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INTRODUCTION

At high currents the longitudinal phase space of the SLC beam is not simply described by gaussian distributions in both position and energy. The distorted ring beam, the curvature of the compressor rf, the limited energy aperture of the RTL, the wakefields in the linac, and the momentum compaction in the arc all contribute to some extent to a distortion of longitudinal phase space. In this paper we present simulation results that describe the phase space of the SLC beam, from the damping rings to the final focus area, and that include all these distorting effects.

SIMULATING LONGITUDINAL PHASE SPACE IN THE SLC, FROM THE DAMPING RINGS TO THE FINAL FOCUS*

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THE RTL**Bunch Compression**

The bunch compression in the RTL transfer line of the SLC is performed in two parts. First the bunch crosses the compressor rf section. The timing is such that the beam sits on a zero crossing of the rf wave, with particles in front gaining energy, those in back losing energy. In this way an energy variation correlated with longitudinal position is induced in the beam. Secondly the bunch traverses a beam line with non-zero momentum compaction. The overall result is a beam with a shorter length at the expense of an increased energy spread. Note that the

properties of bunch compression and its application to the SLC are discussed in Refs. 3-5.

In describing the compression process let us begin with the damping ring beam as it enters the RTL. A point in the initial longitudinal phase space of the beam is described by the position and energy coordinates as (z, δ) . If, as elsewhere in this paper, a more negative value of position is more toward the front of the bunch; in addition, energy coordinates are given in units of relative energy deviation from the mean. After the compressor the point has the new coordinates (u, v) . The compression can be thought of as a coordinate transformation from the variables (z, δ) to the new variables (u, v) . Let us assume that the position $z = 0$ is at the zero crossing of the rf wave. For compression we require that particles at negative values of z gain energy from the rf wave and that particles at positive values of z lose energy. For the moment let us, in addition, assume the compressor rf wave is linear. Then the two parts of the transformation are given by

$$v = -az + \delta \quad \text{and} \quad u = z + bv = \alpha z + b\delta, \quad (1)$$

with $\alpha = 1 - ab$. The compressor rf strength factor a is given by $a = eV_c k_{rf}/E_0$, with V_c the peak amplitude of the compressor rf wave, k_{rf} the rf wave number, and E_0 the average beam energy in the RTL. The compaction factor b transforms an energy variation into a position change.

The two-dimensional bunch distribution function $f(z, \delta)$ transforms to the new distribution function $g(u, v)$ as

$$f(z, \delta) dz d\delta = g(u, v) du dv = f(z, \delta) J(z, \delta; u, v) du dv, \quad (2)$$

with J the Jacobian for the transformation. Our transformation is area preserving and $J(z, \delta; u, v) = 1$. Therefore,

$$g(u, v) = f(z, \delta) = f(u - bv, au + \alpha v). \quad (3)$$

The corresponding position and energy distributions after transformation are obtained by integrations:

$$\lambda_u(u) = \int g(u, v) dv \quad \text{and} \quad \lambda_v(v) = \int g(u, v) du. \quad (4)$$

With no aperture limitations the limits of integration are $\pm\infty$. With an aperture limitation, such as the energy aperture limitation in the RTL, the limits of integration are given by the aperture. The beam distribution in the damping ring is separable as $f(z, \delta) = \lambda_z(z)\lambda_\delta(\delta)$, with λ_z and λ_δ representing respectively the damping ring position and energy distributions. Therefore the distribution after compression (still in the linear rf approximation) is given by

$$g(u, v) = \lambda_z(u - bv)\lambda_\delta(au + \alpha v). \quad (5)$$

Many well known properties of bunch compression can be derived by substituting Eq. (5) into Eq. (4). For example, from these two equations we see that, when $\alpha = 0$, $\lambda_u = a\lambda_\delta(au)$. If, in addition we assume that the beam energy spread is greatly increased in the process of compression (as is normal), then we find that $\lambda_v = b\lambda_z(-bv)$. Thus, we see that with $\alpha = 0$ the shape of the final position distribution is the same as that of the initial energy distribution, and the shape of the final energy distribution is the mirror image of the initial position distribution. In particular, if the initial energy distribution is gaussian (as we will assume in our calculations), so will be the final bunch shape. In the approximation of a linear rf wave the case $\alpha = 0$ results in the shortest final bunch length. Let us designate the compressor amplitude that minimizes the final bunch length as the "full compression" amplitude. The SLC compressor rf normally operates near the full compression amplitude.

For the general case of over- or under-compression, but assuming that both λ_δ and λ_z are gaussian, with the respective lengths σ_δ and σ_z , we see from Eq. (4) and

(5) that the final position and energy distributions are also gaussian. The after compression rms length σ_u and rms energy spread σ_v are given by⁽⁴⁾

$$\sigma_u = \sqrt{\alpha^2 \sigma_z^2 + \beta^2 \sigma_\delta^2} \quad \text{and} \quad \sigma_v = \sqrt{a^2 \sigma_z^2 + \sigma_\delta^2} \quad (6)$$

We are mostly interested in the compressor behavior near full compression, i.e. where $\alpha \sigma_z \lesssim \beta \sigma_\delta$. Note that, near full compression, σ_u is normally sensitive to small changes in a whereas σ_v is not.

The damping ring bunch length increases with current: at higher currents the full length of the bunch approaches half the wavelength of the compressor rf. In order to increase the accuracy of our calculations we will include the effects of the curvature of the compressor rf. We expect that, in the vicinity of their core, the shape of the final distributions will be little affected by this change. The main effect will be seen in the tails of the distributions. Note that the full-compression voltage, i.e. the rf voltage that minimizes the final bunch length, will be slightly larger when the rf curvature is included in the calculation, than when it is not.⁽³⁾ To include the rf curvature in our calculations we need to replace az in the transformation equations Eq. (1) by $(eV_c/E_0) \sin k_{rf} z$. The transformation is still area preserving, the Jacobian is still 1. The after compression distribution becomes

$$g(u, v) = \lambda_z(u - bv) \lambda_\delta(v + [eV_c/E_0] \sin k_{rf}[u - bv]) \quad (7)$$

For our calculations we will substitute this function into Eq. (4) to find the after compression position and energy distributions, λ_u and λ_v .

The Bunch Shape After Compression

We begin our bunch compression calculations with the damping ring distributions, λ_z and λ_δ . To obtain the bunch shape λ_z a modified version of potential well theory is used.⁽²⁾ An essential ingredient in this calculation is the longitudinal wake function that represents the effects of the entire vacuum chamber of the damping

ring, a function that was calculated earlier.⁽⁶⁾ As for the energy distribution λ_δ , it is taken to be gaussian. Below the threshold current for instability N_{th} the bunch rms energy spread σ_δ is taken to be constant, above threshold it is taken to vary as $(N/N_{th})^{1/3}$. We will not go into more detail about the damping ring calculations here. For a complete description of the damping ring wake function and the bunch length calculations see Refs. 2, 6, and 7. We should point out that the wake function that we use here is slightly different from that described in these sources; our function includes, in addition, the effects of the recently installed bellows sleeves. Besides causing a minor change in bunch shape we estimate that these sleeves increase the threshold current slightly. Nominally the ring operates with a ring voltage $V_{ring} = 1$ MV, and in our calculations we will limit ourselves to this voltage. At this voltage the nominal (low current) rms bunch length is 4.42 mm, the nominal rms energy spread is 0.07%. We will take $N_{th} = 1.9 \times 10^{10}$.

To obtain the after compression distributions we substitute the ring distributions λ_z and λ_δ into Eq. (7) and then into Eqs. (4). The integrations are performed numerically. The energy aperture of the RTL transfer line is limited. Measurements performed by P. Morton estimate the full width of the aperture (at least on the North side) to be 4.5%.⁽⁸⁾ We will include this in our calculations by setting the limits of integration over dv to $\pm 2.25\%$. Thus our calculations assume that particles outside this energy aperture are clipped away cleanly. In position we assume there are no aperture limits. Note that for the RTL transfer line the compaction factor $b = 0.603$ m, the average energy $E_0 = 1.15$ GeV, and the rf wave number $k_{rf} = 60 \text{ m}^{-1}$.

As an example, let us consider a beam with population $N = 5 \times 10^{10}$. The initial (i.e. damping ring) energy distribution is gaussian with an rms spread of 0.1%. The initial position distribution is shown in Fig. 1. The front of the distribution is to the left; the position $z = 0$ is the location of the mean. We note that the shape of the distribution is more bulbous than a gaussian, and that the distribution is not symmetric about its mean: its leading edge rises more steeply than its trailing edge. The bunch has lengthened: the rms length $z_{rms} = 8.7$ mm and the full-width-

at-half-maximum $\Delta z_{FWHM} = 25.3$ mm. These values are respectively twice and 2.5 times the nominal values. For compression we position the center of the beam on the zero crossing of the rf wave. For this situation, the dashed vertical bars represent the positions of the nearest maximum and minimum of the rf wave. We see that, at this current, the beam essentially spans the entire distance between the maximum and the minimum of the rf wave. Far from the beam core the tails of the distribution become gaussian. These tails, however, are centered on the nominal synchronous point, which, in this case, is 4.2 mm to the right of the beam centroid.

Fig. 1. The bunch shape before compression, when $N = 5 \times 10^{10}$ and $V_{ring} = 1$ MV.

Let us suppose the compressor voltage is set to a typical value for the SLC, $V_c = 30$ MV, a value which is near full compression. Performing the calculations, we find that, for this example, the after compression distributions and phase space to be those shown in Fig. 2. Fig. 2c shows contours of phase space, with the distance between adjacent contour lines being $1/6^{\text{th}}$ of the peak value. The peak is marked by the symbol '+'; the dashed curve gives the ridge line of the distribution. The rf curvature is clearly visible. The dotted lines mark the limits of the $\pm 2.25\%$ energy aperture. Fig. 2a shows the final energy distribution. This curve is similar

to, though more rectangular than, the mirror image of the damping ring bunch shape (see Fig. 1). The differences between the two shapes are primarily due to the curvature of the rf wave. Due to the curvature tail particles in the initial distribution have moved more toward the core in the final distribution. Note that at 5×10^{10} the distribution essentially fills the energy aperture with no room to spare. The area outside the aperture (indicated by the dashed portion of the curve in Fig. 1) represents 4% of the beam. For the entire distribution the rms width $\sigma_{rms} = 1.17\%$, the full width $\Delta v_{FWHM} = 4.04\%$. In contrast, for a low current beam, for which there is no bunch lengthening nor potential well distortion in the ring, the spectral shape is almost gaussian with $\sigma_v = 0.7\%$ [see Eq. (6)], and therefore $\Delta v_{FWHM} = 1.65\%$.

Fig. 2b displays the bunch shape after compression (the solid curve). As expected the shape is similar to the shape of the energy distribution before compression, i.e. similar to a gaussian. The deviation from a gaussian shape is mostly due to the curvature of the rf, which has slightly depressed the peak and has lengthened the tails. The dotted curve gives a gaussian fit to the bunch shape. The rms length of the fit $\sigma_{ug} = 1.0$ mm. The dashed curve shows the position distribution that is found when we take the energy aperture to be unlimited. We see that it is primarily trailing tail particles that are collimated away by the energy aperture limitation.

We have repeated the calculations for bunch populations of $N = 0, 1, 2, 3, 4, 5, 6 \times 10^{10}$ and compressor settings $V_c = 27, 29, 31, 33, 35$ MV. The length of the gaussian fit to the resulting bunch shapes σ_{ug} is plotted as function of V_c and as function of N in Fig. 3. We note that the bunch length is rather sensitive to the compressor setting. For example, at $N = 5 \times 10^{10}$ and with $V_c = 33$ MV the bunch length is 0.66 mm, whereas with $V_c = 27$ MV it is 1.77 mm. The voltage that gives the minimum bunch length increases slightly with current. This is because as the current increases the bunch lengthens and consequently more of it drapes over the nonlinear part of the rf wave. At zero current the minimum is near 32.8 MV, at $N = 6 \times 10^{10}$ it is near 33.9 MV. The dotted curve in Fig. 3a

Fig. 2. (a) The energy and (b) position distributions, and (c) contours of phase space, after bunch compression. For this example $V_{\text{ring}} = 1$ MV, $N = 5 \times 10^{10}$, and $V_c = 30$ MV.

represents the low current calculation assuming a linear rf wave, which is given by Eq. (6). The minimum of this curve is at $V_c = 31.9$ MV. Finally, recall that under the linear approximation and at full compression ($\alpha = 0$) the final bunch length is given by the initial energy spread. $\sigma_u = b\sigma_\phi$. The dotted curve in Fig. 3b shows this approximation. The length remains at a constant value (0.42 mm) up to the threshold current $N_{th} = 1.9 \times 10^{10}$ and then increases as $N^{1/3}$. Note that this approximation agrees rather well with the full calculation results for $V_c = 33$ MV.

Fig. 3. The bunch length after compression as function of (a) compressor voltage and (b) of bunch population, when $V_{\text{ring}} = 1$ MV. The parameter σ_{ug} is the rms of the gaussian fit to the position distribution λ_u .

The Phase Space Tilt

If there is dispersion at the beginning of the linac a beam tilt in longitudinal phase space at the end of the RTL will translate into an $x - z$ tilt at the front of the linac. An $x - z$ tilt in the beam at the front of the linac, at high currents, can result in large emittance growth by the end of the linac, just as an initial x -offset of the beam can.⁽⁹⁾ Or, conversely, an initial $x - z$ tilt can be used to reduce the emittance growth due to random orbit errors in the linac, just as an initial offset can. As a measure of the phase space tilt of the beam let us take

$$\alpha_{uv} = \int \lambda_u(u) \left[\int g(u, v) v dv - \int g(0, v) v dv \right] \frac{du}{u} \quad (8)$$

The average $x - z$ tilt of the beam at the front of the linac is then given by the product $\eta \alpha_{uv}$, with η the dispersion there. If we consider the approximation of a linear compressor rf wave, and also assume the initial beam shape is gaussian, we

can analytically perform the integration of Eq. (8). The phase space tilt is then given by

$$\alpha_{uv} = \frac{b - a\alpha(\sigma_z/\sigma_\delta)^2}{b^2 + \alpha^2(\sigma_z/\sigma_\delta)^2} \quad (9)$$

We have calculated the phase space tilt, including the rf curvature and the finite energy aperture, for currents of $N = 0$ and 5×10^{10} and for various values of V_c . The results are plotted in Fig. 4. For a low current beam the zero crossing is at $V_c = 31.5$ MV, for $N = 5 \times 10^{10}$ it is at $V_c = 33.6$ MV. The low current, linear rf approximation [Eq. (9)] is given in Fig. 4 by the dots.

Fig. 4. The phase space tilt of the beam after compression as function of compressor voltage, for $N = 0$ and 5×10^{10} , and $V_{ring} = 1$ MV.

Beam Loss in the RTL

As mentioned above, the RTL has a limited energy aperture and will therefore scrape away some of the beam. Suppose the full energy aperture is δ_a . Since the width of the initial energy distribution λ_δ is small compared to that of the

final energy distribution λ_v we can find the maximum RTL transmission by the following method: First we calculate

$$\Delta z_a = \frac{2}{k_{rf}} \arcsin \left(\frac{\delta_a E_0}{2e V_c} \right) \quad (10)$$

The values $\pm \Delta z_a/2$ give the longitudinal points on the rf wave that transform to the limits of the RTL energy acceptance $\pm \delta_a/2$. Therefore, the maximum fraction of the beam that can be transmitted through the RTL is just the maximum fraction of the initial distribution λ_z that will fit into a window of length Δz_a . We have numerically performed this calculation for an energy aperture $\delta_a = 4.5\%$ (which we think is the existing RTL aperture) and for a narrower aperture of $\delta_a = 3.5\%$. Fig. 5 shows the fraction of the beam lost, $(N - N_t)/N$ [N_t is the number of particles transmitted], with the beam optimally positioned on the rf wave. The solid curves represent the case $V_c = 30$ MV, the dashed curves the case $V_c = 33$ MV. We note that at higher currents reducing the aperture by 1% will result in a significant increase in beam loss. At $V_c = 30$ MV and for $N = 5 \times 10^{10}$, 3% of the beam is lost when $\delta_a = 4.5\%$, whereas 16% is lost when $\delta_a = 3.5\%$.

The Population of the Tails of λ_u

Detector backgrounds, especially at higher currents, limit the integrated luminosity that can be achieved in the SLC. It seems to take relatively few stray particles hitting the detector to incapacitate it. It has been suggested that one possible source of these backgrounds are particles originating in the tails of the bunch distribution λ_u that manage to reach the detector. In order to control the tail population adjustable collimators will be installed near the end of the RTL. These collimators will effectively reduce the energy aperture of the RTL. However, collimating in energy v does not remove tails in position u ; it can only reduce the tail population. To remove tails of λ_u we would like to, ideally, collimate in energy somewhere *before* the compressor; however, the dispersion upstream of the compressor is not sufficiently large to make this possible. It is only because there is

Fig. 5. The fraction of beam lost in the RTL as function of current, when $V_{\text{ring}} = 1$ MV and $V_c = 30$ MV. Results are shown for RTL energy apertures $\delta_a = 3.5\%$ and 4.5% . The dashed curves give the calculated loss when $V_c = 33$ MV.

some correlation between u and v in the distribution $g(u, v)$ —due to the rf curvature and to (possibly) being slightly undercompressed—that this scheme can work at all to reduce the tail population. However, as we have seen from the previous section, if at higher currents we reduce the energy aperture much below 4.5% we will not only collimate away some tail particles, but also a significant fraction of the core particles.

To study the effect of energy aperture changes on the tails of the final position distribution λ_u we have again numerically integrated $g(u, v)$ over dv , but this time keeping track far into the tails, to $\pm 10\sigma$, of both λ_z and λ_ϕ . Recall that, far from the core the tails of both λ_z and λ_ϕ are gaussian; however, unlike the energy tails, the tails of λ_z are not centered at the origin. We compare three cases that yield roughly the same bunch population and core shape after compression. For each case the transmitted bunch population $N_t = 5 \times 10^{10}$ and the rms length of the gaussian fit $\sigma_{gt} = 1$ mm. The three cases are: (a) $N = 5.0 \times 10^{10}$, $\delta_a = 4.5\%$.

(b) $N = 5.5 \times 10^{10}$, $\delta_a = 4.0\%$, and (c) $N = 6.5 \times 10^{10}$, $\delta_a = 3.5\%$. For all cases $V_c = 30$ MV.

In Fig. 6 we plot the final particle density $N\lambda_u$ as function of longitudinal position. We show a large ordinate range in order to better display the structure of the curves. Below $N\lambda_u \sim 10^4$ – 10^5 the curves can't really describe the real charge distribution. At a value of 10^3 , for example, the curve represents only 1 particle per millimeter. We see in Fig. 6 that the curves are not symmetric about $u = 0$; there are more particles in the trailing than in the leading tails. This is due to the asymmetry of the tails of the initial distribution λ_z about $z = 0$. We also note that the curves are notched in both leading and trailing tails. This is the result of the energy collimation. To demonstrate this, we have repeated the calculation for Case (a), but this time with no energy aperture limitation. The results are given by the dotted curve. We note that the tails of this curve are linear on the plot, meaning that the particle density in the tails varies exponentially in u (i.e. as $e^{-c_1 u}$, with c_1 a constant) rather than as gaussians. This behavior continues on up to $u \approx \pi/k_{rf} \approx 50$ mm. This exponential dependence can be verified by assuming both initial distributions are gaussian, and then performing a saddle point calculation of the integral $\int g(u, v) dv$.

Whether any tail particles survive to the final focus depends on the linac lattice, on the linac klystron phasings, and, of course, on the collimation system at the end of the linac. One criterion that limits the distance from the core a particle can be and still survive is that the particle's phase advance per cell throughout the linac must be less than π . Normally, in order to induce BNS damping, the klystrons are phased so that the beam rides behind the rf crest near the beginning of the linac and then in front of the crest throughout the rest of the linac. Suppose the beam phase begins at 15° and then later shifts to -15° . In the front half of the linac the phase advance per cell is approximately 70° . Therefore, the limits of stability u_{lim} are given by $\cos(k_{rf} u_{lim} + 15^\circ) \approx \cos 15^\circ$ ($70/180$), or $u_{lim} \approx \pm 15.5$ mm. Note that unlike energy collimation this phenomenon does clearly cut tails of λ_u . We can take advantage of this phenomenon to further shorten the length of the trailing

Fig. 6. The longitudinal particle density after compression when (a) $N = 5.0 \times 10^{10}$, $\delta_a = 4.5\%$, (b) $N = 5.5 \times 10^{10}$, $\delta_a = 4.0\%$, and (c) $N = 6.5 \times 10^{10}$, $\delta_a = 3.5\%$. In all cases $V_c = 30$ MV.

tail that survives to the final focus.⁽¹⁰⁾ We can do this by phasing the beam way behind crest in the first few cells of the linac. If, for example, the beam phase were initially 45° then the stability limit behind the bunch would be reduced to $u_{lim} = 8.5$ mm.

Let us return to Fig. 6 to compare the tail populations for the different cases. Let us assume that the stability limits are $u_{lim} = \pm 15.5$ mm. The shapes of the distributions beyond these limits do not concern us. We have computed the areas under the trailing tails. The areas between $u = 5\sigma_{ug} = 5$ mm and $u_{lim} = 15.5$ mm are 6×10^8 , 7×10^7 , 3×10^6 , and 4×10^5 , for respectively the dotted curve, Curve a, Curve b, and Curve c. The areas between $u = 10\sigma_{ug}$ and $u_{lim} = 15.5$ mm are 3×10^7 , 3×10^4 , 6, and 0, for respectively the same curves. The number of particles in the leading tails are less than these values. Clearly at $N = 5 \times 10^{10}$ it is better to have a 4.5% energy aperture limitation than none at all, since the limitation significantly reduces the tail population at the expense of only 3% of the core. If we reduce the aperture to 4.0% (Case b) and then to 3.5% (Case c) we continue to significantly reduce the tail population. However, it is not clear that

these gains compensate for the extra 10% (Case b) or 25% (Case c) beam current that we need to store in the damping rings, if we want to be left with the same 5×10^{10} particles in the linac.

THE LINAC

The shape of the beam spectrum at the end of the SLC linac has been studied by many authors (see Refs. 11-17). Ref. 17, for example, presents a detailed description of the spectral properties at the end of the linac as function of current, bunch length, and rf phase. In this paper we will limit ourselves to briefly describing the theory and presenting the results of one example calculation.

The transformation to the linac position and energy coordinates (u', v') is described by

$$u' = u \quad \text{and} \quad v' = \frac{E_0}{E_f} v + \tilde{v}(u) \quad (11)$$

with E_f the final average energy. The component of energy correlated with longitudinal position $\tilde{v}(u)$ is given by

$$\tilde{v}(u) = \frac{1}{E_f} [E_0 + E_a \cos(k_{rf}u + \phi) + eV_{ind}] - 1 \quad (12)$$

with E_a the total peak rf energy gain, ϕ the beam phase with respect to the rf crest, and V_{ind} the induced voltage in the linac. The induced voltage can also be written as the product of the total charge in the beam times the bunch wakefield. For this transformation the Jacobian is E_f/E_0 and the final position and energy distributions are given by

$$\lambda_{u'} = \lambda_u \quad \text{and} \quad \lambda_{v'} = \frac{E_f}{E_0} \int \lambda_u(u') \lambda_v \left(\frac{E_f}{E_0} [v' - \tilde{v}(u')] \right) du' \quad (13)$$

Note that parentheses in the above equation indicate the argument of a function, whereas brackets merely indicate multiplication.

If E_f/E_0 is very large then the function λ_v in the integral of Eq. (13) is much more sharply peaked as function of u' than is λ_u . We can, therefore, replace λ_v by a delta function. With this approximation we find that

$$\lambda_{v'} = \sum_w \lambda_u(w) \left| \frac{d\tilde{v}}{dw}(w) \right| \quad \text{with} \quad w = \tilde{v}^{-1}(v') \quad (14)$$

The sum in this equation indicates that if the inverse function $\tilde{v}^{-1}$ is multivalued at v' then the contributions for all values must be summed. According to Eq. (14) whenever $d\tilde{v}/du' = 0$ the energy distribution has an infinitely high spike. In reality, however, the distribution must be everywhere finite, and the minimum width of any spikes is given by the width of the uncorrelated component of the energy variation. Given a solution of Eq. (14) with infinite spikes we can, if need be, estimate the true height of these spikes by convolving the distribution with the uncorrelated component of energy variation $[E_f/E_0] \lambda_v(E_f v'/E_0)$.

It is normally the method of Eq. (14), with λ_u taken to be a gaussian, that is used to find the spectrum at the end of the SLC linac. From Eq. (14) we see that the spectrum depends on the bunch shape and the slope of the correlated component of the energy variation $d\tilde{v}/du'$. The function $d\tilde{v}/du'$ itself depends on the slope of the rf wave and on the slope of the induced voltage V'_{ind} . Going one more step we can write this last function as

$$V'_{ind}(u') = -eN \int_0^\infty W'(x) \lambda_u(u' - x) dx \quad (15)$$

with W' the slope of the wakefield for the linac structure. Therefore, the accuracy of the spectrum calculation depends on the accurate knowledge of the wakefield. In our simulations we will use the calculated wakefield for the SLAC structure given in Ref. 20. Note that in Appendix A we describe a method of measuring both the linac bunch shape and the induced voltage on the SLC with the existing hardware.

Fig. 7. The induced voltage for the entire SLAC linac, when $N = 5 \times 10^{10}$ and $V_c = 30$ MV. The dotted curve gives the results for a gaussian bunch with a 1 mm rms length.

Let us find the spectrum at the end of the linac for the case $N = 5 \times 10^{10}$ and $V_c = 30$ MV. The total length of accelerating structure (needed for finding V'_{ind}) is 2744 m. The acceleration gradient is chosen to give $E_f = 47$ GeV when $\phi = -10^\circ$. We will use the integration method, Eq. (13) rather than the approximation Eq. (14). Since we already know λ_u and λ_v from our earlier calculation, we can obtain the final energy spectrum by performing the integration of Eq. (13). In Fig. 7 we display the induced voltage for this example. The dotted curve gives the results assuming the bunch shape is gaussian with a 1 mm rms length. Since the bunch shape is nearly gaussian [see Fig. 2b] the curves appear to be nearly identical. The average slopes of the two curves, when weighted by the charge distribution, differ by 10%.

Fig. 8 displays the spectral changes when, beginning at the top of the rf crest, we move the beam forward on the wave in steps of 5° . The dashed curves, with their scales on the right, give the function $\tilde{v}(u')$. At the top of the crest, the spectrum has two, widely-separated spikes. As the beam is moved ever more forward the core of the spectrum narrows, and tails begin to grow. At its narrowest the core

Fig. 8. The bunch spectrum at the end of the linac $\lambda_{v'}$ for several values of rf phase. The dotted curves give the component of energy correlated with longitudinal position $\bar{v}(u')$. For this example $N = 5 \times 10^{10}$ and $V_c = 30$ MV.

full width is approximately given by $(E_0/E_f)\delta_{FWHM} = 0.1\%$. Beyond about $\phi = -20^\circ$ there is no longer a spike in the energy distribution of the beam. The widths of the examples given are respectively $\Delta v'_{FWHM} = 3.00\%$, 1.76% , 0.87% , 0.27% , 0.13% , 0.78% , for the phases $\phi = 0^\circ$, -5° , -10° , -15° , -20° , and -25° . If we can assume that detector backgrounds are not a problem, then to optimize the luminosity we want to maximize the number of particles within an energy window of $\pm 0.5\%$.⁽¹⁵⁾ For our six example calculations we find that respectively 21% , 44% , 87% , 74% , 52% , and 33% of the beam is within this window. Therefore, at $N = 5 \times 10^{10}$ and $V_c = 30$ MV, assuming backgrounds are not a consideration, the optimal phase is near -10° .

THE FINAL FOCUS

The SLC arcs have non-zero momentum compaction. Therefore, since the beam is not mono-energetic, the bunch shape will change as it passes through the arcs. (The width of the spectrum will also increase slightly, by 0.08% ,⁽¹⁶⁾ due to synchrotron radiation. However, we will ignore this effect.) Note that the beam compression in the arcs has been studied in Ref. 19.

The transformation of a particle position at the end of the linac (u', v') to the corresponding position at the final focus (u'', v'') is given by

$$u'' = u' + b_{arc}v' \quad \text{and} \quad v'' = v' \quad (16)$$

with b_{arc} the compaction factor of the arcs. The Jacobian for the transformation is 1 and the distribution is given by

$$\lambda_{u''} = \int \lambda_{u'}(u'' - b_{arc}v'')\lambda_{v'}(v'')dv'' \quad \text{and} \quad \lambda_{v''} = \lambda_{v'} \quad (17)$$

If the beam energy at the end of the linac is large compared to that at the beginning then we can approximate the position distribution at the final focus by

$$\lambda_{u''} = \sum_s \lambda_u(s) / \left| \frac{dh}{ds}(s) \right| \quad \text{with} \quad s = h^{-1}(u'') \quad (18)$$

and $h(u) = u + b_{arc}\tilde{v}(u)$. The sum in this equation indicates that if the inverse function $\tilde{h}^{-1}$ is multivalued at u'' then the contributions for all values must be summed. The bunch shape will have a spike wherever the denominator in Eq. (18) is zero. Note also that wherever the slope of $\tilde{v}(u)$ is negative the bunch will compress, wherever it is positive the bunch will expand. If $\tilde{v}(u)$ is a linear function of u , with c_1 the constant of proportionality, then the final distribution is given by the initial distribution, but with its width scaled by the factor $(1 + b_{arc}c_1)$.

The compaction parameter for the arcs b_{arc} is 0.145 m. For $N = 5 \times 10^{10}$ and $V_c = 30$ MV, and for the same six values of linac phase described in the previous section we have numerically solved Eq. (17) to obtain the bunch shape at the final focus. The results are shown in Fig. 9. The dashed curves, with their scales on the right, give the function $\tilde{v}(u'') \equiv \tilde{v}(u'' - b_{arc}v')$. This function gives the energy/position correlation at the final focus. From the plots, we note that the bunch shapes at high currents are, in general, not at all gaussian. At the top of the crest, the bunch shape has two sharp spikes that narrow as the beam is moved more forward. At the phase with the optimal energy spectrum, $\phi = -10^\circ$, the bunch shape has one very large spike and very long tails. The peak value of the spike, which is off scale in the plot, is 3660 m^{-1} . From Fig. 9 we see that as the beam is moved still farther off-crest the core broadens and the tails lengthen. For our example calculations we find that the widths of the bunch shape $\Delta u''_{FWHM}$ are 1.94, 0.79, 0.11, 0.57, 1.79, and 3.21 mm for the respective phases $\phi = 0^\circ, -5^\circ, -10^\circ, -15^\circ, -20^\circ$, and -25° . The width is a very sensitive function of phase.

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Fig. 9. The bunch shape at the final focus λ_u'' for several values of rf phase. The dotted curves give the component of energy correlated with longitudinal position $\tilde{v}(u'')$. For this example $N = 5 \times 10^{10}$ and $V_c = 30$ MV.

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APPENDIX A: MEASURING THE BUNCH SHAPE AND THE INDUCED VOLTAGE IN THE SLC LINAC

Introduction

The shape of the bunch and the slope of the induced voltage determine the shape of the spectrum in the SLC linac. The bunch shape, in addition, is an important factor in the transverse instabilities in the linac. To date the bunch length, but not the shape, has been measured in the SLC linac. Although the measurement errors were large, the measured length as function of current appears to agree with theory. The average value of the induced voltage has also been measured. These measurement results agree extremely well with predictions that were based on the calculated longitudinal wake function for the SLC accelerating structure. We propose here a measurement technique that will allow us to measure both the bunch shape and the shape of the induced voltage. These measurements will give us a more stringent test of the correctness of the calculations.

The longitudinal charge distribution of the SLC Damping Ring bunch has been measured very accurately in the following manner:⁽¹⁾ The beam is first extracted into the RTL transfer line. In the compressor rf section, which is phased 90° to the bunch, a monotonic correlation between longitudinal position and energy is induced in the bunch. Then the beam is intercepted by a fluorescent screen, positioned at a point of high dispersion. Finally, the spot on the screen is digitized, and from this data the longitudinal bunch shape is obtained.

A similar method has been used to obtain the bunch length, though not the bunch shape, of the SLC linac beam.^(21,22) In these measurements the beam traverses the first 8 sectors of the linac (Sectors 2-9). At the end of Sector 9 the bunch is kicked into a beam line with a known dispersion and is observed on a screen. The actual measurement is performed in two parts: For the first part the Sector 9 rf is turned off. The overall rf phase of the first 8 sectors is adjusted to obtain a small spot on the screen, indicating a narrow energy spread in the beam. For the second part of the measurement the Sector 9 rf is turned on again, but phased

at 90° to the beam. The spot on the screen becomes larger and the rms width is measured again. Finally, the two measured widths are subtracted in quadrature to give an estimate of the rms bunch length. Note that it is more difficult to obtain the shape of the linac bunch than it is to obtain the shape of the damping ring bunch. The main reason is that the linac bunch is 5-10 times shorter than the damping ring bunch, and consequently its longitudinal wakefield is much stronger. Therefore, in the measurement of the linac bunch, unlike in that of the damping ring bunch, the wakefield contribution to the spectrum tends not to be negligible and cannot be ignored.

In this chapter we propose again measuring the spectrum at the end of Sector 9 of the linac. We propose measuring it twice, with the bunch at two different settings of overall rf phase. For the method to work it is important that the two phases are such that the correlation between longitudinal position and energy is monotonic for each measurement. Therefore, in our choices of phase we must avoid a "forbidden" region in front of the rf crest for which the beam spectrum contains one or more spikes. If this condition is satisfied, then from the two measured spectra we will be able to obtain both the longitudinal charge distribution and the induced voltage.

Theory

Consider a bunch of charged particles that pass the portion of the linac used in the measurement. The peak energy gain of the rf in the linac is E_a ; the bunch phase, with respect to the crest, is ϕ . We take the convention that a more negative value of phase is a position more forward on the rf wave. Let us assume that the energy gain in the linac is sufficiently high so that we can ignore the component of energy variation that is uncorrelated with longitudinal position. Then the relative energy of a particle at position z within the bunch is given by

$$\delta(z) = [E_0 + E_a \cos(k_{rf}z + \phi) + eV_{ind}(z)]/E_f - 1, \quad (A.1)$$

with E_0 the initial energy, k_{rf} the rf wave number, $V_{ind}(z)$ the induced voltage, and E_f the final energy. As discussed in the main part of this paper [see Eq. (14)] by

knowing both the charge distribution λ_z and the dependence of energy on position $\delta(z)$ we can compute the energy distribution λ_δ . It is also true that if ϕ is properly chosen so that $\delta(z)$ is monotonic over the bunch then we can calculate λ_z from λ_δ . Let us assume that this is the case. Then the longitudinal charge distribution is given by (see Eq. (14))

$$\lambda_z(z) = \lambda_\delta(\delta(z)) |\delta'(z)| \quad (A.2)$$

In Eq. (A.2) we display the arguments of the three functions explicitly. Suppose we know E_0 , E_a , E_f , k_{rf} , and ϕ . Without knowing the induced voltage we cannot, in general, obtain λ_z from λ_δ , since δ depends on V_{ind} . Only if eV_{ind} is small compared to δE_f over the bunch does a single measurement of λ_δ suffice to give λ_z .

Suppose we measure the bunch spectrum twice: with the beam at phase ϕ^a we obtain λ_δ^a , and then with the beam at ϕ^b we obtain λ_δ^b . We assume the phases are chosen so that $\delta(z)$ is monotonic for both measurements. For the first measurement Eq. (A.2) becomes

$$\lambda_z(z) = \lambda_\delta^a |E_a k_{rf} \sin(kz + \phi^a) - eV_{ind}^a(z)| / E_f \quad (A.3)$$

and a similar equation, with superscript b replacing superscript a , holds for the second measurement. Combining these two equations we find that

$$V_{ind}^a(z) = \frac{E_a k_{rf}}{\lambda_\delta^a \pm \lambda_\delta^b} \left[\lambda_\delta^a \sin(kz + \phi^a) \pm \lambda_\delta^b \sin(kz + \phi^b) \right] \quad (A.4)$$

In Eq. (A.4) the upper symbol of $\pm$ applies if the sign of $\delta'(z)$ is different for the two measurements, otherwise the lower symbol applies. The right hand side of Eq. (A.4) is a function both of z and—through the argument of λ_δ —of $V_{ind}(z)$. Eq. (A.4) is therefore a first order non-linear differential equation which we can solve numerically for the unknown $V_{ind}(z)$. As initial condition we take V_{ind} at the front of the bunch to be zero.

We can write the equation for λ_z as

$$\lambda_z(z) = \frac{E_a k_{rf} \lambda_\delta^a \lambda_\delta^b}{E_f |\lambda_\delta^a \pm \lambda_\delta^b|} \left| \sin(kz + \phi^a) - \sin(kz + \phi^b) \right| \quad (A.5)$$

In Eq. (A.5) again the upper symbol of $\pm$ applies if the sign of δ' is different for the two measurements, otherwise the lower symbol applies. Once we know $V_{ind}(z)$, and therefore also $\delta(z)$, we can obtain λ_z using Eq. (A.5).

Simulated Examples

We give two simulated examples to see what we might expect from the measurements. For each of two phases we first generate spectra that we will take to represent the measured data. These calculations follow the methods described in the body of this paper: Beginning with a damping ring bunch we simulate the compression and energy collimation in the RTL. The compressor voltage is $V_c = 31$ MV, the energy aperture is $\pm 2.25\%$. Longitudinal phase space is then propagated through 8 linac sectors giving us the final energy spectrum. For the linac the parameters are $E_0 = 1.15$ GeV, $k_{rf} = 60 \text{ m}^{-1}$; the structure length is 800 m, the peak gradient is 17 MeV/m, and therefore $E_a = 13.6$ GeV. Finally, in order to study the sensitivity to random measurement errors, we add a uniformly distributed random contribution to the calculated spectrum. Having obtained the "measured" spectra we then calculate V_{ind} by solving Eq. (A.4), and then λ_z by solving Eq. (A.5). Note that the calculation is redundant. We already know λ_z after the compressor simulation and can directly calculate V_{ind} using it and the wake function. We will use these results as a check on the measurement method.

As the first example we consider a bunch with $N = 1 \times 10^{10}$. We choose the phases so that the slope of δ' is different for the two measurements: $\phi^a = 10^\circ$ and $\phi^b = -20^\circ$. The results are shown in Fig. 10. Fig. 10a,b displays the calculated spectra. A random component with amplitude ± 4 units, which is equivalent to $\pm 5\%$ of the distribution peak, was added to the calculated spectra. (We have connected the "measured" points with straight lines in the plot.) Fig. 10c displays

Fig. 10. Simulation for $N = 1 \times 10^{10}$, $V_c = 31$ MV, and $E_a = 13.6$ GeV: (a,b) The "measured" spectra λ_g^a and λ_g^b that correspond to beam phases $\phi^a = 10^\circ$ and $\phi^b = -20^\circ$; (c,d) the induced voltage V_{ind} and the calculated bunch shape λ_z . The dots give the expected results.

the calculated induced voltage. Note that the curve is smooth. Since the solution of Eq. (A.4) is essentially an integration, random fluctuations in the spectra will be smoothed out. The expected form of V_{ind} is given by the dotted curve in the figure. The agreement is quite good. Finally Fig. 10d displays the calculated bunch shape. This function agrees well with the expected results, again given by the dotted curve. For this example the rms length is 0.50 mm.

Could we have obtained a good approximation to the bunch shape from the single "measurement" λ_g^a , by using Eq. (A.2) and just ignoring the wakefield effect? Fig. 11 shows the results of this process. Clearly the shape and the width of the curve are very different from the expected bunch shape (given by the dots). The rms width of the solid curve is 0.90 mm. For this single measurement method to work we need eV'_{ind} to be small compared to $\delta' E_f$ in the bunch. Thus, we need $\sin \phi$ to be large compared to $eV'_{ind}/(E_a k_{rf})$. From Fig. 10c we note that a typical value of V'_{ind} for this bunch is 0.15 GV/mm. Therefore, with $E_a = 13.6$ GeV, for the single measurement method to work, ϕ would need to be large compared to 10° .

Fig. 11. The calculated bunch shape using the spectrum of Fig. 10a but assuming the contribution of the induced voltage is negligible and can be ignored. The dots give the expected bunch shape.

At higher currents it will be impractical to perform the measurement at phases so that the sign of δ' is different for the two measurements. At high current the beam needs to be stabilized against single bunch beam break-up. With the beam behind the rf crest ($\phi > 0$) BNS damping is induced, and the beam is therefore stabilized. With the beam in front of the rf crest ($\phi < 0$), however, this is no longer

the case, and a good measurement will be difficult to make. Therefore, for our second example we make both "measurements" with the beam behind the rf crest. We let $N = 3 \times 10^{10}$. For phases we choose $\phi^a = 30^\circ$ and $\phi^b = 15^\circ$. The results are shown in Fig. 12. Fig. 12a,b displays the calculated spectra. A random component with amplitude ± 0.8 units, which is again equivalent to $\pm 5\%$ of the distribution peak, was added to the calculated spectra. From these spectra we calculate the induced voltage and bunch shape (see Fig. 12c,d). Again the fluctuations in V_{ind} are small, and the function agrees well with the expected results. The bunch shape, however, has large fluctuations, on the order of $\pm 30\%$ of the peak value. This is expected from Eq. (A.5). With the two measurements on the same side of the rf crest the two spectra are subtracted in the denominator. Whenever these two functions are nearly equal the error in their difference can be large. However, the calculated bunch shape, on average, agrees well with the expected result. For both functions the rms width is 0.70 mm.

Fig. 12. Simulation for $N = 3 \times 10^{10}$, $V_c = 31$ MV, and $E_a = 13.6$ GeV: (a,b) The "measured" spectra λ_g^a and λ_g^b that correspond to beam phases $\phi^a = 30^\circ$ and $\phi^b = 15^\circ$; (c,d) the induced voltage V_{ind} and the calculated bunch shape λ_z . The dots give the expected results.

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