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CHAPTER 3

ON THE MOTION OF THE CENTER OF MASS OF A SPHERICAL TURBULENT PREMIXED FLAME*

ABSTRACT

The movement of the center of mass of a premixed statistically spherical flame in the wrinkled-laminar flame regime has been examined. When the flame size is small (or comparable) to the integral scale of the turbulence, the flame ball is convected as a whole by the turbulent eddy. When the flame grows to a size large compared to the integral scale, the flame center of motion is not affected by the turbulence. This phenomenon has been explained in terms of the phase coherence of the local turbulent convection velocity at the flame front. When the flame is small, the turbulent velocity is coherent over the entire flame surface; as a result, the flame is convected as a whole by the turbulent eddy. When the flame is large, the velocity at the different area elements of the flame front is independent of each other. The center of mass velocity, which is an aggregate of the velocities at the different elements, therefore, tends to the mean velocity and is independent of the turbulence. A theory for predicting the variance of the flame center velocity has been developed. The prediction is in good agreement with the experimental results.

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Nomenclature

Symbol

I	normalized variance of the flame center of mass velocity, see Eq. (14) for definition
f, g	longitudinal and transverse correlation functions
F, G	Greens functions, see Eq. (16) for definition
$\hat{n}$	unit normal of the flame front area element
$\underline{q}$	local flame front velocity with reference to a fixed frame
r	radial coordinate of flame surface; also nondimensional argument to correlation functions f and g
S_L	laminar flame speed
$\underline{X}$	flame center of mass position vector, has components (X, Y, Z)
$\underline{x}$	flame surface area element position vector, has components (x, y, z)
$\underline{u}$	turbulent velocity vector, has components (u, v, w)
$\underline{U}$	mean flow velocity
V	volume of flame ball
Y_{lm}	spherical harmonics
$\underline{\xi}$	vector between two points on the flame ball
ξ	magnitude of $\underline{\xi}$
ζ	$(= \xi/R)$
ν	kinematic viscosity

Subscript

i, j	indices
r	radical component
θ	polar component
ϕ	azimuthal component

Introduction

Because of its importance as an energy conversion process, the turbulent premixed flame has been the subject of many theoretical and experimental studies^{1,2}. In many practical devices, for example, in the spark ignition engines, which operate at high pressure and temperature, the reaction zone consists of surfaces whose thickness is small compared to the size of the energy containing turbulent eddies. Then, the combustion is in the so-called wrinkled-laminar flame regime³. In this regime, the flame is locally laminar, and the "flame sheet" separates the burned from the unburned mixture. The propagation behavior of these flames has been studied extensively^{1,2}. There is, however, one important aspect which has not been examined in detail so far. This concerns the movement of the center of mass of a propagating flame ball, and such is the subject of the present paper.

The center of mass movement of the flame ball is of special concern to spark ignition engines because it has important implications to the heat release profile. It has been observed⁴ that during the early stage of development, the displacement of the flame kernel in a spark ignition engine shows a pronounced cycle to cycle variation. This phenomenon has been explained in terms of the convection of the kernel by the large scale turbulent eddies, which are statistical in nature. The motion of the flame center of mass is important because it sets the stage for the heat release profile. If the flame center is pushed against the wall of the combustion chamber, for example, a significant part of the flame entrainment front would be restricted by the wall and a slower burning cycle would result. The fluid velocities at the vicinity of the spark plug have been correlated to cycle performance⁴ and the importance of flame center location on burn time has been demonstrated^{5,6}.

In this paper, we first describe the experiment from which the flame ball center of mass velocities were obtained. Then a theory for predicting the variance of the flame-center velocity variance is developed. Finally the prediction of the theory is compared to the experimental results.

Experiment

The measurements were carried out in a 10 cm x 10 cm square cross-section combustion wind tunnel. The flow of premixed gas was supplied by air from the laboratory compressor, and methane from high pressure gas bottles. The mixture flowed through a long pipe ($L/D \sim 300$) to ensure good mixing, and through a plenum and contraction section (area ratio 9:1) to obtain a flat velocity profile. The flow then went through a perforated plate, which served as the turbulence generator, and entered the test section. Plates with hole-sizes of 0.6, 1.3 and 2.5 cm., and solidity (blocked area/total area) of 0.3 and 0.6 were used. It was not possible to "tailor" the turbulence generator to produce the desired velocity fluctuation and scale, but rather to take what was produced by the generator and to characterize the turbulence by hot wire measurements. The Reynolds number based on the hole diameter and the mean flow velocity was $\sim 10^4$.

The methane and air were metered by sonic orifices, and the resultant equivalence ratio was measured at the test section by a hydrocarbon flame ionization analyser which had been calibrated with methane/air mixtures of known equivalence ratios. The measurements were consistent with values calculated by choked orifice calculations.

The turbulence properties of the flow were measured by hot wire anemometry in air flow only. The anemometer signal was stored and analyzed digitally. Using Taylor hypothesis, the energy spectrum and longitudinal

correlations were calculated. The spectrum indicates a power law dependence with an exponent equal to -2 rather than the -5/3 dependence of inertial subrange. This dependence⁷ is typical of grid generated turbulence with Reynolds number less than 10^5 . The Reynolds number based on the turbulent velocity u and the integral scale λ for this case was $\sim 10^3$. Detailed characterization of the turbulent flow generated by the various perforated plates may be found in the reference^{8,9}.

The flame balls were initiated by a pair of spark electrodes made out of 500 μm tungsten wire. Care was taken to ensure that the resulting flame balls were not influenced by the wakes of the electrodes.

The turbulence properties u and λ were functions of the downstream distance. The classical correlation¹⁰ for grid turbulence decay in terms of the mesh size M is in the form of

$$\left(\frac{u}{u_0}\right)^2 \propto \left(\frac{x}{M} - B\right)^n, \quad n = 1.0 - 1.39; \quad \lambda \propto (x_1 - x)^m, \quad m = 0.30 - 0.53 \quad (1)$$

Using the hole diameters of the perforated plate as M , the values for n measured in this experiment ranged from 1.4 to 1.8 and m from 0.15 to 0.36. The differences between these values and those in Eq. (1) are attributed to the low x/M values in this experiment (in particular, for the plates with large d) so that the jets had not mixed out yet, as well as the finite cross-sectional dimension so that wall effects were not negligible.

The spark location was chosen with the conflicting requirement to obtain a high u (small x/d) and a small decrease of u along the test section (large x/d). The turbulent velocity u has to be comparable or higher than the laminar flame speed for meaningful measurement, and the variation of u and λ along the test section has to be small for unambiguous interpretation.

Measurements were obtained with the spark located at 15.4 cm and at 45 cm. downstream of the plate. At the 15.4 cm location, the variation of turbulence was acceptable (u between 2.0 to 1.7 m/s and λ between 8.4 to 11 mm) over the length of the test section in which the flame ball was observed. At the 45 cm spark location, u was much lower (< 1 m/s) and the variations of u and λ were within 20 percent.

The measurement points in this experiment are shown in Fig. 1. The Reynolds number Re and Damkohler number Ω are based on the measured values of λ and u at the center of the measurement region. The laminar flame speed S_L is based on the values given in the literature¹¹. To examine the statistical behavior of the flame, a large number of flame balls under the same global conditions (same turbulence generator, mean flow and equivalence ratio) had been recorded on high speed Schlieren movie. The outlines of these flame balls were digitized for statistical analysis.

Flame Image

The image of the growing flame ball was recorded by high speed Schlieren movie at 7,000 frames per second. The timing information was obtained from the image of a timing wheel which had been recorded on the film simultaneously. The outlines of the flame balls were digitized, frame by frame, to analyze the flame growth behavior. In the following discussion, the flame radius R is the radius of a circle having the same area as the area encompassed by the flame outline. Likewise, the flame center of mass is that of the outline. It should be noted that the outline is really the silhouette of the Schlieren image of the flame, and thus, it does not contain the three dimensional information. This point will be further discussed in the theoretical section.

Fig. 1 Measurement conditions of the turbulent premixed flame experiment
 Re is the Reynolds no. based on the turbulent velocity and the integral scale ($Re = u\lambda/\nu$), Ω is the Damkohler no. ($\Omega = \lambda S_L^2/\nu u$).

The digitization was done manually using a bit-pad and cross-hair digitizing system. The flame image was magnified and 36 points at equal angular intervals were marked and read into the computer. The flame area and equivalent radius R were calculated from the area of the inscribed polygon. Two reference points giving the dimension and the location of the electrodes were also digitized. The total error in the flame outline position (including the uncertainty of determining the outline of the image and the digitizing error) was estimated to be ~ 0.2 mm in actual dimension. The uncertainty in R , however, was harder to estimate because of the polygon approximation to the outline. An estimate of this error was obtained by comparing the values of R obtained from different sets of inscribed points and from different number of points used in constructing the polygon. The error in R is of the order of 2 percent.

The position of the center of mass of the flame outline is calculated based on the inscribed polygon. The error in this position is similar to that of R . The uncertainty in the center of mass velocity is more difficult to estimate. The velocity is calculated from differencing the center of mass position, and therefore, the uncertainty is inversely proportional to the velocity itself. This uncertainty is ~ 15 percent at a velocity of 1 m/s for a 1 cm radius flame ball.

Experimental Results

The data base in this study consists of 77 high speed Schlieren movies of the flame ball development. This data set includes experiments at a range of equivalence ratios (0.7, 0.9, 1.0 and 1.1), and turbulence levels, as indicated in Fig. 2. We are interested in the movement of the flame center of mass here. Other aspects of the flame behavior have been reported

Fig. 2 Typical flame center of mass position (measured from the wind tunnel center-line) as a function of the flame radius.

elsewhere^{8,9}.

The typical behavior of the flame center of mass is shown in Fig. 2. In this figure, the displacement y , of the center of mass measured from the wind tunnel center-line is plotted against the flame radius R for a typical realization. When the flame is small ($\sim$ integral scale), the flame center exhibits a finite drift velocity $\dot{y}$, which decreases as R increases. At large R , $\dot{y}$ decreases to zero.

This behavior may be explained qualitatively as follows. (A quantitative theory will be given in the next section.) The movement of the flame center of mass is the cumulative effect of the movement of the individual area elements of the flame front, which are influenced by the local turbulence velocity. When the flame size is small compared to the turbulent eddy size, the turbulent velocity is coherent over the entire flame surface area and the flame ball is convected as a whole by the turbulent eddy. When the flame ball grows to a size large compared to the integral scale, the turbulent velocities at the flame front area elements are no longer in phase. As a result, the area elements move randomly with respect to each other, and the center of mass velocity, which is an average over the velocity of the area elements, tends to zero.

The center of mass drift velocities $\dot{y}$ as a function of the flame radii R for 77 flame balls are plotted in Fig. 3. Because these experiments were done over a range of turbulence conditions, the results have been normalized by the measured turbulence intensity and longitudinal length scale. These results qualitatively support the foregoing explanation. Quantitative statistical analysis and comparison with a theory will be given in a later section.

Fig. 3 Center of mass velocity (normalized by the turbulent velocity) as a function of flame radius (normalized by the longitudinal integral scale) for 77 different flame balls. These flames include a range of turbulence conditions and equivalence ratios.

Theoretical Analysis of the Flame Center Motion

In this section, we shall formalize the above qualitative explanation of the motion of the center of mass of a statistically spherical flame. An expression for the variance of the center of mass velocity is derived, and then the computed variance is compared to the experimental results. It is reiterated here that the experimental results concern the center of mass of the silhouette of the flame outline as seen in the Schlieren movies. The theory, however, calculates the center of mass of the three dimensional flame ball, assuming that the flame ball is a solid body occupying the volume within the entrainment front of the flame. In general, these two definitions of the flame center of mass are not necessarily the same. The theory, as seen below, assumes that the flame shape is spherical, which may be considered as the lowest order term in a spherical expansion of the flame shape. In this context, the two dimensional projection of the center of mass of the three dimensional sphere, and that of the circular silhouette are the same. Therefore, meaningful comparisons of the theory and experiment are possible.

We shall assume that the flame ball surface is not severely wrinkled so that there is no substantial isolated islands of unburned mixture behind the flame front, and the flame front may be described as a simply connected surface. The equation of motion of the flame center of mass $\underline{X}$ may be obtained from the definition

$$\underline{VX} = \int \underline{x} \cdot dV \quad (2)$$

where V is the volume of the flame ball, and the integration on the right hand side is over the entire volume within the flame front. Differentiating

Eq. (2), we obtain

$$\begin{aligned}
 \frac{d(V\bar{X})}{dt} &= \lim_{\delta t \rightarrow 0} \left[\int (\bar{x} dV)_{t+\delta t} - \int (\bar{x} dV)_t \right] \\
 &= \lim_{\delta t \rightarrow 0} \left[\int \bar{x} (\bar{q} \cdot d\bar{A}) + O(\delta t) \right] \\
 &= \int \bar{x} (\bar{q} \cdot d\bar{A})
 \end{aligned} \tag{3}$$

where $\bar{q}$ is the flame-front velocity with respect to a fixed frame. Using $dV/dt = \int \bar{q} \cdot d\bar{A}$, Eq. (3) may be rearranged to obtain $d\bar{X}/dt$ explicitly,

$$\frac{d\bar{X}}{dt} = \frac{1}{V} \int (\bar{x} - \bar{X}) (\bar{q} \cdot d\bar{A}) \tag{4}$$

Furthermore, if the origin is located at the instantaneous flame center of mass, Eq. (4) is further simplified to

$$\frac{d\bar{X}}{dt} = \frac{1}{V} \int \bar{x} (\bar{q} \cdot d\bar{A}) \tag{5}$$

with the integration carried out over the instantaneous flame-front surface.

In the wrinkled laminar flame regime, the flame front velocity is

$$\bar{q} = \bar{u} + \bar{U} + S_L \hat{n} \tag{6}$$

where $\bar{u}$ is the turbulent velocity, $\bar{U}$ is the mean velocity (a constant), S_L the local laminar flame speed, and $\hat{n}$ the unit outward normal to the flame front. The component $\bar{U}$, when substituted into Eq. (5), will result in a constant convection velocity of the flame center. Without loss of generality, $\bar{U}$ is assumed to be zero,

The contribution of the $\bar{q} \cdot d\bar{A}$ term in Eq. (3) depends on the phase coherence of $\bar{q}$ over the entire flame front surface. The flame front surface may be represented by using a spherical harmonic expansion about an origin

centered at the center of mass of the flame. The instantaneous radial coordinate, r , of the flame front is

$$r(\theta, \phi, t) = R(t) + \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m}(t) Y_{\ell m}(\theta, \phi) \quad (7)$$

The equation of motion of the center of mass is then

$$\frac{d\mathbf{X}}{dt} = \frac{1}{V} \int (\mathbf{x} - \mathbf{X}) \frac{(q_r + q_\theta \frac{1}{r} \frac{\partial r}{\partial \theta} + q_\phi \frac{1}{r \sin \theta} \frac{\partial r}{\partial \phi}) r \sin \theta d\phi d\theta}{[1 + (\frac{1}{r} \frac{\partial r}{\partial \theta})^2 + (\frac{1}{r \sin \theta} \frac{\partial r}{\partial \phi})^2]^{1/2}} \quad (8)$$

In the following, we shall consider only the lowest order term ($r = R$) of the expansion in (8). To this order, the integration is over the sphere of radius R , and the contribution due to the laminar flame speed S_L integrates to zero. Therefore

$$\frac{d\mathbf{X}}{dt} = \frac{1}{V} \int \mathbf{x} (\mathbf{u} \cdot d\mathbf{A}) \quad (9)$$

The x component of the flame center of mass velocity is

$$\frac{dX}{dt} = \frac{1}{V} \int x (\mathbf{u} \cdot d\mathbf{A}) \quad (10)$$

and the mean square value of this velocity is

$$\begin{aligned} \left\langle \left(\frac{dX}{dt} \right)^2 \right\rangle &= \frac{1}{V^2} \left\langle \int x (\mathbf{u} \cdot d\mathbf{A}) \int x' (\mathbf{u}' \cdot d\mathbf{A}') \right\rangle \\ &= \frac{1}{V^2 R^2} \int d\mathbf{A} \int d\mathbf{A}' \langle (x^2 u_x^2 + xy u_y + xz u_z) (x'^2 u_x'^2 + x'y' u_y' + x'z' u_z') \rangle \end{aligned} \quad (11)$$

Similar expressions may be obtained for $(dY/dt)^2$ and $(dZ/dt)^2$.

If the turbulence is homogeneous and isotropic, the cross-correlation

terms $\langle u_x u_y' \rangle$ etc. all vanish

$$\begin{aligned} \left\langle \left(\frac{dX}{dt} \right)^2 \right\rangle &= \frac{1}{V_R^2} \int dA \int dA' (x^2 x'^2 \langle u_x u_x' \rangle + xyx'y' \langle u_y u_y' \rangle \\ &\quad + xzx'z' \langle u_z u_z' \rangle) \end{aligned} \quad (12)$$

Furthermore, the correlations may be represented by a longitudinal correlation function $f(\xi)$, and a transverse correlation function $g(\xi)$, where ξ is the vector connecting the two correlation points, $\xi = x' - x$, whence,

$$\langle u_x u_x' \rangle = \langle u^2 \rangle (f(\xi) (\hat{\xi} \cdot \hat{x})^2 + g(\xi) [1 - (\hat{\xi} \cdot \hat{x})^2]) \quad (13)$$

Similar expressions may be obtained for $\langle u_y u_y' \rangle$ and $\langle u_z u_z' \rangle$. Then the mean square value of the flame center of mass x - velocity is

$$\begin{aligned} I &= \left\langle \left(\frac{dX}{dt} \right)^2 \right\rangle / \langle u^2 \rangle \\ &= \frac{1}{V_R^2} \int dA \int dA' \{ f(\xi) [(\hat{\xi} \cdot \hat{x})^2 x^2 x'^2 + (\hat{\xi} \cdot \hat{y})^2 xyx'y' + (\hat{\xi} \cdot \hat{z})^2 xzx'z'] \\ &\quad + g(\xi) [(1 - (\hat{\xi} \cdot \hat{x})^2) x^2 x'^2 + (1 - (\hat{\xi} \cdot \hat{y})^2) xyx'y' + (1 - (\hat{\xi} \cdot \hat{z})^2) xzx'z'] \} \end{aligned} \quad (14)$$

Since the mean value of the velocity from Eq. (9) is zero, the above expression is also the variance of the velocity.

To evaluate the above integral for arbitrary functions $f(\xi)$ and $g(\xi)$, we shall first construct the Greens function as follows. The integral in Eq. (14) is solved numerically for top-hat profiles of f and g , with cut-off at ζR

$$f(\xi), g(\xi) = \begin{cases} 1 & \xi \leq \zeta R \\ 0 & \xi \geq \zeta R \end{cases} \quad (15)$$

The maximum value of interest for ζR is $2R$, since this is the maximum separation between the points on the surface of a sphere. Let the integrals

be F and G , corresponding to the first and second part of Eq. (14). The values of F and G will be a function of ζ . The functions $F(\zeta)$ and $G(\zeta)$ (or, more precisely, their derivatives) are the Greens functions in the sense that for arbitrary $f(\xi)$ and $g(\xi)$, the integral in Eq. (14) becomes

$$I = \int d\zeta \left[f(\zeta R) \frac{dF}{d\zeta} + g(\zeta R) \frac{dG}{d\zeta} \right] \quad (16)$$

The numerical evaluations of $F(\zeta)$ and $G(\zeta)$ are tedious but straight forward. The actual four dimensional integration of Eq. (14) is done in polar coordinates using Simpson's rule with $(40 \times 80)^2$ points. The results have been compared to those computed with $(25 \times 50)^2$ points, and there is not much difference (only in the third significant figure). These results are shown in Fig. 4. The shapes of these functions are strictly a consequence of the geometric properties of the integrand.

For homogeneous isotropic turbulence, the functions f and g are related. Furthermore, these functions are usually tabulated in terms of the non-dimensional separation r

$$r = \xi/\lambda \quad (17)$$

where λ is the longitudinal integral scale. The relationship between f and g is⁴

$$g(r) = f + \frac{r}{2} \frac{\partial f}{\partial r} \quad (18)$$

Using integration by parts on Eq. (16)

$$I = [F(2) f(2R/\lambda) + G(2) g(2R/\lambda)] - \frac{R}{\lambda} \int_0^2 \left(F(\zeta) \frac{\partial f}{\partial r} \right)_{\zeta R/\lambda} + G(\zeta) \frac{\partial g}{\partial r} \bigg|_{\zeta R/\lambda} d\zeta \quad (19)$$

Fig. 4 The Greens functions F and G (see Eq. (16)).

For $R/\lambda \rightarrow 0$, the flame ball is infinitely small compared to the eddy size, and it should be convected as a whole by the turbulent velocity. Therefore, $I \rightarrow 1$. The limiting value of Eq. (19) is

$$I \rightarrow F(2) + G(2) \quad (20)$$

or $F(2) + G(2) = 1$. This limit serves as a check on the numerical calculation, which indeed has the correct limit.

Equation (19) has been evaluated using the experimental data of Comte-Bellot and Corrsin¹² for values of f and g . Their experimental values for f and g were replotted in terms of ξ/λ and cubic splines were fitted to the data. The non-dimensional center of mass velocity variance, I , has been evaluated as a function of the flame ball radius (normalized by the integral length scale). The result is shown in Fig. 5. The variance decreases rapidly as R increases, and is essentially zero at $R/\lambda \sim 6$.

That the variance goes to zero and not to a finite value at large R/λ can be explained as the following. Equation (10) may be evaluated by dividing the surface area A into N elements, each of area $\delta A = A/N$. We look at the limit as $N \rightarrow \infty$. The partial sum for Eq. (10) is

$$\frac{dX_N}{dt} = \frac{1}{V} \sum_{i=1}^N x_i (\underline{u}_i \cdot \underline{\delta A}_i)$$

and

$$\begin{aligned} \left\langle \left(\frac{dX_N}{dt} \right)^2 \right\rangle &= \frac{1}{V^2} \left\langle \left(\sum_{i=1}^N x_i \underline{u}_i \cdot \underline{\delta A}_i \right) \left(\sum_{j=1}^N x_j \underline{u}_j \cdot \underline{\delta A}_j \right) \right\rangle \\ &= \frac{1}{V^2} \left(\sum_{i=1}^N x_i^2 \langle u_i^2 \rangle \cos^2 \theta_i \cdot \delta A_i \right) \frac{A}{N} \\ &\quad + \frac{1}{V^2} \sum_{i=1}^N \sum_{j=i+1}^N x_i x_j \cdot \langle u_i u_j \rangle \cos \theta_i \cdot \cos \theta_j \cdot \delta A_i \cdot \delta A_j \end{aligned}$$

(21)

Fig. 5 Comparison of theoretical prediction and experimental results for the normalized flame-center velocity variance.

Here the subscripted quantities refer to those evaluated at the individual area elements and θ_i is the angle between the velocity $\underline{u}$ and the outward normal $\hat{n}$ of the area element δA_i . For small λ ($\lambda \ll R$), the velocity $\underline{u}$ at each area element may be considered to be uncorrelated with that of all other elements; then the second term in Eq. (21) goes to zero by virtue of the $\langle u_i u_j \rangle$. The first term in Eq. (21) is a sum of the squares, and may be considered as a consequence of the central limit theorem. The sum within the parenthesis of the first term is $O(N\delta A) = O(A)$, and is finite. Therefore, the first term is $O(A/N)$ or $O(\delta A/A)$. For large N , this term goes to zero.

Comparison of Theory and Experiment

The data on flame center velocity in Fig. 4 has been analyzed as follows. The data points were grouped according to their values of R/λ , using a bin size $\Delta R/\lambda = 0.5$. Within each bin, the average values of the normalized flame center velocity, $X/\sqrt{u^2}$ and its variance, I , are calculated. (The notation has been changed to conform with the theoretical section so that the flame center displacement from the tunnel axis is denoted by X instead of y .) The results are plotted in Fig. 5 together with the theoretical curve. The abscissa of the data point is the mean value of the original data in the bin. The horizontal error bar denotes the range of R/λ in the bin. The vertical error bars are estimated from the fourth-order moment of the center velocity (see appendix). Information from any bin with less than five data points has been discarded.

The experimental results compare very favorably with the theory, considering the simplicity of the theory. Furthermore, to the order of

approximation in the theory, the theoretical result does not depend on S_L . This is supported by the experimental data, which include flames at equivalence ratios of 0.7 to 1.1 (S_L from 22 to 43 cm/s).

Conclusion

The movement of the center of mass of a statistically spherical turbulent premixed flame in the wrinkled-laminar flame regime has been examined. When the flame radius is small ($\approx$ integral scale of the turbulence), the flame ball is convected as a whole by the turbulent eddy. As the flame increases in size, this flame center drift velocity decreases and tends to zero. This effect has been explained as a result of the phase coherence of the flame front velocity. When the flame is small, the turbulent velocities at the flame front area elements are coherent over the whole flame, thus the flame is convected as a whole. When, the flame is large (compared to the integral scale), the turbulent velocity is no longer coherent over the entire flame surface. The flame center velocity, which is the aggregate over the flame front velocity of the individual flame area elements, therefore tends to the mean flow velocity and is independent of the turbulence. A theory based on the turbulent velocity correlation has been developed to predict the variance of the flame center velocity as a function of the flame radius. To the lowest order approximation, the phenomenon is purely kinematic, and is independent of the laminar flame speed. The prediction of the theory has been compared to experimental data obtained over a range of turbulence conditions and equivalence ratios, as well as over many realizations of the flame growth under the same global conditions. There is good agreement between theory and experiment.

Appendix

Estimation of the uncertainty in I

The error bars of the normalized flame center velocity variance, I, in Fig. 5 have been calculated as follows. Let the ordinates of the data points in Fig. 4 be V_i . For a bin with N data points, we are interested in the probability distribution of I_N , where

$$I_N = \frac{1}{N} \sum_{i=1}^N V_i^2 - (\bar{V})^2$$

The mean value $\bar{V}$ is estimated from

$$\bar{V} = \frac{1}{N} \sum_{i=1}^N V_i$$

The bins used in the analysis have been chosen so that each of the data points within one bin corresponds to the data from a different movie.

Therefore the V_i 's are independent random variables. From the central limit theorem, for a large enough N, I_N would be distributed normally, and the width of the distribution (at the 1/e point) is

$$\frac{1}{N} \sqrt{\sum V_i^4 - (\sum V_i^2)^2 / N}$$

This width is used as the error bar of I in Fig. 5.

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