MASTER

SYSTEMS, SCIENCE AND SOFTWARE

SSS-R-81-4749

**Geothermal Reservoir Engineering Computer
Code Comparison and Validation Calculations
Using MUSHRM and CHARGR Geothermal
Reservoir Simulators**

J. W. Pritchett

**FINAL REPORT
WORK PERFORMED UNDER CONTRACT**

DE-AC03-80SF11451

NOVEMBER 1980

PREPARED FOR

DEPARTMENT OF ENERGY

SAN FRANCISCO OPERATIONS OFFICE

DISCLAIMER

This book was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

MAIL ADDRESS: P. O. BOX 1620, LA JOLLA, CALIFORNIA 92038 TELEPHONE: (714) 453-0060
STREET ADDRESS: 3398 CARMEL MOUNTAIN ROAD, SAN DIEGO, CALIFORNIA 92121

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

TABLE OF CONTENTS

SECTION	PAGE
ABSTRACT	i
I. INTRODUCTION	1
II. THE <u>MUSHRM</u> AND <u>CHARGR</u> GEOTHERMAL RESERVOIR SIMULATIONS	2
A. Basic Governing Equations	2
B. Constitutive Relations	5
C. Source and Sink Terms	8
D. Geometric Representation	9
E. The Wellbore Sub-Model	10
F. Numerical Solution Techniques	15
G. Time-Discretization	17
H. Spatial Discretization	19
III. SOLUTIONS TO THE PROBLEM SET	
A. Problem 1: "1-D Avdonin Solution"	26
B. Problem 2: "1-D Well Test Analysis"	40
C. Problem 3: "2-D Flow to a Well in Fracture/Block Media"	56
D. Problem 4: "Expanding Two-Phase System with Drainage"...	70
E. Problem 5: "Flow in a 2-D Areal Reservoir"	88
F. Problem 6: "Flow in a 3-D Reservoir"	107
IV. REFERENCES	125

I. INTRODUCTION

Since the mid-1970's, several independent research groups within the United States have developed reservoir simulation computer programs to describe two-phase (water/steam) flow and heat transfer in geothermal systems. These geothermal reservoir simulators are large computer programs which solve the non-linear system of partial differential equations which express the conservation of mass, momentum and energy in the motion of water, steam and water/steam mixtures through porous rock. The general problem is unsteady and multi-dimensional. All of these various simulators employ discretized methods (finite-difference or finite-element), permitting proper representation of reservoir geometry and heterogeneity.

In order to increase the confidence of the geothermal community in numerical reservoir simulation as a useful engineering research tool, the U. S. Department of Energy has undertaken a program to test several of these simulation programs. A suite of specific problems was selected; some of these problems have approximate analytic solutions, others do not. Then, contracts were issued to a number of different groups; each group was to solve the set of problems using their own geothermal reservoir simulator and report the results. Once the computed results from the various research groups have been assembled, comparisons will be made among them; the results of this comparison will be presented at the Sixth Workshop on Geothermal Reservoir Engineering at Stanford University in December, 1980.

In this report, the computations made by the group at Systems, Science and Software are presented. For all of the problems, our newest reservoir simulator ("CHARGR") was employed; for one of them, computations were also carried out using the earlier "MUSHRM" simulator. The first part of the report describes both of these simulators in detail. Then, the results obtained for the suite of problems are presented.

II. THE MUSHRM AND CHARGR GEOTHERMAL RESERVOIR SIMULATORS

Since 1975, the MUSHRM (Multi-Species Hydrothermal Reservoir Model) program has been S-Cubed's standard reservoir simulator. MUSHRM is capable of treating unsteady multiphase fluid and heat flow in multidimensional geometries (including 3-D). The MUSHRM code is also compositional; in addition to water/steam systems, it is capable of treating fluids with dissolved solids (including precipitation effects) and dissolved incondensable gases such as CH_4 and CO_2 (including outgassing phenomena). MUSHRM also has the capability to treat systems with substantial rock compressibility; thus, porosity and permeability may change in response to pressure and temperature changes in a nonlinear fashion. MUSHRM has been used to study several geothermal systems, including the Wairakei field in New Zealand, the Salton Sea and East Mesa fields in the Imperial Valley of California, the Brazoria geopressured field on the Texas gulf coast, and (most recently) the Baca prospect in New Mexico.

Within the last year, a major undertaking at S³ has been the development of a new reservoir simulator which is fully-implicit and, therefore, is not restricted in the time-step which may be used (as MUSHRM is). The new simulator (CHARGR; "Compositional Hydrothermal Aalysis of the Response of a Geothermal Reservoir) possesses all of the basic capabilities of the MUSHRM code, and shares many of the same numerical solution techniques. The CHARGR code was employed throughout in this study; one case (Problem 5A) was also run on MUSHRM for comparison purposes. In this section, the essential features of CHARGR and MUSHRM are described.

A. BASIC GOVERNING EQUATIONS

Both MUSHRM and CHARGR solve the following set of equations:

Mass Conservation:

$$\frac{\partial}{\partial t} [\phi(V_S \rho_S + V_L \rho_L + V_V \rho_V)] + \nabla \cdot [\phi(V_L \rho_L \vec{u}_L + V_V \rho_V \vec{u}_V)] = \sum_{\ell} \dot{m}_{\ell} \quad (1)$$

Energy Conservation:

$$\begin{aligned} & \frac{\partial}{\partial t} [(1-\phi)\rho_R E_R + \phi(V_S \rho_S E_S + V_L \rho_L E_L + V_V \rho_V E_V)] \\ & + \nabla \cdot \left\{ \phi[(P + \rho_L E_L) V_L \vec{u}_L + (P + \rho_V E_V) V_V \vec{u}_V] \right\} \\ & = \nabla \cdot (K \nabla T) + \phi(V_L \rho_L \vec{u}_L + V_V \rho_V \vec{u}_V) \cdot \vec{g} + \dot{e} + \sum_{\ell} (\dot{m}_{\ell} H_{\ell}) \end{aligned} \quad (2)$$

Note that this expression includes the effects of pressure-work and viscous dissipation (see Garg and Pritchett [1977]).

Species α Conservation:

$$\begin{aligned} & \frac{\partial}{\partial t} [\phi(V_S \rho_S C_{\alpha S} + V_L \rho_L C_{\alpha L} + V_V \rho_V C_{\alpha V})] + \nabla \cdot [\phi(V_L \rho_L C_{\alpha L} \vec{u}_L + V_V \rho_V C_{\alpha V} \vec{u}_V)] \\ & = \sum_{\ell} (\dot{m}_{\ell} C_{\alpha \ell}) \end{aligned} \quad (3)$$

For a system with a fluid containing N species (for instance H_2O , $NaCl$, CH_4 , etc.) the above amount to $N+1$ equations (one equation (1), one equation (2), and $N-1$ equations (3)). In these expressions,

t = time

ϕ = porosity

- V_V = volume fraction of vapor phase in pores
 V_L = volume fraction of liquid phase in pores
 V_S = volume fraction of "pore solid" (i.e., solid precipitates) in pores. (Note that $V_L + V_V + V_S = 1$ always)
 ρ_V = density of vapor phase
 ρ_L = density of liquid phase
 ρ_S = density of "pore solid" phase
 ρ_R = density of rock matrix
 E_V = internal energy per mass of vapor phase
 E_L = internal energy per mass of liquid phase
 E_S = internal energy per mass of pore solid phase
 E_R = internal energy per mass of rock matrix
 $C_{\alpha V}$ = mass fraction of species α in vapor phase
 $C_{\alpha L}$ = mass fraction of species α in liquid phase
 $C_{\alpha S}$ = mass fraction of species α in pore solid
 (Note that $\sum_{\alpha} C_{\alpha S} = \sum_{\alpha} C_{\alpha L} = \sum_{\alpha} C_{\alpha V} = 1$)
 $\vec{u}_V$ = vapor phase velocity
 $\vec{u}_L$ = liquid phase velocity
 P = pressure
 T = temperature
 $\vec{g}$ = gravity acceleration
 $\bar{K}$ = effective thermal conductivity
 $\dot{e}$ = local heat source rate (power per unit volume)
 $\dot{m}_\ell$ = local mass source rate (mass per volume per time)
 H_ℓ = enthalpy associated with $\dot{m}_\ell$
 $C_{\alpha \ell}$ = mass fraction of species α associated with $\dot{m}_\ell$.

The velocities are given by Darcy's law:

$$\vec{u}_v = \frac{k R_v}{\phi V_v \mu_v} (\rho_v \vec{g} - \nabla P) \quad (4)$$

$$\vec{u}_L = \frac{k R_L}{\phi V_L \mu_L} (\rho_L \vec{g} - \nabla P) \quad (5)$$

where

- k = absolute rock permeability
- R_v = relative permeability for vapor
- R_L = relative permeability for liquid
- μ_v = viscosity of vapor phase
- μ_L = viscosity of liquid phase

B. CONSTITUTIVE RELATIONS

The constitutive data for the fluid mixture and for the rock are required to close the set of equations. The fluid constitutive data provide a relationship between the following quantities:

$$P, T, V_s, V_L, V_v, \rho_s, \rho_L, \rho_v, C_{as}, C_{aL}, C_{av}, \\ \mu_L, \mu_v, E_s, E_L, E_v, K_s, K_L, K_v.$$

Both CHARGR and MUSHRM utilize subroutines which provide the relevant constitutive relationships for the fluid mixture in the pores. The library for the MUSHRM code is more extensive, however:

Available MUSHRM Constitutive Packages:

1. Single-phase liquid water — 1 specie, 1 phase.
2. Water/steam — 1 specie, 2 phases
3. Brine — 2 species (H_2O , $NaCl$), 3 phases (steam, liquid, salt precipitate)

4. Water-methane — 2 species (H_2O , CH_4), 2 phases (free CH_4 , liquid with dissolved CH_4)
5. Brine-methane — 3 species (H_2O , CH_4 , $NaCl$), 2 phases (free CH_4 , liquid with dissolved CH_4 and $NaCl$)
6. Water/ CO_2 — 2 species (H_2O , CO_2), 2 phases (free gas mixed with steam, liquid water with dissolved gas; presently being formulated)

For the CHARGR code, we have:

Available CHARGR Constitutive Packages:

1. Single-phase liquid water — 1 specie, 1 phase.
2. Water/steam — 1 specie, 2 phases.
3. Water with dissolved ideal gas — 2 species, 2 phases (free gas, liquid with dissolved gas)
4. Water/ CO_2 — 2 species (H_2O , CO_2), 2 phases (free gas mixed with steam, liquid water with dissolved gas; presently under development).

As time goes on, the library will be extended and the MUSHRM packages will be adopted to CHARGR as required.

For the rock, the user provides the constitutive data. These include the density of the rock (ρ_R) and the rock heat capacity (c_{VR}); we assume that the rock internal energy is given by:

$$E_R = C_{VR} T$$

Also to be provided are the rock thermal conductivity (K_R), the initial porosity (ϕ_0), the absolute permeability of the rock in the various coordinate directions (k_x , k_y , k_z for example), and the mathematical forms of the relative permeability functions:

$$R_L, R_V = \text{functions of } (V_S, V_L, V_V). \quad (7)$$

All of these rock properties may be arbitrary functions of position.

It should be noted that the porosity is included within the time-differentiation in Equations (1-3). This permits the local porosity to change with time so that rock deformation and compressibility effects may be taken into account. Rock deformation effects are treated using a non-linear uniaxial compaction model including thermal expansion effects:

$$\dot{\phi} = \frac{1-\phi}{A + \frac{4}{3} B} \left[\dot{P} + \dot{T}(3A \epsilon_p - 3(A + \frac{4}{3} B) \epsilon_g) \right]$$

where

$$A = A_{\text{LOAD}} \text{ for } \dot{P} < 0$$

$$= A_{\text{UNLOAD}} \text{ for } \dot{P} \geq 0$$

$$A_{\text{LOAD}} = \text{bulk modulus on loading}$$

$$A_{\text{UNLOAD}} = \text{bulk modulus on unloading}$$

$$B = \text{shear modulus}$$

$$\epsilon_p = \text{linear thermal expansion coefficient of porous dry rock}$$

$$\epsilon_g = \text{linear thermal expansion coefficient of rock grain material}$$

Of course, if the porosity at a point changes with time, the absolute permeability of the rock also changes. To represent this effect, a generalized form of the well-known Karman-Kozeny law is employed:

$$k(\phi) = k_0(\phi_0) \times \left(\frac{\phi}{\phi_0} \right)^{n_3} \times \left(\frac{1-\phi_0}{1-\phi} \right)^{n_2} \quad (9)$$

where n_2 and n_3 (neither negative) are additional rock properties. Note that if $n_3 = 3$, $n_2 = 2$ the usual Karman-Kozeny formula results. These additional rock properties (A_{LOAD} , A_{UNLOAD} , B , ϵ_p , ϵ_g , n_3 and n_2) are likewise arbitrary functions of position.

Note that, in Equation (2), the "effective thermal conductivity" ($\bar{K}$) is required. The fluid constitutive package provides the separate conductivities of the various phases within the pores (K_S , K_L , K_V) and the thermal conductivity of the rock matrix (K_R) is a specified function of position. To obtain $\bar{K}$, we first employ a mixture formula due to Budiansky [1970] to obtain the net conductivity of the material within the pore space (K_p):

$$\frac{V_S}{K_S + 2K_p} + \frac{V_L}{K_L + 2K_p} + \frac{V_V}{K_V + 2K_p} - \frac{1}{3K_p} = 0 \quad (10)$$

Then, the overall effective conductivity is found by a second application of Budiansky's formula:

$$\frac{\phi}{2\bar{K} + K_p} + \frac{(1-\phi)}{2\bar{K} + K_R} - \frac{1}{3\bar{K}} = 0 \quad (11)$$

C. SOURCE AND SINK TERMS

The mass source and sink terms in Equations (1-3) warrant some discussion. The energy source rate ($\dot{e}$) may be arbitrarily prescribed by the user as a function of position, and may also vary with local temperature and pressure. The mass source/sink terms ($\dot{m}_\ell$'s) may also be user-specified functions of position, pressure and temperature. The parameters describing both the mass and energy sources and sinks may be changed from time to time during the course of a run. The $\dot{m}_\ell$'s are also used to couple the effects of wells into the simulators, as will be discussed later. Since more than

one mass source/sink may be present at a given location, the net effect is determined by summation. The transported quantities ($C_{\alpha l}$, H_l) associated with $\dot{m}_l$ are determined by mobility weighting, as follows. For $\dot{m}_l > 0$ (i.e., fluid injection), the internal energy (E_{INl}) and composition ($C_{\alpha INl}$) of the fluid entering the system are provided by the user. The constitutive package is then consulted to provide ρ_{IN} where

$$\rho_{IN} = V_S \rho_S + V_L \rho_L + V_V \rho_V \quad (12)$$

associated with the local instantaneous pressure. Then $C_{\alpha l}$ and H_l are given by:

$$\left. \begin{aligned} C_{\alpha l} &= C_{\alpha INl} \\ H_l &= E_{INl} + \frac{P}{\rho_{INl}} \end{aligned} \right\} \quad \text{for } \dot{m}_l > 0 \quad (13)$$

$$(14)$$

If $\dot{m}_l < 0$, of course, the user cannot specify the enthalpy and composition of the transported fluid; these are determined by events taking place within the system:

$$C_{\alpha} = \frac{C_{\alpha V}(R_V \rho_V u_L) + C_{\alpha L}(R_L \rho_L u_V)}{(R_V \rho_V u_L) + (R_L \rho_L u_V)} \quad (15)$$

$$H = \frac{(E_V \rho_V + P)(R_V u_L) + (E_L \rho_L + P)(R_L u_V)}{(R_V \rho_V u_L) + (R_L \rho_L u_V)} \quad (16)$$

D. GEOMETRIC REPRESENTATION

Both MUSHRM and CHARGR employ finite-difference techniques to solve the set of equations described above. Considerable geometric flexibility is provided; available coordinate systems are:

- (1) 1-D slab
- (2) 1-D radial
- (3) 1-D spherical

- (4) 2-D planar or areal
- (5) 2-D axisymmetric
- (6) 3-D Cartesian

In all of these geometries, provision is made to allow a variable mesh (i.e., Δx is a function of x , and similarly for y and z). In 1-D slab geometry, the cross-section area may be an arbitrary function of position; similarly, in 1-D radial and 2-D areal geometries the formation thickness may be arbitrarily distributed. Provision is made to allow both the magnitude and direction of the gravity acceleration ($\vec{g}$) to vary with position; this permits formations with variable dip to be treated in areal mode, for example. Any zone within the grid may be flagged "void", which facilitates the treatment of irregularly shaped regions. Any face of any zone may represent a physical boundary of any of several types:

- (1) Impermeable, insulated.
- (2) Impermeable, prescribed temperature.
- (3) Impermeable, prescribed heat flux.
- (4) Permeable, prescribed pressure.
- (5) Permeable, prescribed mass flux.

E. THE WELLBORE SUB-MODEL

Both the CHARGR and MUSHRM programs incorporate a well-bore "sub-model"; in the following, the principal features of the well model will be outlined. It is frequently necessary in reservoir simulation to employ computational zones (or "blocks") which are much larger in physical size than the diameter of a well. Under such circumstances, it is necessary to approximate the behavior on the sub-grid scale in some fashion. Since events on the sub-grid scale reach equilibrium much more rapidly than in the reservoir as a

whole, it is permissible to treat the sub-grid flow as if it were essentially steady. Now, for steady single-phase liquid flow in the vicinity of a well, it may easily be shown that:

$$P_W = P_B + \frac{Q\mu}{2\pi k\phi L} (\ln(r_B/r_W) + S) \quad (17)$$

where Q is the injection rate (mass per unit time; positive for an injection well, negative for a production well), k is rock permeability, L is the length of open section in the well, r_W is the wellbore radius, S is the "skin factor", P_B is the "well-block pressure" and r_B is the "effective well-block radius". If $Q > 0$, then ρ and μ denote the density and viscosity of the injected fluid; otherwise, they represent those characterizing the well-block. The effective well-block radius r_B remains to be determined. Pritchett [1980] has shown that, for the axisymmetric case with uniform zoning, $r_B \approx 0.56 \Delta r$. Peaceman [1978] and Pritchett and Garg [1980] have demonstrated that for the areal case with a uniform grid of square zones, $r_B \approx 0.2 \Delta x$. Pritchett and Garg [1980] also evaluated the effect of non-square wellblocks upon the proper value of r_B . Thus, for single phase flow, Equation (17) provides the relationship between well-block pressure, the pressure at the sandface of the well, and the flow rate.

Of course, many problems of geothermal interest involve multi-phase flow. Pritchett [1979] has extended the theory to treat two-phase (water/steam or water/gas) flows. The linear algebraic expression (17) is replaced by a non-linear function in the multi-phase case, but the treatment is straightforward. Pritchett's method is incorporated in both the CHARGR and MUSHRM simulators.

For the moment, attention will be confined to wells that penetrate only a single block in the computational grid; the more general case will be discussed below. Essentially, to use the well model, the engineer specifies the location and physical properties

of the well (r_w , S , L) and also provides the operating parameters Q^* and P^* . Provision is made in the simulators for changing any or all of these parameters from time. The physical significance of Q^* and P^* is as follows. Q^* is the "desired" injection rate, and is negative for a production well. Note that if $Q^* > 0$, the composition and internal energy of the injected fluid must also be provided. For $Q^* > 0$, injection will take place at the rate Q^* unless or until injection at that rate would cause the sandface pressure for the well (P_w) to exceed P^* . Then, the injection rate will be reduced to a value such that the sandface pressure equals P^* . For $Q^* < 0$, production takes place at $|-Q^*|$ unless this would cause $P_w < P^*$, in which case the production rate becomes pressure-controlled, with $P_w = P^*$.

The treatment is somewhat more complicated if a well penetrates more than one grid block, as is often the case in axisymmetric and three-dimensional problems. For such problems, a "datum level" is defined as the height within the well of the top of the open interval (y_D); if there is more than one such interval, the datum level is located at the top of the highest interval. The instantaneous pressure at the datum level within the well is denoted by P_D . The pressure distribution below the datum level in the well is taken as hydrostatic:

$$P_{well}(y) = P_D + |g| \int_{y_D}^y \rho \, dy \quad (18)$$

Here, $\rho (= \rho_s V_s + \rho_L V_L + \rho_v V_v)$ is the bulk fluid density within the well, which depends on height, and of course $\rho = \rho(P_{well}, E_w, C_{aw})$ according to the fluid constitutive description, where E_w and C_{aw} are the internal energy and composition of the fluid within the well. For an injection well, of

course, these latter quantities are supplied by the user; for a production well, they are determined by mobility-weighting of the values for the various well-blocks associated with the well, as follows:

$$C_{aw} = \frac{\sum_i \left[k_i L_i \left(\frac{C_{aL_i} \rho_{L_i} R_{L_i}}{\mu_{L_i}} + \frac{C_{aV_i} \rho_{V_i} R_{V_i}}{\mu_{V_i}} \right) \right]}{\sum_i \left[k_i L_i \left(\frac{\rho_{L_i} R_{L_i}}{\mu_{L_i}} + \frac{\rho_{V_i} R_{V_i}}{\mu_{V_i}} \right) \right]} \quad (19)$$

$$E_w = \frac{\sum_i \left[k_i L_i \left(\frac{E_{L_i} \rho_{L_i} R_{L_i}}{\mu_{L_i}} + \frac{E_{V_i} \rho_{V_i} R_{V_i}}{\mu_{V_i}} \right) \right]}{\sum_i \left[k_i L_i \left(\frac{\rho_{L_i} R_{L_i}}{\mu_{L_i}} + \frac{\rho_{V_i} R_{V_i}}{\mu_{V_i}} \right) \right]} \quad (20)$$

The summations extend over all well-blocks (i) associated with the well. In either case, given a value of P_D , Equation (18) is numerically integrated to provide the pressure distribution within the well. This permits the sandface pressure (P_{wi}) to be determined for each wellblock.

We first assume that the well is pressure-controlled (i.e., $P_D = P^*$). Using (18), we then determine the sand face pressure (P_{wi}) for each wellblock. Next, knowing the sandface pressure, we may compute the mass flow rate into (or out of) the well for that block (using Equation (17) for the single-phase case or the more general treatment of Pritchett [1979] for multiphase flow). We may then determine the total flow rate for the well by summation over all blocks. For an injection well, if this total flowrate is less than the "desired" rate Q^* , then the well is indeed

pressure-controlled and the process is complete. Likewise, for a production well, if the total flowrate is less than Q^* , the well is pressure controlled. Otherwise, however, our assumption that $P_D = P^*$ was erroneous and we must proceed further. Essentially, the above process is iterated upon until a value of P_D is found for which the total flowrate is equal to the "desired" flowrate Q^* . In this way, the so-called "allocation" problem is solved automatically. Note that this technique allows for the possibility that a well which is producing very slowly may actually be injecting fluid into some strata and producing fluid from others.

The above description of the wellbore model in MUSHRM and CHARGR is necessarily somewhat sketchy; the reader is referred particularly to Pritchett [1979] and Pritchett and Garg [1980] for more details. Note that the model does not predict wellhead conditions; conditions at the "datum level" are used to characterize the well, at the top of the open interval. In principle, it is possible to employ one or another of the experimental correlations in the literature for two-phase flow in wells (see, for example, Duckler, Wicks and Cleveland [1964]; Hagedorn and Brown [1964]; Hughmark [1962]; Orkiszewski [1967]; Ros [1961]) to extend the treatment to the wellhead. Unfortunately, our test calculations have shown that these empirical models are quite different one from another, and frequently produce very different results for the same problem. Until the state-of-the-art in this area improves, it seemed prudent to us to defer implementation of such a treatment to a future date. Of course, the engineer can use correlations of this type to "predict" wellhead conditions from results computed by MUSHRM or CHARGR a posteriori. For a production well, since the flowing enthalpy and fluid composition are computed as functions of time by either simulator for each well, if the wellhead pressure can be estimated, the steam quality and other thermodynamic properties of the flow stream can be obtained by consulting the steam tables.

F. NUMERICAL SOLUTION TECHNIQUES

In the remainder of this discussion, the numerical procedures used to solve the basic system of Equation (1-3) will be outlined. These equations amount to $N+1$ partial differential equations in $N+1$ unknown functions of position and time, where N is the number of species in the fluid mixture. One major difference between MUSHRM and CHARGR is the choice of basic field variables. IN MUSHRM, the unknown functions are taken to be the bulk fluid density,

$$\rho = V_S \rho_S + V_L \rho_L + V_V \rho_V \quad (21)$$

the bulk fluid internal energy,

$$E = (V_S \rho_S E_S + V_L \rho_L E_L + V_V \rho_V E_V) / (V_S \rho_S + V_L \rho_L + V_V \rho_V)$$

and the bulk composition C_α :

$$C_\alpha = (V_S \rho_S C_{\alpha S} + V_L \rho_L C_{\alpha L} + V_V \rho_V C_{\alpha V}) / (V_S \rho_S + V_L \rho_L + V_V \rho_V)$$

so that pressure, temperature, etc. become dependent variables. In CHARGR, E and C_α remain principal field variables, but the density (ρ) is eliminated and replaced by the pressure (P). Thus, ρ is a dependent variable in CHARGR.

The fundamental field equations (1-3) may be combined with Darcy's law (Equations 4,5) to yield $N+1$ equations, all of the following general form:

$$\frac{\partial A}{\partial t} + \nabla \cdot (F_L \vec{U}_L + F_V \vec{U}_V) = \nabla \cdot (K \nabla T) + (W_L \vec{U}_L + W_V \vec{U}_V) \cdot \vec{g} + \text{source terms} \quad (24)$$

In Equation (24),

$$\vec{U}_L \equiv k (\rho_L g - \nabla P)$$

$$\vec{U}_V \equiv k (\rho_V g - \nabla P)$$

The accumulation terms are:

<u>EQUATION</u>	<u>"A"</u>
MASS EQN.	$\phi(V_S \rho_S + V_L \rho_L + V_V \rho_V)$
SPECIES EQNS.	$\phi(V_S \rho_S C_{aS} + V_L \rho_L C_{aL} + V_V \rho_V C_{aV})$
ENERGY EQNS.	$(1-\phi)\rho_R C_{VR} T + \phi(V_S \rho_S E_S + V_L \rho_L E_L + V_V \rho_V E_V)$

The fluxed quantities are:

<u>EQUATION</u>	<u>"F_L"</u>	<u>"F_V"</u>
MASS EQN.	$R_L \rho_L / \mu_L$	$R_V \rho_V / \mu_V$
SPECIES EQNS.	$R_L \rho_L C_{aL} / \mu_L$	$R_V \rho_V C_{aV} / \mu_V$
ENERGY EQN.	$R_L (P + \rho_L E_L) / \mu_L$	$R_V (P + \rho_V E_V) / \mu_V$

The conduction coefficients are:

<u>EQUATION</u>	<u>"K"</u>
MASS EQN.	zero
SPECIES EQNS.	zero
ENERGY EQN.	$\bar{K}$

The displacement-work rates are:

<u>EQUATION</u>	<u>"W_L"</u>	<u>"W_V"</u>
MASS EQN.	zero	zero
SPECIES EQNS.	zero	zero
ENERGY EQN.	$\rho_L R_L / \mu_L$	$\rho_V R_V / \mu_V$

The essential problem addressed by each simulator (MUSHRM and CHARGR) is as follows. Consider that we know the state of the system at a time $t = t^n$. How do we find the state of the system a short time later, at time $t = t^{n+1} = t^n + \Delta t^{n+1/2}$? Note that we desire a simultaneous solution of $N+1$ equations of form (24) for the $N+1$ new space functions (denoted by superscript $n+1$) in terms of their earlier values (superscript n). The second major difference between MUSHRM and CHARGR is in the character of the time discretization employed.

G. TIME-DISCRETIZATION

The time-discretization technique used by MUSHRM may be represented as follows (compare to Equation (24)):

$$\begin{aligned} \frac{A^{n+1} - A^n}{\Delta t^{n+1/2}} + \nabla \cdot \left(F_L^n \vec{U}_L^* + F_V^n \vec{U}_V^* \right) = \nabla \cdot (K^n \nabla T^{n+1}) \\ + \left(w_L^n \vec{U}_L^* + w_V^n \vec{U}_V^* \right) \cdot \vec{g} + \text{source terms} \end{aligned} \quad (25)$$

where

$$\vec{U}_L^* = k^n \left(\rho_L^n \vec{g} - \nabla p^{n+1} \right) \quad (26)$$

$$\vec{U}_V^* = k^n \left(\rho_V^n \vec{g} - \nabla p^{n+1} \right) \quad (27)$$

In CHARGR, the following time-discretization is used:

$$\frac{A^{n+1} - A^n}{\Delta t^{n+1/2}} + \nabla \cdot \left(F_L^{n+1} \vec{U}_L^{n+1} + F_V^{n+1} \vec{U}_V^{n+1} \right) = \nabla \cdot (K^n \nabla T^{n+1})$$

$$+ \left(W_L^{n+1} \vec{U}_L^{n+1} + W_V^{n+1} \vec{U}_V^{n+1} \right) \cdot \vec{g} + \text{source terms} \quad (28)$$

where

$$\vec{U}_L^{n+1} = k^n \left(\rho_L^{n+1} \vec{g} - \nabla p^{n+1} \right) \quad (29)$$

$$\vec{U}_V^{n+1} = k^n \left(\rho_V^{n+1} \vec{g} - \nabla p^{n+1} \right) \quad (30)$$

As can be seen by inspection, the essential difference is that CHARGR is fully implicit, whereas MUSHRM is semi-implicit. In practical terms, this means that MUSHRM is limited in the size of the time step ($\Delta t^{n+1/2}$) which may be used by the Courant condition. The Courant condition states, in effect, that the maximum "throughput ratio" in the computational grid during a single time step must be less than unity. The "throughput ratio" may be defined as the mass of fluid which enters or leaves a computational zone during the time step divided by the fluid mass in that zone at the beginning of the time step. Of course, for many problems of practical interest, throughput ratios will generally be far less than unity for reasonable time steps, and the Courant condition does not impose a serious limitation. Certain special classes of problems (such as well-test simulation, for example) do, however, occasionally present a combination of small zone sizes, large flow rates, and long times of interest. Under these circumstances, the Courant condition can cause calculations with MUSHRM to be rather expensive due to the many time steps required. Problems of this type provided the motivation for the development of CHARGR. Test calculations with CHARGR have proved successful with problems involving throughput ratios as high as 10^4 .

Thus, the essential differences between MUSHRM and CHARGR may be summarized as follows. MUSHRM is semi-implicit and, therefore, imposes a Courant limitation on the time step size. CHARGR is fully implicit, and has no such limitation. Of course, due to the more complicated fully implicit treatment, CHARGR is somewhat slower on the computer, in terms of CPU time per zone per time step. MUSHRM uses (ρ, E, C_α) as principal field variables, whereas CHARGR uses (P, E, C_α) . Therefore, the fluid constitutive packages for the two codes are not interchangeable due to the difference in calling arguments. MUSHRM, being several years older, has a more extensive library of constitutive packages and a more extended performance history, and has been more thoroughly documented (a selection of applications of MUSHRM and its progenitors includes Garg, Pritchett and Brownell [1975]; Pritchett, Garg and Riney [1977]; Riney, Pritchett and Garg [1977]; Garg [1978]; and Garg, Rice and Pritchett [1979]). In other respects, the two codes are very similar. The remainder of this discussion on numerical methods applies to both codes.

H. SPATIAL DISCRETIZATION

To illustrate the spatial discretization scheme, it is expedient to first treat a much simpler system of equations as an example. Consider for the moment the single-phase isothermal flow of a pure liquid through a rigid, undeformable rock matrix, and consider only relatively small disturbances. This reduces the problem to a single field equation (mass conservation):

$$\phi \frac{\partial \rho}{\partial t} + \nabla \cdot (\phi \rho \vec{u}) = \sum_l \dot{m}_l \quad (31)$$

Note that Darcy's law for single-phase flow is:

$$\vec{u} = \frac{k}{\phi \mu} (\rho \vec{g} - \nabla P) \quad (32)$$

Next, we confine our attention to two-dimensional horizontal flow in the x-y plane, treat the rock matrix as homogeneous, and consider the following simple constitutive relations for the fluid:

$$\rho = \rho_0 + \alpha P \quad (33)$$

$$\mu = \mu_0 \quad (34)$$

where μ_0 , ρ_0 and α are constants. Since disturbances are small,

$$\rho \approx \rho_0$$

but

$$d\rho = \alpha dP \quad (35)$$

Combining the above, we obtain the following governing equation for pressure:

$$\frac{\partial P}{\partial t} = \left(\frac{\rho_0 k}{\mu_0 \phi \alpha} \right) \left(\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} \right) + \frac{\sum \dot{m}_\ell}{\phi \alpha} \quad (30)$$

Discretized in the time domain as before,

$$\frac{P^{n+1} - P^n}{\Delta t} = \left(\frac{\rho_0 k}{\mu_0 \phi \alpha} \right) \left(\frac{\partial^2 P^{n+1}}{\partial x^2} + \frac{\partial^2 P^{n+1}}{\partial y^2} \right) + \frac{\sum \dot{m}_\ell}{\phi \alpha} \quad (37)$$

Now, we may discretize (37) in space, using for simplicity a uniform grid with square zones of size Δx by Δx :

$$\begin{aligned} P_{ij}^{n+1} - P_{ij}^n = & \left[\frac{\rho_0 k \Delta t}{\mu_0 \phi \alpha (\Delta x)^2} \right] \times \left[\left(P_{i+1,j}^{n+1} + P_{i-1,j}^{n+1} - 2P_{ij}^{n+1} \right) \right. \\ & \left. + \left(P_{ij+1}^{n+1} + P_{ij-1}^{n+1} - 2P_{ij}^{n+1} \right) \right] + \frac{\Delta t}{\phi \alpha} \sum \dot{m}_\ell \end{aligned} \quad (38)$$

Here p_{ij}^n denotes the value of the pressure at the center of zone i, j ; $x = (i-1/2)\Delta x$, $y = (j-1/2)\Delta x$, and at time $= t^n = n\Delta t$.

One approach to the solution of Equation (38) is the now classical ADI ("alternating-direction-implicit") iterative technique. Let pre-superscript ℓ denote an iteration counter, so that ${}^\ell p_{ij}^{n+1}$ means the ℓ -th approximation to p_{ij}^{n+1} . The first approximation ${}^0 p_{ij}^{n+1}$ may be taken to be equal to p_{ij}^n . Then, the technique is as follows. We first obtain the $(\ell+1)$ -th approximation from:

$$\begin{aligned} {}^{\ell+1} p_{ij}^{n+1} - p_{ij}^n = & \left[\frac{\rho_0 k \Delta t}{\mu_0 \phi_\alpha (\Delta x)^2} \right] \times \left[{}^{\ell+1} p_{i+1 j}^{n+1} + {}^{\ell+1} p_{i-1 j}^{n+1} - 2^{\ell+1} p_{ij}^{n+1} \right] \\ & + \left[\frac{\rho_0 k \Delta t}{\mu_0 \phi_\alpha (\Delta x)^2} \right] \times \left[{}^\ell p_{ij+1}^{n+1} + {}^\ell p_{ij-1}^{n+1} - 2^\ell p_{ij}^{n+1} \right] + \frac{\Delta t}{\phi_\alpha} \sum \dot{m}_\ell \end{aligned} \quad (39)$$

Then, with ${}^{\ell+1} p_{ij}^{n+1}$ known, we find ${}^{\ell+2} p_{ij}^{n+1}$ from:

$$\begin{aligned} {}^{\ell+2} p_{ij}^{n+1} - p_{ij}^n = & \left[\frac{\rho_0 k \Delta t}{\mu_0 \phi_\alpha (\Delta x)^2} \right] \times \left[{}^{\ell+2} p_{ij+1}^{n+1} + {}^{\ell+2} p_{ij-1}^{n+1} - 2^{\ell+2} p_{ij}^{n+1} \right] \\ & + \left[\frac{\rho_0 k \Delta t}{\mu_0 \phi_\alpha (\Delta x)^2} \right] \times \left[{}^{\ell+1} p_{i+1 j}^{n+1} + {}^{\ell+1} p_{i-1 j}^{n+1} - 2^{\ell+1} p_{ij}^{n+1} \right] + \frac{\Delta t}{\phi_\alpha} \sum \dot{m}_\ell \end{aligned} \quad (40)$$

Next, we find ${}^{\ell+3} p_{ij}^{n+1}$ using Equation (39) again but with ℓ larger by 2, then obtain ${}^{\ell+4} p_{ij}^{n+1}$ from (40) with $\ell = \ell+2$, and so

on until convergence is attained. Note, however, that the problem posed by (39) is one-dimensional in space. That is, the unknowns

$${}^{l+1}p_{i-1 j}^{n+1}, {}^{l+1}p_{ij}^{n+1}, {}^{l+1}p_{i+1 j}^{n+1}$$

all share a constant value of j (or y). Likewise, Equation (40) is also one-dimensional, but in the y -direction. In other words, Equation (39) may be solved separately for each value of j , so that solving (39) amounts to the solution of a series of one-dimensional problems, and similarly for Equation (40). Thus, we have replaced the original two-dimensional problem (38) with an iterative sequence of one-dimensional problems (39, 40). Since these one-dimensional problems amount to the solution of a diagonally-banded matrix, they may be solved very efficiently on a computer. The technique may easily be extended to three space dimensions as well.

A different method for solving Equation (38) which is closely related to ADI is the LSR ("line-successive-relaxation") technique. For LSR, Equation (39) becomes:

$$\begin{aligned} & {}^{l+1}p_{ij}^{n+1} - p_{ij}^n = \\ & \left[\frac{\rho_0 k \Delta t}{\mu_0 \phi_\alpha (\Delta x)^2} \right] \times \left[{}^{l+1}p_{i+1 j}^{n+1} + {}^{l+1}p_{i-1 j}^{n+1} - 4 {}^{l+1}p_{ij}^{n+1} \right] \\ & \left[\frac{\rho_0 k \Delta t}{\mu_0 \phi_\alpha (\Delta x)^2} \right] \times \left[{}^{l+1}p_{ij+1}^{n+1} + {}^{l+1}p_{ij-1}^{n+1} \right] + \frac{\Delta t}{\phi_\alpha} \sum m_\ell \end{aligned} \quad (41)$$

and (40) becomes:

$$\begin{aligned}
 & \rho_{ij}^{n+1} - \rho_{ij}^n = \\
 & \left[\frac{\rho_0 k \Delta t}{\mu_0 \phi_\alpha (\Delta x)^2} \right] \times \left[\rho_{ij+1}^{n+1} + \rho_{ij-1}^{n+1} - 4 \rho_{ij}^{n+1} \right] \\
 & + \left[\frac{\rho_0 k \Delta t}{\mu_0 \phi_\alpha (\Delta x)^2} \right] \times \left[\rho_{i+1,j}^{n+1} + \rho_{i-1,j}^{n+1} \right] + \frac{\Delta t}{\phi_\alpha} \sum m_\ell \quad (42)
 \end{aligned}$$

For practical problems, the two techniques (ADI and LSR) may be compared as follows. ADI requires somewhat less computer storage and CPU time per iteration than does LSR. For small values of the dimensionless time step group $(\rho_0 k \Delta t / \mu_0 \phi_\alpha (\Delta x)^2)$ in the present simplified example), the convergence rates for both schemes are about the same. For large values of the dimensionless time step group, however, LSR converges more rapidly than ADI. Since for problems of geothermal interest the analogous dimensionless time step group for the more general problem associated with the advection terms in the field equations (see Equation (24)):

$$\nabla \cdot (F_L \vec{U}_L + F_V \vec{U}_V) + (W_L \vec{U}_L + W_V \vec{U}_V) \cdot \vec{g} \quad (63)$$

is likely to be large, the LSR procedure is used for treating these terms in both MUSHRM and CHARGR. The dimensionless time-step group associated with heat conduction:

$$\nabla \cdot (K \nabla T) \quad (44)$$

is almost always small, however. Therefore, ADI is employed for the heat conduction terms in both simulators.

Of course, the implementation of the mixed ADI/LSR scheme for the general equations is far more complicated than for the above simple case. First, instead of a single equation in a single

unknown function, we have $N+1$ equations in $N+1$ unknowns. Second, the resulting one-dimensional problems are highly non-linear in the general case. Nonetheless, the technique reduces the general multi-dimensional problem to a series of one-dimensional problems (albeit non-linear, in several unknown functions). Owing to the non-linear character of the one-dimensional case, a second level of iteration is required to obtain solutions.

The spatial discretization of the one-dimensional analogue of (24) follows conventional practice. In slab geometry with constant cross-section for example, we obtain:

$$\begin{aligned}
 & \left(\frac{\partial A}{\partial t} \right)_i + \frac{1}{\Delta x_i} \left(F_{L,i+1/2} U_{L,i+1/2} + F_{V,i+1/2} U_{V,i+1/2} \right. \\
 & \quad \left. - F_{L,i-1/2} U_{L,i-1/2} - F_{V,i-1/2} U_{V,i-1/2} \right) \\
 & = \frac{1}{\Delta x_i} \left(K_{i+1/2} \frac{2(T_{i+1} - T_i)}{\Delta x_i + \Delta x_{i+1}} + K_{i-1/2} \frac{2(T_{i-1} - T_i)}{\Delta x_i + \Delta x_{i-1}} \right) \\
 & \quad + \frac{1}{2} \left(W_{L,i+1/2} U_{L,i+1/2} g_{i+1/2} + W_{L,i-1/2} U_{L,i-1/2} g_{i-1/2} \right. \\
 & \quad \left. + W_{V,i+1/2} U_{V,i+1/2} g_{i+1/2} + W_{V,i-1/2} U_{V,i-1/2} g_{i-1/2} \right) \\
 & \quad + \text{source terms} \tag{45}
 \end{aligned}$$

The quantities $F_{L,i+1/2}$, $F_{V,i+1/2}$, $W_{L,i+1/2}$, $W_{V,i+1/2}$ are determined using the "donor-cell" or "upwind" technique:

$$\text{For } U_{L,i+1/2} \geq 0: F_{L,i+1/2} = F_{L,i}, W_{L,i+1/2} = W_{L,i} \tag{46}$$

$$\text{For } U_{L,i+1/2} < 0: F_{L,i+1/2} = F_{L,i+1}, W_{L,i+1/2} = W_{L,i+1} \tag{47}$$

and similarly for F_V , W_V and U_V . As is well-known, the donor-cell technique produces a method which is exceedingly stable computationally.

III. SOLUTIONS TO THE PROBLEM SET

In this section, solutions obtained for the problem set posed by the Department of Energy are presented. In connection with this problem set, it should be noted that the scope of the problems in no way exercises all of the capabilities possessed by S-Cubed's simulators. In particular, in all cases the rock matrix is treated as rigid, so that the nonlinear compaction model is inoperative, and absolute rock permeability at a point does not change with time. Furthermore, all of the problems involve only pure H_2O as the pore fluid; thus, no compositional effects are present. Therefore, liquid water and steam are the only materials present within the pore spaces; no salt dissolution/precipitation effects occur, and the effects of dissolved and free incondensable gases are absent. Only one of the problem involves the use of the near-wellbore sub-model. Effects of this kind are likely to be significant in many problems involving analysis of systems of practical importance. The problem set was chosen in such a way as to maximize the number of simulators which could solve it; many existing simulators lack one or another of these augmented capabilities.

It should be further noted that the problems in the suite all involve fairly small computational efforts; the number of zones and the number of time-steps are quite restricted compared to a typical engineering analysis of a real geothermal system. Thus, numerical errors associated with poor space/time resolution tend to be vastly exaggerated in these results compared to what would be encountered in a normal engineering application. This restriction on space/time resolution was deliberate in the problem set formulation, both to minimize costs (and thereby maximize the number of problems and separate contracts involved), and, in fact, to emphasize the effects of numerical methodology upon results.

A. PROBLEM NUMBER 1: "ONE-DIMENSIONAL AVDONIN SOLUTION"

PROBLEM DESCRIPTION

Consider a cylindrical region with vertical axis. The radius is 1000 m and the thickness is 100 m. The rock properties within the region are as follows:

$$\text{Permeability} = 10^{-12} \text{ m}^2$$

$$\text{Rock density} = 2500 \text{ kg/m}^3$$

$$\text{Rock specific heat} = 1 \text{ j/g-}^\circ\text{C}$$

$$\text{Porosity} = 0.2$$

At zero time, the system contains water at a uniform pressure (50 bars) and temperature (170°C). The upper and lower faces of the region are insulated and impermeable; the problem is to be treated as one-dimensional. At the outer perimeter, the pressure is maintained at 50 bars. For $t > 0$, water at 160°C is injected at the axis at a rate of 10 kg/sec.

The requested fluid properties (specific heat, viscosity and density) at the average temperature (165°C) and initial pressure (50 bars), accordingly to the constitutive package used, were

$$\text{density} = 904.906 \text{ kg/m}^3$$

$$\text{viscosity} = 1.5808 \times 10^{-4} \text{ Pa-sec}$$

$$\text{heat capacity} = 4.3329 \text{ kJ/kg-}^\circ\text{C}$$

At these same conditions, the thermal conductivity of water is $0.6908 \text{ W/m-}^\circ\text{C}$. The problem specifications indicated a bulk effective conductivity of $20 \text{ W/m-}^\circ\text{C}$, so the rock matrix thermal conductivity was set at $28.076 \text{ W/m-}^\circ\text{C}$ so that the Budiansky formula would yield $\bar{K} = 20.000 \text{ W/m-}^\circ\text{C}$.

The specified spatial zoning is uniform; the 1000 m radius is to be represented by 40 zones with $\Delta r = \text{constant} = 25$ meters, so that the node location (i.e., zone center) for zone i is $r_i = 25(i - 1/2)$ meters. The upper bound on the time step size is 1.6667×10^7 seconds. Particular results required are the temperature versus radial distance at $t = 10^9$ seconds, and the temperature history for node $i = 2$ ($r = 37.5$ meters).

NUMERICAL REPRESENTATION:

The problem was zoned as specified for the CHARGR code employing the two-phase water/steam equation-of-state. The problem remains single-phase liquid, however. The above-specified problem parameters were used; 10 kg/sec of 160°C water were injected into zone 1. The CHARGR code automatically selects a small time-step to use for the first two cycles, then doubles the time-step each cycle until the desired value (1.6667×10^7 sec in the present case) is reached (see Table 1.1). The problem was run out to $t = 1.5 \times 10^9$, half again as long as required.

COMPUTED RESULTS:

Temperature values for each node with $T \neq 170^\circ\text{C}$ obtained for $t = 10^9$ seconds are listed in Table 1-2. Also shown are the values for $t = 2.5 \times 10^8$. These results are shown plotted in Figure 1.1 and compared with the Avdonin analytic solution. Time-histories of temperature for node $i = 2$ ($r = 37.5$ m), $i = 4$ ($r = 87.5$ m) and $i = 8$ ($r = 187.5$ m) are listed in Table 1-3 and shown graphically (together with the analytic solution) in Figure 1.2.

DISCUSSION:

Comparison of the computed results with the analytic solution illustrates the following general features. As Figure 1.1 shows, the agreement with analytic results is reasonably good at large

TABLE 1-1

PROBLEM 1: TIME DISCRETIZATION

<u>Cycle, n</u>	<u>$\Delta t^{n-1/2}(\text{sec})$</u>	<u>$t^n(\text{sec})$</u>
1	5.0863×10^2	5.0863×10^2
2	5.0863×10^2	1.0172×10^3
3	1.0172×10^3	2.0345×10^3
4	2.0345×10^3	4.0690×10^3
5	4.0690×10^3	8.1380×10^3
6	8.1380×10^3	1.6276×10^4
7	1.6276×10^4	3.2552×10^4
8	3.2552×10^4	6.5104×10^4
9	6.5104×10^4	1.3021×10^5
10	1.3021×10^5	2.6042×10^5
11	2.6042×10^5	5.2083×10^5
12	5.2083×10^5	1.0417×10^6
13	1.0417×10^6	2.0833×10^6
14	2.0833×10^6	4.1667×10^6
15	4.1667×10^6	8.3333×10^6
16	8.3333×10^6	1.6667×10^7
17	1.6667×10^7	3.3333×10^7
18	1.6667×10^7	5.0000×10^7
19	1.6667×10^7	6.6667×10^7
20	1.6667×10^7	8.3333×10^7
.	.	.
.	.	.
.	.	.
105	1.6667×10^7	1.5000×10^9

TABLE 1-2
PROBLEM 1: TEMPERATURE DISTRIBUTION WITH RADIUS AT FIXED TIMES

<u>Zone</u>	<u>Radius</u>	<u>$t = 10^9 \text{ sec}$</u>		<u>$t = 2.5 \times 10^8 \text{ sec}$</u>	
		<u>T(°C)</u>	<u>R/√t</u> <u>(m/√sec)</u>	<u>T(°C)</u>	<u>R/√t</u> <u>(m/√sec)</u>
1	12.5	160.043	.00039	160.264	.00079
2	37.5	160.187	.00119	161.108	.00237
3	62.5	160.489	.00198	162.620	.00395
4	87.5	160.987	.00277	164.528	.00553
5	112.5	161.692	.00356	166.387	.00712
6	137.5	162.579	.00435	167.865	.00870
7	162.5	163.593	.00514	168.861	.01028
8	187.5	164.663	.00593	169.446	.01186
9	212.5	165.716	.00672	169.752	.01344
10	237.5	166.686	.00751	169.897	.01502
11	262.5	167.531	.00830	169.960	.01660
12	287.5	168.227	.00909	169.985	.01818
13	312.5	168.772	.00988	169.995	.01976
14	337.5	169.179	.01067	169.998	.02135
15	362.5	169.470	.01146	170.000	.02293
16	387.5	169.669	.01225	170.000	.02451
17	412.5	169.800	.01304		
18	437.5	169.883	.01383		
19	462.5	169.934	.01463		
20	487.5	169.963	.01542		
21	512.5	169.980	.01621		
22	537.5	169.990	.01700		
23	562.5	169.995	.01779		
24	587.5	169.998	.01858		
25	612.5	169.999	.01937		
26	637.5	169.999	.02016		
27	662.5	170.000	.02095		
28	687.5	170.000	.02174		

TABLE 1-3

PROBLEM 1: TEMPERATURE HISTORY AT FIXED RADII

<u>r = 37.5 m</u> <u>(Zone 2)</u>				<u>r = 87.5 m</u> <u>(Zone 4)</u>		<u>r = 187.5 m</u> <u>(Zone 8)</u>	
<u>Cycle</u>	<u>t(sec)</u>	<u>T(°C)</u>	<u>r/√t</u> <u>(m/√sec)</u>	<u>T(°C)</u>	<u>r/√t</u> <u>(m/√sec)</u>	<u>T(°C)</u>	<u>r/√t</u> <u>(m/√sec)</u>
16	1.667×10^7	168.281	.00919	169.950	.02143	170.000	.04593
17	3.333×10^7	166.534	.00650	169.707	.01516	170.000	.03248
18	5.000×10^7	165.212	.00530	169.342	.01237	169.998	.02652
19	6.667×10^7	164.238	.00459	168.897	.01072	169.993	.02296
20	8.333×10^7	163.518	.00411	168.412	.00959	169.984	.02054
21	1.000×10^8	162.976	.00375	167.916	.00875	169.970	.01875
23	1.333×10^8	162.233	.00325	166.965	.00758	169.918	.01624
25	1.667×10^8	161.759	.00290	166.122	.00678	169.830	.01452
27	2.000×10^8	161.435	.00265	165.401	.00619	169.703	.01326
30	2.500×10^9	161.108	.00237	164.528	.00553	169.446	.01186
33	3.000×10^8	160.892	.00217	163.852	.00505	169.126	.01083
39	4.000×10^8	160.626	.00188	162.901	.00438	168.381	.00938
45	5.000×10^8	160.471	.00168	162.281	.00391	167.605	.00839
51	6.000×10^8	160.372	.00153	161.852	.00357	166.872	.00765
57	7.000×10^8	160.303	.00142	161.542	.00331	166.210	.00709
63	8.000×10^8	160.254	.00133	161.309	.00309	165.625	.00663
75	1.000×10^9	160.187	.00119	160.987	.00277	164.663	.00593
87	1.200×10^9	160.145	.00108	160.778	.00253	163.926	.00541
105	1.500×10^9	160.106	.00097	160.577	.00226	163.114	.00484

Figure 1.1. Problem 1: Results --- Temperature versus Radius at Fixed Times.

Figure 1.2. Problem 1: Results --- Temperature versus Time at Fixed Radii.

radii, but is poorer at small radii; the temperature profiles seem somewhat "over-diffused" at small r . This effect is even more dramatic in Figure 1.2; results are reasonably good for zone 8 ($r = 187.5$ m), poorer for zone 4 ($r = 87.5$ m) and terrible for zone 2 ($r = 37.5$ m).

The basic explanation for the poor results at small radii arises from the exceedingly poor space and time resolution specified for this region. For instance, let us define the spatial resolution for zone i as $\Delta r/r_i$; the size of the zone divided by the midpoint radius. Similarly, we may define the temporal resolution for this problem as the time step ($\Delta t = 1.6667 \times 10^7$ sec) divided by the time (t_i) that the mean temperature ($T = 165^\circ\text{C}$) should arrive at r_i according to the analytic solution. Results for the various zones in the grid are as follows:

<u>i</u>	<u>$\Delta r/r_i$</u>	<u>$\Delta t/t_i^*$</u>
1	2.0	4.3
2	0.67	0.48
3	0.40	0.17
4	0.29	0.088
6	0.18	0.036
8	0.13	0.019
10	0.11	0.012
15	0.069	0.005
20	0.051	0.003
30	0.034	0.001
40	0.025	0.001

The above list shows that both the spatial and temporal resolution are very poor for the first few zones. Thus, in this region, good agreement with analytical results should not be expected.

The question of the impact of space and time resolution can be put on a firmer quantitative basis in the following way. As is well-known, numerical diffusion errors are an inevitable consequence

of discretization. Numerical diffusivity arises both from space-discretization and from time-discretization. For problems of the present type, Taylor-series analysis shows that, for the donor-cell/backward-time scheme used in CHARGR and MUSHRM, the "numerical thermal conductivity" is as follows:

$$K_{\text{NUMERICAL}} = K_{\text{SPACE}} + K_{\text{TIME}}$$

K_{SPACE} arises from poor spatial resolution; K_{TIME} is due to poor time resolution. K_{SPACE} and K_{TIME} are:

$$K_{\text{SPACE}} = \frac{Q c_{vf}}{2\pi H(2i-1)}$$

$$K_{\text{TIME}} = \frac{\Delta t Q^2 c_{vF}^2}{2\pi^2 H^2 [\phi \rho_F c_{vF} + (1-\phi) \rho_R c_{vR}] (2i-1)^2 \Delta r^2}$$

where

- Q = injection rate (10 kg/sec)
- c_{vF} = fluid heat capacity (4333 j/kg-°C)
- c_{vR} = rock heat capacity (1000 j/kg-°C)
- ρ_F = fluid density (905 kg/m³)
- ρ_R = rock density (2500 kg/m³)
- ϕ = porosity (0.2)
- H = thickness (100 m)
- Δr = space interval (25 m)
- Δt = time interval (1.667 x 10⁷ sec)
- i = zone index

Values for K_{SPACE} , K_{TIME} , and the total effective thermal conductivity ($K_{\text{TOTAL}} = K_{\text{REAL}} + K_{\text{SPACE}} + K_{\text{TIME}}$ where $K_{\text{REAL}} = 20 \text{ W/m-°C}$) are shown in Figure 1.3. Selected values are as follows:

Figure 1.3. Distribution of Numerical Thermal Conductivity for Problem 1.

<u>i</u>	<u>K_{SPACE} (W/m-°C)</u>	<u>K_{TIME} (W/m-°C)</u>	<u>$\frac{K_{\text{NUMERICAL}}}{K_{\text{REAL}}} \times 100\%$</u>
1	69	91	800%
2	23	10.1	166%
3	13.8	3.6	87%
4	9.9	1.86	59%
6	6.3	0.75	35%
8	4.6	0.40	25%
10	3.6	0.25	19%
15	2.4	0.11	12%
20	1.77	0.06	9%
30	1.17	0.03	6%
40	0.87	0.01	4%

It may easily be shown that the analytic solution for this problem may be expressed as:

$$T = f(r/\sqrt{t})$$

that is, the temperature depends on the similarity parameter (radius/ $\sqrt{\text{time}}$). Figure 1.4 shows (as solid lines) the analytic solution for $K = 20 \text{ W/m-°C}$ (the actual value for Problem 1) and for several higher values of thermal conductivity as well (25, 35 and 50 W/m-°C). Also shown are the numerical results for $T(r)$ at $t = 2.5 \times 10^8 \text{ sec}$ and $t = 10^9 \text{ sec}$ reported earlier. As the figures show, the computed distribution at $t = 10^9$ seconds matches the theoretical distribution for $K = 25 \text{ W/m-°C}$ reasonably well. The earlier data seems to fit best $K = 30$ to 35 W/m-°C . At $t = 10^9$ seconds, the "thermal front" (the radius at which $T = 165^\circ\text{C}$) is about $r = 200 \text{ m}$ according to the theoretical solution; at that point, $K_{\text{TOTAL}} (= K_{\text{REAL}} + K_{\text{SPACE}} + K_{\text{TIME}})$ is about 25 W/m-°C . At 2.5×10^8 seconds, the thermal front is at 100 m , where K_{TOTAL} is about 31 W/m-°C .

Figure 1.4. Problem 1: Reduced Results - Temperature versus Radius at a Fixed Time.

Figure 1.5 shows the same analytical solutions (for $K = 20, 25, 30$ and $50 \text{ W/m-}^\circ\text{C}$). Also shown, in terms of $r/\sqrt{t}$, are the time-history data $T(t)$ reported earlier. For $i = 8$, the history matches the $K = 25 \text{ W/m-}^\circ\text{C}$ solution fairly closely. For $i = 4$, the computed results generally fall between $K = 25 \text{ W/m-}^\circ\text{C}$ and $K = 35 \text{ W/m-}^\circ\text{C}$. For $i = 2$, the closest match seems to be $K \approx 50 \text{ W/m-}^\circ\text{C}$. For small i , it is not surprising that the results do not match any of the curves exactly, since the analytic solution assumes uniform K , and as Figure 1.3 shows, a substantial gradient in K_{TOTAL} is present for small radii. Nonetheless, the above behavior is consistent with the K_{TOTAL} 's derived above for the various zones ($i = 2, 53 \text{ W/m-}^\circ\text{C}$; $i = 4, 32 \text{ W/m-}^\circ\text{C}$; $i = 8, 25 \text{ W/m-}^\circ\text{C}$).

The poor agreement with analytical results for the temperature history at $i = 2$ could clearly be improved by improved space and time resolution in this vicinity.

Figure 1.5. Problem 1: Reduced Results - Temperature versus Time at a Fixed Radius.

B. PROBLEM NUMBER 2: "1-D WELL TEST ANALYSIS"

PROBLEM DESCRIPTION

Problem 2 actually consists of four different but related problems. Problems 2A, 2B and 2C all involve the simulation of transient well-tests in one-dimensional radial geometry. Problem 2D considers the depletion of a single computational grid block, and is therefore in effect a zero-dimensional problem.

For problems 2A - 2C, consider a uniform aquifer of unbounded radial extent and of constant thickness (100 meters). At zero time, fluid withdrawal begins from a line-sink at $r = 0$. Problems 2A, 2B and 2C differ in the rock properties, the initial fluid state, and the production rate. The rock properties are as follows:

	Problems 2A, 2C	Problem 2B
Porosity	0.2	0.15
Permeability	$1 \times 10^{-14} \text{ m}^2$	$2.4 \times 10^{-13} \text{ m}^2$
Heat Capacity	$1 \text{ j/g-}^\circ\text{C}$	$1 \text{ j/g-}^\circ\text{C}$
Thermal Conductivity	0	0
Grain Density	2650 kg/m^3	2000 kg/m^3

In all cases, the relative permeability functions are given by the Corey relations with residual liquid saturation = 0.3 and residual steam saturation = 0.05:

$$\left. \begin{array}{l} R_L = 0 \\ R_S = 1 \end{array} \right\} \text{ for } 0 \leq S_L \leq 0.3$$

$$\left. \begin{array}{l} R_L = Z^4 \\ R_S = (1-Z^2) (1-Z)^2 \end{array} \right\} \text{ for } 0.3 < S_L < 0.95$$

$$\left. \begin{array}{l} R_L = 1 \\ R_S = 0 \end{array} \right\} \text{ for } 0.95 \leq S_L \leq 1$$

where S_L is liquid saturation and Z is given by:

$$Z = (S_L - 0.3)/0.65$$

The fluid production rate for problems 2A and 2C is 14 kg/sec; that for problem 2B is 16.7 kg/sec. The initial state of the fluid in the system is as follows:

	<u>2A</u>	<u>2B</u>	<u>2C</u>
Initial Pressure	90 bars	30 bars	90 bars
Initial Temperature	260°C	233.886°C	300°C
Initial Liquid Saturation	1	0.65	1

Problem 2A involves single-phase liquid initially as does problem 2C; problem 2B is initially two-phase everywhere ($T_{\text{sat}}(30 \text{ bars}) = 233.886^\circ\text{C}$). For problem 2A, the saturation pressure associated with the initial temperature (260°C) is 46 bars, so the problem remains single-phase for radii of interest over the time scale under study ($0 \leq t \leq 1$ day). In problem 2C, the saturation pressure is 85.917 bars at 300°C ; in this case, a boiling front propagates radially outward into the system.

Problem 2D involves a single grid block of total volume 314.0 m^3 , initially at the same conditions as problem 2B and with the same rock properties. Starting at $t = 0$, fluid is withdrawn at a rate of 16.7 kg/sec. The time scale of interest is 0.01 day.

For problem 2A, the desired output information are the pressure histories at $r = 0.5, 1/\sqrt{2}$, and 1 meter as functions of the similarity parameter t/r^2 . For problems 2B and 2C, the liquid

saturation and flowing enthalpy histories at the same points are also desired. Output desired for problem 2D is the history of pressure, liquid saturation and discharge enthalpy for the block.

NUMERICAL REPRESENTATION

The CHARGR code with the water/steam equation-of-state package was used for all four cases. For problems 2A - 2C, the same spatial grid was employed. Grid spacings were assigned as follows:

$$\Delta r_1 = 1/(1+\sqrt{2}) \text{ meters}$$

$$\Delta r_i = 2^{[(i-2)/2]} \text{ meters for } 2 \leq i \leq 27$$

As shown in Table 2-1, this yields a grid extending from $r = 0$ to $r = 3.393$ km with node points (r_i) located at 0.5, $1/\sqrt{2}$, and 1 meter radii as desired. The outer radius is sufficient that the influence of production is not felt there by $t = 1$ day, so that the results will be the same as if the grid were unbounded in radial extent. For problem 2D, a single radial zone with $\Delta r = 1$ meter and height = 99.9493 meters (yielding a volume of 314.000 m^3) was used.

For problem 2D, 100 time-steps of duration 10^{-4} days (8.64 seconds) were employed. For problem 2A, 2B and 2C the time step started at a very small value ($\Delta t = [1/49152] \text{ day} = 1.7578 \text{ sec}$). Periodically, the time step was doubled, reaching a maximum value of $1/24$ day (1 hour) at the end of the problem (see Table 2-2). Total calculation time for problems 2A - 2C was 1 day, and 200 time-steps were used. The line-source was simulated using a mass-sink in zone 1.

COMPUTED RESULTS

Tables 2-3, 2-4 and 2-5 list respectively, the calculated results for problems 2A - 2C, in terms of pressure, and, for problems 2B and 2C, liquid saturation and flowing enthalpy, as

TABLE 2-1
PROBLEMS 2A, 2B AND 2C SPATIAL DISCRETIZATION

<u>i</u>	<u>Δr_i (m)</u>	<u>$r_{i-1/2}$ (m)</u>	<u>r_i (m)</u>	<u>$r_{i+1/2}$ (m)</u>
1	0.414	0	0.207	0.414
2	0.172	0.414	0.500	0.586
3	0.243	0.586	0.707	0.828
4	0.343	0.828	1.00	1.17
5	0.485	1.17	1.41	1.66
6	0.686	1.66	2.00	2.34
7	0.971	2.34	2.83	3.31
8	1.37	3.31	4.00	4.69
9	1.94	4.69	5.66	6.63
10	2.75	6.63	8.00	9.37
11	3.88	9.37	11.3	13.3
12	5.49	13.3	16.0	18.7
13	7.77	18.7	22.6	26.5
14	11.0	26.5	32.0	37.5
15	15.5	37.5	45.3	53.0
16	22.0	53.0	64.0	75.0
17	31.1	75.0	90.5	106
18	43.9	106	128	150
19	62.1	150	181	212
20	87.8	212	256	300
21	124	300	362	424
22	176	424	512	600
23	248	600	724	848
24	351	848	1024	1200
25	497	1200	1448	1697
26	703	1697	2048	2399
27	994	2399	2896	3393

TABLE 2-2
PROBLEMS 2A, 2B AND 2C TIME-DISCRETIZATION

<u>Cycle n</u>	<u>$\Delta t^{n-1/2}$ (Day)</u>	<u>t^n (Day)</u>
1	1/49,152	1/49,152
2	1/49,152	1/24,576
⋮	⋮	⋮
32	1/49,152	1/1,536
33	1/24,576	17/24,576
⋮	⋮	⋮
48	1/24,576	1/768
49	1/12,288	17/12,288
⋮	⋮	⋮
64	1/12,288	1/384
65	1/6,144	17/6,144
⋮	⋮	⋮
80	1/6,144	1/192
81	1/3,072	17/3,072
⋮	⋮	⋮
96	1/3,072	1/96
97	1/1,536	17/1536
⋮	⋮	⋮
112	1/1,536	1/48
113	1/768	17/768
⋮	⋮	⋮
128	1/768	1/24
129	1/384	17/384
⋮	⋮	⋮
144	1/384	1/12
145	1/192	17/192
⋮	⋮	⋮
160	1/192	1/6
161	1/96	17/96
⋮	⋮	⋮
176	1/96	1/3
177	1/48	17/48
⋮	⋮	⋮
192	1/48	2/3
193	1/24	17/24
⋮	⋮	⋮
200	1/24	1

TABLE 2-3
PROBLEM 2A COMPUTED RESULTS

Pressure (Bars) at R = ...

t/r^2 <u>Days/m²</u>	<u>0.5 m</u>	<u>0.7071 m</u>	<u>1 m</u>
1/49,152	-	-	89.079
1/24,576	-	88.467	88.318
1/12,288	87.718	87.490	87.348
1/8,192	-	86.857	86.746
1/6,144	86.574	86.403	86.316
1/4,096	85.981	85.769	85.709
1/2,048	84.766	84.703	84.673
1/1,024	83.686	83.655	83.644
1/512	82.631	82.621	82.618
1/256	81.594	81.592	81.591
1/128	80.564	80.563	80.563
1/64	79.534	79.534	79.534
1/32	78.505	78.505	78.505
1/16	77.475	77.475	77.475
1/8	76.446	76.446	76.446
1/4	75.416	75.416	75.416
1/2	74.386	74.386	74.386
1	73.356	73.356	73.356
2	72.327	72.327	-
4	71.297	-	-

TABLE 2-4
PROBLEM 2B COMPUTED RESULTS

t/r^2 Days/m ²	Pressure (Bars) at R=...			Liquid Saturation at R=...			Flowing Enthalpy (J/G) at R=...		
	0.5 m	0.7071 m	1 m	0.5 m	0.7071 m	1 m	0.5 m	0.7071 m	1 m
1/49152	-	-	30.000	-	-	0.6500	-	-	1335
1/24576	-	30.000	30.000	-	0.6500	0.6500	-	1335	1335
1/12288	29.998	30.000	30.000	0.6498	0.6500	0.6500	1336	1335	1335
1/8192	-	29.999	30.000	-	0.6499	0.6500	-	1336	1335
1/6144	29.994	29.998	29.999	0.6494	0.6498	0.6500	1338	1336	1335
1/4096	29.988	29.996	29.998	0.6489	0.6496	0.6498	1341	1337	1336
1/2048	29.961	29.978	29.984	0.6465	0.6480	0.6486	1354	1346	1343
1/1024	29.871	29.888	29.884	0.6396	0.6408	0.6404	1394	1387	1389
1/512	29.599	29.552	29.503	0.6242	0.6218	0.6197	1494	1511	1526
1/256	28.983	28.895	28.887	0.6058	0.6041	0.6044	1630	1643	1640
1/128	28.163	28.172	28.184	0.5941	0.5950	0.5952	1720	1711	1710
1/64	27.397	27.414	27.417	0.5893	0.5895	0.5894	1751	1749	1750
1/32	26.606	26.609	26.609	0.5857	0.5856	0.5855	1771	1772	1773
1/16	25.773	25.773	25.773	0.5827	0.5827	0.5827	1785	1785	1785
1/8	24.913	24.913	24.913	0.5804	0.5804	0.5804	1792	1792	1792
1/4	24.029	24.029	24.029	0.5783	0.5783	0.5783	1795	1795	1795
1/2	23.120	23.120	23.120	0.5763	0.5763	0.5763	1797	1797	1797
1	22.186	22.186	22.186	0.5742	0.5742	0.5742	1798	1798	1798
2	21.224	21.224	-	0.5721	0.5721	-	1798	1798	-
4	20.231	-	-	0.5699	-	-	1799	-	-

TABLE 2-5
PROBLEM 2C COMPUTED RESULTS

t/r^2 Days/m ²	Pressure (Bars) at R=...			Liquid Saturation at R=...			Flowing Enthalpy (J/G) at R=...		
	0.5 m	0.7071 m	1 m	0.5 m	0.7071 m	1 m	0.5 m	0.7071 m	1 m
1/49152	-	-	89.383	-	-	1.0000	-	-	1345
1/24576	-	88.885	88.869	-	1.0000	1.0000	-	1345	1345
1/12288	88.240	88.209	88.395	1.0000	1.0000	1.0000	1345	1345	1345
1/8192	-	87.930	88.179	-	1.0000	1.0000	-	1345	1345
1/6144	87.460	87.777	88.044	1.0000	1.0000	1.0000	1345	1345	1345
1/4096	87.246	87.601	87.873	1.0000	1.0000	1.0000	1345	1345	1345
1/2048	87.013	87.353	87.597	1.0000	1.0000	1.0000	1345	1345	1345
1/1024	86.829	87.116	87.286	1.0000	1.0000	1.0000	1345	1345	1345
1/512	86.633	86.823	86.837	1.0000	1.0000	1.0000	1345	1345	1345
1/256	86.359	86.362	86.381	1.0000	1.0000	1.0000	1345	1345	1345
1/128	85.885	85.901	85.858	1.0000	1.0000	0.9999	1345	1345	1345
1/64	85.421	85.362	85.311	0.9199	0.9091	0.8996	1343	1343	1343
1/32	84.259	84.119	84.129	0.7849	0.7810	0.7815	1374	1377	1376
1/16	81.071	81.120	81.128	0.7477	0.7476	0.7480	1407	1407	1407
1/8	77.121	77.146	77.143	0.7278	0.7280	0.7277	1428	1428	1428
1/4	72.426	72.405	72.397	0.7128	0.7123	0.7121	1442	1444	1444
1/2	67.000	66.985	66.980	0.6988	0.6986	0.6986	1455	1456	1456
1	60.905	60.897	60.986	0.6859	0.6859	0.6859	1464	1464	1664
2	54.086	54.086	-	0.6729	0.6729	-	1470	1470	-
4	46.405	-	-	0.6558	-	-	1474	-	-

functions of t/r^2 for zones 2 ($r = 0.5$ m), 3 ($r = 1/\sqrt{2}$ m) and 4 ($r = 1$ m). Table 2-6 lists the calculated results for problem 2D. These results are plotted in Figures 2.1 through 2.6.

DISCUSSION

The results for Problem 2A can be compared directly to analytic results. At late times, it can be shown that the pressure response will obey:

$$\frac{dP}{d\eta} = \frac{-\dot{M}_v}{4\pi kH}$$

where P is pressure, η is $[\ln(t/r^2)]$, $\dot{M}$ is production rate, ν is the fluid kinematic viscosity, k is permeability, and H is thickness. Using the parameters $\dot{M} = 14$ kg/sec, $\nu = 1.31 \times 10^{-7}$ m²/sec, $k = 10^{-14}$ m² and $H = 100$ m, this yields:

$$\left(\frac{dP}{d\eta}\right)_{\text{THEORETICAL}} = -1.46 \text{ bars}$$

Differentiation of the numerical data in Table 2-3 at late times yields an empirical value of:

$$\left(\frac{dP}{d\eta}\right)_{\text{EMPIRICAL}} = -1.48 \text{ bars to } -1.49 \text{ bars}$$

Considering the crude spatial grid employed, this agreement is considered reasonably good.

TABLE 2-6
PROBLEM 2D COMPUTED RESULTS

<u>Cycle</u>	<u>Time (days)</u>	<u>Pressure (Bars)</u>	<u>Liquid Saturation</u>	<u>Discharge Enthalpy (J/G)</u>
0	0.0000	30.000	0.6500	1335
5	0.0005	29.747	0.6306	1451
10	0.0010	29.417	0.6110	1593
15	0.0015	28.997	0.5914	1758
20	0.0020	28.479	0.5717	1938
25	0.0025	27.857	0.5518	2120
30	0.0030	27.120	0.5320	2290
35	0.0035	26.308	0.5121	2438
40	0.0040	25.441	0.4923	2557
45	0.0045	24.542	0.4725	2646
50	0.0050	23.628	0.4528	2709
55	0.0055	22.694	0.4332	2749
60	0.0060	21.755	0.4137	2774
65	0.0065	20.836	0.3944	2788
70	0.0070	19.944	0.3751	2794
75	0.0075	19.080	0.3560	2797
80	0.0080	18.205	0.3371	2797
85	0.0085	17.363	0.3183	2796
90	0.0090	16.552	0.2996	2795
95	0.0095	15.771	0.2811	2793
100	0.0100	14.996	0.2624	2792

Figure 2.1. Problem 2A: Pressure Histories

Figure 2.2. Problem 2B: Pressure Histories

Figure 2.3. Problem 2B: Histories of Liquid Saturation, Flowing Enthalpy.

Figure 2.4. Problem 2C: Pressure Histories.

Figure 2.5. Problem 2C: Histories of Liquid Saturation, Flowing Enthalpy.

Figure 2.6. Results for Problem 2D

C. PROBLEM NUMBER 3: "2-D FLOW TO A WELL IN FRACTURE/BLOCK MEDIA"

PROBLEM DESCRIPTION

Consider an axisymmetric region of space of unbounded radial extent with a vertical axis of symmetry and of vertical thickness 1.1 meters (see Figure 3.1). From $r = 0$ to $r = 0.16$ meters a well is present. From $r = 0.16$ meters outwards, a "fracture" is present extending from $z = 0$ to $z = 0.1$ meters. Above the fracture, from $z = 0.1$ meters to $z = 1.1$ meters, is the "block" material. The well is to be represented by a fictitious "well" material extending from $r = 0$ to $r = 0.16$ m and from $z = 0$ to $z = 0.1$ meters; all fluid production comes from this region. The upper ($z = 1.1$ meters) and lower ($z = 0$) faces are impermeable and insulated, as is the cylindrical surface at $r = 0.16$ m from $z = 0.1$ meters to $z = 1.1$ meters. The rock properties for the "well", "fracture", and "block" regions are as follows:

	<u>"Well"</u>	<u>"Fracture"</u>	<u>"Block"</u>
Grain Density	1 kg/m ³	2570 kg/m ³	2570 kg/m ³
Porosity	0.99999	0.1	0.1
Radial Permeability	3×10^{-11} m ²	3×10^{-13} m ²	0
Vertical Permeability	3×10^{-11} m ²	3×10^{-13} m ²	3×10^{-17} m ²
Heat Capacity	0.001 j/g-°C	1 j/g-°C	1 j/g-°C
Thermal Conductivity	0	0	0

For all three media, the relative permeability to steam is unity and that to liquid water is zero for all values of the liquid saturation.

Figure 3.1. Problem 3 Geometry

Starting at $t = 0$, production begins at a rate of 0.028 kg/second from the upper surface of the "well" region at $z = 0.1$ m from $r = 0$ to $r = 0.16$ m. The initial fluid pressure is uniform at $P = 30.5$ bars everywhere in the system and the effects of gravity are neglected. In "Case A", the entire system is full of saturated steam at $P = 30.5$ bars at zero time, so that the liquid saturation S_L is zero everywhere but the temperature is uniform at $T = T_{\text{sat}}(30.5 \text{ bars}) = 234.8^\circ\text{C}$. In "Case B", the "well" and "fracture" regions ($0 \leq r \leq \infty$, $0 \leq z \leq 0.1$ m) are initially full of steam at the same conditions ($P = 30.5$ bars, $T = 234.8^\circ\text{C}$, $S_L = 0$), but the "block" region contains a steam/water mixture ($P = 30.5$ bars, $T = 234.8^\circ\text{C}$, $S_L = 0.2$; $0.16 \text{ m} \leq r \leq \infty$, $0.1 \text{ m} \leq z \leq 1.1 \text{ m}$). Note that the liquid water is immobile, however.

The output desired consists of the histories of pressure in the well-block and at a point within the "block" located at $r = 2.5$ m, $z = 0.35$ m (0.25 m above the upper surface of the fracture) for both cases (A and B). In addition, for case B, the time-history of liquid saturation at the same point in the block ($r = 2.5$ m, 0.25 m above the fracture) is desired. The time-scale of interest extends from 1 to 10^4 seconds.

NUMERICAL REPRESENTATION

The spatial zoning employed for both problems (3A and 3B) is shown in Table 3-1. In the vertical direction, uniform zoning was used ($\Delta z = 0.1$ m; 11 zones). In the radial direction, the first zone (representing the wellblock) was 0.16 m in size. The second zone was 0.411 m in size. The remaining zone spacings were given by the relation:

$$\Delta r_i = \frac{6 (2.5)^{i-3}}{7} \text{ meters}$$

TABLE 3-1
PROBLEM 3 SPATIAL DISCRETIZATION

RADIAL DIRECTION

<u>i</u>	<u>$\Delta r_i(m)$</u>	<u>$r_{i-1/2}(m)$</u>	<u>$r_i(m)$</u>	<u>$r_{i+1/2}(m)$</u>
1	0.160	0.000	0.080	0.160
2	0.411	0.160	0.366	0.571
3	0.857	0.571	1.000	1.429
4	2.143	1.429	2.500	3.571
5	5.357	3.571	6.250	8.929
6	13.393	8.929	15.625	22.321
7	33.482	22.321	39.062	55.804
8	83.705	55.804	97.656	139.509
9	209.263	139.509	244.141	348.772
10	523.158	348.772	610.352	871.931
11	1307.896	871.931	1525.879	2179.827

VERTICAL DIRECTION

<u>j</u>	<u>$\Delta z_j(m)$</u>	<u>$z_{j-1/2}(m)$</u>	<u>$z_j(m)$</u>	<u>$z_{j+1/2}(m)$</u>
1	0.1	0	0.05	0.1
2	0.1	0.1	10.15	0.2
3	0.1	0.2	0.25	0.3
4	0.1	0.3	0.35	0.4
5	0.1	0.4	0.45	0.5
6	0.1	0.5	0.55	0.6
7	0.1	0.6	0.65	0.7
8	0.1	0.7	0.75	0.8
9	0.1	0.8	0.85	0.9
10	0.1	0.9	0.95	1.0
11	0.1	1.0	1.05	1.1

for $3 \leq i \leq 11$. This yields a node ($i = 4$) located at $r = 2.5$ meters and provides a total outer radius of 2.18 km which is sufficient that no signal will reach the outer boundary by 10^4 seconds. Thus, results will be the same as if the grid were infinite in radial extent.

The time-discretization employed for both cases (A and B) is shown in Table 3-2. For the first several cycles, the time step is quite small. At cycle 13, the time step reaches one second. Thereafter, it periodically doubles. This scheme is compared to that which would result if an initial time-step of 1 second were used, and thereafter $\Delta t^{n+1} = 1.05 \Delta t^n$, in Figure 3.2.

COMPUTED RESULTS

The results for Problem 3A (no liquid) are listed in Table 3-3; those for Problem 3B (two-phase) are shown in Table 3-4. These results are shown in graphical form in Figures 3.3 and 3.4.

DISCUSSION

For $t < 10$ sec or so, the pressure histories are the same for Cases A and B. After $t = 100$ sec, pressures are higher in Case B than in Case A. This results from the pressure support arising from the boiling of liquid water to steam in Case B.

TABLE 3-2
PROBLEM 3 TIME DISCRETIZATION

<u>Cycle, n</u>	<u>$\Delta t^{n-1/2}(\text{sec})$</u>	<u>$t^n(\text{sec})$</u>
1	1/2048	1/2048
2	1/2048	1/1024
3	1/1024	1/512
4	1/512	1/256
5	1/256	1/128
6	1/128	1/64
7	1/64	1/32
8	1/32	1/16
9	1/16	1/8
10	1/8	1/4
11	1/4	1/2
12	1/2	1
13	1	2
.	.	.
.	.	.
.	.	.
22	1	11
23	2	13
.	.	.
.	.	.
.	.	.
44	2	55
45	4	59
.	.	.
.	.	.
.	.	.
66	4	143
67	8	151
.	.	.
.	.	.
.	.	.
88	8	319
89	16	335
.	.	.
.	.	.
.	.	.
110	16	671
111	32	703
.	.	.
.	.	.
.	.	.
132	32	1375
133	64	1439
.	.	.
.	.	.
.	.	.

TABLE 3-2
PROBLEM 3 TIME DISCRETIZATION (Continued)

<u>Cycle, n</u>	<u>$\Delta t^{n-1/2}$(sec)</u>	<u>t^n(sec)</u>
154	64	2783
155	128	2911
.	.	.
.	.	.
.	.	.
176	128	5599
177	256	5855
.	.	.
.	.	.
.	.	.
198	256	11231

TABLE 3-3
PROBLEM 3A PRESSURE HISTORIES

<u>Cycle</u>	<u>Time (sec)</u>	<u>Well Pressure (Bars)</u>	<u>Block Pressure (Bars)</u>
12	1	27.569	30.500
13	2	26.568	30.500
14	3	25.933	30.500
15	4	25.487	30.500
16	5	25.152	30.500
18	7	24.674	30.500
20	9	24.340	30.500
22	11	24.085	30.500
24	15	23.731	30.500
26	19	23.418	30.500
30	27	22.965	30.500
36	39	22.538	30.499
44	55	22.178	30.497
48	71	21.928	30.494
52	87	21.728	30.488
58	111	21.489	30.476
66	143	21.242	30.453
70	175	21.050	30.422
74	207	20.888	30.386
80	255	20.689	30.322
88	319	20.477	30.228
92	383	20.312	30.127
96	447	20.169	30.027
102	543	19.990	29.880
110	671	19.797	29.696
114	799	19.646	25.529

TABLE 3-3
PROBLEM 3A PRESSURE HISTORIES (Continued)

<u>Cycle</u>	<u>Time (sec)</u>	<u>Well Pressure (Bars)</u>	<u>Block Pressure (Bars)</u>
118	927	19.511	29.377
124	1119	19.341	29.175
132	1375	19.156	28.944
136	1631	19.015	28.748
140	1887	18.885	28.578
146	2399	18.674	28.293
154	2783	18.545	28.116
158	3295	18.413	27.915
162	3807	18.287	27.741
168	4575	18.129	27.516
176	5599	17.953	27.260
180	6623	17.799	27.040
184	7647	17.666	26.842
190	9183	17.488	26.577
198	11231	17.276	26.269

"Well Pressure" is the pressure in zone $i = 1, j = 1$.

"Block Pressure" is the pressure in zone $i = 4, j = 4$.

TABLE 3-4
PROBLEM 3B PRESSURE, LIQUID SATURATION HISTORIES

<u>Cycle</u>	<u>Time (sec)</u>	<u>Well Pressure (Bars)</u>	<u>Block Pressure (Bars)</u>	<u>Block Liquid Saturation</u>
12	1	27.569	30.500	0.20000
13	2	26.568	30.500	0.20000
14	3	25.934	30.500	0.20000
15	4	25.487	30.500	0.20000
16	5	25.153	30.500	0.20000
18	7	24.675	30.500	0.20000
20	9	24.342	30.500	0.20000
22	11	24.089	30.500	0.20000
24	15	23.739	30.500	0.20000
26	19	23.431	30.500	0.20000
30	27	22.992	30.500	0.20000
36	39	22.593	30.500	0.20000
44	55	22.279	30.500	0.20000
48	71	22.081	30.500	0.20000
52	87	21.940	30.500	0.20000
58	111	21.792	30.500	0.20000
66	143	21.666	30.500	0.20000
70	175	21.590	30.500	0.20000
74	207	21.539	30.500	0.20000
80	255	21.490	30.500	0.20000
88	319	21.449	30.500	0.20000
92	383	21.422	30.500	0.20000
96	447	21.400	30.500	0.20000
102	543	21.371	30.500	0.20000
110	671	21.337	30.500	0.19999
114	799	21.308	30.500	0.19999
118	927	21.277	30.499	0.19998

TABLE 3-4
PROBLEM 3B PRESSURE, LIQUID SATURATION HISTORIES
(Continued)

<u>Cycle</u>	<u>Time (sec)</u>	<u>Well Pressure (Bars)</u>	<u>Block Pressure (Bars)</u>	<u>Block Liquid Saturation</u>
124	1119	21.233	30.499	0.19997
132	1375	21.177	30.498	0.19994
136	1631	21.129	30.497	0.19990
140	1887	21.078	30.495	0.19985
146	2399	20.985	30.490	0.19971
154	2783	20.921	30.486	0.19957
158	3295	20.850	30.478	0.19934
162	3807	20.777	30.469	0.19906
168	4575	20.678	30.452	0.19855
176	5595	20.563	30.424	0.19771
180	6623	20.474	30.392	0.19673
184	7647	20.384	30.356	0.19562
190	9183	20.268	30.296	0.19381
198	11231	20.137	30.210	0.19122

"Well Pressure" is the pressure in zone $i = 1, j = 1$.

"Block Pressure" and "Block Liquid Saturation" are conditions in zone $i = 4, j = 4$.

Figure 3.2. Time-Discretization for Problem 3.

Figure 3.3. Problem 3A, 3B: Pressure Histories in Well and in Block ($R=2.5$ m, 0.25 m Above Upper Fracture Surface)

Figure 3.4. Problem 3B: Conditions at $R = 2.5$ m, 0.25 m Above Upper Fracture Surface

D. PROBLEM NUMBER 4: "EXPANDING TWO-PHASE SYSTEM WITH DRAINAGE"

PROBLEM DESCRIPTION

Consider a geothermal reservoir wherein all properties are functions of depth only. The problem is to be treated as one-dimensional (vertical). Two types of rock are present in the system. Layer "A" extends from the surface at zero depth (D) to D = 1 km. Layer "B" lies from D = 1 km to D = 2 km (the bottom of the region of interest). In both layers, the relative permeability functions are the same as those used in Problem 2. The remaining rock properties are:

	<u>Layer "A"</u>	<u>Layer "B"</u>
Grain Density	2500 kg/m ³	2500 kg/m ³
Porosity	0.15	0.25
Permeability	5x10 ⁻¹⁵ m ²	100x10 ⁻¹⁵ m ²
Heat Capacity	1 j/g-°C	1 j/g-°C
Rock Thermal Conductivity	1 w/m-°C	1 w/m-°C

At $t = 0$, the temperature distribution is linear with depth from $D = 0$ (10°C) to $D = 1$ km (290°C), and is also linear from $D = 1$ km (290°C) to $D = 2$ km (310°C). Pressure at the surface ($D = 0$) is one atmosphere (1.013 bars) and is maintained at that value throughout. The initial pressure distribution for $D > 0$ is hydrostatic. This yields single-phase (liquid) conditions throughout the column at $t = 0$.

Starting at $t = 0$, fluid production begins at the bottom of the column ($D = 2$ km) at 100 kg/sec - km². The time-scale of interest is $0 < t \leq 40$ years. The problem is to be uniformly zoned (20 zones, each 100 m thick). Desired output information is (1) the discharge enthalpy vs. time, (2) the surface recharge rate and cumulative recharge vs. time, (3) the histories of pressure, temperature and liquid saturation at $D = 0.55$ km (zone 6), $D = 1.05$ km (zone 11), $D = 1.55$ km (zone 16), and $D = 1.95$ km (zone 20).

NUMERICAL REPRESENTATION

The CHARGR code with the water/steam constitutive package was employed; spatial zoning was as described above/ As shown in Table 4-1, after the first few cycles a time-step of 0.2 years (6.3072×10^6 sec) was used. The calculation was carried out to $t = 48.8$ years (250 cycles).

COMPUTED RESULTS

Table 4-2 tabulates the surface recharge rate and cumulative surface recharge as functions of time. These data are plotted in Figures 4.1 and 4.2. Note that, for most of the history, the recharge rate lies in the range 25-33 kg/sec-km². This is much less than the production rate (100 kg/sec-km²).

The discharge enthalpy history is tabulated in Table 4-3 and plotted in Figure 4.3. In Tables 4-4, 4-5, 4-6 and 4-7 the histories of pressure, temperature, and liquid saturation for $D = 0.55$ km (zone 6), $D = 1.05$ km (zone 11), $D = 1.55$ km (zone 16) and $D = 1.95$ km (zone 20) are tabulated. In Figures 4.4, 4.5 and 4.6 these data are plotted vs. time.

DISCUSSION

As the recharge data shows, recharge rates are typically about 25 percent - 33 percent of discharge rates: at $t = 48.8$ years, 1.539×10^{11} kg/km² have been withdrawn, whereas recharge has only replaced 0.438×10^{11} kg/km² by that same time. Until about 38 years, the recharge slowly declines with time; during the same period the pressure at $D = 1.05$ km remains essentially constant. At early times, the fluid near the Layer A/Layer B interface becomes two-phase, stabilizing pressures there. Thus, the pressure gradient driving recharge remains essentially fixed during this period, whereas cold-water surface recharge causes the average fluid viscosity in the upper layer to

TABLE 4-1
PROBLEM 1. TIME - DISCRETIZATION

<u>Cycle</u>	<u>Δt^n ^{1/2} (yr)</u>	<u>t^n (yr)</u>
1	0.003125	0.003125
2	0.003125	0.00625
3	0.00625	0.0125
4	0.0125	0.025
5	0.025	0.05
6	0.05	0.1
7	0.1	0.2
8	0.2	0.4
9	0.2	0.6
10	0.2	0.8
.	.	.
.	.	.
.	.	.
249	0.2	48.6
250	0.2	48.8

TABLE 4-2
PROBLEM 4. RECHARGE HISTORY

<u>Cycle</u>	<u>t (year)</u>	Surface Recharge	Cumulative Recharge
		<u>Rate (kg/sec-km²)</u>	<u>(10¹⁰ kg/km²)</u>
0	0	0	0
5	0.05	2.6	0.0002
7	0.2	15.8	0.0063
10	0.8	25.6	0.0535
15	1.8	27.3	0.1373
20	2.8	30.9	0.2296
25	3.8	32.8	0.3327
30	4.8	31.8	0.4344
40	6.8	30.7	0.6294
50	8.8	30.4	0.8237
60	10.8	30.4	1.0140
70	12.8	29.9	1.2047
80	14.8	29.8	1.3924
90	16.8	29.3	1.5786
100	18.8	29.0	1.7624
110	20.8	28.7	1.9439
120	22.8	28.1	2.1230
130	24.8	27.9	2.2998
140	26.8	27.3	2.7434
150	28.8	26.9	2.6446
160	30.8	26.4	2.8122
170	32.8	25.8	2.9771
180	34.8	25.1	3.1374
190	36.8	24.8	3.2946
200	38.8	25.2	3.4517
210	40.8	27.5	3.6178
220	42.8	28.1	3.7930
230	44.8	29.6	3.9748
240	46.8	32.1	4.1696
250	48.8	35.2	4.3828

TABLE 4-3
PROBLEM 4. DISCHARGE ENTHALPY HISTORY

<u>Cycle</u>	<u>Time (yr)</u>	<u>Discharge Enthalpy (kj/kg)</u>
0	0	1389
10	0.8	1389
20	2.8	1388
30	4.8	1387
40	6.8	1386
50	8.8	1384
60	10.8	1383
70	12.8	1382
80	14.8	1381
90	16.8	1380
100	18.8	1378
110	20.8	1377
120	22.8	1376
130	24.8	1375
140	26.8	1373
150	28.8	1369
160	30.8	1359
170	32.8	1339
180	34.8	1316
185	35.8	1313
190	36.8	1376
195	37.8	1650
200	38.8	2067
205	39.8	2356
210	40.8	2524
215	41.8	2624
220	42.8	2688
225	43.8	2730
230	44.8	2756
240	46.8	2782
250	48.8	2792

TABLE 4-4

PROBLEM 4. ZONE 6 HISTORY (D = 0.55 km) (LIQUID SATURATION = 1)

<u>Cycle</u>	<u>Time (yr)</u>	<u>P (Bars)</u>	<u>T (°C)</u>
0	0	52.9	164.0
5	0.05	50.9	164.0
7	0.2	43.1	163.9
10	0.8	37.9	163.7
15	1.8	36.8	163.4
20	2.8	34.6	162.9
30	4.8	34.0	162.1
40	6.8	34.6	161.2
50	8.8	34.6	160.4
60	10.8	34.5	159.5
70	12.8	34.8	158.7
80	14.8	34.7	157.9
90	16.8	34.9	157.1
100	18.8	35.0	156.3
110	20.8	35.1	155.5
120	22.8	35.4	154.8
130	24.8	35.4	154.0
140	26.8	35.7	153.2
150	28.8	35.9	152.5
160	30.8	36.1	151.8
170	32.8	36.4	151.1
180	34.8	36.8	150.4
190	36.8	37.0	149.7
200	38.8	36.6	149.0
210	40.8	35.0	148.3
220	42.8	34.5	147.6
230	44.8	33.4	146.8
240	46.8	31.6	145.9
250	48.8	29.3	145.0

TABLE 4-5
PROBLEM 4. ZONE 11 HISTORY (D = 1.05 km)

<u>Cycle</u>	<u>Time (yr)</u>	<u>P (bars)</u>	<u>T (°C)</u>	<u>S_L</u>
0	0	93.2	291.0	1.000
5	0.05	89.7	290.9	1.000
7	0.2	80.2	290.8	1.000
10	0.8	74.7	290.2	0.928
15	1.8	73.4	289.0	0.789
20	2.8	70.5	286.3	0.670
30	4.8	67.7	283.6	0.660
40	6.8	65.7	281.6	0.568
50	8.8	66.0	281.9	0.519
60	10.8	65.2	281.1	0.494
70	12.8	65.8	281.7	0.494
80	14.8	65.5	281.4	0.492
90	16.8	65.8	281.7	0.492
100	18.8	66.0	281.8	0.492
110	20.8	66.0	281.9	0.491
120	22.8	66.3	282.2	0.491
130	24.8	66.5	282.3	0.490
140	26.8	66.7	282.6	0.489
150	28.8	67.0	282.9	0.490
160	30.8	67.2	283.1	0.488
170	32.8	67.6	283.5	0.489
180	34.8	68.0	283.9	0.488
190	36.8	68.1	284.0	0.486
200	38.8	67.3	283.2	0.481
210	40.8	64.9	280.7	0.472
220	42.8	61.7	277.4	0.490
230	44.8	58.2	273.7	0.481
240	46.8	54.8	269.7	0.466
250	48.8	51.4	265.7	0.458

TABLE 4-6
PROBLEM 4. ZONE 16 HISTORY (D = 1.55 km)

<u>Cycle</u>	<u>Time (yr)</u>	<u>P (bars)</u>	<u>T (°C)</u>	<u>S_L</u>
0	0	128.7	301.0	1.000
5	0.05	125.0	300.9	1.000
7	0.2	115.3	300.8	1.000
10	0.8	109.5	300.7	1.000
15	1.8	107.9	300.5	1.000
20	2.8	104.6	300.4	1.000
30	4.8	101.1	300.1	1.000
40	6.8	98.3	299.9	1.000
50	8.8	94.5	299.6	1.000
60	10.8	92.1	299.3	1.000
70	12.8	88.4	299.0	1.000
80	14.8	85.4	298.6	1.000
85	15.8	83.6	298.1	0.970
90	16.8	82.4	297.1	0.876
100	18.8	78.9	294.0	0.855
110	20.8	76.3	291.7	0.838
120	22.8	72.9	288.6	0.832
130	24.8	70.4	286.3	0.721
140	26.8	69.1	285.0	0.588
150	28.8	69.7	285.6	0.517
160	30.8	69.7	285.5	0.497
170	32.8	70.3	286.1	0.497
180	34.8	70.6	286.4	0.496
190	36.8	70.4	286.2	0.491
200	38.8	68.8	284.7	0.476
210	40.8	65.6	281.4	0.449
220	42.8	61.8	277.6	0.419
230	44.8	58.1	273.5	0.389
240	46.8	54.4	269.3	0.360
250	48.8	50.9	265.0	0.330

TABLE 4-7.
PROBLEM 4. ZONE 20 HISTORY (D = 1.95 km)

<u>Cycle</u>	<u>Time (yr)</u>	<u>P (bars)</u>	<u>T (°C)</u>	<u>S_L</u>
0	0	156.6	309.0	1.000
5	0.05	152.5	308.9	1.000
7	0.2	142.7	308.8	1.000
10	0.8	136.7	308.6	1.000
15	1.8	135.2	308.5	1.000
20	2.8	131.8	308.3	1.000
30	4.8	128.4	308.0	1.000
40	6.8	125.6	307.8	1.000
50	8.8	121.8	307.5	1.000
60	10.8	119.4	307.2	1.000
70	12.8	115.7	306.9	1.000
80	14.8	112.7	306.7	1.000
90	16.8	109.6	306.4	1.000
100	18.8	106.1	306.1	1.000
110	20.8	103.5	305.9	1.000
120	22.8	100.4	305.6	1.000
130	24.8	97.1	305.3	1.000
140	26.8	94.5	304.9	1.000
145	27.8	92.7	304.7	1.000
150	28.8	91.0	304.1	0.957
155	29.8	89.8	303.2	0.880
160	30.8	88.2	301.9	0.865
170	32.8	83.5	298.0	0.848
180	34.8	78.0	293.3	0.837
190	36.8	72.5	288.2	0.739
200	38.8	68.9	284.8	0.603
210	40.8	64.6	280.5	0.532
220	42.8	60.6	276.3	0.485
230	44.8	56.8	272.0	0.444
240	46.8	53.2	267.8	0.406
250	48.8	49.6	263.5	0.371

Figure 4.1. Problem 4: Surface Recharge Rate.

Figure 4.2. Problem 4: Cumulative Recharge

Figure 4.3. Problem 4: Discharge Enthalpy History

Figure 4.4. Problem 4: Pressure Histories.

Figure 4.5. Problem 4: Temperature Histories.

Figure 4.6. Problem 4: Liquid Saturation Histories

increase with time. This explains the declining recharge rate. Figures 4.7 and 4.8 illustrate profiles with depth of pressure and liquid saturation with time. As production goes on (exceeding recharge), the spatial extent of the two-phase region increases and deepens with time. Eventually, at about 38 years, substantial quantities of steam begin to be produced from the (bottom) discharge surface. This results in a large increase in the discharge enthalpy. Also, the presence of a significant steam fraction in the discharge stream forces the pressures within the two-phase region (which is by now at essentially constant pressure) to decline rapidly. This, in turn, causes the recharge rate to begin to increase after about 38 years.

Figure 4.7. Problem 4: Pressure Profiles

Figure 4.8. Problem 4: Liquid Saturation Profiles

E. PROBLEM NUMBER 5: "FLOW IN A 2-D AREAL RESERVOIR"

PROBLEM DESCRIPTION

Consider a horizontal homogeneous aquifer of uniform thickness. The region under consideration extends over

$$0 \leq x \leq 300 \text{ meters}$$

$$0 \leq y \leq 200 \text{ meters}$$

The properties of the reservoir rock are as follows:

Grain density	2500 kg/m ³
Porosity	0.35
Permeability	$2.5 \times 10^{-14} \text{ m}^2$
Heat capacity	1 J/g-°C
Thermal Conductivity	1 W/m-°C
Residual liquid saturation	0.3
Residual steam saturation	0.1

As in Problem 2, Corey relative permeabilities are used, but note that the residual steam saturation is greater (0.1 vs. 0.05). The relative permeability functions for Problem 5 may be written:

$$\left. \begin{array}{l} R_L = 1 \\ R_S = 0 \end{array} \right\} \text{ for } 0 \leq S \leq 0.1$$

$$\left. \begin{array}{l} R_L = Z^4 \\ R_S = (1-Z^2) (1-Z)^2 \end{array} \right\} \text{ for } 0.1 < S < 0.7$$

$$\left. \begin{array}{l} R_L = 0 \\ R_S = 1 \end{array} \right\} \text{ for } 0.7 \leq S \leq 1$$

where S is the steam saturation ($= 1$ minus liquid saturation), and where

$$Z = (0.7 - S)/0.6$$

Initially, the fluid pressure is uniform everywhere in the system, and equal to 35.98 bars. Note that this pressure is equal to the saturation pressure at 240°C plus 2.5 bars. The initial temperature distribution is non-uniform, however; it is given by:

$$T = 240^\circ\text{C} \quad \text{for} \quad r \leq 100 \text{ meters}$$

$$T = \left[240 - 160 \left(\frac{r-100\text{m}}{200\text{m}} \right)^2 + 80 \left(\frac{r-100\text{m}}{200\text{m}} \right)^4 \right] ^\circ\text{C}$$

$$\text{for } 100 \text{ meters} < r < 300 \text{ meters}$$

$$T = 160^\circ\text{C} \quad \text{for} \quad r \geq 300 \text{ meters}$$

where r is the distance from the origin:

$$r = \sqrt{x^2 + y^2}$$

The boundaries at $x = 0$, at $y = 0$, and at $y = 200$ meters are impermeable and insulated. The boundary at $x = 300$ meters retains the initial pressure (35.98 bars) and temperature (160°C) throughout, so fluid recharge may occur through this boundary.

Problem 5 actually consists of two similar problems. In problem 5A, a fully-penetrating production well, located at $x = 62.5$ m, $y = 62.5$ m begin producing fluid at $t = 0$ at a constant rate of 0.05 kilograms/second-meter of thickness. The well radius is 0.15 meter and no skin effect is present. The production well is also present in problem 2B, but in addition an injection well is also located at $x = 162.5$ m, $y = 137.5$ m. The injection well is also of radius 0.15 m, has no skin effect, and is fully penetrating. The

injection well is shut-in until $t = 1 \text{ year} = 3.1536 \times 10^7 \text{ sec.}$ Then, it begins injecting water at $T = 80^\circ\text{C}$ at a rate of 0.03 kilograms/second-meter of thickness. In both cases, the time-domain of interest is $0 \leq t \leq 10 \text{ years.}$

The desired output consists of the pressure-histories in the grid blocks containing the wells, the temperature history in the grid block containing the production well, the discharge enthalpy history for the production well, and the variation with time of the total mass of steam in the system per meter of thickness. All of these histories are desired for both problems (5A, 5B). In addition, the histories of sandface pressure and sandface steam saturation for the production well are desired for both cases, and the sandface pressure history for the injection well is desired for problem 5B.

NUMERICAL REPRESENTATION

Spatial zoning is specified as follows: in the x-direction, 12 zones of size $\Delta x = 25 \text{ m;}$ in the y-direction, 8 zones of size $\Delta y = 25 \text{ m.}$ Thus, the production well is located in the center of zone $i = 3, j = 3$ and the injection well is in zone $i = 7, j = 6.$ The total grid size is $8 \times 12 = 96 \text{ zones.}$ Problems 5A and 5B were run using the CHARGR code, with the time-discretization shown in Table 5-1. In addition, for comparison purposes, Problem 5A was also run using the older MUSHRM code. As discussed earlier, MUSHRM, being semi-explicit, is constrained in the time-step size which may be used. The MUSHRM time-step fluctuated somewhat during the run, but the average value was about 0.032 years. As Table 5-1 shows, the CHARGR time step chosen was slightly larger (0.05 years).

COMPUTED RESULTS:

Table 5-2 lists the desired histories for Problem 5A; Table 5-3 contains the results for Problem 5B. Table 5-4 lists results for the MUSHRM calculation of Problem 5A. Figure 5.1 shows the history

TABLE 5-1
TIME-DISCRETIZATION FOR CHARGE PROBLEM 5 CALCULATIONS

<u>Cycle, n</u>	<u>$\Delta t^{n-1/2}$ (YR)</u>	<u>t^n (YR)</u>
PROBLEM 5A		
1	0.000390625	0.000390625
2	0.000390625	0.00078125
3	0.00078125	0.0015625
4	0.0015625	0.003125
5	0.003125	0.00625
6	0.00625	0.0125
7	0.0125	0.025
8	0.025	0.05
9	0.05	0.10
10	0.05	0.15
.	.	.
.	.	.
.	.	.
220	0.05	10.65
PROBLEM 5B		
1	0.000390625	0.000390625
2	0.000390625	0.00078125
3	0.00078125	0.0015625
4	0.0015625	0.003125
5	0.003125	0.00625
6	0.00625	0.0125
7	0.0125	0.025
8	0.025	0.05
9	0.05	0.10

TABLE 5-1
 PROBLEM 5 CALCULATIONS TIME-DISCRETIZATION FOR CHARGR
 (Continued)

<u>Cycle, n</u>	<u>$\Delta t^{n-1/2}$ (YR)</u>	<u>t^n (YR)</u>
PROBLEM 5B (continued)		
10	0.05	0.15
.	.	.
.	.	.
.	.	.
27	0.05	1.00
28	0.000390625	1.000390625
29	0.000390625	1.00078125
30	0.00078125	1.0015625
31	0.0015625	1.003125
32	0.003125	1.00625
33	0.00625	1.0125
34	0.0125	1.025
35	0.025	1.05
36	0.05	1.10
.	.	.
.	.	.
.	.	.
220	0.05	10.30

TABLE 5-2
PROBLEM 5A RESULTS COMPUTED BY CHARGR CODE

		Zone i=3, j=3		Production Well			Zone i=7, j=6	
<u>Cycle</u>	<u>Time (YR)</u>	<u>P_{3,3}</u> (Bars)	<u>T_{3,3}</u> (°C)	<u>Discharge</u> <u>Enthalpy</u> (J/G)	<u>Sandface</u> <u>Pressure</u> (Bars)	<u>Sandface</u> <u>Steam</u> <u>Saturation</u>	<u>P_{7,6}</u> (Bars)	<u>Steam</u> <u>Mass</u> (10 ⁶ G/m)
0	0	35.98	240.0	1038	35.98	0	35.98	0
10	0.15	32.72	238.7	1043	28.49	0.282	34.42	1.50
15	0.40	32.15	237.7	1036	26.79	0.297	34.22	4.68
20	0.65	31.87	237.3	1035	26.47	0.298	34.17	6.74
25	0.90	31.59	236.8	1033	25.98	0.301	34.02	8.57
30	1.15	31.33	236.3	1032	25.61	0.303	33.90	9.97
35	1.40	31.10	235.9	1030	25.28	0.305	33.80	10.99
40	1.65	30.88	235.5	1028	25.01	0.305	33.68	11.66
45	1.90	30.69	235.1	1025	25.00	0.303	33.58	11.97
50	2.15	30.51	234.8	1020	25.16	0.299	33.48	11.99
55	2.40	30.40	234.6	1014	25.58	0.290	33.42	11.81
60	2.65	30.21	234.3	1011	25.69	0.285	33.34	11.36
65	2.90	30.24	233.4	1006	26.75	0.269	33.26	10.82
70	3.15	30.27	232.0	1000	28.19	0.225	33.27	10.17
75	3.40	30.31	230.7	994	28.75	0	33.27	9.44
80	3.65	30.30	229.6	989	28.76	0	33.23	8.61
90	4.15	30.34	227.4	979	28.75	0	33.21	7.19
100	4.65	30.45	225.1	968	28.86	0	33.30	5.87
110	5.15	30.46	223.0	958	28.87	0	33.30	4.72
120	5.65	30.47	220.9	948	28.87	0	33.30	3.79
140	6.65	30.55	216.5	928	28.93	0	33.37	2.33
160	7.65	30.58	212.0	907	28.93	0	33.41	1.46
180	8.65	30.58	207.6	887	28.91	0	33.43	0.90
200	9.65	30.54	203.4	868	28.85	0	33.43	0.59
220	10.65	30.55	199.4	850	28.83	0	33.47	0.38

TABLE 5-3
PROBLEM 5B RESULTS COMPUTED BY CHARGR CODE

Cycle	Time (YR)	Zone i=3, j=3		Production well			Zone i=7	Injection	Steam Mass (10 ⁶ G/m)
		P _{3,3} (Bars)	T _{3,3} (°C)	Discharge Enthalpy (J/G)	Sandface Pressure (Bars)	Sandface Steam Saturation	j=6 P _{7,6} (Bars)	Well Sandface Pressure (Bars)	
0	0	35.98	240.0	1038	35.98	0	35.98	35.98	0
10	0.15	32.72	238.7	1043	28.49	0.282	34.42	34.42	1.50
15	0.40	32.15	237.7	1036	26.79	0.297	34.22	34.22	4.68
20	0.65	31.87	237.3	1035	26.47	0.298	34.17	34.17	6.74
25	0.90	31.59	236.8	1033	25.98	0.301	34.02	34.02	8.57
35	1.05	31.44	236.5	1029	25.77	0.301	35.28	37.59	9.38
40	1.30	31.73	237.0	1024	27.24	0.282	35.45	37.83	8.41
45	1.55	31.73	236.8	1023	27.26	0.281	35.59	37.99	7.63
50	1.80	31.84	236.0	1019	28.90	0.254	35.71	38.10	7.12
55	2.05	31.92	235.2	1015	29.86	0.222	35.84	38.23	6.67
60	2.30	31.94	234.4	1011	30.40	0	35.93	38.33	6.08
65	2.55	31.93	233.6	1007	30.38	0	35.95	38.36	5.59
70	2.80	31.92	232.7	1003	30.37	0	35.99	38.39	5.18
80	3.30	31.99	230.8	994	30.43	0	36.09	38.50	4.70
90	3.80	31.97	228.7	985	30.41	0	36.14	38.56	4.14
100	4.30	31.95	226.4	974	30.37	0	36.15	38.56	3.85
120	5.30	31.83	220.8	948	30.24	0	36.19	38.60	3.15
140	6.30	31.79	214.5	919	30.16	0	36.26	38.67	2.68
160	7.30	31.71	207.8	888	30.06	0	36.29	38.71	2.31
180	8.30	31.58	200.9	857	29.86	0	36.29	38.70	1.97
200	9.30	31.51	194.2	827	29.76	0	36.33	38.75	1.62
220	10.30	31.38	187.7	798	29.58	0	36.32	38.74	1.20

TABLE 5-4
PROBLEM 5A RESULTS COMPUTED BY MUSHRM CODE

<u>Zone i=3, j=3</u>				<u>Steam Mass</u>
<u>Cycle</u>	<u>Time (YR)</u>	<u>P_{3,3} (Bars)</u>	<u>T_{3,3} (°C)</u>	<u>(10⁶ G/M)</u>
0	0	36.01	240.0	0
10	0.094	32.99	239.1	0.93
20	0.406	32.18	237.7	4.80
30	0.719	31.80	237.1	7.49
40	1.031	31.46	236.5	9.73
50	1.344	31.13	235.9	11.27
60	1.656	30.82	235.3	12.22
80	2.281	30.38	234.5	12.51
100	2.906	29.97	233.7	11.25
120	3.531	30.18	230.4	9.44
140	4.156	30.22	227.6	7.34
160	4.781	30.36	224.7	5.68
180	5.406	30.39	222.0	4.33
200	6.031	30.45	219.4	3.22
220	6.656	30.47	216.6	2.38
240	7.281	30.48	213.8	1.68
260	7.906	30.52	211.0	1.26
280	8.531	30.51	208.2	0.99
300	9.156	30.48	205.5	0.72
330	10.094	30.49	201.6	0.46

Figure 5.1. Problem 5: Time History of Total Steam Mass (CHARGR Calculations).

of total steam mass per meter of thickness present in the system for Problems 5A and 5B as computed by CHARGR. Figure 5.2 illustrates the CHARGR pressure histories in the zones containing the wells for both problems. Figure 5.3 plots the CHARGR-computed temperature history in the production well-block for both cases. Figure 5.4 shows the discharge enthalpy history and Figure 5.5 illustrates the histories of sandface steam saturation and sandface pressure for both cases based on CHARGR results. Figure 5.6 shows the sandface pressure history for the injection well for Problem 5B. In Figures 5.7, 5.8 and 5.9, results are compared between CHARGR and MUSHRM for Problem 5A.

DISCUSSION

Agreement between the CHARGR and MUSHRM results is reasonably good. Figure 5.7 indicates that the maximum steam mass in the system (at about 2 years) is about 4 percent higher in the MUSHRM results; Figure 5.8 shows that the pressure drop (relative to initial pressure - 36 bars) is from 1 percent to 3 percent greater for the production well-blocks in the MUSHRM calculation than that using CHARGR. As Figure 5.9 shows, the production well-block temperature histories are in excellent agreement.

For practical engineering purposes, discrepancies of this magnitude are of little importance. It is worthwhile, nonetheless, to consider the possible source of the deviations, even though they are quite small. Both MUSHRM and CHARGR solve exactly the same conservation equations, using similar methods. Although the time-discretization treatment is different between the two, a sufficiently small time-step was used in both runs that time-discretization effects are unlikely to be responsible for the discrepancies.

There is another major difference between the two runs, however. The water/steam equation-of-state package used with the MUSHRM simulator was written several years ago, and was designed to

Figure 5.2. Problem 5: Well-Block Pressure Histories (CHARGR Calculations)

Figure 5.3. Problem 5: Temperature Histories in Production Well-Block (CHARGEK Calculations).

Figure 5.4. Problem 5: Discharge Enthalpy Histories (CHARGR Calculations)

Figure 5.5. Problem 5: Production Well Sandface Conditions (CHARGR Calculations).

Figure 5.6. Problem 5B: Injection Well Sandface Pressure (CHARGR Calculation).

Figure 5.7. Problem 5A: Total Steam Mass --- Comparison of MUSHRM and CHARGR.

Figure 5.8. Problem 5A: Pressure History in Production Well-Block --- Comparison of MUSHRM and CHARGR.

Figure 5.9. Problem 5A: Temperature History in Production Well-Block --- Comparison of MUSHRM and CHARGR.

be applicable to a very wide range of temperatures and pressures. When the CHARGR code was written, a new constitutive package was developed for water/steam which is more restricted in range (pressures to 500 bars, temperatures to 350°C), but is probably more accurate within that range. Furthermore, with the new CHARGR package, advantage was taken of new data presented in the latest (1977) edition of the ASME Steam Tables, which had been previously unavailable. Therefore, the author believes that the slight disparity between the results from CHARGR and MUSHRM arises primarily from small differences in the representation of water/steam constitutive behavior. In this connection, the experimental values presented in the Steam Tables for viscosity (for example) typically have experimental tolerances in the 1 - 3 percent range. Since the pressure drop required to induce a fixed flowrate is proportional to viscosity, this effect alone could explain the differences between the two calculations. This matter should be kept in mind when comparing results provided by various contractors for the problem set.

F. PROBLEM NUMBER 6: "FLOW IN A 3-D RESERVOIR"

PROBLEM DESCRIPTION

Consider a three-dimensional system whose dimensions are as follows: length (x) - 5 km; width (y) - 4 km; and height (z) - 1.8 km. The system is sub-divided into five vertical layers, as follows:

<u>LAYER</u>	<u>HEIGHT RANGE</u>	<u>ROCK TYPE</u>
5	1500-1800 m	A
4	1200-1500 m	B
3	900-1200 m	B
2	600- 900 m	B
1	0- 600 m	A

The rock properties are as follows:

	<u>TYPE "A"</u>	<u>TYPE "B"</u>
Grain Density	2500 kg/m ³	2500 kg/m ³
Porosity	0.20	0.25
Horizontal Permeability	100 x 10 ⁻¹⁵ m ²	200 x 10 ⁻¹⁵ m ²
Vertical Permeability	2 x 10 ⁻¹⁵ m ²	50 x 10 ⁻¹⁵ m ²
Heat Capacity	1 J/g-°C	1 J/g-°C
Thermal Conductivity	1 W/m-°C	1 W/m-°C
Residual Liquid Saturation	0.3	0.3
Residual Steam Saturation	0.1	0.1

Corey relative permeabilities are assumed for both rock types,
so:

$$\left. \begin{array}{l} R_L = 1 \\ R_S = 0 \end{array} \right\} \text{ for } 0 \leq S \leq 0.1$$

$$\left. \begin{array}{l} R_L = Z^4 \\ R_S = (1-Z^2) (1-Z)^2 \end{array} \right\} \text{ where } 0.1 \leq S \leq 0.7$$

where
 $Z = (0.7 - S)/0.6$

$$\left. \begin{array}{l} R_L = 0 \\ R_S = 1 \end{array} \right\} \text{ for } 0.7 \leq S \leq 1$$

where R_L and R_S are relative permeabilities to liquid and steam, and S is steam saturation.

In layers 1 to 4, the initial temperature is uniform and equal to 280°C; in layer 5, the temperature is taken to be 160°C. In layer 4, the initial pressure is equal to the saturation pressure at 280°C (64.2 bars), and the initial value of the steam saturation (S) is 0.1. The other layers are initially single-phase liquid. Initial pressures in layer 5 and in layers 1 to 3 are liquid-hydrostatic with respect to the pressure in layer 4, and boundary pressures are applied at the top ($Z = 1.8$ km; 25.9 bars) and bottom ($Z = 0$; 164.4 bars) of the system which are also liquid-hydrostatic, so that initially no fluid motion occurs. (Note that the steam phase in layer 4 is immobile, since $S = S_{RS} = 0.1$.)

As mentioned above, pressures are maintained at their initial values throughout at the top and bottom faces of the system. The top and bottom boundary fluid temperatures are 100°C and 280°C, respectively. The vertical faces at $x = 0$, at $y = 0$, and at $y = 4$

km are impermeable and insulated. At the vertical face at $x = 5$ km, the initial conditions (pressure, temperature, and steam saturation) are maintained.

Fluid production is all taken uniformly from the region bounded by $0 \leq x \leq 1$ km, $0 \leq y \leq 0.8$ km, and $0.6 \text{ km} \leq z \leq 0.9$ km (i.e., layer 2). The fluid production rate increases with time, as follows:

<u>TIME</u>	<u>PRODUCTION RATE</u>
$t \leq 0$	0
$0 < t \leq 2$ years	1000 kg/sec
2 years $< t \leq 4$ years	2500 kg/sec
4 years $< t \leq 6$ years	4000 kg/sec
$t > 6$ years	6000 kg/sec

The time-scale of interest is $0 \leq t \leq 10$ years. As will be seen, however, the high production rate after $t = 6$ years causes the absolute pressure in the production area to pass through zero at $t \approx 6.8$ years. Thus, calculations were terminated at that time.

Zoning is specified as uniform in the x- and y-directions, so that the spatial extent of zone i, j, and k is given by:

<u>i</u>	<u>x_{LO}</u> to <u>x_{HI}</u>	<u>j</u>	<u>y_{LO}</u> to <u>y_{HI}</u>
1	0 to 1 km	1	0 to 0.8 km
2	1 km to 2 km	2	0.8 km to 1.6 km
3	2 km to 3 km	3	1.6 km to 2.4 km
4	3 km to 4 km	4	2.4 km to 3.2 km
5	4 km to 5 km	5	3.2 km to 4.0 km

<u>k</u>	<u>z_{LO}</u>	to	<u>z_{HI}</u>
1	0		0.6 km
2	0.6 km		0.9 km
3	0.9 km		1.2 km
4	1.2 km		1.5 km
5	1.5 km		1.8 km

As can be seen, all fluid production takes place from zone $i = 1$, $j = 1$, $k = 2$.

The output data desired is the discharge enthalpy history, and the histories of pressure, temperature, and steam saturation in the vertical column of zones including the production zone; $i = 1$, $j = 1$, and $1 \leq k \leq 5$.

NUMERICAL REPRESENTATION

The CHARGR code with the water/steam package was used; the problem was zoned as described above. The time-step was generally 0.05 years (1.576×10^6 sec) except for the first few cycles (as before), a short time-interval after each change in flow-rate, and near the end of the run (see Table 6-1). The small time-steps used at the end ($t > 6.4$ years) were automatically selected by the code to prevent $\Delta P/P$ for the production zone from becoming too large in a single cycle. As noted earlier, after six years the pressure drops catastrophically in the production zone; it reaches 0.9 bars at cycle 148 (6.684 years) and 0.1 bars at cycle 156 (6.806 years). The run automatically terminated at cycle 156, since the water/steam equation-of-state cannot treat states with $P < 0.1$ bar.

TABLE 6-1
PROBLEM 6: TIME - DISCRETIZATION

<u>CYCLE, n</u>	$\Delta t^{n-1/2}$ (Year)	t^n (Year)	<u>CYCLE, n</u>	$\Delta t^{n-1/2}$ (Year)	t^n (Year)
1	0.0125	0.0125	128	0.0125	6.025
2	0.0125	0.025	129	0.025	6.05
3	0.025	0.05	130	0.05	6.10
4	0.05	0.10	131	0.05	6.15
5	0.05	0.15	132	0.05	6.20
6	0.05	0.20	.	.	.
.	.	.	136	0.05	6.40
41	0.05	1.95	137	0.0125	6.4125
42	0.05	2.00	138	0.025	6.4375
43	0.0125	2.0125	139	0.05	6.4875
44	0.0125	2.025	140	0.0125	6.50
45	0.025	2.05	141	0.025	6.525
46	0.05	2.10	142	0.05	6.575
47	0.05	2.15	143	0.05	6.625
48	0.05	2.20	144	0.0125	6.6375
.	.	.	145	0.025	6.6625
84	0.05	4.00	146	0.0125	6.675
85	0.0125	4.0125	147	0.00625	6.68125
86	0.0125	4.025	148	0.003125	6.684375
87	0.025	4.05	149	0.00625	6.690625
88	0.05	4.10	150	0.0125	6.703125
89	0.05	4.15	151	0.025	6.728125
90	0.05	4.20	152	0.0125	6.740625
.	.	.	153	0.025	6.765625
125	0.05	5.95	154	0.0125	6.778125
126	0.05	6.00	155	0.025	6.803125
127	0.0125	6.0125	156	0.003125	6.80625

COMPUTED RESULTS

The discharge enthalpy history is tabulated in Table 6-2, and plotted in Figure 6.1. Pressure, steam saturation, and temperature histories are tabulated in Tables 6-3, 6-4, and 6-5 for the five zones of interest; Figures 6.2, 6.3, and 6.4 are plots of these quantities versus time.

DISCUSSION

As Figure 6.3 shows, two-phase behaviour is confined to layer 4 until about 3.8 years; a definite discontinuity in the slope of the saturation/time curve for this zone at $t = 2$ years is a consequence of the change in production rate at that time. After $t = 4$ years, two-phase behaviour begins in the production zone; this results in a departure of the discharge enthalpy from its previously-constant value. The pressure-time histories (Figure 6.2) show pressure-jumps for layers 1, 2, and 3 at $t = 0$ and $t = 2$ years, due to the flow-rate changes at those times, but no such jumps for layers 4 and 5. This occurs because layer 4 is two-phase with very high compressibility and, hence, very low pressure diffusivity; layer 5, while single-phase, is "shielded" from the pressure disturbance by layer 4. The flow-rate change at $t = 4$ years causes pressure jumps for layers 1 and 2, but not 3; it has become two-phase in the meantime. Finally, the last flow-rate change at $t = 6$ years causes the pressure to drop precipitously in the production zone, as noted earlier, culminating in a decline to zero pressure prior to 7 years. Figure 6.5 shows pressure profiles with depth through the five zones of interest at $t = 0$, $t = 0.9$ years (during the first production interval), $t = 3.0$ years (during the second), $t = 5.1$ years (during the third), and $t = 6.5$ years (during the fourth). This plot shows that even the third pumping rate (4000 kg/sec; 4 years to 6 years) caused a "pressure inversion", wherein pressures both above and below the production area exceed that of the production zone itself.

TABLE 6-2
PROBLEM 6: DISCHARGE ENTHALPY HISTORY

CYCLE	TIME (Years)	DISCHARGE ENTHALPY	CYCLE	TIME (Years)	DISCHARGE ENTHALPY
		(J/G)			(J/G)
0	0	1235	110	5.2	1405
8	0.3	1235	112	5.3	1308
16	0.7	1235	114	5.4	1251
24	1.1	1235	116	5.5	1232
32	1.5	1235	118	5.6	1231
40	1.9	1235	120	5.7	1244
48	2.2	1235	122	5.8	1264
56	2.6	1235	124	5.9	1284
64	3.0	1235	126	6.0	1301
72	3.4	1235	128	6.025	1480
80	3.8	1235	130	6.1	2079
88	4.1	1235	132	6.2	2256
92	4.3	1234	134	6.3	1964
96	4.5	1232	136	6.4	1701
98	4.6	1236	140	6.5	1816
100	4.7	1264	144	6.638	2138
102	4.8	1360	148	6.684	2245
104	4.9	1510	152	6.741	2340
106	5.0	1577	156	6.806	2440
108	5.1	1524			

TABLE 6-3

PROBLEM 6: PRESSURE HISTORIES

<u>CYCLE</u>	<u>Time</u> <u>(Years)</u>	<u>P_{1,1,1}</u> <u>(Bars)</u>	<u>P_{1,1,2}</u> <u>(Bars)</u>	<u>P_{1,1,3}</u> <u>(Bars)</u>	<u>P_{1,1,4}</u> <u>(Bars)</u>	<u>P_{1,1,5}</u> <u>(Bars)</u>
0	0	142.0	108.5	86.3	64.2	39.7
4	0.1	141.2	101.0	83.2	64.1	39.6
8	0.3	140.9	100.3	82.5	63.9	39.6
12	0.5	140.9	100.1	82.1	63.8	39.5
16	0.7	140.9	99.8	81.6	63.6	39.5
20	0.9	140.8	99.5	81.3	63.5	39.4
24	1.1	140.7	99.2	80.9	63.4	39.4
28	1.3	140.7	99.0	80.6	63.2	39.3
32	1.5	140.7	98.8	80.4	63.1	39.3
36	1.7	140.6	98.6	80.1	63.0	39.3
40	1.9	140.6	98.4	79.9	62.9	39.2
44	2.025	139.6	84.8	71.8	62.8	39.2
48	2.2	138.3	82.3	69.2	62.6	39.2
52	2.4	138.0	81.7	68.5	62.3	39.2
56	2.6	138.0	81.0	67.6	62.1	39.1
60	2.8	137.9	80.2	66.8	61.9	39.1
64	3.0	137.6	79.5	66.0	61.7	39.0
68	3.2	137.4	78.9	65.3	61.4	39.0
72	3.4	137.3	78.4	64.7	61.2	38.9
76	3.6	137.1	77.8	64.2	61.0	38.9
80	3.8	136.9	77.5	64.0	60.7	38.8
84	4.0	136.9	77.4	64.0	60.5	38.7
88	4.1	135.9	68.5	63.6	60.4	38.7
92	4.3	134.9	63.7	63.1	60.2	38.6
96	4.5	134.6	63.3	62.9	59.9	38.5
100	4.7	134.5	62.3	62.8	59.7	38.4

TABLE 6-3

PROBLEM 6: PRESSURE HISTORIES (Continued)

<u>CYCLE</u>	<u>Time</u> <u>(Years)</u>	<u>P_{1,1,1}</u> <u>(Bars)</u>	<u>P_{1,1,2}</u> <u>(Bars)</u>	<u>P_{1,1,3}</u> <u>(Bars)</u>	<u>P_{1,1,4}</u> <u>(Bars)</u>	<u>P_{1,1,5}</u> <u>(Bars)</u>
104	4.9	133.8	55.7	62.6	59.5	38.3
108	5.1	132.1	45.2	61.9	59.3	38.2
112	5.3	131.5	41.7	60.9	59.1	38.1
116	5.5	131.4	42.2	59.9	58.9	38.0
120	5.7	131.4	42.9	59.0	58.6	37.9
124	5.9	131.2	42.4	58.3	58.2	37.8
128	6.025	130.8	40.5	57.8	58.0	37.6
132	6.2	128.4	15.7	56.4	57.6	37.5
136	6.4	126.7	6.6	53.6	57.0	37.3
140	6.5	126.3	4.3	52.1	56.5	37.2
144	6.638	126.3	1.6	50.0	55.9	36.8
148	6.684	126.3	0.9	49.3	55.6	36.9
152	6.741	126.2	0.4	48.4	55.3	36.8
156	6.806	126.0	0.1	47.5	54.9	36.7

TABLE 6-4
PROBLEM 6: STEAM SATURATION HISTORIES

<u>Cycle</u>	<u>Time (Years)</u>	<u>$S_{1,1,1}$</u>	<u>$S_{1,1,2}$</u>	<u>$S_{1,1,3}$</u>	<u>$S_{1,1,4}$</u>	<u>$S_{1,1,5}$</u>
0	0	0	0	0	0.100	0
4	0.1	0	0	0	0.120	0
8	0.3	0	0	0	0.157	0
12	0.5	0	0	0	0.189	0
16	0.7	0	0	0	0.215	0
20	0.9	0	0	0	0.237	0
24	1.1	0	0	0	0.256	0
28	1.3	0	0	0	0.273	0
32	1.5	0	0	0	0.287	0
36	1.7	0	0	0	0.300	0
40	1.9	0	0	0	0.312	0
44	2.025	0	0	0	0.320	0
48	2.2	0	0	0	0.344	0
52	2.4	0	0	0	0.367	0
56	2.6	0	0	0	0.386	0
60	2.8	0	0	0	0.401	0
64	3.0	0	0	0	0.414	0
68	3.2	0	0	0	0.425	0
72	3.4	0	0	0	0.434	0
76	3.6	0	0	0	0.443	0
80	3.8	0	0	0.003	0.450	0
84	4.0	0	0	0.011	0.456	0
88	4.1	0	0	0.091	0.459	0
92	4.3	0	0.030	0.202	0.464	0
96	4.5	0	0.133	0.245	0.468	0
100	4.7	0	0.260	0.267	0.472	0
104	4.9	0	0.361	0.295	0.475	0
108	5.1	0	0.379	0.342	0.478	0

TABLE 6-4
PROBLEM 6: STEAM SATURATION HISTORIES (Continued)

<u>Cycle</u>	<u>Time (Years)</u>	<u>S_{1,1,1}</u>	<u>S_{1,1,2}</u>	<u>S_{1,1,3}</u>	<u>S_{1,1,4}</u>	<u>S_{1,1,5}</u>
112	5.3	0	0.345	0.372	0.481	0
116	5.5	0	0.321	0.386	0.484	0
120	5.7	0	0.323	0.390	0.487	0
124	5.9	0	0.337	0.389	0.490	0
128	6.025	0	0.379	0.389	0.492	0
132	6.2	0	0.502	0.406	0.495	0
136	6.4	0	0.485	0.439	0.500	0
140	6.5	0	0.506	0.455	0.504	0
144	6.638	0	0.556	0.473	0.509	0
148	6.684	0	0.574	0.479	0.510	0
152	6.741	0	0.598	0.485	0.513	0
156	6.806	0	0.626	0.492	0.515	0

TABLE 6-5
PROBLEM 6: TEMPERATURE HISTORIES

<u>Cycle</u>	<u>Time (Years)</u>	<u>T_{1,1,1} (°C)</u>	<u>T_{1,1,2} (°C)</u>	<u>T_{1,1,3} (°C)</u>	<u>T_{1,1,4} (°C)</u>	<u>T_{1,1,5} (°C)</u>
0	0	280.0	280.0	280.0	280.0	160.0
4	0.1	280.0	279.9	280.0	279.9	160.0
8	0.3	280.0	279.9	280.0	279.7	160.0
12	0.5	280.0	279.9	280.0	279.6	160.0
16	0.7	280.0	279.9	280.0	279.4	160.0
20	0.9	280.0	279.9	280.0	279.3	160.0
24	1.1	280.0	279.9	280.0	279.2	160.1
28	1.3	280.0	279.9	280.0	279.0	160.1
32	1.5	280.0	280.0	280.0	278.9	160.1
36	1.7	280.0	280.0	280.0	278.8	160.2
40	1.9	280.0	280.0	280.0	278.6	160.2
44	2.025	280.0	279.8	279.9	278.5	160.3
48	2.2	280.0	279.8	279.9	278.3	160.3
52	2.4	280.0	279.8	279.8	278.1	160.4
56	2.6	280.0	279.8	279.8	277.9	160.5
60	2.8	280.0	279.8	279.8	277.6	160.7
64	3.0	279.9	279.8	279.8	277.4	160.8
68	3.2	279.9	279.8	279.8	277.2	161.0
72	3.4	279.9	279.8	279.8	276.9	161.2
76	3.6	279.9	279.8	279.8	276.7	161.4
80	3.8	279.9	279.8	279.8	276.4	161.5
84	4.0	279.9	279.8	279.8	276.2	161.7
88	4.1	279.9	279.7	279.4	276.0	161.8
92	4.3	279.9	279.5	278.9	275.8	162.1
96	4.5	279.9	279.1	278.7	275.5	162.3
100	4.7	279.9	278.1	278.6	275.3	162.5
104	4.9	279.9	270.7	278.4	275.1	162.7
108	5.1	279.9	257.7	277.7	274.8	162.9

TABLE 6-5
PROBLEM 6: TEMPERATURE HISTORIES
(Continued)

<u>Cycle</u>	<u>Time (Years)</u>	<u>T_{1,1,1} (°C)</u>	<u>T_{1,1,2} (°C)</u>	<u>T_{1,1,3} (°C)</u>	<u>T_{1,1,4} (°C)</u>	<u>T_{1,1,5} (°C)</u>
112	5.3	279.9	252.9	276.6	274.6	163.1
116	5.5	279.9	253.6	275.5	274.4	163.3
120	5.7	279.9	254.5	274.5	274.0	163.6
124	5.9	279.9	253.9	273.7	273.6	163.8
128	6.025	279.8	251.0	273.2	273.4	163.9
132	6.2	279.8	200.5	271.6	272.9	164.1
136	6.4	279.8	162.7	268.4	272.2	164.4
140	6.5	279.8	146.7	266.6	271.7	164.5
144	6.638	279.8	112.7	263.9	271.0	164.6
148	6.684	279.8	98.0	263.0	270.7	164.7
152	6.741	279.8	76.4	261.9	270.3	164.7
156	6.806	279.8	49.9	260.7	269.9	164.8

Figure 6.1. Problem 6: Discharge Enthalpy History.

Figure 6.2. Problem 6: Pressure Histories.

Figure 6.3. Problem 6: Steam Saturation Histories.

Figure 6.4. Problem 6: Temperature Histories

Figure 6.5. Problem 6: Pressure Profiles

REFERENCES

- BUDIANSKY, B [1970], "Thermal and Thermoelastic Properties of Isotropic Composites," J. Composite Materials, Vol. 4, pp. 286-295.
- DUCKLER, A. E., M. Wicks, III and R. G. Cleveland [1964], "Frictional Pressure Drop in Two-Phase Flow - B. An Approach through Similarity Analysis," AIChE J., Vol. 10.
- GARG, S. K., J. W. Pritchett and D. H. Brownell, Jr. [1975], "Transport of Mass and Energy in Porous Media," Proceedings 2nd U.N. Geothermal Symposium, San Francisco, pp. 1651-1656, May.
- GARG, S. K. and J. W. Pritchett [1977], "On Pressure-Work, Viscous Dissipation and the Energy Balance Relation for Geothermal Reservoirs," Advances in Water Resources, Vol. 1, No. 1, pp. 41-47.
- GARG, S. K. [1978], "Pressure Transient Analysis for Two-Phase (Liquid Water/Steam) Geothermal Reservoirs," Proc. Geothermal Resources Council 1978 Annual Meeting, pp. 203-206, July. Also Society of Petroleum Engineers Journal, Vol. 20, pp. 206-214, 1980.
- HAGEDORN, A. R. and K. E. Brown [1964], "The Effect of Liquid Viscosity in Two-Phase Vertical Flow," J. Petrol. Tech., Vol. 16.
- HUGHMARK, G. A. [1962], "Holdup in Gas-Liquid Flow," Chem. Engineering Progr., Vol. 58.
- ORKISZEWSKI, J. [1967], "Predicting Two-Phase Pressure Drops in Vertical Pipe", J. Petrol. Tech., Vol. 19.
- PEACEMAN, D. W. [1978], "Interpretation of Well-Block Pressures in Numerical Reservoir Simulations," Soc. Petrol. Eng. J., Vol. 18, pp. 183-194, June.
- PRITCHETT, J. W., S. K. Garg and T. D. Riney [1977], "Numerical Simulation of the Effects of ReInjection upon the Performance of a Geopressured Geothermal Reservoir," Geothermal: State-of-the-Art - Transactions Geothermal Resources Council Annual Meeting, San Diego, pp. 245-247, May 9-11. Also, Proceedings of Third Geopressured Geothermal Energy Conference, pp. ES-73 to ES-83, Univ. of Southwestern Louisiana, 1977.
- PRITCHETT, J. W. [1979], "A Semi-Analytical Description of Two-Phase Flow near Production Wells in Hydrothermal and Geopressured Reservoirs," Proc. Fifth Workshop on Geothermal Reservoir Engineering (H. J. Ramey and P. Kruger, eds.), Stanford University, pp. 173-182.
- PRITCHETT, J. W. and S. K. Garg [1980], "Determination of Effective Well Block Radii for Numerical Reservoir Simulations," Water Resources Research, Vol. 16, No. 4, pp. 665-674, August.

RINEY, T. D., J. W. Pritchett and S. K. Garg [1977], "Salton Sea Geothermal Reservoir Simulations," Proc. Third Workshop on Geothermal Reservoir Engineering, Stanford University, pp. 178-184, December. Also in Proc. of the Geothermal Resources Council 1978 Annual Meeting, pp. 571-574, Hilo, Hawaii, July 1978.

ROS, N. C. M. [1961], "Simultaneous Flow of Gas and Liquid as Encountered in Well Tubing," J. Petrol. Tech., Vol. 13.