Stability

In order to understand the derivation of the scaling rule for I_{lim} , it is first necessary to understand the explanation given in ref. 1 of how multiple stability can occur. We summarize that explanation here.

The point B in Fig. 2 corresponds to $v = 0$, i.e., to zero externally imposed flow. When there is no steady imposed flow, transient, heating-induced flow is responsible for recovery. Early conductive heating of the helium induces a strong flow because of thermal expansion, and the induced flow greatly enhances heat transfer in subsequent stages of recovery. So on the one hand, the induced flow is caused by the thermal expansion arising from heat transfer from the metal, and on the other hand, the heat transfer from the metal is promoted by the induced flow over the metal surface. This coupling makes a rigorous solution of the stability problem difficult. But we can at least see how multivalued stability is possible by artificially breaking this coupling.

First, we consider what the stability margin is in an artificial problem in which the heat transfer coefficient h is externally imposed. Second, we consider what the induced flow and associated heat transfer are in another artificial problem in which the heating rate of the helium is externally imposed. By combining the results of these two "uncoupled" problems, we shall be able to understand the experimental results.

When the heat transfer coefficient h is externally imposed, a theory² based on constant thermophysical properties shows that the curve ΔH versus h resembles the curve sketched in Fig. 3. We call this curve the stability curve.

A theory³ also exists for calculating the velocity induced in initially stagnant, supercritical helium in

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a tube by uniform heating at a constant rate. The helium in the tube undergoes a thermally induced flow during which its velocity varies in both space and time. The magnitude of the induced velocity turns out to be proportional to the heating rate. The induced velocities are quite high — of the order of $1-10 \text{ ms}^{-1}$ — and the induced flow is expected to have a turbulent boundary layer. The heat transfer should then be connected to the velocity by the Dittus-Boelter-Giarratano correlation; thus, h should be proportional to the 0.8 power of the velocity and, therefore, of the heating rate. Sketched in Fig. 4 is curve OAB showing this dependence of h on ΔH , the heat input (assuming a fixed heat pulse duration).

The Dittus-Boelter-Giarratano correlation is for steady-state heat transfer to a turbulent coolant. We do not expect it to apply for very short times. According to the work of Schmidt* and Steward² for boiling helium, heat transfer during the first moments of pulse heating is limited only by the interfacial Kapitza resistance. After a short time, during which the surface temperature remains low, the surface goes over to film boiling and the surface temperature rises ("takeoff"). Schmidt's measurements show that the time to takeoff is inversely proportional to the square of the interfacial heat flux. The Giarratano correlation applies after takeoff.

When the coolant is supercritical helium near the critical or pseudocritical line, the analogue of film boiling is the blanketing of the surface with low-density helium. A simple extension of Schmidt's model shows that the time to takeoff should vary inversely as the square of the interfacial heat flux also for single-phase, near-critical helium.

When the pulse duration is fixed, a large heat pulse ΔH corresponds to a large interfacial heat flux and therefore a short takeoff time. Thus, most of the heat is transferred after takeoff where the Giarratano correlation is valid. This corresponds to arc AB of curve OAB in Fig. 4. A small ΔH corresponds to a small interfacial heat flux and a long takeoff time. Most (if not all) of the heat is transferred before takeoff when the heat transfer coefficient is very high (arc CD). The real heat transfer curve then looks like curve CDAB.

Let us now combine Figs. 3 and 4 (see Fig. 5). We can see from this diagram that there will be four regions of alternating stability and instability. If we add a heat pulse ΔH , the magnitude of which exceeds that of point R, then we induce a heat transfer coefficient corresponding to that at, say, point 1. That heat transfer coefficient is insufficient to cause recovery since the stability curve lies below point 1. Hence, the region of the ΔH axis above R corresponds to instability.

For an energy pulse between Q and R, we induce a heating transfer coefficient corresponding, say, to point 2. Since the stability curve lies above point 2, the induced heat transfer coefficient is more than adequate to promote recovery. Thus, region QR corresponds to stability. Similar reasoning shows region QP to be unstable and region OP to be stable.

Fig. 5 must, therefore, correspond to abscissas like p_0 or l_0 in Fig. 1a or v_0 in Fig. 1b. What happens if we decrease the transport current? The theory of ref. 2 indicates that the foot of the curve in Fig. 3 moves to the left, the plateau moves up, and the knee moves up and to the left. The heat transfer curve does not change. Thus, the R intersection in Fig. 5 moves to higher ΔH , while the Q and P intersections approach one another. Eventually, the lower band of instability disappears. Then we have conditions like those at l_1 in Fig. 1a. If I is increased, the stability curve moves down and to the right. The R and Q intersections approach one another,

and the upper band of stability eventually disappears. Then conditions like those at l_2 in Fig. 1a prevail.

We can also move the heat transfer curve while the stability curve remains fixed by imposing an external flow on the helium. If this external flow is large enough, it will surely shift the heat transfer curve to the right in Fig. 5. The Q and P intersections disappear and with them the lower band of instability. This is exactly what happens when we go from v_0 to v_1 in Fig. 1b.

Derivation of the Scaling Rule

From the discussion given above we can see that the condition determining the limiting current is the confluence of the Q and P intersections in Fig. 5 in a point of tangency. The theory of ref. 2 indicates that the foot of the stability curve occurs when

$$I^2 \rho = f f_{co} A Ph (T_c - T_b) \quad (1a)$$

or

$$J_{co}^2 = \frac{4f(1 - f_{co})}{f_{co}} h D^{-1} (T_c - T_b) \rho^{-1} \quad (1b)$$

If we knew what value of h to use in (1), we could calculate $I = I_{lim}$. We estimate h as follows.

The heat flux into the helium $\dot{q}$ is related to the time t to takeoff of the surface temperature according to

$$\dot{q} = Ct^{-1/2} \quad (2)$$

where C is a constant depending on the ambient helium state. The lower stability margin (intersection P in Fig. 5) is characterized by the condition that the duration of the normalizing pulse just equals the time to takeoff. Thus t in (2) is the pulse duration.

A heat flux of magnitude $\dot{q}$ will induce a flow velocity v of the order of³

$$v \sim \frac{\dot{q} p^2}{(1 - f_{co}) A} \cdot \frac{\beta}{S_p} \quad (3)$$

Combining (2) and (3), and ignoring thermodynamic quantities and numerical constants, we find

$$v \sim t^{-1/2} D^{-1} \quad (4)$$

The upper branch of the heat transfer curve represents steady-state heat transfer, which we take to be described by the Dittus-Boelter equation

$$Nu = \text{const} \times (Re)^{0.8} (Pr)^{0.4} \quad (5a)$$

or

$$h \sim v^{4/5} D^{-1/5} \quad (5b)$$

when we again ignore thermodynamic quantities and numerical constants. Inserting (4) into (5b), we find

$$h \sim t^{-2/5} D^{-4/5} \quad (6)$$

Combining (6) and (1b), we see that $J_{co,lim}$ scales as

$$J_{co,lim} \sim \left[\frac{f(1 - f_{co})}{f_{co}} \right]^{1/2} \cdot (T_c - T_b)^{1/2} \rho^{-1/2} \cdot D^{-1} \cdot t^{-1/5} \cdot l^{2/5} \quad (7)$$

Discussion

This scaling rule, stated without justification, was used earlier⁶ to estimate I_{lim} for the Nb₃Sn/Cu W/LCP conductor from the data on NbTi/Cu triplets reported in ref. 1. The estimates showed I_{lim} to be quite large for the W/LCP conductor so no multiple stability was expected. None was found,⁶ and this was the first crude check on (7).

In a companion paper at this conference,⁷ Lue and Miller report tests of certain of the dependences given in (7). By varying the external magnetic field they changed T_c and ρ (magnetoresistive effect) and checked the dependence of I_{lim} on $(T_c - T_b)^{1/2} \rho^{-1/2}$. Agreement is excellent (see Fig. 6). Agreement is also excellent with the $t^{-1/5}$ dependence on pulse duration (see Fig. 7). However, agreement with the $l^{2/5}$ dependence on heated length l is bad (see Fig. 8).

The heated length l enters the scaling rule through (6), which depends through (5) on (3). Since the powers of t and l are determined by (5) and that of t appears to be right, we assume provisionally that the fault lies with (3). Furthermore, the q -dependence of (3) appears correct, for from it and (2) come the dependence on t . So our best guess is that the l -dependence of (3) is wrong.

The l -dependence in (3) refers to a short heated length in a long conductor, whereas the data of ref. 7 refer to a short heated length open at both ends to an atmosphere of supercritical helium. In the latter case,⁷ the induced velocity oscillates with the period $2l/c$. The space- and time-averaged induced velocity over the heating interval t rises from zero at $t = 0$ and then oscillates with decreasing amplitude and period $2l/c$ around one-fourth the value given in (3). Thus the right-hand side of (3) should be modulated by a damped periodic function of $ct/2l$. These oscillations show up clearly in the dependence of I_{lim} on l , and, to a lesser extent, in the dependence of I_{lim} on t . Furthermore, longer lengths l , which correspond to smaller values of $ct/2l$, correspond to larger amplitudes of oscillation, exactly as observed in Fig. 8. The disagreement of the l -dependence of (7) with experiment now seems less ominous. A better check would be in an actual coil where the heated zone is connected not to an open atmosphere, but to an unheated conductor of the same geometry. All in all, the scaling rule seems reliable for making design estimates.

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Symbols

A	area of cable space
C	sound speed
D	hydraulic diameter of helium-filled space $\{4A(l - f_{co})/P\}$
f	volume fraction of copper in the metal
f_{co}	volume fraction of metal in the cable space
h	heat transfer coefficient
ΔH	stability margin
I	transport current
I_{lim}	limiting current
J_{co}	current density in the metal ($I/f_{co}A$)
l	length of the heated zone
p	cooled perimeter of the cable
q	heat flux into the helium
S_p	volumetric heat capacity of helium at constant pressure
$t(\tau_h)$	duration of heating pulse
T_b	ambient helium temperature
T_c	critical temperature
v	helium flow velocity in the cable interstices
β	volume coefficient of thermal expansion of helium
ρ	resistivity of copper

Fig. 1. Sketches of the stability margin ΔH as a function of the ambient pressure p , the transport current I , or the helium flow velocity v . The open areas are stable; the hatched areas are unstable.

Fig. 2. Schematic representation of the stability margin as a function of imposed helium flow and transport current. The fold in the surface is connected with the occasional multivaluedness of the stability.

Fig. 3. The variation of the stability margin ΔH with imposed heat transfer coefficient h .

Fig. 4. The dependence of the induced heat transfer coefficient h on the size of the heat pulse ΔH , assuming a fixed pulse duration.

Fig. 5. Combined plot of the stability curve (Fig. 3) and the heat transfer curve (Fig. 4).

Fig. 6. Scaling of I_{lim} with $(T_c - T_b)^{1/2} p^{-1/2}$.

Fig. 7. Scaling of I_{lim} with pulse duration.

Fig. 8. Scaling of I_{lim} with the length of the heated zone.